中正中学 义顺
CHUNG CHENG HIGH SCHOOL YISHUN

Mid-Year Examination (2017)
Secondary 4 Express/ 5 Normal (Academic)

Candidate Name		Register No	Class

MATHEMATICS
4048/01

For examiner's use
/ 80

Date: 2 May 2017
Duration: 2 hours

Candidates answer on the Question Paper.

READ THESE INSTRUCTIONS FIRST

Write your name, index number and class on all the work you hand in.
Write in dark blue or black pen.
You may use an HB pencil for any diagrams or graphs.
Do not use staples, paper clips, highlighters, glue or correction fluid.

Answer **all** questions.
If working is needed for any question, it must be shown with the answer.
Omission of essential workings and units will result in loss of marks.
The use of an approved scientific calculator is expected, where appropriate.
Give your answer in the simplest form. Leave your answer in fraction where applicable or correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.
For π, use your calculator value, unless the question requires the answer in terms of π.

At the end of the examination, fasten all your work securely together.
The number of marks is given in brackets [ ] at the end of each question or part question.
The total number of marks for this paper is 80.

This paper consists of **19** printed pages, INCLUDING the cover page.

[TURN OVER

Mathematical Formulae

Compound interest

$$\text{Total amount} = P\left(1+\frac{r}{100}\right)^n$$

Mensuration

Curved Surface area of a cone $= \pi rl$

Surface area of a sphere of radius $= 4\pi r^2$

$$\text{Volume of a cone} = \frac{1}{3}\pi r^2 h$$

$$\text{Volume of a sphere} = \frac{4}{3}\pi r^3$$

$$\text{Area of triangle } ABC = \frac{1}{2}ab\sin C$$

Arc length $= r\theta$, where θ is in radians

Sector area $= \frac{1}{2}r^2\theta$, where θ is in radians

Trigonometry

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$a^2 = b^2 + c^2 - 2bc\cos A$$

Statistics

$$\text{Mean} = \frac{\sum fx}{\sum f}$$

$$\text{Standard Deviation} = \sqrt{\frac{\sum fx^2}{\sum f} - \left(\frac{\sum fx}{\sum f}\right)^2}$$

1 (a) Calculate $\frac{-1.4^2 + 2\pi^3}{4 - \sqrt{17}}$, giving your answer to 2 significant figures.

(b) Express 0.35% as a fraction in its simplest form.

Answer: (a) _____ [1]

(b) _____ [1]

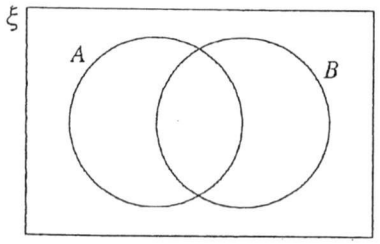
2 The universal set ξ is the set of natural numbers less than 14.

Given that $B = \{x: 2x - 7 < 19 \leq x + 10\}$.

(a) List all elements in B'.

Answer: (a) $B' =$ _____ [2]

(b) On the Venn diagram, shade the region which represents $A' \cap B'$. [1]



3 A metal rod A has a length of 47 m, correct to the nearest m.

A metal rod B has a length of 63 m, correct to the nearest m.

Find

(a) the least possible length of metal rod A,

(b) the greatest possible difference in their lengths.

Answer: (a) _____m [1]

(b) _____m[1]

4 A charity carnival sells tickets to adults and children. The total cost of 3 adult tickets and 1 child ticket is \$29. The total cost of 1 adult ticket and 3 child tickets is \$19. Find the cost of one adult ticket and the cost of one child ticket.

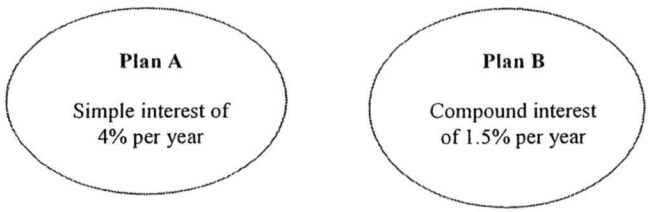
Answer: *Cost of one adult ticket* = \$_____

Cost of one child ticket = \$_____ [3]

5 The mean, median, and modal height of 4 men are 178 cm, 176 cm, and 173 cm respectively. Write down the heights of each man in ascending order.

Answer: _____cm, _____cm, _____cm, _____cm [2]

6 Benjamin wants to invest $30000 in a savings account for 5 years. He finds information about two savings plans. Which of these savings plans should he choose in order to have more money at the end of 5 years? Show your working clearly.

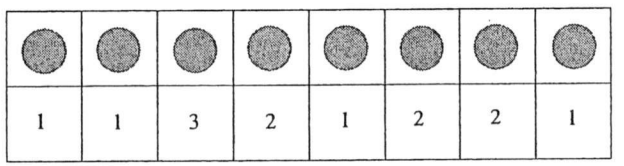


Answer: Plan_____ because_____ [3]

7 It is given that y is directly proportional to the cube root of x. Find the percentage increase in y when x is increased by 400%.

Answer: _____% [3]

8 A ball is dropped at random into one of the eight holes, numbered as shown in the diagram below. The number under each hole gives the score obtained when the ball drops into that hole. Each hole can only occupy one ball.



(a) State the probability of getting a score of 1.

Answer: (a) _____[1]

(b) Given that two balls are dropped at the same time, find the probability of scoring
(i) a total of 2,
(ii) a total of 6.

Answer: (b)(i) _____ [1]

(b)(ii) _____ [1]

9 (a) Express 315 as a product of its prime factors.

(b) On Youth Day, the teacher distributed 315 bookmarks and 90 pens equally among the students for her classes. Given that each student received the same number of bookmarks and pens with no leftover,

(i) find the largest possible number of students the teacher distributed for her classes,

(ii) find the number of bookmarks each student received.

Answer: (a) _____ [1]

(b)(i) _____ [1]

(b)(ii) _____ [1]

10 (a) Simplify $\frac{2a^4}{5bc} \div \frac{14a}{15c}$.

Answer: (a) _____ [2]

(b) The distance from Earth to the Moon is 384 400 km while the distance from Earth to Uranus is 2.72394×10^{12} m. By how many times is the distance of Earth from Uranus more than the distance of Earth from the Moon? Leave your answer in standard form.

Answer: (b) _____ [2]

11 An area of 8 cm^2 on a map represents an actual area of 0.03 km^2. Calculate

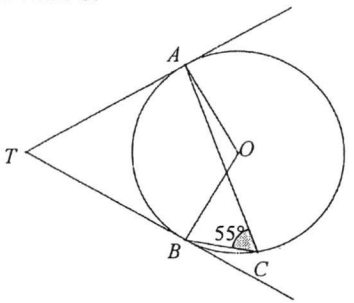
(a) the area, in cm^2, on the map , which represents an actual area of 6000 m^2,

(b) the actual distance, in km, represented by a length of 7.9 cm on the map.

Answer: (a) _____ cm^2 [2]

(b) _____ km [2]

12 The diagram shows a circle, ABC with centre O.
TA and *TB* are tangents to the circle.
$\angle ACB = 55°$.



Find

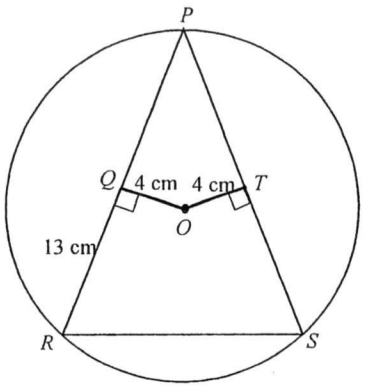
(a) $\angle AOB$,

Answer (a) $\angle AOB$ = _____ ° [1]

(b) $\angle ATB$,

Answer (b) $\angle ATB$ = _____ ° [2]

13 In the figure, O is the centre of the circle, $OQ = OT = 4$ cm and $QR = 13$ cm.
P, S and R are points on the circle.



(a) State the length of PS.

Answer: (a) _____cm [1]

(b) Find the length of the shortest chord.

Answer: (b) _____cm [3]

14 The coordinates of A is $(3, -5)$. Another point B is such that $\overrightarrow{AB} = \begin{pmatrix} 1 \\ 9 \end{pmatrix}$.

(a) Find $\left|\overrightarrow{AB}\right|$.

Answer: (a) _____ units [1]

(b) Find the coordinates of B.

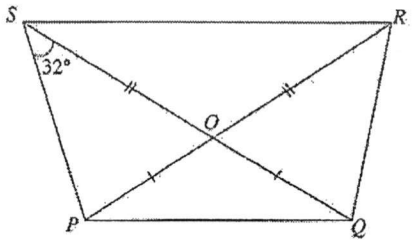
Answer: (b) (_____, _____) [1]

(c) Given the coordinates of C is $(-1, m)$ and A, B and C are collinear.
Find (i) $\overrightarrow{AC}$,
(ii) m.

Answer: (c)(i) _____ [1]

(ii) _____ [2]

15 In the diagram, $OR = OS$, $QO = PO$ and $\angle PSQ = 32°$.



(a) Show that triangles POS and QOR are congruent, stating your reasons clearly.
Answer (a)

[2]

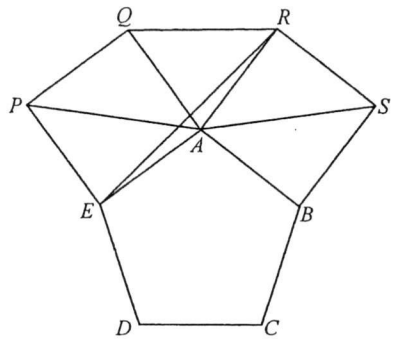
(b) Name another pair of congruent triangles.

Answer: (b) _____ [1]

16 (a) Calculate the size of each interior angle of a regular pentagon.

Answer: (a) _____ ° [1]

(b)



In the diagram above, $ABCDE$ is a regular pentagon. $EPQA$ and $ARSB$ are two squares.

Calculate

(i) reflex $\angle PED$,

Answer: (b)(i) _____ ° [1]

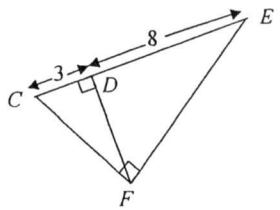
(ii) obtuse $\angle PAR$,

Answer: (b)(ii) _____ ° [1]

(iii) acute $\angle ERA$.

Answer: (b)(iii) _____ ° [1]

17 In the diagram, CFE is a right-angled triangle. D is a point on CE such that $\angle CFE = \angle CDF = 90°$.
All the measurements are in cm.



Show that triangles CFE and CDF are similar.
Answer (a)

[2]

(b) Calculate CF.

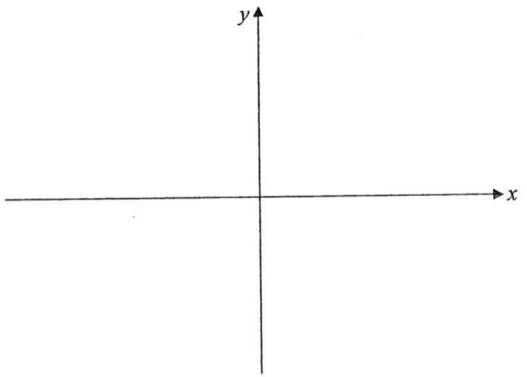
Answer: (b) $CF =$ _____ cm [2]

18 (a) Express $-x^2 + 3x - 5$ in the form $-(x-a)^2 - b$.

Answer: (a) _____ [2]

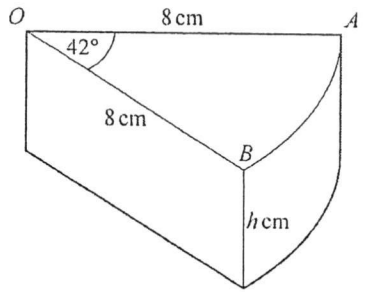
(b) Hence, sketch the graph of $y = -x^2 + 3x - 5$, showing clearly the turning point and the intercepts, if any.

Answer (b)(ii)



[2]

19 A wedge of cheese in the shape of a prism is cut from a cylinder of cheese of height h cm. The radius of the cylinder, OA, is 8 cm and the $\angle AOB = 42°$.

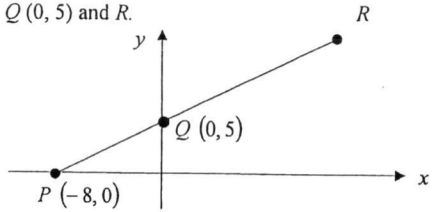


(a) The volume of the wedge of cheese is 100 cm^3.
Show that the value of h is 4.26 cm correct to 2 decimal places.
Answer: (a) [2]

(b) Calculate the total surface area of the wedge of cheese.

Answer: (b) _____ cm^2 [3]

20 The diagram below shows the sketch of a straight line passing through three points P (-8, 0), Q (0, 5) and R.



(a) Write down the equation of the line.

Answer: (a) _____ [1]

(b) Given that another line that is parallel to the line PQR, passes through the point (4, 5), find the coordinates of the y-intercept of the line.

Answer: (b) (_____ , _____) [2]

(c) The point R is such that PQ: QR is 1 : 2. Find the coordinates of R.

Answer: (c) _____ [2]

21 A team of 4 NPCC cadets took part in an orienteering competition.

The diagram below shows a scale drawing representing 3 checkpoints A, B and C.
B is due south of A.

In the drawing, 1 cm represents 0.5 km.

(a) Using ruler and compasses only, construct on the diagram,

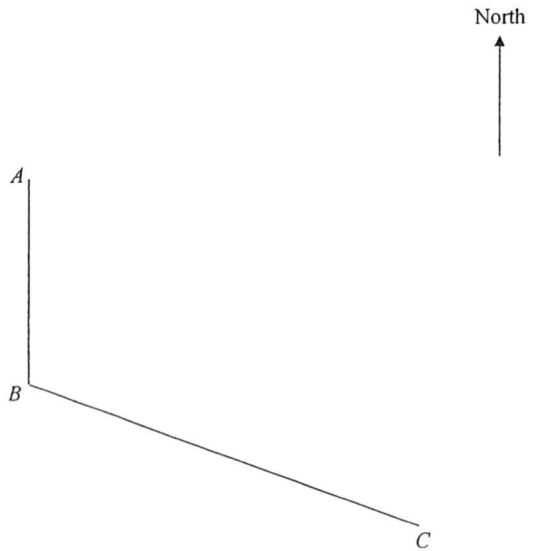
(i) the bisector of angle ABC, [1]

(ii) the perpendicular bisector of BC. [1]

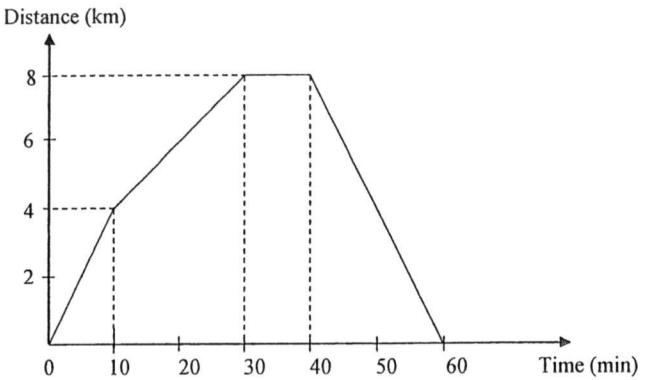
(b) A referee, R, is to be stationed equidistant from B and C and equidistant from AB and BC. Find and label the position of R on the diagram. [1]

(c) The cadets are instructed to find a building D which is on a bearing of 210° and 2.5 km from B. Find and label the position of D on the diagram. [1]

Answer



22 The diagram below shows the distance-time graph of a cyclist's journey.



The cyclist started from rest.

(a) Find the average speed for the whole journey.

Answer: (a) _____km/min [1]

(b) Find the distance travelled in the first 24 minutes.

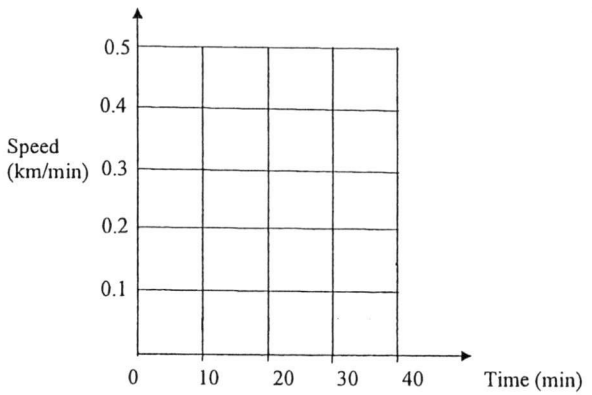
Answer: (b) _____km [2]

(c) Find the acceleration during the last 5 minutes.

Answer: (c) _____ km/min^2 [1]

(d) On the grid given below, draw the speed-time graph for the first 40 minutes of the journey.

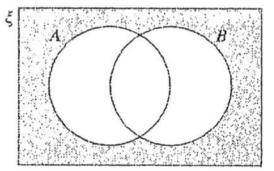
Answer (b)(ii)



[3]

~End of Paper~

2017 CCHY Mid-year Exam 4E5N EMP1 Marking Scheme

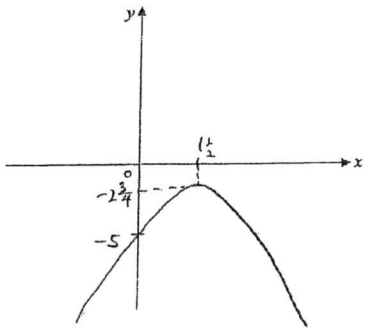
Qn	Solution	Marks
1(a)	-490	B1
1(b)	$\frac{7}{2000}$	B1
2(a)	$2x-7<19 \quad 19\le x+10$ $2x<26 \quad 9\le x$ $x<13 \quad x\ge 9$ $B' = \{1, 2, 3, 4, 5, 6, 7, 8, 13\}$	M1 A1
2(b)	 $A' \cap B'$	B1
3(a)	46.5	B1
3(b)	$46.5 \le$ length A < 47.5 $62.5 \le$ length B < 63.5 Greatest difference in length = 63.5 – 46.5 m = 17 m	B1
4	Let x be the no. of adult tickets. Let y be the no. of child tickets. $3x + y = 29$ ----------(1) $x + 3y = 19$ ----------(2) (1)×3: $9x + 3y = 87$ -----(3) (3)−(2): $8x = 68$ $x = \frac{68}{8}$ $= 8.50$ subst $x = 8.50$ into (1), $3(8.50) + y = 29$ $y = 29 - 3\ (8.50)$ $= 3.50$ ∴ Cost of adult ticket is \$8.50 and cost of child ticket is \$3.50.	M1 A1 A1
5	Third height : $176 \times 2 - 173 = 179$ Fourth height : $4(178) - 179 - 2(173) = 187$ Height in order : 173, 173, 179, 187	M1 A1

6	At the end of 5 years, Plan A: $30000(1+(0.04\times 5))=36000$ Plan B: $30000(1+0.015)^5=32318.52$ Benjamin should choose Plan A as it gives him more money at the end of the five years or Benjamin should choose Plan A because \$36000 >\$32318.52	M1 M1 A1
7	$y=k\sqrt[3]{x}$ $x_1=5x,\ y_1=k\sqrt[3]{5x}$ $=\sqrt[3]{5}\left(k\sqrt[3]{x}\right)$ $=\sqrt[3]{5}y$ % increase in $y=\frac{y_1-y}{y}\times 100\%$ $=\frac{\sqrt[3]{5}y-y}{y}\times 100\%$ $=\frac{y\left(\sqrt[3]{5}-1\right)}{y}\times 100\%$ $=\left(\sqrt[3]{5}-1\right)\times 100\%$ $=70.9976\%$ $=71.0\%$ (3sf)	M1 M1 A1
8(a)	$\frac{1}{2}$	B1
8(b)(i)	P(total is 2) $=\frac{4}{8}\times\frac{3}{7}$ $=\frac{3}{14}$	B1
8(b)(ii)	P(total is 6) = 0	B1
9(a)	$315=3^2\times 5\times 7$	B1
9(b)(i)	$315=3^2\times 5\times 7$ $90=2\times 3^2\times 5$ HCF $=3^2\times 5$ $=45$ $\therefore$ greatest possible number of students the teacher distributed the bookmarks and pens = 45	B1

9(b)(ii)	Number of books each student gets $=\frac{3^2\times 5\times 7}{3^2\times 5}$ $=7$	B1
10(a)	$\frac{2a^4}{5bc}\div\frac{14a}{15c}$ $=\frac{2a^4}{5bc}\times\frac{15c}{14a}$ $=\frac{3a^3}{7b}$	M1 A1
10(b)	Number of times Earth is further from Uranus compared to the Moon $=\frac{2.72394\times 10^{12}}{384\,400\times 1000}$ $=7.0862\times 10^3$ $=7.09\times 10^3$ (3 s.f.)	M1 A1
11(a)	$0.03\ \text{km}^2=0.03\times(1000\text{m})^2$ $=30000\ \text{m}^2$ $30000\ \text{m}^2 : 8\ \text{cm}^2$ $\div 5,\quad 6000\ \text{m}^2 : 1.6\ \text{cm}^2$ $\therefore$ area on map $=1.6\ \text{cm}^2$	M1 A1
11(b)	$8\ \text{cm}^2 : 0.03\ \text{km}^2$ $8\ \text{cm}^2 : \frac{3}{100}\ \text{km}^2$ $\div 8,\quad 1\ \text{cm}^2 : \frac{3}{800}\ \text{km}^2$ take sq root, $1\ \text{cm} : \sqrt{\frac{3}{800}}\ \text{km}$ actual distance on ground $=7.9\ \text{cm}\times\sqrt{\frac{3}{800}}\ \text{km}$ $=0.48377\ \text{km}$ $\sim 0.484\ \text{km}$	M1 A1
12(a)	$\angle AOB=2\times 55°$ ($\angle$at centre $=2\times\angle$ at circumference) $=110°$	B1
12(b)	$\angle OAT=90°$ (tangent $\perp$ radius) $\angle ATB=360°-90°-90°-110°$ ($\angle$ sum of quadrilateral) $=70°$	M1 A1

13(a)	$PS = 26cm$	B1
13(b)	In right-angled ΔQPO, $\tan \angle QPO = \frac{4}{13}$ $\angle QPO = 17.1027°$ Let M = foot of the perpendicular from P to RS In right-angled ΔPMR, $\sin \angle RPM = \frac{RM}{RP}$ $\sin 17.1027° = \frac{RM}{26}$ $RM = 26 \sin 17.1027°$ $\therefore$ RS = 2 x 26 sin 17.1027° cm = 15.2924 cm ~ 15.3 cm (3sf) $\therefore$ the shortest chord, RS = 15.3 cm	M1 M1 A1
14(a)	$\overrightarrow{AB} = \begin{pmatrix} 1 \\ 9 \end{pmatrix}$ $\left\|\overrightarrow{AB}\right\| = \sqrt{1^2 + 9^2}$ units = 9.05539 units = 9.06 units (3sf)	B1
14(b)	$\overrightarrow{AB} = \overrightarrow{OB} - \overrightarrow{OA}$ $\overrightarrow{OB} = \overrightarrow{AB} + \overrightarrow{OA}$ $= \begin{pmatrix} 1 \\ 9 \end{pmatrix} + \begin{pmatrix} 3 \\ -5 \end{pmatrix}$ $= \begin{pmatrix} 4 \\ 4 \end{pmatrix}$ $\therefore$ coordinates of B = (4, 4)	B1
14c(i)	$\overrightarrow{AC} = \overrightarrow{OC} - \overrightarrow{OA}$ $= \begin{pmatrix} -1 \\ m \end{pmatrix} - \begin{pmatrix} 3 \\ -5 \end{pmatrix}$ $= \begin{pmatrix} -4 \\ m+5 \end{pmatrix}$	B1
14c(ii)	$\begin{pmatrix} -4 \\ m+5 \end{pmatrix} = k \begin{pmatrix} 1 \\ 9 \end{pmatrix}$	M1

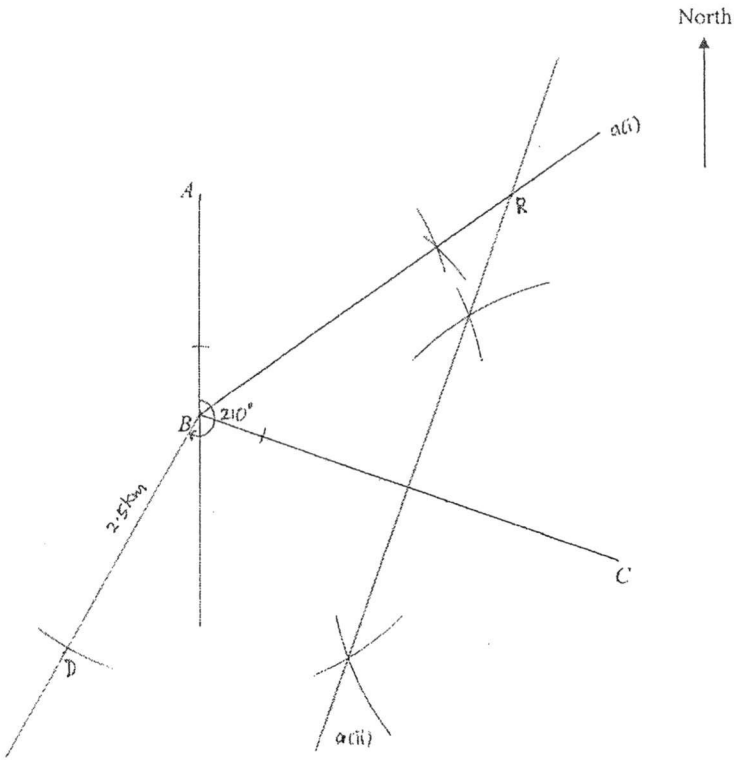
	comparing terms, $k = -4$ $m + 5 = 9k$ $m = 9(-4) - 5$ $= -41$	A1
15(a)	$QO = PO$ (given) (S) $RO = SO$ (given) (S) $\angle ROQ = \angle SOP$ (vertically opposite angles) (A) $\therefore \Delta POS$ and ΔQOR are congruent (SAS)	M1 R1
15(b)	triangle SPR and triangle RQS Or triangle PSQ and triangle QRP	B1
16(a)	Each interior angle = $\frac{(5-2) \times 180^0}{5}$ $= 108°$	B1
16(b)(i)	reflex $\angle PED = 90° + 108°$ $= 198°$	B1
16(b)(ii)	In triangle QAR, $\angle QAR = 360° - 90° - 90° - 108°$ $= 72°$ $\therefore$ obtuse $P\hat{A}R = 72° + 45°$ $= 117°$	B1
16(b)(iii)	$\angle ERA = \frac{180° - (72° + 90°)}{2}$ (EA = AR, triangle EAR is isosceles) $= 9°$ $\therefore$ acute $\angle ERA = 9°$	B1
17(a)	$\angle FCE = \angle DCF$ (Common Angle) $\angle CFE = \angle CDF = 90°$ $\therefore$ Triangles CFE and CDF are similar since 2 pairs of corresponding angles are equal.	M1 R1
17(b)	ΔCFE and ΔCDF are similar, $\frac{CF}{CD} = \frac{FE}{DF} = \frac{CE}{CF}$ =scale factor $\frac{CF}{CD} = \frac{CE}{CF}$ $\frac{CF}{3} = \frac{11}{CF}$ $CF^2 = 33$	M1

	$CF = \sqrt{33}$ $= 5.74456$ ~ 5.74 $\therefore$ CF = 5.74 cm	A1
18(a)	$-x^2 + 3x - 5$ $= -\left(x^2 - 3x + 5\right)$ $= -\left[x^2 - 3x + \left(\frac{3}{2}\right)^2 - \left(\frac{3}{2}\right)^2 + 5\right]$ $= -\left[\left(x - \frac{3}{2}\right)^2 + 2\frac{3}{4}\right]$ $= -\left(x - 1\frac{1}{2}\right)^2 - 2\frac{3}{4}$	M1 A1
18(b)	 B1 (Shape & curve) B1 (correct pts of turning pts & y-intercept.)	
19(a)	Cross sectional area of prism $= \frac{42°}{360°} \times \pi \times 8^2$ Volume of prism $= \frac{42°}{360°} \times \pi \times 8^2 \times h$ $100 = \frac{42°}{360°} \times \pi \times 8^2 \times h$ $h = \frac{360° \times 100}{42° \times \pi \times 8^2}$ $= 4.26308$ $\therefore$ $h = 4.26$ (correct to 2 decimal places)	M1 A1

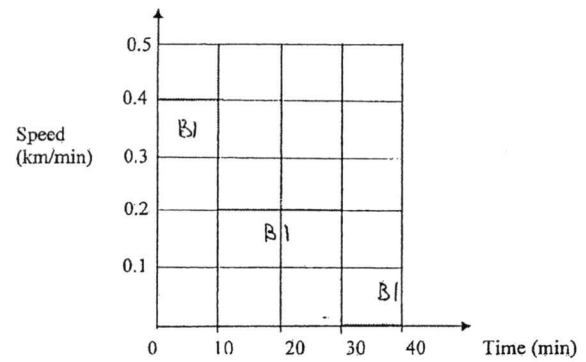
19(b)	Arc length $= \frac{42°}{360°} \times 2\pi \times 8 = 5.86431$ cm Curve surface area $= 5.86431 \times 4.26308 = 25.00002267$ cm^2 Total surface area $= \left(2 \times \frac{42°}{360°} \times \pi \times 8^2\right) + (2 \times 8 \times 4.26308) + 25.00002267$ cm^2 $= 140.1238$ cm^2 ~ 140 cm^2 (3sf)	M1 M1 A1
20(a)	gradient $= \frac{5}{8}$ $\therefore$ equation of line is $y = \frac{5}{8}x + 5$	 B1
20(b)	Equation of parallel line is $y = \frac{5}{8}x + c$ At (4,5), $5 = \frac{5}{8}(4) + c$ $c = 5 - \frac{5}{2}$ $= \frac{5}{2}$ $\therefore$ co-ordinates of y-intercept $= \left(0, 2\frac{1}{2}\right)$	M1 A1
20(c)	$x : y = 8 : 5$ $= 16 : 10$ $\therefore$ R = (16, 5+10) $= (16,15)$	M1 A1

21

Answer



22(a)	Average speed $= \frac{8+8}{60}$ km/min $= \frac{4}{15}$ km/min	B1
22(b)	Method 1 Let distance from $t = 10$ min to $t = 24$ min be h km Using similar triangle, $\frac{h}{4} = \frac{14}{20} = scale\ \ factor$ $h = \frac{4 \times 14}{20}$	M1

	$= 2.8$ $\therefore$ distance travelled in the first 24 min $= 4 + 2.8$km $= 6.8$km	A1
	Method 2 Equation of line from $t = 10$ min to $t = 24$ min : gradient of line $= \frac{8-4}{30-10}$ $= 0.2$ Equation of line is $y = 0.2x + c$ At (10,4), $4 = 0.2(10) + c$ $= 2 + c$ $c = 4 - 2$ $= 2$ subst $x = 24$ into y, $y = 0.2(24) + 2$ $= 6.8$ $\therefore$ distance travelled in the first 24 min = 6.8 km	M1 A1
22(c)	0 km/min^2	B1
22(d)		

1. (a) It is given that $S = \frac{a}{2}\sqrt{n^2 - b}$.

(i) Find S when $a = 8$, $b = -7$ and $n = 3$. [1]

(ii) Express n in terms of a, b and S. [3]

(b) Solve the equation $\frac{5}{\sqrt[3]{5}} = 5^{x-1}$. [2]

(c) Factorise $9x^2 - 25 + 12xy - 20y$. [2]

(d) Simplify $\frac{x^2 + 3x - 7}{x - 4} + \frac{9 + 3x}{4 - x}$. [3]

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2. Nancy is planning a holiday to United States. On 1 March 2017, she exchanged S$3000 into US dollars (US$) at Kumar's Money Exchange at a rate of US$1 = S$$x$.

(a) Find an expression, in terms of x, for the amount of US$ she received from Kumar's Money Exchange. [1]

On 15 March 2017, she decided to exchange another S$2100 into US$ at Lee's Money Exchange at a rate of US$1 = S$$(x - 0.1)$.

(b) Find an expression, in terms of x, for the amount of US$ she received from Lee's Money Exchange. [1]

(c) Given that Nancy received a total of US$3500 from the two Money Exchanges, form an equation in x and show that it simplifies to $70x^2 - 109x + 6 = 0$. [3]

(d) (i) Solve the equation $70x^2 - 109x + 6 = 0$, giving your answers correct to 4 decimal places. [2]

(ii) Suggest a reason why one of the answers has to be rejected. [1]

(iii) Hence, find the exchange rate between S$ and US$ offered by Lee's Money Exchange. [1]

(e) Is it better for Nancy to change her currency on 1 March or 15 March? Justify your answer with appropriate workings. [2]

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3. A cake shop sells 3 different types of muffins. The table below shows the numbers of muffins sold over 2 days and the price of each type of muffin.

	Chocolate	Blueberry	Cheese
Day 1	12	25	10
Day 2	15	24	9
Price of each muffin, $	2.00	1.50	2.50

It is given that $\mathbf{P} = \begin{pmatrix} 12 & 25 & 10 \\ 15 & 24 & 9 \end{pmatrix}$ and $\mathbf{Q} = \begin{pmatrix} 2.0 \\ 1.5 \\ 2.5 \end{pmatrix}$.

(a) Evaluate the matrix $\mathbf{M} = \mathbf{PQ}$. [1]

(b) State what the elements of $\mathbf{M}$ represent. [1]

(c) The cake shop is having a promotion by giving a 25% discount on chocolate muffins, a 30% discount on blueberry muffins and a 40% discount on cheese muffins.

(i) Write down a 3×3 matrix $\mathbf{R}$, where the product of $\mathbf{RQ}$ will give the discounted price of each type of muffin. [1]

(ii) Evaluate the matrix $\mathbf{N} = \mathbf{RQ}$. [1]

(d) Evaluate the matrix $\mathbf{PN}$ and state what the elements of $\mathbf{PN}$ represent. [2]

(e) Given that $\mathbf{T} = \begin{pmatrix} 1 & 1 \end{pmatrix}$, evaluate the matrix $\mathbf{TPN}$ and state what the elements of $\mathbf{TPN}$ represent. [2]

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4. (a) Bag A contains 1 gold medal, 4 silver medals and 2 bronze medals. Bag B contains 2 gold medals and 5 silver medals. Peter draws a medal at random from bag A, took note of the colour and placed it into bag B. He then draws a medal at random from bag B.

(i) Draw a tree diagram to show the probabilities of the possible outcomes. [2]

(ii) Find, as a fraction in its simplest form, the probability that

(a) the medal drawn from bag B is a gold medal, [2]

(b) the medal drawn from bag B is a bronze medal. [1]

(b) (i) The nth term of a sequence is given by $T_n = n^2 + 2n - 5$.

(a) Write down the first 5 terms of the sequence. [2]

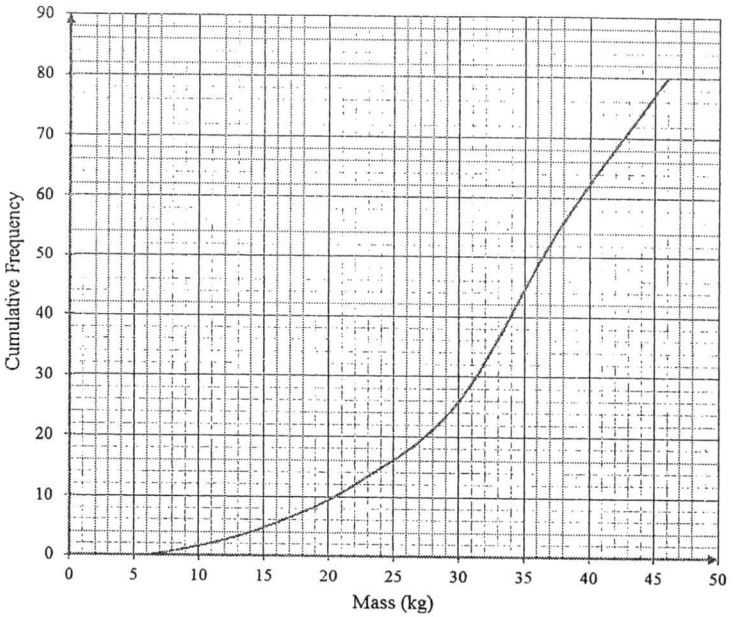
(b) Which term of the sequence has value 163? [2]

(ii) The first five terms of another sequence are 1, 6, 13, 22, 33, ...

(a) By comparing this sequence with the sequence in part (i), write down the nth term of the sequence 1, 6, 13, 22, 33, ... [1]

(b) Hence, find the 99th term. [1]

5. In Plantation A, the total mass of the mangoes produced by each of 80 mango trees were measured. The cumulative frequency curve below shows the distribution of the masses.



(a) Use the graph to find the median mass. [1]

(b) In the following grouped frequency table of the mass of mangoes in the plantation, write down the values of p and of q. [2]

Mass (x kg)	$6 \le x < 14$	$14 \le x < 22$	$22 \le x < 30$	$30 \le x < 38$	$38 \le x < 46$
Frequency	4	p	14	q	24

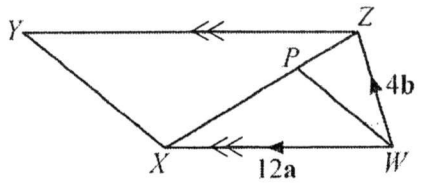
(c) Using your grouped frequency table, calculate an estimate of

(i) the mean mass, [2]

(ii) the standard deviation. [1]

(d) In Plantation B, the total mass of the mangoes produced by each of 80 mango trees were measured. Their mean and standard deviation were found to be 29.1 kg and 10.4 kg respectively. Make two comparisons between the mass of the mangoes produced by the trees in both plantations. [2]

6. In the diagram, $WXYZ$ is a trapezium and WX is parallel to ZY. The point P on XZ is such that $ZP : PX = 1 : 3$ and $WX : ZY = 3 : 4$.



(a) Give that $\overrightarrow{WX} = 12\mathbf{a}$ and $\overrightarrow{WZ} = 4\mathbf{b}$, express in terms of $\mathbf{a}$ and/or $\mathbf{b}$,

(i) $\overrightarrow{ZX}$, [1]

(ii) $\overrightarrow{WP}$, [1]

(b) Determine with justification, if the line XY is parallel to the line WP. [2]

(c) Find

(i) $\dfrac{\text{Area of } \Delta WZP}{\text{Area of } \Delta WXP}$, [1]

(ii) $\dfrac{\text{Area of } \Delta WZP}{\text{Area of } \Delta YXZ}$. [2]

(d) Hence, find the area of $WXYZ$ if the area of ΔWZP is 12 units^2. [1]

7. Diagram I shows a trough where the base $BCGF$ and the top $ADHE$ are horizontal rectangles. Each of the vertical sides $ABCD$ and $EFGH$ is a trapezium. It is given that $AD = 15$ cm, $BC = 11$ cm, $BX = 9$ cm and $CG = 20$ cm. It is fully filled with water.

Take $\pi = \dfrac{22}{7}$.

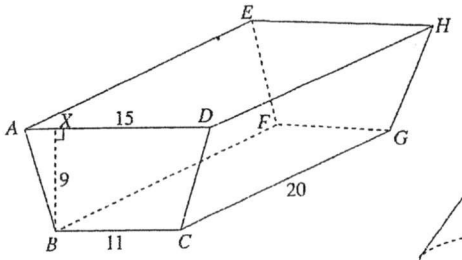


Diagram I

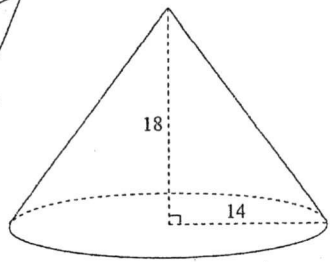


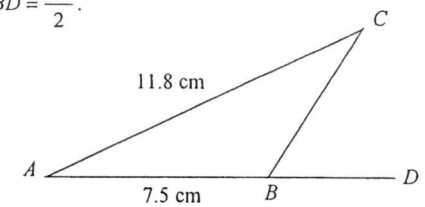
Diagram II

(a) Calculate the volume of the water in the trough in Diagram I. [2]

(b) All the water in Diagram I is made to flow down through a hole at the top of the cone in Diagram II. The water flows at a rate of 0.2 litres per minute. Calculate the time taken for the water in the trough to be transferred, giving your answer in minutes and seconds. [2]

(c) The cone in Diagram II is of base radius 14 cm and height 18 cm. The cone is made to stand on its base. Calculate the height of water in the cone. [4]

8. (a) In the diagram below, ABD is a straight line. $AB = 7.5$ cm, $AC = 11.8$ cm and $\sin \angle CBD = \frac{\sqrt{3}}{2}$.



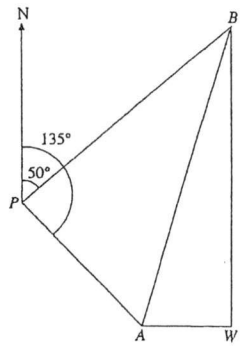
Calculate

(i) $\sin \angle ACB$, giving your answer correct to 3 significant figures, [2]

(ii) $\angle BAC$, [2]

(iii) the area of ΔABC. [1]

(b) Two military ships, Amaze and Brave left port P at 1000. Amaze sailed at 12 km/h on a bearing of 135°. Brave sailed at 18 km/h on a bearing of 050°. After sailing for 3 hours, Amaze is at Island A and Brave is at Island B.



Calculate

(i) the length of AB. [3]

(ii) the bearing of A from B. [2]

(iii) Amaze later sailed to Island W which is due east of Island A. It is given that Island B is due north of Island W. Find the distance BW. [2]

(iv) A helicopter is hovering at a height of 10 km vertically above B. Find the greatest angle of elevation Amaze can have of the helicopter as it sailed towards W. [2]

9. **Answer the whole of this question on a sheet of graph paper.**

The variables x and y are connected by the equation $y = \frac{1}{2}x^2 + \frac{18}{x} - 10$.

Some corresponding values of x and y, correct to 1 decimal place, are given in the table below.

x	1	1.5	2	2.5	3	4	4.5	5
y	8.5	3.1	1	0.3	0.5	k	4.1	6.1

(a) Find the value of k. [1]

(b) Using a scale of 2 cm to represent 1 unit on both axes, draw the graph of $y = \frac{1}{2}x^2 + \frac{18}{x} - 10$ for $0 \le x \le 5$. [3]

(c) Use your graph to solve $\frac{1}{2}x^2 + \frac{18}{x} = 15$. [2]

(d) By drawing a tangent, find the gradient of the curve at $x = 3$. [2]

(e) On the same axes, draw the graph of $y = 6 - x$ for $0 \le x \le 5$. [1]

(f) (i) Write down the x-coordinates of the points where the two graphs intersect. [2]

(ii) These values of x are solutions of the equation $x^3 + Ax^2 + Bx + C = 0$. Find the value of A, the value of B and the value of C. [2]

10. The table below shows the admission rates for three different tourist locations in Singapore.

	Per adult ($)	Per child ($)
Zoological Gardens	33	22
Bird Park	29	19
Night Safari	30	20
3-parkhopper package	69	49

Note: The 3-parkhopper package allows admission into all 3 parks within a period of 10 days.

(a) Mr Tan, a Malaysian, brought his wife and three children below 12 years old to visit Singapore in June 2016. First, they visited the Zoo and the Bird Park. Later, they decided to go to the Night Safari.

(i) Calculate the amount Mr Tan spent visiting the 3 parks with his family. [1]

(ii) If he had planned earlier and purchased the 3-parkhopper package, how much would he have saved? [2]

(b) His 3 children enjoyed their excursions at the parks and requested Mr Tan to bring them to the parks during subsequent school holidays. Mr Tan came to know about Wildlife Membership package. The details are as follows:

Membership Package Type	Annual Fee ($)
Individual	
Adult	112
Child	72
Family	
2 adults and 1 child	260

* Add $15 for every additional child up to a maximum of 5 children in a family Membership. Family membership is limited to a maximum of 2 adults and 5 children.

Membership benefits:

- Unlimited admission to Bird Park, Night Safari and Singapore Zoo;
- Complimentary tram rides at Bird Park and Singapore Zoo;
- Complimentary English language commentary tram rides at Night Safari on all nights;
- 10% discount at all F&B outlets at all parks;
- 10% discount off regular-priced products at all retail outlets at all parks.

Should Mr Tan take up a membership package? If so, which package should he buy? Justify the decision you make and show your calculations clearly. [5]

[End of Paper]

1. (a) It is given that $S = \frac{a}{2}\sqrt{n^2 - b}$.

(i) Find S when $a = 8$, $b = -7$ and $n = 3$. [1]

(ii) Express n in terms of a, b and S. [3]

(b) Solve the equation $\frac{5}{\sqrt[3]{5}} = 5^{x-1}$. [2]

(c) Factorise $9x^2 - 25 + 12xy - 20y$. [2]

(d) Simplify $\frac{x^2 + 3x - 7}{x - 4} + \frac{9 + 3x}{4 - x}$. [3]

(a)(i) $S = \frac{8}{2}\sqrt{(3)^2 - (-7)}$
$= 16$ B1

(ii) $S = \frac{a}{2}\sqrt{n^2 - b}$
$\frac{2S}{a} = \sqrt{n^2 - b}$ M1
$n^2 - b = \frac{4S^2}{a^2}$ M1
$n = \pm\sqrt{\frac{4S^2}{a^2} + b}$ A1

(b)) $\frac{5}{\sqrt[3]{5}} = 5^{x-1}$
$x - 1 = 1 - \frac{1}{3}$ M1
$x = 1\frac{2}{3}$ A1

(c)
$9x^2 - 25 + 12xy - 20y$
$= (3x + 5)(3x - 5) + 4y(3x - 5)$ M1
$= (3x - 5)(3x + 5 + 4y)$ A1

(d) $\frac{x^2 + 3x - 7}{x - 4} + \frac{9 + 3x}{4 - x}$
$= \frac{x^2 + 3x - 7}{x - 4} - \frac{9 + 3x}{(x - 4)}$ M1
$= \frac{x^2 + 3x - 7 - 9 - 3x}{x - 4}$ B1
$= \frac{x^2 - 16}{x - 4}$
$= x + 4$ A1

2. Nancy is planning a holiday to United States. On 1 March 2017, she exchanged S$3000 into US dollars (US$) at Kumar's Money Exchange at a rate of US$1 = S$$x$.

(a) Find an expression, in terms of x, for the amount of US$ she received from Kumar's Money Exchange. [1]

On 15 March 2017, she decided to exchange another S$2100 into US$ at Lee's Money Exchange at a rate of US$1 = S$$(x - 0.1)$.

(b) Find an expression, in terms of x, for the amount of US$ she received from Lee's Money Exchange. [1]

(c) Given that Nancy received a total of US$3500 from the two Money Exchanges, form an equation in x and show that it simplifies to $70x^2 - 109x + 6 = 0$. [3]

(d) (i) Solve the equation $70x^2 - 109x + 6 = 0$, giving your answers correct to 4 decimal places. [2]

(ii) Suggest a reason why one of the answers has to be rejected. [1]

(iii) Hence, find the exchange rate between S$ and US$ offered by Lee's Money Exchange. [1]

(e) Is it better for Nancy to change her currency on 1 March or 15 March? Justify your answer with appropriate workings. [2]

(a) $\frac{3000}{x}$ B1

(b) $\frac{2100}{x - 0.1}$ B1

(c) $\frac{3000}{x} + \frac{2100}{x - 0.1} = 3500$ M1
$\frac{3000(x - 0.1) + 2100x}{x(x - 0.1)} = 3500$ M1
$5100x - 300 = 3500(x^2 - 0.1x)$
$3500x^2 - 5450x + 300 = 0$ B1
$70x^2 - 109x + 6 = 0$ *(shown)*

(d)(i)
$x = \frac{-(-109) \pm \sqrt{(-109)^2 - 4(70)(6)}}{2(70)}$ M1
$= \frac{109 \pm \sqrt{10201}}{140}$
$= 1.5$ (exact value) or 0.0571 A1 (for both answers)

(ii) The answer 0.0571 has to be rejected as substituting it into $(x-0.1)$ will make the value negative, and thus inapplicable. R1

(iii) The exchange rate offered by Lee is US$1 = S$1.40. A1

(e) On 1 Mar:

$S\$3000 = US\$\left(\frac{3000}{1.5}\right) = US\$2000$ | M1; evaluation step needed |

On 15 Mar:

$S\$3000 = US\$\left(\frac{3000}{1.5-0.1}\right) = US\$2142.86$

Since Nancy can exchange for more US$ on 15 Mar as compared to 1 Mar for the same S$3000, she should changed her money on 15 Mar. | R1 |

3. A cake shop sells 3 different types of muffins. The table below shows the numbers of muffins sold over 2 days and the price of each type of muffin.

	Chocolate	Blueberry	Cheese
Day 1	12	25	10
Day 2	15	24	9
Price of each muffin, $	2.00	1.50	2.50

It is given that $\mathbf{P} = \begin{pmatrix} 12 & 25 & 10 \\ 15 & 24 & 9 \end{pmatrix}$ and $\mathbf{Q} = \begin{pmatrix} 2.0 \\ 1.5 \\ 2.5 \end{pmatrix}$.

(a) Evaluate the matrix $\mathbf{M} = \mathbf{PQ}$. [1]

(b) State what the elements of $\mathbf{M}$ represent. [1]

(c) The cake shop is having a promotion by giving a 25% discount on chocolate muffins, a 30% discount on blueberry muffins and a 40% discount on cheese muffins.

(i) Write down a 3×3 matrix $\mathbf{R}$, where the product of $\mathbf{RQ}$ will give the discounted price of each type of muffin. [1]

(ii) Evaluate the matrix $\mathbf{N} = \mathbf{RQ}$. [1]

(d) Evaluate the matrix $\mathbf{PN}$ and state what the elements of $\mathbf{PN}$ represent. [2]

(e) Given that $\mathbf{T} = \begin{pmatrix} 1 & 1 \end{pmatrix}$, evaluate the matrix $\mathbf{TPN}$ and state what the elements of $\mathbf{TPN}$ represent. [2]

(a) $\mathbf{M} = \begin{pmatrix} 12 & 25 & 10 \\ 15 & 24 & 9 \end{pmatrix}\begin{pmatrix} 2.0 \\ 1.5 \\ 2.5 \end{pmatrix}$

$= \begin{pmatrix} 86.5 \\ 88.5 \end{pmatrix}$ | A1 |

(b) The elements of $\mathbf{M}$ represent the total amount of money collected from the sale of 3 types of muffins in Day 1 and Day 2 respectively. | A1 | (accept alternatives with keywords)

(c)(i) $\mathbf{R} = \begin{pmatrix} 0.75 & 0 & 0 \\ 0 & 0.7 & 0 \\ 0 & 0 & 0.6 \end{pmatrix}$ A1

(c)(ii) $\mathbf{N} = \begin{pmatrix} 0.75 & 0 & 0 \\ 0 & 0.7 & 0 \\ 0 & 0 & 0.6 \end{pmatrix}\begin{pmatrix} 2.0 \\ 1.5 \\ 2.5 \end{pmatrix} = \begin{pmatrix} 1.5 \\ 1.05 \\ 1.5 \end{pmatrix}$ A1

(d) $\mathbf{PN} = \begin{pmatrix} 12 & 25 & 10 \\ 15 & 24 & 9 \end{pmatrix}\begin{pmatrix} 1.5 \\ 1.05 \\ 1.5 \end{pmatrix} = \begin{pmatrix} 59.25 \\ 61.2 \end{pmatrix}$ A1

The elements of **PN** represent the total amount of money collected from the sale of muffins during the promotion period on Day 1 and Day 2 respectively. A1

(e) $\mathbf{TPN} = \begin{pmatrix} 1 & 1 \end{pmatrix}\begin{pmatrix} 59.25 \\ 61.2 \end{pmatrix} = (120.45)$ A1

The elements of **TPN** represent the total amount of money collected from the sale of muffins during the promotion period over 2 days. A1

4. (a) Bag A contains 1 gold medal, 4 silver medals and 2 bronze medals. Bag B contains 2 gold medals and 5 silver medals. Peter draws a medal at random from bag A, took note of the colour and placed it into bag B. He then draws a medal at random from bag B.

(i) Draw a tree diagram to show the probabilities of the possible outcomes. [2]

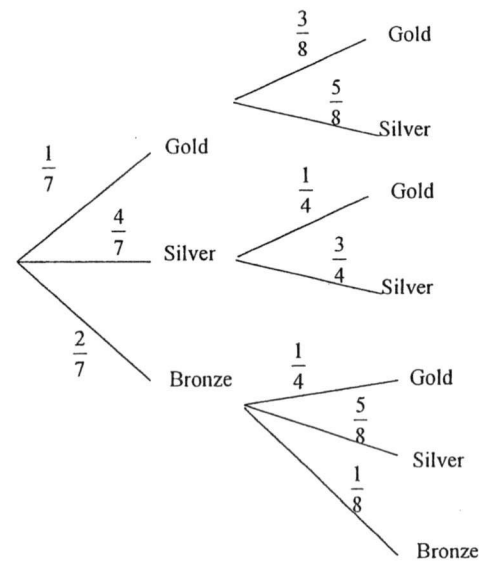
(ii) Find, as a fraction in its simplest form, the probability that

(a) the medal drawn from bag B is a gold medal, [2]

(b) the medal drawn from bag B is a bronze medal. [1]

(a)(i) Answer:

A2; deduct 1 mark for each error



(b)(i) P(medal from B is a gold medal)

$= \frac{1}{7}\times\frac{3}{8} + \frac{4}{7}\times\frac{2}{8} + \frac{2}{7}\times\frac{2}{8}$ M1

$= \frac{15}{56}$ A1

(b) P(medal from B is a bronze medal)

$= \frac{2}{7}\times\frac{1}{8}$

$= \frac{1}{28}$ A1

4(b) (i) The nth term of a sequence is given by $T_n = n^2 + 2n - 5$.

(a) Write down the first 5 terms of the sequence. [2]

(b) Which term of the sequence has value 163? [2]

(ii) The first five terms of another sequence are 1, 6, 13, 22, 33, ...

(a) By comparing this sequence with the sequence in part (i), write down the nth term of the sequence 1, 6, 13, 22, 33, ... [1]

(b) Hence, find the 99th term. [1]

(b)(i)(a) First 5 terms: $-2, 3, 10, 19, 30$ | A2; deduct 1 mark for each error

(b) $n^2 + 2n - 5 = 163$ | M1

$n^2 + 2n - 168 = 0$

$(n-12)(n+14) = 0$

$n = 12 \quad or \quad -14 \quad (rejected)$

The 12th term has value 163. | A1

(ii)(a) $T_n = (n^2 + 2n - 5) + 3 = n^2 + 2n - 2$ | A1

(b)

$T_n = (99)^2 + 2(99) - 2$

$= 9997$ | A1

5(a) Use the graph to find the median mass. [1]

(b) In the following grouped frequency table of the mass of mangoes in the plantation, write down the values of p and of q. [2]

Mass (x kg)	$6 \le x < 14$	$14 \le x < 22$	$22 \le x < 30$	$30 \le x < 38$	$38 \le x < 46$
Frequency	4	p	14	q	24

(c) Using your grouped frequency table, calculate an estimate of

(i) the mean mass, [2]

(ii) the standard deviation. [1]

(d) In Plantation B, the total mass of the mangoes produced by each of 80 mango trees were measured. Their mean and standard deviation were found to be 29.1 kg and 10.4 kg respectively. Make two comparisons between the mass of the mangoes produced by the trees in both plantations. [2]

(a) Median mass = 34 kg | A1

(b) $p = 12 - 4 = 8$ | A1 $\quad q = 56 - 26 = 30$ | A1

(c)(i) Estimated mean mass

$= \dfrac{4\times10 + 8\times18 + 14\times26 + 30\times34 + 24\times42}{80}$ kg | M1; for mid-values into formula

$= 32.2$ kg | A1

(ii) Standard deviation

$= \sqrt{\dfrac{89472}{80} - \left(\dfrac{2576}{80}\right)^2}$ kg

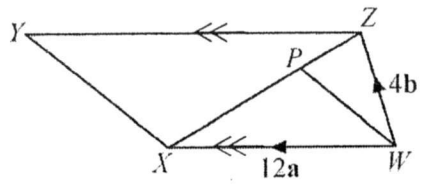
$= 9.031$ kg

$= 9.03$ kg | A1 (to 3 s.f.)

(d) - Since mean mass of A is higher than mean mass of B, the average mass of mangoes produced in plantation A is heavier than that in plantation B. | A1

- Since standard deviation of A is less than standard deviation of B, the mass of mangoes produced in plantation A is more consistent than that in plantation B. | A1

6. In the diagram, $WXYZ$ is a trapezium and WX is parallel to ZY. The point P on XZ is such that $ZP : PX = 1 : 3$ and $WX : ZY = 3 : 4$.



(a) Give that $\overrightarrow{WX} = 12\mathbf{a}$ and $\overrightarrow{WZ} = 4\mathbf{b}$, express in terms of **a** and/or **b**,

(i) $\overrightarrow{ZX}$, [1]

(ii) $\overrightarrow{WP}$, [1]

(b) Determine with justification, if the line XY is parallel to the line WP. [2]

(c) Find

(i) $\dfrac{\text{Area of } \Delta WZP}{\text{Area of } \Delta WXP}$, [1]

(ii) $\dfrac{\text{Area of } \Delta WZP}{\text{Area of } \Delta YXZ}$. [2]

(d) Hence, find the area of $WXYZ$ if the area of ΔWZP is 12 units^2. [1]

(a)(i) $\overrightarrow{ZX} = \overrightarrow{ZW} + \overrightarrow{WX}$
$= -4\mathbf{b} + 12\mathbf{a}$ A1

(ii) $\overrightarrow{ZP} = \dfrac{1}{4}\overrightarrow{ZX}$
$= -\mathbf{b} + 3\mathbf{a}$

$\overrightarrow{WP} = \overrightarrow{WZ} + \overrightarrow{ZP}$
$= 4\mathbf{b} + (-\mathbf{b} + 3\mathbf{a})$
$= 3\mathbf{b} + 3\mathbf{a}$ A1

Note: 1 mark overall is penalized if students missed out the vector symbol

(b) $\overrightarrow{ZY} = \dfrac{4}{3}\overrightarrow{WX} = 16\mathbf{a}$

$\overrightarrow{XY} = \overrightarrow{XZ} + \overrightarrow{ZY}$
$= (4\mathbf{b} - 12\mathbf{a}) + 16\mathbf{a}$ M1
$= 4\mathbf{b} + 4\mathbf{a}$
$= \dfrac{4}{3}\overrightarrow{WP}$

Since $\overrightarrow{XY} = \dfrac{4}{3}\overrightarrow{WP}$, XY is parallel to WP. R1

(c)(i) $\dfrac{\text{Area of } \Delta WZP}{\text{Area of } \Delta WXP} = \dfrac{\frac{1}{2} \times ZP \times h}{\frac{1}{2} \times PX \times h} = \dfrac{1}{3}$ A1

(ii) $\dfrac{\text{Area of } \Delta WZP}{\text{Area of } \Delta YXZ}$

$= \dfrac{\text{Area of } \Delta WZP}{\text{Area of } \Delta WZX} \times \dfrac{\text{Area of } \Delta WZX}{\text{Area of } \Delta YXZ}$

$= \dfrac{1}{4} \times \dfrac{\frac{1}{2} \times WX \times h}{\frac{1}{2} \times ZY \times h}$ M1

$= \dfrac{1}{4} \times \dfrac{WX}{ZY}$

$= \dfrac{1}{4} \times \dfrac{3}{4}$

$= \dfrac{3}{16}$ A1

(d) Area of $\Delta WZX = 4 \times 12 = 48$ units^2

Area of $\Delta YXZ = \dfrac{16}{3} \times 12$ units^2
$= 64$ units^2

Area of $WXYZ = 48 + 64$ units^2
$= 112$ units^2 A1

7. Diagram I shows a trough where the base $BCGF$ and the top $ADHE$ are horizontal rectangles. Each of the vertical sides $ABCD$ and $EFGH$ is a trapezium. It is given that $AD = 15$ cm, $BC = 11$ cm, $BX = 9$ cm and $CG = 20$ cm. It is fully filled with water.

Take $\pi = \dfrac{22}{7}$.

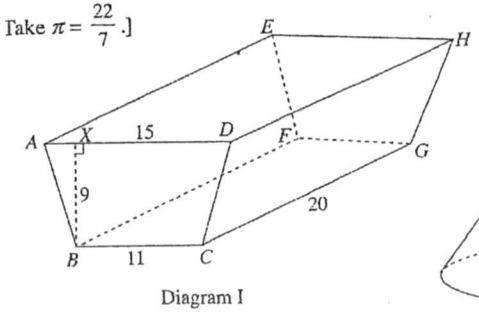


Diagram I

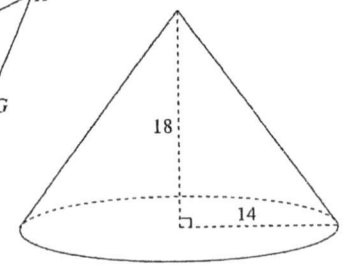


Diagram II

(a) Calculate the volume of the water in the trough in Diagram I. [2]

(b) All the water in Diagram I is made to flow down through a hole at the top of the cone in Diagram II. The water flows at a rate of 0.2 litres per minute. Calculate the time taken for the water in the trough to be transferred, giving your answer in minutes and seconds. [2]

(c) The cone in Diagram II is of base radius 14 cm and height 18 cm. The cone is made to stand on its base. Calculate the height of water in the cone. [4]

(a) Volume of water

$= \left[\frac{1}{2}\times(11+15)\times 9\right]\times 20\,\text{cm}^3$ M1

$= 2340\ \text{cm}^3$ A1

(b) 0.2 litres $= 200\ \text{cm}^3$

Time taken $= \frac{2340}{200}$ min M1

$= 11.7$ min $= 11$ min 42 sec A1

(c) Volume of cone

$= \frac{1}{3}\times\frac{22}{7}\times 14^2\times 18\ \text{cm}^3 = 3696\ \text{cm}^3$

Volume of cone not occupied by water

$= (3696 - 2340)\ \text{cm}^3 = 1356\ \text{cm}^3$ B1

By similar triangles, $\frac{h}{18} = \frac{r}{14}$

$r = \frac{7}{9}h$ B1

$\frac{1}{3}\times\frac{22}{7}\times(\frac{7}{9}h)^2\times h = 1356$

$h^3 = 2139\frac{51}{77}$

$h = 12.8859$ cm

≈ 12.9 cm A1

Height of water in cone $= 18 - 12.8859$ cm $= 5.1141$ cm ≈ 5.11 cm A1

Alternative method for (c):

Volume of cone not occupied by water
$= (3696 - 2340)\ \text{cm}^3$
$= 1356\ \text{cm}^3$ B1

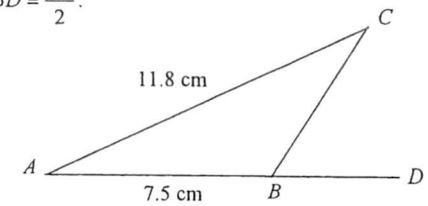
$\left(\frac{h}{18}\right)^3 = \frac{1356}{3696}$ M1

$h = 12.8859$

$h \approx 12.9$ A1

Height of water in cone
$= 18 - 12.8859$ cm
$= 5.1141$ cm A1
≈ 5.11 cm

8. (a) In the diagram below, ABD is a straight line. $AB = 7.5$ cm, $AC = 11.8$ cm and $\sin\angle CBD = \frac{\sqrt{3}}{2}$.



Calculate

(i) $\sin\angle ACB$, giving your answer correct to 3 significant figures, [2]

(ii) $\angle BAC$, [2]

(iii) the area of ΔABC. [1]

(a)(i) $\frac{\sin\angle ACB}{7.5} = \frac{\frac{\sqrt{3}}{2}}{11.8}$ M1

$\sin\angle ACB = 0.550439$

≈ 0.550 (to 3 s.f.) A1

(ii) $\sin\angle CBD = \frac{\sqrt{3}}{2}$

$\angle CBD = 60°$

$\angle CBA = 120°$ M1

$\sin\angle ACB = 0.550439$

$\angle ACB = 33.3971°$

$\angle BAC = 180° - 120° - 33.3971°$ (angle sum of triangle)

$= 26.6029°$

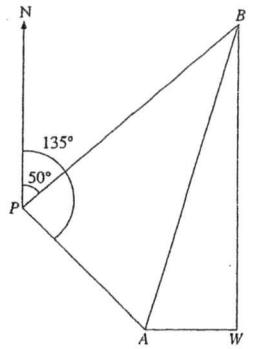
$\sim 26.6°$ (correct to 1 dp) A1

(iii) Area of $\Delta ABC = \frac{1}{2}\times 11.8\times 7.5\times\sin 26.6029°\ \text{cm}^2$

$= 19.8153\ \text{cm}^2$

$\approx 19.8\ \text{cm}^2$ (to 3 s.f.) A1

8. (b) Two military ships, Amaze and Brave left port P at 1000. Amaze sailed at 12 km/h on a bearing of 135°. Brave sailed at 18 km/h on a bearing of 050°. After sailing for 3 hours, Amaze is at Island A and Brave is at Island B.



Calculate

(i) the length of AB. [3]

(ii) the bearing of A from B. [2]

(iii) Amaze later sailed to Island W which is due east of Island A. It is given that Island B is due north of Island W. Find the distance BW. [2]

(iv) A helicopter is hovering at a height of 10 km vertically above B. Find the greatest angle of elevation Amaze can have of the helicopter as it sailed towards W. [2]

(b)(i) Derive: $PB = 54$ km and $PA = 36$ km B1

$AB^2 = 36^2 + 54^2 - 2(36)(54)\cos 85°$ M1

$= 62.2345$

$AB \approx 62.2$ km (to 3 s.f.) A1

(ii) $\dfrac{\sin \angle ABP}{36} = \dfrac{\sin 85°}{62.2345}$

$\sin \angle ABP = \dfrac{36 \sin 85°}{62.2345}$ B1

$\angle ABP = 35.1876°$

Bearing of A from $B = 360° - (180° - 050°) - 035.1876°$

$= 194.8124°$

$\sim 194.8°$ (correct to 1 d.p.) A1

(iii) $\angle ABW = 180° - 130° - 35.1876° = 14.8124°$

$BW = (\cos 14.8124°) \times 62.2345$ M1 * $BW = \cos 14.81° \times 62.234$ has no mark

$= 60.1663$ km

~ 60.2 km (to 3 s.f.)

(iv) Let the greatest angle of elevation be α.

$\tan \alpha = \dfrac{10}{60.166}$

$\alpha = 9.4°$ (to 1 d.p.)

10.

(a)(i) Amount = 2 x \$(33+29+30) + 3 x \$(22+19+20)

= \$367 A1

(ii) Amount (under 3-parkhopper package)

= \$(2 x 69) + \$(3 x 49) M1

= \$285

He would save (\$367 - \$285) = \$82 A1

(b) Membership Individual (MI):
(2 x \$112) + (3 x \$72) = \$440

Membership Family (MF):
\$260 + (\$15 x 2) = \$290

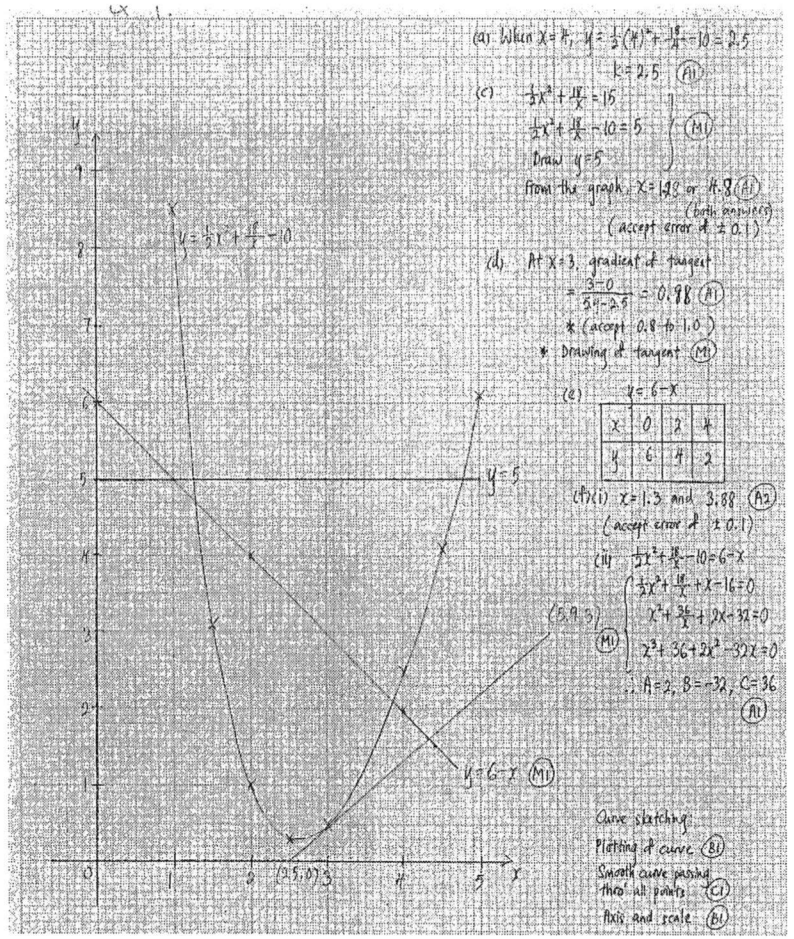
M1

Comparing MI and MF:
Savings = \$440 - \$290 = \$150

Mr Tan should take up Membership Family package as he would save \$150. A1 + R1

From part (a)(ii): 3-parkhopper package = \$285
Although the 3-parkhopper package is cheaper than the MF by \$5, but Mr Tay only needs to pay this extra \$5 to have unlimted admission into all 3 parks in a year, which is more value for money.
[M1] for comparison between 3-parkhopper package and MF
[R1] for logical reasoning to justify the extra \$5 spent

9



(a) When $x = 4$, $y = \frac{1}{2}(4)^2 + \frac{18}{4} - 10 = 2.5$

$k = 2.5$ (A1)

(c) $\frac{1}{2}x^2 + \frac{18}{x} = 15$

$\frac{1}{2}x^2 + \frac{18}{x} - 10 = 5$ (M1)

Draw $y = 5$

from the graph, $x = 1.48$ or 11.8 (A1)

(both answers)

(accept error of ± 0.1)

(d) At $x = 3$, gradient of tangent

$= \frac{5-0}{5.4-2.5} = 0.98$ (A1)

* (accept 0.8 to 1.0)

* Drawing of tangent (M1)

(e)

x	0	2	4
y	6	4	2

(f)(i) $x = 1.3$ and 3.88 (A2)

(accept error of ± 0.1)

(ii) $\frac{1}{2}x^2 + \frac{18}{x} - 10 = 6 - x$

(M1)
$\frac{1}{2}x^2 + \frac{18}{x} + x - 16 = 0$

$x^3 + 36 + 2x^2 - 32x = 0$

$x^3 + 36 + 2x^2 - 32x = 0$

$\therefore A = 2, B = -32, C = 36$ (A1)

Curve sketching:

Plotting of curves (B1)

Smooth curve passing through all points (C1)

Axis and scale (B1)

Answer **all** the questions.

1 **(a)** Simplify $9-5(2x+3)$.

Answer: ..... [1]

(b) Factorise $30xy^2-6xy$.

Answer: ..... [1]

---

2 **(a)** These are the first five terms in a sequence.

1 4 9 16 25

Write down an expression for the nth term of this sequence.

Answer: ..... [1]

(b) Hence, write down an expression for the nth term of this sequence.

4 7 12 19 28

Answer: ..... [1]

---

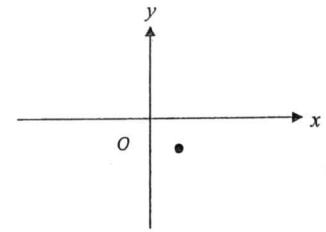
3 Determine if 3^{400} or 8^{200} is greater. Explain your answer.

Answer:

[2]

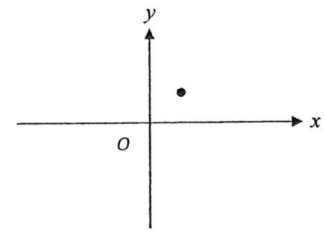
---

4 **(a)** On the diagram, sketch the graph of $y=-\frac{1}{x}$. The point $(1,-1)$ is marked.



[1]

(b) On the diagram, sketch the graph of $y=\frac{1}{x^2}$. The point $(1,1)$ is marked.



[1]

---

5 **(a)** Express $7-4x+x^2$ in the form $p+(x+q)^2$ where p and q are constants.

Answer: ..... [2]

(b) Hence, explain why $7-4x+x^2=0$ has no real solution.

Answer: .....

..... [1]

---

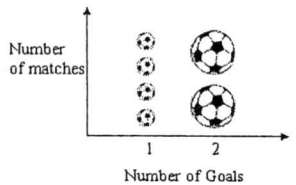
6 The table shows the number of goals scored in the soccer matches played by a group of boys in S League.

Number of Goals	1	2	3	4
Number of Matches	4	2	7	x

(a) The boys scored 4 goals in x number of matches. Given that the modal number of goals is 3, find the greatest possible number of matches played by the boys during the league.

Answer: ..... [1]

(b) Part of the above table is represented in the pictogram.



State one aspect of the graph that may be misleading and explain how this may lead to a misinterpretation of the pictogram.

Answer: .....

.....

..... [2]

---

7 Megan is playing with 594 cubes. Megan uses all 594 cubes to make a cuboid. Each of the sides of the cuboid is made up of more than 3 cubes. Find the number of cubes on each side of the cuboid.

Answer: ..... × ..... × ..... [3]

---

8 When an object travels in a circular motion, there are two equations which govern the physics of the motion.

Equation **I**: $F = \frac{Mv^2}{r}$

Equation **II**: $F = \frac{GMN}{r^2}$

(a) Given that $M = 7\times10^{22}$, $v = 1.2\times10^{3}$ and $r = 2.6\times10^{7}$, using equation **I**, evaluate F. Leave your answers in standard notation.

Answer: $F =$ ..... [1]

(b) Using both equations **I** and **II**, express v in terms of N, G and r.

Answer: ..... [2]

---

9 (a) Solve the inequality $17 - 4x < x + 5 \leq 3x - 6$.

Answer: ..... [2]

(b) Hence, write down the smallest value of x if x is a prime number.

Answer: ..... [1]

---

10 The freezing point of a liquid is $-7\,°C$. The temperature difference between its freezing point and boiling point is 25°C.

(a) Find the boiling point of the liquid.

Answer: ..... °C [1]

(b) An addition of a small quantity of salt into the liquid decreased its freezing point by x°C and increased its boiling point by y°C.

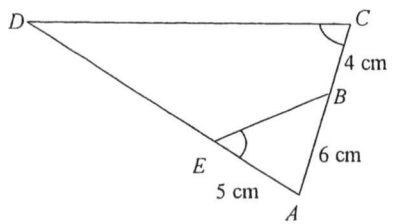
Find the temperature difference between the freezing point and the boiling point of this liquid-salt mixture, in terms of x and y.

Answer: ..... °C [2]

---

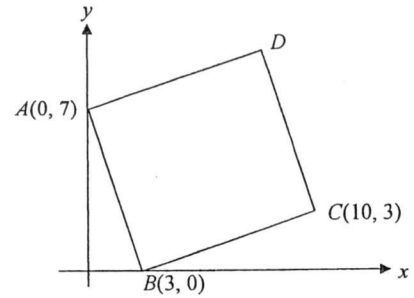
11 In the diagram below, $AB = 6$ cm, $BC = 4$ cm, $AE = 5$ cm and $\angle AEB = \angle ACD$.

Find the length of DE.



Answer: ..... cm [3]

12 In the diagram below, not drawn to scale, $ABCD$ is a square.
A is (0, 7), B is (3, 0) and C is (10, 3).



(a) Find the coordinates of D.

Answer: D (..... , ...) [1]

(b) Find the area of triangle BCS, where S is the centre of the square.

Answer: ..... units^2 [2]

13 $\varepsilon = \{x : 6 < x < 20\}$
$A = \{x : x \text{ is an even number}\}$
$B = \{x : x \text{ is a multiple of } 3\}$

(a) List the elements of

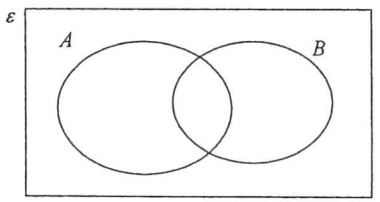
(i) A',

Answer: ..... [1]

(ii) $A' \cap B$.

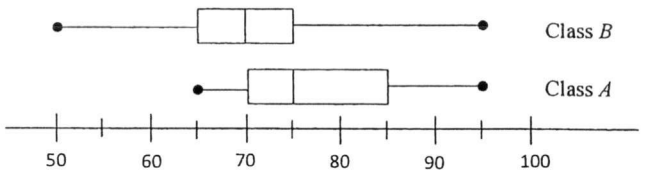
Answer: ..... [1]

(b) On the Venn diagram shown, shade the set $A' \cup B'$.



[1]

14 Class A and class B have 40 students each. The box-and-whisker plot below shows the distribution of their marks in a Math test.



(a) Find the number of students in, class A, who scored less than 85 marks.

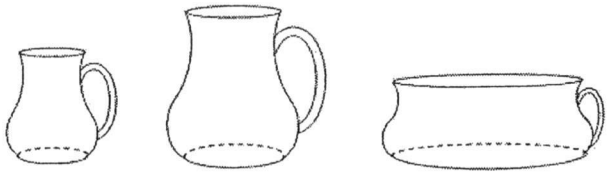
Answer: ..... [1]

(b) Below are two statements comparing the marks for Class A and B.
For each one, write whether you agree or disagree, giving a reason for each answer.

Statement	Agree/ Disagree	Reason
Students in Class A score better		
Greater number of students in Class A score at least 70 marks		

[2]

15 There are three jugs X, Y and Z. Jugs X and Y are geometrically similar. The volume of X and Y are 216 cm^3 and 512 cm^3 respectively.



Jug X Jug Y Jug Z

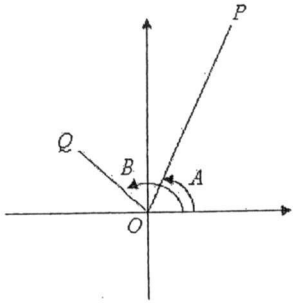
(a) Find the ratio of the height of X to Y.

Answer: ..... : ..... [1]

(b) The volume of X is given by the formula $V = \frac{7}{15}r^2h$ where h is the height of the jug and r the radius of the circular base. Find the volume of Z which has $\frac{3}{4}$ the height of X and thrice the radius of the circular base of X.

Answer: ..... cm^3 [2]

16 In the diagram, the coordinates of P (8, 15) and Q (– 3, 4) are drawn.



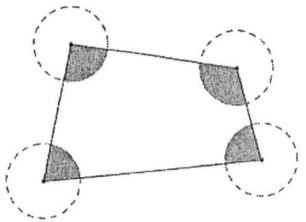
(a) Find the value of tan A.

Answer: ..... [1]

(b) Find the value of cos B.

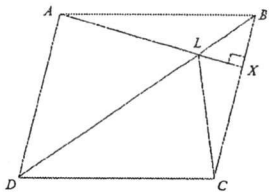
Answer: ..... [2]

17 **(a)** The figure below shows a quadrilateral. Identical circles with radius of 2 cm are drawn such that the centres of the circles are at the vertices of the quadrilateral. Calculate the area of the shaded region if each of the side of the quadrilateral is at least 5 cm. Leave your answer in π.



Answer: ..... cm^2 [2]

(b) In the diagram, $ABCD$ is a rhombus.
AX is perpendicular to BC and intersects BD at L.



Prove that ΔALD is congruent to ΔCLD.

Answer:

[2]

---

18 An area of 324 km^2 is represented on a map by an area of 36 cm^2.

(a) Find the scale of the map in the form 1 : n.

Answer: ..... : ..... [2]

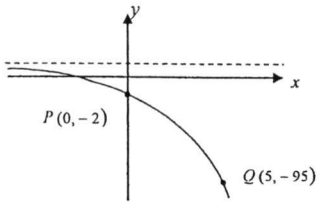
(b) Find the length of a road on the map with an actual distance of 85 km, leaving your answer to the nearest centimeters.

Answer: ..... cm [2]

---

19 The sketch shows the graph of $y = ka^x + 1$. The graph passes through the points $P(0, -2)$ and $Q(5, -95)$.

(a) Find the values of k and a.



Answer: $k =$ .....

$a =$ ..... [2]

(b) A straight line is drawn from P to Q.

Find the equation of the line PQ.

Answer: ..... [2]

---

20 Alvin rented a 696 square feet apartment in Washington D.C. for 1800 USD.
Benjamin rented a 60 m^2 apartment in Beijing for 6500 CNY.

1.00 USD = 6.81 CNY.
1 square feet = 0.093 m^2.

Which apartment is cheaper to rent? You must show your calculations.

Answer: ..... [4]

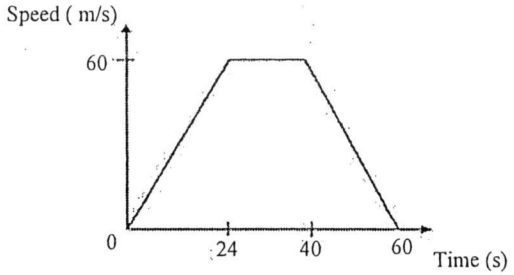
21 (a) Factorise completely $5ax - 5ay - 25dx + 25dy$.

Answer: ..... [2]

(b) Write as a single fraction in its simplest form $\frac{5}{x-2} + \frac{11}{x^2-4}$.

Answer: ..... [2]

22 The diagram shows the speed-time graph of a car's journey.



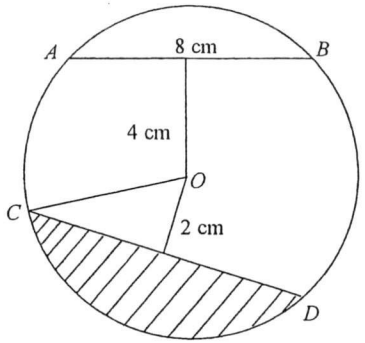
(a) Calculate the acceleration when $t = 15$ s.

Answer: ..... m/s^2 [1]

(b) Calculate the time taken by the car to travel 1.62 km.

Answer: ..... s [3]

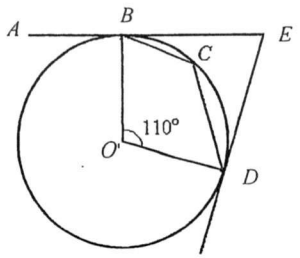
23 The diagram shows four points A, B, C and D on the circumference of a circle centre O.



AB is a chord of length 8 cm and is 4 cm from O. CD is a chord 2 cm from O. Find the area of the shaded segment.

Answer: ..... cm^2 [4]

24 The diagram shows a circle BDC, with centre O. $\angle BOD = 110°$, AE and DE are tangents to the circle at B and D respectively.



(a) Showing all reasons clearly, find $\angle BCD$.

Answer: ..... ° [2]

(b) Explain why $OBED$ lie on the circumference of another circle.

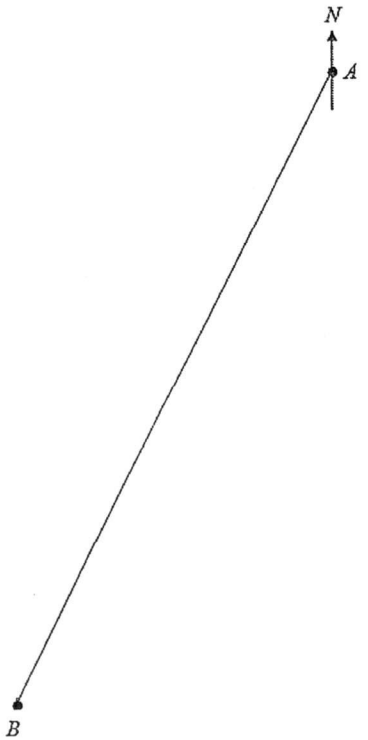
Answer:

[2]

25 Points A, B and C are three checkpoints on flat ground. Points A and B are given below.

(a) Point C is located 7 km away from Point A, at a bearing of 160°.
Using a scale of 1 cm to represent 1 km, construct and label the position of checkpoint C clearly. [1]

(b) A checkpoint D is to be built equidistant from
I. A and B.
II. AB and AC.
Using ruler and compasses only, find and label the position of the checkpoint D. [3]



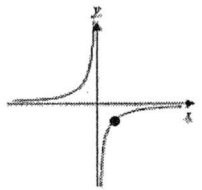
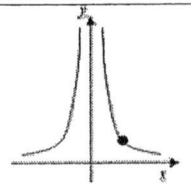
End of Paper

Secondary 4 Express/ 5NA Mid Year Examination 2017
Mathematics 4048 Paper 1 Marking Scheme

1	(a)	$-10x-6$	B1	1
	(b)	$6xy(5y-1)$	B1	1
Marker's comment:				

2	(a)	n^2	B1	1
	(b)	n^2+3	B1	1
Marker's comment:				

3		$3^{400}=3^{2(200)}$ $=9^{200}$ $9^{200}>8^{200}$ Hence 3^{400} is greater.	M1 A1	M1 for showing 9^{200} No A1 if no explanation. 2
Marker's comment:				

4	(a)		B1	1
	(b)		B1	1
Marker's comment:				

5	(a)	$(x-2)^2+7-4$ $=3+(x-2)^2$	M1 A1	2
	(b)	$(x-2)^2$ **is always positive** or zero hence $3+(x-2)^2$ **is always greater than zero or does not cut the x axis, hence no solution.**	B1	1
Marker's comment:				

6	(a)	19	B1	1
	(b)	The **size of the football is not the same** hence it may mislead that **bigger football means more matches.** **Or** Pictogram of **same height** may mislead that **both have same number of matches.**	B1 B1 B1 B1	B1 for different size B1 for bigger football is misleading as it means more matches. B1 for same height 2 B1 for it misleads that both have same number of matches.
Marker's comment:				

7		$594=2\times3^3\times11$ $=(2\times3)\times3^2\times11$ (each side made up of more than 3) $=6\times9\times11$	M1 M1 A1	 3
Marker's comment:				

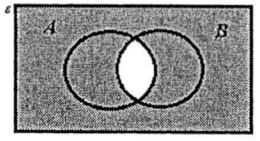
8	(a)	$F=3.88\times10^{21}$	B1	1 Rounded off to 3sf.
	(b)	$\frac{GMN}{r^2}=\frac{Mv^2}{r}$ $v^2=\frac{GN}{r}$ $v=\pm\sqrt{\frac{GN}{r}}$	M1 A1	2
Marker's comment:				

9	(a)	$x+5\le3x-6$ $-2x\le-11$ $x\ge5.5$ $17-4x<x+5$ $-5x<-12$ $x>2.4$ Hence $x\ge5.5$.	 M1 A1	M1 for both $x\ge5.5$ and $x>2.4$ correct. 2
	(b)	7	B1	1
Marker's comment:				

10	**(a)**	18°C	B1	1
	(b)	$18+y-(-7-x)$ $=25+y+x$	M1√ A1	ecf for their (a) 2
Marker's comment:				

11		$\frac{AD}{6}=\frac{10}{5}$ $AD=12$ $DE=12-5$ $=7$ cm	M1 M1 A1	 3
Marker's comment:				

12	**(a)**	$D(7, 10)$	B1	1
	(b)	length $AB=\sqrt{7^2+3^2}$ $=\sqrt{58}$ Area of $\Delta BCS=\frac{\left(\sqrt{58}\right)^2}{4}$ $=14.5$ units^2	M1 A1	M1 for length of BC= AB 2
Marker's comment:				

13	**(ai)**	{7, 9, 11, 13, 15, 17, 19}	B1	1
	(aii)	{9, 15}	B1	1
	(b)		B1	1
Marker's comment				

14	**(a)**	$\frac{75}{100}\times 40=30$ students	B1	1
	(b)	**Agree** because the students in class A has **higher median score.**	B1	1 B1 for agree and correct reason.
		Agree because **$Q2$, 50th percentile of students in class B score 70 marks or less** while **$Q1$, 25th percentile of students in class A score 70 marks or less**	B1	1 B1 for agree and correct reason.
Marker's comment:				

15	**(a)**	$\frac{\text{height of } X}{\text{height of } Y}=\sqrt[3]{\frac{216}{512}}$ $=\frac{3}{4}$ Ratio 3 : 4	 B1	 1
	(b)	$V=\frac{7}{15}(3r)^2\frac{3}{4}h$ $=\frac{27}{4}\times\frac{7}{15}r^2h$ $=\frac{27}{4}\times 216$ $=1458$ cm^3	M1 A1	M1 for substituting into formula 2
Marker's comment:				

16	**(a)**	$\frac{15}{8}$	B1	1
	(b)	$OQ=\sqrt{3^2+4^2}$ $=5$ $\cos B=-\frac{3}{5}$	 M1 A1	 2
Marker's comment:				

17	**(a)**	Sum of int angle = 360° 360° is the same as angle at a point. Hence area of shaded region = $\pi(2)^2$ $=4\pi$ cm^2		area of shaded region = area of circle.
			B2	2
	(b)	$AD=CD$ (sides of a rhombus) $\angle ADL=\angle LDC$ (LD bisects $\angle ADC$) DL is common.	M1	
		By SAS, ΔALD is congruent to ΔCLD.	A1	2
Marker's comment:				

18	**(a)**	36 cm^2 on map rep 324 km^2 on ground 6 cm on map rep 18 km on ground 1 cm on map rep 3 km on ground $\therefore$ 1 : 300 000	M1 A1	M1 accept 1 cm^2 on map rep 9 km^2 on ground 2
	(b)	$\frac{85}{3}=28.333$ ≈ 28 cm (nearest cm)	M1√ A1	ecf for their (a) 2
Marker's comment:				

19 **(a)**	Sub. $(0,-2)$, $-2 = ka^0 + 1$ $k = -3$ Sub. $(5,-95)$ and $k=-3$, $-95 = -3a^5 + 1$ $-96 = -3a^5$ $32 = a^5$ $a^5 = 2^5$ $a = 2$	 B1 B1 2	
(b)	$m = \dfrac{-95-(-2)}{5}$ $= -\dfrac{93}{5}$ $y = -\dfrac{93}{5}x - 2$	 M1 A1 2	Accept m as -18.6
Marker's comment:			

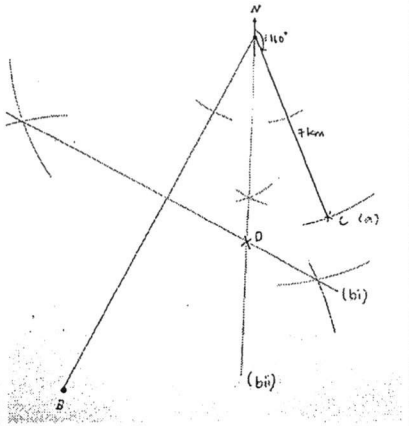
20	1800USD = 1800×6.81CNY = 12 258 CNY 696 sq ft = $696 \times 0.093\,m^2$ = $64.728\,m^2$ Alvin's apartment cost 189.38 CNY/m² Benjamin's apartment cost 108.33 CNY/m² So it is **cheaper to rent Benjamin's apartment**	M1 M1 M1 A1 4	Accept alternative comparison with common basis. Eg in USD. Accept alternative comparison with common basis. Eg . in Sq feet.
Marker's comment:			

21 **(a)**	$5a(x-y) - 25d(x-y)$ $5(a-5d)(x-y)$	M1 A1 2	
(b)	$\dfrac{5(x+2)}{(x-2)(x+2)} + \dfrac{11}{(x-2)(x+2)}$ $\dfrac{5x+10+11}{(x-2)(x+2)}$ $\dfrac{5x+21}{(x-2)(x+2)}$	M1 A1 2	
Marker's comment:			

22 **(a)**	$\dfrac{60}{24} = 2.5\,m/s^2$	B1 1	
(b)	Let t be the time taken to travel 1.62 km. $1620 - \dfrac{1}{2}(24)(60) = 900$ km $60(t-24) = 900$ $t - 24 = 15$ $t = 39s$	 M1 M1 A1 3	
Marker's comment:			

23	Radius $= \sqrt{4^2 + 4^2}$ $= \sqrt{32}$ $= 5.66$ cm (3 sig. fig.) $\angle COD = 2 \times \cos^{-1}\left(\dfrac{2}{\sqrt{32}}\right)$ $= 138.590°$ Area of shaded region $= \left[\dfrac{138.590}{360} \times \pi \times \left(\sqrt{32}\right)^2\right] -$ $\left(\dfrac{1}{2} \times \sqrt{32} \times \sqrt{32} \times \sin 138.590\right)$ $= 28.118$ $= 28.1\,cm^2$ (3 sig. fig.)	 M1 M1 M1 A1	
Marker's comment:			

24 **(a)**	*reflex*$\angle BOD = 360 - 110$ $= 250°$ (angle at a point) $\angle BCD = \dfrac{250}{2}$ $= 125°$ ($\angle$ at centre= 2 $\angle$ at circumf)	 M1 A1	
(b)	$\angle BED = 360 - 110 - 90 - 90$ $= 70°$ (sum of int $\angle$ of quad) Since $\angle BED + \angle BOD = 180°$ (**properties of cyclic quad or $\angle$ in the opp segment**), *OBED* lie on the circumference of a circle.	 M1 A1	
Marker's comment:			

25	(a)		B1	1	B1 for labelling of checkpoint C correctly.
	(b)	Angle bisector $\angle BNC$	B1√		ecf for their (a)
		Perpendicular bisector of AB	B1		
		Point D labelled	B1	3	
Marker's comment:					

Answer **all** the questions.

1 **(a)** Simplify $\left(\frac{2}{xy^2}\right)^{-3} \div \left(\frac{2x}{3y}\right)$. [2]

(b) Simplify $\frac{x^2+4x-21}{2x^2-18}$. [2]

(c) Solve the equation $(p-2)(2p-7)=9$. [3]

---

2 John has three 50 cent coins and two 10 cent coins in his pocket. He takes two coins out of his pocket, at random, one after another. The coins are not replaced.

(a) Draw a complete probability tree diagram to show the possible outcomes and their probabilities. [2]

(b) Find the probability that the total value of the two coins taken out is

(i) 20 cents, [1]

(ii) 60 cents. [2]

(c) John takes out a third coin from his pocket. Find the probability that the total value of the three coins taken out is 70 cents. [2]

---

3 **(a)** **(i)** Express 2025 as a product of its prime factors. [1]

(ii) Using your answer to **part a(i)**, explain why 2025 is a perfect square. [1]

(iii) m and n are both prime numbers.

Find the values of m and n so that $2025 \times \frac{m}{n}$ is a perfect cube. [1]

(b) A gift shop sells three types of goodie bags.

Bag A contains 3 bottles of soft drink and 5 boxes of chocolates.
Bag B contains 2 bottles of soft drink, 3 boxes of chocolates and 5 boxes of candies.
Bag C contains 1 bottle of soft drink, 2 boxes of chocolates and 7 boxes of candies.

The cost price of a bottle of soft drink is \$2, a box of chocolates is \$12 and a box of candies is \$5.

The numbers of each type of item in each goodie bag are represented by the matrix

$$\mathbf{Q} = \begin{pmatrix} 3 & 5 & 0 \\ 2 & 3 & 5 \\ 1 & 2 & 7 \end{pmatrix}.$$

(i) Represent the cost price for each type of item in the goodie bag by the matrix **P**. [1]

(ii) Evaluate $\mathbf{M} = \mathbf{QP}$. [1]

(iii) State what the elements of **M** represent. [1]

(iv) The shop intends to make a profit of 15% on goodie bag A, 20% on goodie bag B and 30% on goodie bag C.

Write down a matrix **N** such that the product **NM** gives the selling price of each goodie bag. [1]

(v) Evaluate **NM**. [1]

---

4 (a) The cost, c dollars, of the electricity bill is given by the formula $c = p + qn$, where n is the number of units of power used.
Mrs Tan has to pay \$54 if she uses 300 units of power and \$78 if she uses 500 units of power.

(i) Write down a pair of simultaneous equations in terms of p and q to represent this information. [2]

(ii) Solve these simultaneous equations to find the values of p and q. [2]

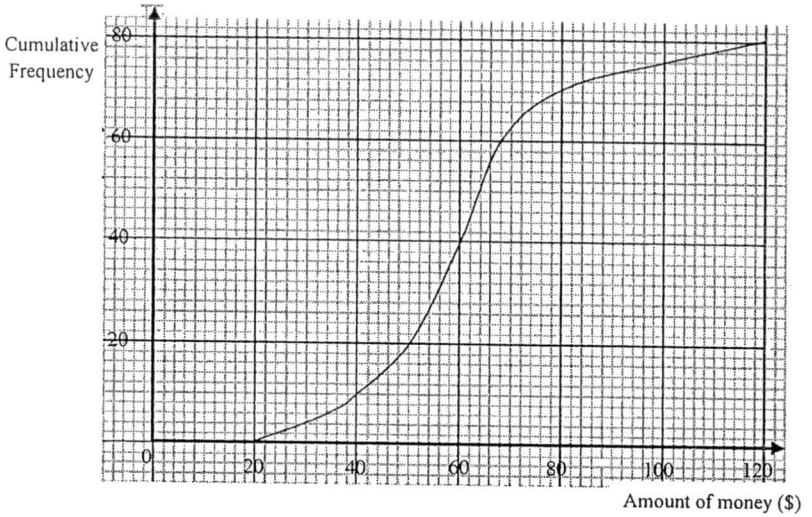
(iii) Find the number of units of power used by Mrs Tan if she has to pay \$68.40. [1]

(b) A shopkeeper raises the prices of his goods by 10%. He then starts the annual sales by offering his customers a discount of 10%.

Did the customers actually receive any discount? Explain your answer showing clear working. [2]

(c) If the height of a triangle is decreased by 20% while its area remained unchanged, find the percentage change in the length of the base. [2]

5 The amount of money collected by 80 members of the school soccer club for a fundraising event is distributed as shown in the cumulative frequency curve below.



(a) Use the cumulative frequency curve to estimate

(i) the median amount of money raised, [1]

(ii) the interquartile range. [2]

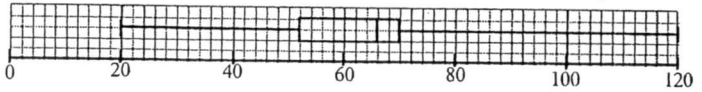
(b) The same information can be represented using a grouped frequency table as shown below.

Amount of money ($)	$20 < x \leq 40$	$40 < x \leq 60$	$60 < x \leq 80$	$80 < x \leq 100$	$100 < x \leq 120$
Frequency	p	30	30	q	4

(i) Obtain the values of p and q. [2]

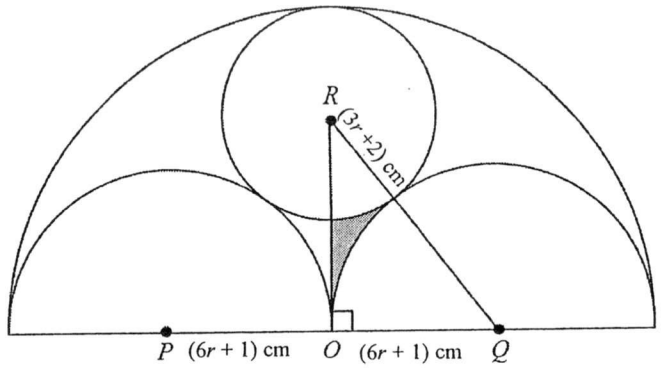
(ii) Using your grouped frequency table, calculate an estimate of the mean and standard deviation. [3]

(c) 80 members from the school outdoor club also raised funds for the same event. The box-and-whisker plot shows the distribution of the amount collected by them.



Using the data from the box-and-whisker plot, make 2 comments about the amount of money collected by the two different clubs. [3]

6 In the diagram, O is the centre of the largest semicircle. The circle with centre R has a radius of $(3r + 2)$ cm. Two identical semicircle, with centres P and Q, each has a radius of $(6r + 1)$ cm.



(a) Write down an expression, in terms of r, for

(i) QR, [1]

(ii) OR. [2]

(b) Form an equation in r and show that it reduces to $18r^2 - 21r - 4 = 0$. [2]

(c) Solve the equation $18r^2 - 21r - 4 = 0$. [3]

(d) Find the area of the shaded region. [4]

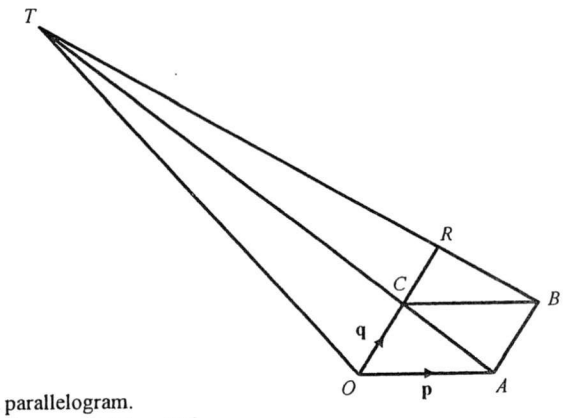
7 **(a)** P is the point $(7, 1)$ and Q is the point $(2, 6)$.

(i) Find $\left|\overrightarrow{PQ}\right|$. [2]

(ii) If $\overrightarrow{SP} = 3\overrightarrow{PQ}$, find the coordinates of S. [2]

(iii) Given that $\overrightarrow{OR} = \begin{pmatrix} h+2 \\ 5 \end{pmatrix}$, find the value of h if $\overrightarrow{OR}$ is parallel to $\overrightarrow{PQ}$. [1]

(b)



$OABC$ is a parallelogram.
$\overrightarrow{OA} = \mathbf{p}$, $\overrightarrow{OC} = \mathbf{q}$ and $\overrightarrow{CT} = 4\overrightarrow{AC}$.
ACT, BRT and OCR are straight lines.

(i) Express each of the following, as simply as possible, in terms of $\mathbf{p}$ and/or $\mathbf{q}$,

(a) $\overrightarrow{OB}$, [1]

(b) $\overrightarrow{OT}$, [2]

(c) $\overrightarrow{BT}$. [1]

(ii) Given that $\overrightarrow{BR} = \frac{4}{5}\mathbf{q} - \mathbf{p}$, find k if $\overrightarrow{OC} = k\overrightarrow{CR}$. [2]

(iii) Find the value of $\frac{\text{area of } \Delta BCR}{\text{area of } \Delta CRT}$. [1]

8 **Answer the whole of this question on a sheet of graph paper.**

The variables x and y are connected by the equation $y = \frac{1}{2}x^3 - 3x^2 + 7$.

Some corresponding values of x and y are given in the table below.

x	–2	–1	0	1	2	3	4	5	6
y	–9	p	7	4.5	–1	–6.5	–9	–5.5	7

(a) Find the value of p. [1]

(b) Using a scale of 2 cm to represent 1 unit, draw a horizontal x-axis for $-2 \le x \le 6$.
Using a scale of 1 cm to represent 1 unit, draw a vertical y-axis for $-9 \le y \le 7$.
On your axes, plot the points given in the table and join them with a smooth curve. [3]

(c) The equation $\frac{1}{2}x^3 - 3x^2 = -12$ has three solutions.
Explain how this can be seen from your graph. [2]

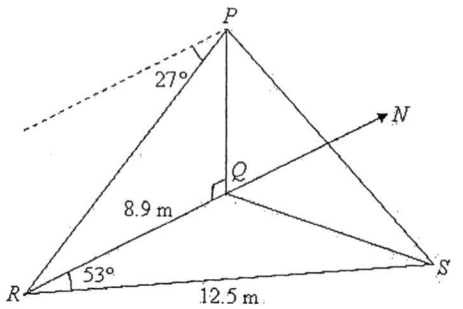
(d) By drawing a tangent, find the gradient of the curve at $x = 0.5$. [2]

(e) **(i)** On the same axes, draw the graph of $y = 2 - x$ for $-2 \le x \le 6$. [1]

(ii) Write down the x-coordinate of the points where this line intersects the curve. [1]

(iii) The x-coordinates of the points where the two graphs intersect are solutions of the equation $x^3 + ax^2 + bx + c = 0$. Find the values of a, b and c. [2]

9. In the diagram below, Q, R and S are three points on horizontal ground.
$RQ = 8.9$ m and $SR = 12.5$ m.
A vertical flag pole PQ stands at Q and the angle of depression of R from P is $27°$.
R is due south of Q and the bearing of S from R is $053°$.



(a) Find

(i) the length of QS, [3]

(ii) the angle QSR, [2]

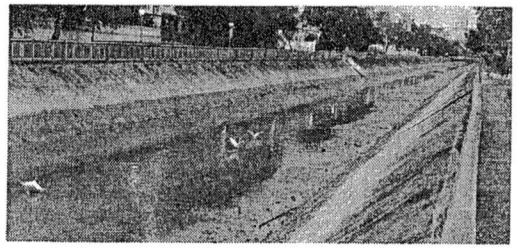
(iii) the bearing of S from Q, [1]

(iv) the area of triangle QRS, [2]

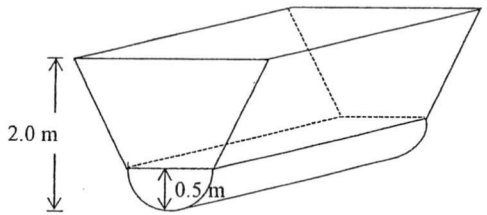
(v) the height of flag pole PQ. [2]

(b) A man walks from R to S.
Find the greatest angle of elevation of P from a point on RS. [2]

10. Large canals are used in Singapore to regulate water flow to prevent floods from occurring.



In this question, the canal can be modelled as a trapezoid attached to the top of a half- cylinder as shown below.



The cross section of the drain is made up of a trapezium and a semicircle. The radius of the semicircle is 0.5 m and the vertical height measured from the bottom of the semicircle to the top of the trapezium is 2 m. The length of one of the parallel sides of the trapezium is twice the length of the other.

(a) Find the lengths of the parallel sides and the vertical height of the trapezium. [2]

(b) Calculate the volume of the drain, in cubic metres, which stretches for 10 m. [4]

(c) A drain must be able to channel away 90% of the rain water within 30 seconds. If not, preventive measures need to be set up to curb the flood.

Useful Information

- The rate of flow of water for this drain during a particular rainstorm is 48000 litres per minute
- 1 m^3 is equivalent to 1000 litres

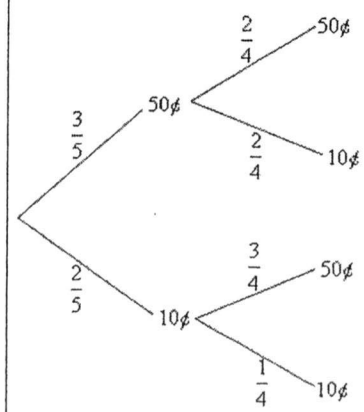
Determine whether preventive measures need to be set up for that particular rainstorm. Show your working and give reasons to justify your answer. [4]

End of Paper 2

Secondary 4 Express/ 5NA Mid Year Examination 2017
Mathematics 4048 Paper 2 Marking Scheme

Deduct 1 mark overall for missing or incorrect units.

1	(a)	$\left(\frac{xy^2}{2}\right)^3 \times \frac{3y}{2x}$ $= \frac{3x^2y^7}{16}$	B1 B1	2	
	(b)	$\frac{(x+7)(x-3)}{2(x+3)(x-3)}$ $= \frac{x+7}{2(x+3)}$	M1 A1	2	M1 for both expression factorised
	(c)	$2p^2 - 7p - 4p + 14 = 9$ $2p^2 - 11p + 5 = 0$ $(2p-1)(p-5) = 0$ $p = \frac{1}{2}$ or 5	M1 M1 A1	2	Correct expansion Either using factorisation or quadratic formula
Marker's comment:					

2	(a)	Complete correct tree diagram for 2 selections with all 6 probabilities correct.  	B2	2	Award B1 for branches if first selection correct. Probability: $\frac{2}{4}$ o.e.
	(bi)	P(20 cents) $= \frac{2}{5} \times \frac{1}{4} = \frac{1}{10}$	B1	1	
	(bii)	P(60 cents) $= \frac{3}{5} \times \frac{2}{4} + \frac{2}{5} \times \frac{3}{4}$ $= \frac{3}{5}$	M1 A1	2	√ their tree diagram
	(c)	P(70 cents) $= \left(\frac{3}{5} \times \frac{2}{4} \times \frac{1}{3}\right) + \left(\frac{2}{5} \times \frac{3}{4} \times \frac{1}{3}\right) + \left(\frac{2}{5} \times \frac{1}{4} \times \frac{3}{3}\right)$ $= \frac{3}{10}$	M1 A1	2	√ their tree diagram
Marker's comment:					

3	(ai)	$3^4 \times 5^2$	B1	1	
	(aii)	Indices of the prime factors are all mutiples of 2/ divisible by 2.	B1	1	
	(aiv)	$m = 5, n = 3$	B1	1	B1 for both correct values
	(bi)	$\mathbf{P} = \begin{pmatrix} 2 \\ 12 \\ 5 \end{pmatrix}$	B1	1	
	(bii)	$\mathrm{M} = \begin{pmatrix} 3 & 5 & 0 \\ 2 & 3 & 5 \\ 1 & 2 & 7 \end{pmatrix} \begin{pmatrix} 2 \\ 12 \\ 5 \end{pmatrix}$ $= \begin{pmatrix} 66 \\ 65 \\ 61 \end{pmatrix}$	B1	1	√ for their (bi)
	(biii)	The elements represent the **cost price** of **each type of goodie bag** respectively.	B1	1	
	(biv)	$\begin{pmatrix} 1.15 & 0 & 0 \\ 0 & 1.2 & 0 \\ 0 & 0 & 1.3 \end{pmatrix}$	B1	1	
	(bv)	$\begin{pmatrix} 75.90 \\ 78 \\ 79.30 \end{pmatrix}$	B1	1	√ for their (biv) Do not award if elements are not to 2 d.p. for non-exact answers
Marker's comment:					

4	(ai)	$p + 300q = 54 \quad -(1)$ $p + 500q = 78 \quad -(2)$	B1 B1	2	
	(aii)	$p = 18$ and $q = 0.12$.	B2	2	or M1 for correct method to substitute or eliminate one variable. B1 for 1 correct solution.
	(aiii)	420	B1	1	√ their p and q
	(b)	Let the original price of the goods be $\$x$. Price after the discount $= \frac{90}{100} \times 1.1x$ $= \$0.99x$ **Yes**, actual discount of 1%.	M1 A1	2	Correct method to calculate discount
Marker's comment:					

	(c)	$\frac{1}{2}\times b'\times 0.8h=\frac{1}{2}\times b\times h$ $\frac{b'}{b}=1.25$ % change = 25%	M1 A1	 2	Correct attempt to compare the area of the triangle
Marker's comment:					

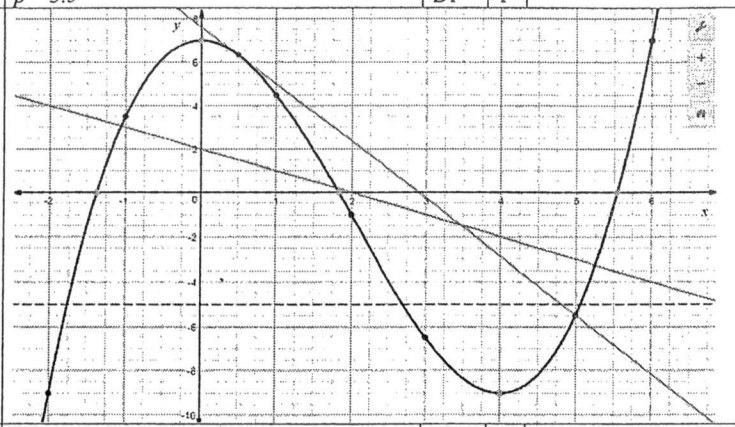
5	(ai)	\$60	B1	1	
	(aii)	Interquartile range = \$68 – \$50 = \$18	M1 A1	2	
	(bi)	$p = 10$ $q = 6$	B1 B1	2	
	(bii)	Mean =\$ 61 Std Deviation = $\sqrt{\frac{328000}{80}-(61)^2}$ = \$19.50	B1 M1, A1 **Or** B2	 3	 Penalise one mark from question 5 if answers (non-exact) are not to 2d.p.
	(c)	Median = \$66 IQR = 70 – 52 = \$18 Students from **outdoor club raised more money** than soccer club as their **median** amount is **higher**. Amount of money collected by both groups of students are **equally consistent** as their **IQR is the same**.	 B1 B1 B1	 3	For obtaining median and IQR √ their **(ai)** and **(aii)** √ their **(ai)** and **(aii)**
Marker's comment:					

6	(ai)	$RQ=(3r+2)+(6r+1)$ $=9r+3$	B1	1	
	(aii)	$OR=2(6r+1)-(3r+2)$ $=9r$	M1 A1	2	Use radius of large semicircle minus radius of small circle
	(b)	$(9r+3)^2=(6r+1)^2+(9r)^2$ $81r^2+54r+9=36r^2+12r+1+81r^2$ $36r^2-42r-8=0$ $18r^2-21r-4=0$ (shown)	B1 B1	 2	Form appropriate equation Simplify the equation
	(c)	$r=\frac{-(-21)\pm\sqrt{(-21)^2-4(18)(-4)}}{2(18)}$ $=\frac{21\pm\sqrt{729}}{36}$ $=1\frac{1}{3}$ or $-\frac{1}{6}$ or $(3r-4)(6r+1)=0$ $r=1\frac{1}{3}$ or $-\frac{1}{6}$	B1 B1 B1 B1 B1 B1	 3	Apply quadratic formula Both answers correct, no rejection

	(d)	$RQ=9\left(1\frac{1}{3}\right)+3=15$ cm $OR=9\left(1\frac{1}{3}\right)=12$ cm $OR=9$ cm $\angle OQR=\sin^{-1}\left(\frac{12}{15}\right)$ $=53.1301^\circ$ $\angle ORQ=\cos^{-1}\left(\frac{12}{15}\right)$ $=36.8699^\circ$ Area of small sector $=\frac{36.8699}{360}\times(3.142)\times(6)^2=11.5845$ cm^2 Area of large sector $=\frac{53.1301}{360}\times(3.142)\times 9^2$ $=37.5603$ cm^2 Area of triangle $OQR=\frac{1}{2}\times 9\times 12=54$ cm^2 Area of shaded region $=54-37.5603-11.5845=4.8552$ $=4.86$ cm^2	 M1 M1 A1	 4	 Finding one accurate angle in the triangle by use of trigonometry. √ their (c) Finding the areas of both sector Finding the area of the triangle √ their (c)
Marker's comment:					

7	(ai)	$\overrightarrow{PQ}=\overrightarrow{PO}+\overrightarrow{OQ}=\begin{pmatrix}-5\\5\end{pmatrix}$ $\left\lvert\overrightarrow{PQ}\right\rvert=\sqrt{(-5)^2+(5)^2}=7.07$ units		M1 A1	 2	
	(aii)	$\overrightarrow{SP}=3\overrightarrow{PQ}$ $\overrightarrow{SO}+\overrightarrow{OP}=3\overrightarrow{PQ}$ $\overrightarrow{SO}=3\begin{pmatrix}-5\\5\end{pmatrix}-\begin{pmatrix}7\\1\end{pmatrix}=\begin{pmatrix}-22\\14\end{pmatrix}$ The coordinates of S is (22 , -14).		 M1 A1	 2	 √ their **(ai)** Answer in coordinate form
	(aiii)	$\begin{pmatrix}h+2\\5\end{pmatrix}=k\overrightarrow{PQ}=k\begin{pmatrix}-5\\5\end{pmatrix}$ $k=1$ $h=-7$		 B1	 1	√ their **(ai)**
	(bi)(a)	$\overrightarrow{OB}=\overrightarrow{OA}+\overrightarrow{AB}$ $=\overrightarrow{OA}+\overrightarrow{OC}$ $=\underset{\sim}{p}+\underset{\sim}{q}$		 B1	 1	

	(bi)(b)	$\overrightarrow{AC} = \overrightarrow{OC} - \overrightarrow{OA}$ $= \underset{\sim}{q} - \underset{\sim}{p}$ $\overrightarrow{OT} = \overrightarrow{OA} + \overrightarrow{AT}$ $= \underset{\sim}{p} + 5\overrightarrow{AC}$ $= \underset{\sim}{p} + 5(\underset{\sim}{q} - \underset{\sim}{p})$ $= 5\underset{\sim}{q} - 4\underset{\sim}{p}$		M1 A1	 2	For finding $\overrightarrow{AC}$
	(bi)(c)	$\overrightarrow{BT} = \overrightarrow{OT} - \overrightarrow{OB}$ $= 5\underset{\sim}{q} - 4\underset{\sim}{p} - \underset{\sim}{p} - \underset{\sim}{q}$ $= 4\underset{\sim}{q} - 5\underset{\sim}{p}$		 B1	 1	√ their **(bi)(b)**
	(bii)	$\overrightarrow{OR} = \overrightarrow{OA} + \overrightarrow{AB} + \overrightarrow{BR}$ $= \frac{9}{5}q$ $\overrightarrow{OC} = \frac{5}{4}\overrightarrow{CR}$ $k = \frac{5}{4}$		M1 A1	 2	For finding $\overrightarrow{OR}$ or $\overrightarrow{CR}$
	(biii)	$\overrightarrow{BR} = \frac{1}{5}\overrightarrow{BT}$ $\frac{\text{area of } \Delta BCR}{\text{area of } \Delta CRT} = \frac{1}{4}$		 B1	 1	
	Marker's comment:					

8	**(a)**	$p = 3.5$	B1	1	
	(b)				
		All points plotted correctly Smooth curve through plotted points Correct scale and axes labelled	P1 C1 B1	3	
	(c)	$\frac{1}{2}x^3 - 3x^2 + 7 = -5$ At y = - 5, there are **3 points of intersection**, indicating 3 solutions to the equation.	B1 B1	2	
	(d)	Draws tangent at $x = 0.5$ and estimates (change in y)/ (change in x) -2.63 (± 0.3)	M1 A1	2	
	(ei)	Line of $y = 2 - x$ drawn	B1	1	
	(eii)	$x = -1.05$, 1.8 , 5.25 (all values ± 0.1)	B1	1	For all 3 correct answers
	(eiii)	$\frac{1}{2}x^3 - 3x^2 + 7 = 2 - x$ $\frac{1}{2}x^3 - 3x^2 + x + 5 = 0$ $x^3 - 6x^2 + 2x + 10 = 0$ $a = -6$, $b = 2$, $c = 10$	M1 A1	2	
Marker's comment:					

9	(ai)	$QS^2 = 8.9^2 + 12.5^2 - 2(8.9)(12.5)\cos 53°$ $QS = 10.0775$ $= 10.1$ m	M2 A1	3	
	(aii)	By Sine Rule, $\frac{\sin \angle QSR}{8.9} = \frac{\sin 53°}{10.0775}$ $\sin \angle QSR = \frac{8.9 \sin 53°}{10.0775}$ $\angle QSR = 44.8554°$ $\approx 44.9°$	M1 A1	2	Finding angle either using sine or cosine rule
	(aiii)	Bearing of S from $Q = 53° + 44.9°$ $= 097.9°$	B1	1	√ their (ai) No mark if answer is not expressed to 3 digit
	(aiv)	$\frac{1}{2} \times 8.9 \times 12.5 \sin 53°$ $= 44.4$ m^2	M1 A1	2	
	(av)	$\tan 27° = \frac{PQ}{8.9}$ $PQ = 4.53$ m	M1 A1	2	Appropriate trigo ratio
	(b)	Shortest distance from Q to RS $= \frac{44.424}{\frac{1}{2} \times 12.5} = 7.10785$ m or $= 8.9 \sin 53° = 7.10785$ m $\tan \theta = \frac{4.5347}{7.10785}$ $\theta = 32.5°$	M1 A1	2	For finding perpendicular distance √ their (av)
Marker's Comments:					

10	(a)	Vertical height of trapezium $= 2 - 0.5$ $= 1.5$ m Length of shorter side $= 0.5 \times 2 = 1$ m Length of longer side $= 1.0 \times 2 = 2$ m	B1 B1	2	For both correct parallel lengths
	(b)	Volume of half of cylinder $= \pi(0.5)^2 \times 10 \times \frac{1}{2}$ $= 3.9275$ m^3 Volume of the trapezoid $= \frac{1}{2} \times (1+2) \times 1.5 \times 10$ $= 22.5$ m^3 Volume of the drain $= 22.5 + 3.9275$ $= 26.4275$ $= 26.4$ m^3	M1 M1 M1 A1	4	√ for their length in (a) √ for their length in (a) Apply volumes of cylinder and trapezium
	(c)	90% volume of water $= 26.4275 \times \frac{90}{100}$ $= 23.785$ m^3 48000 litres per min $= \frac{48000}{1000 \times 60}$ $= 0.8$ m^3/s Time taken to drain the volume of water $= \frac{23.785}{0.8}$ $= 29.73125$ s Since the time taken to channel water is 29.73s which is < 30s, there is **no** need to set up preventive measures. *Alternative method of determination can be based on whether the canal can hold the stated volume of water during the storm.*	M1 M1 M1 A1	4	√ for their volume in (b) Use the appropriate non-linear info such as 90% For comparing the rate of flow based on same time interval Using either time taken to drain or volume it can hold as a means of checking Make judgement based on sound mathematical calculations * *All M1 awarded for correct method regardless of accuracy.*
Marker's Comments:					

Candidate Name: _____ Class _____ Index No. _____

FUHUA SECONDARY SCHOOL

Secondary Four Express / Five Normal (Academic)

4E & 5NA

Mid-Year Examination 2017

Fuhua Secondary Fuhua Secondary Fuhua Secondary Fuhua Secondary Fuhua Secondary Fuhua Secondary Fuhua Secondary Fuhua Secondary Fuhua Secondary
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MATHEMATICS **4048/01**

PAPER 1

DATE: **8 May 2017**

TIME: **10 30 – 12 30**

DURATION: **2 hours**

READ THESE INSTRUCTIONS FIRST

Write your class, index number and name on all the work you hand in.
Write in dark blue or black pen.
You may use an HB pencil for any diagrams or graphs.
Do not use staples, paper clips, highlighters, glue or correction fluid.

Answer **all** questions.
If working is needed for any question it must be shown with the answer.
Omission of essential working will result in loss of marks.
The use of an approved scientific calculator is expected, where appropriate.
If the degree of accuracy is not specified in the question, and if the answer is not exact, give the answer to 3 significant figures. Give answers in degrees to one decimal place.
For π, use either your calculator value of 3.142, unless the question requires the answer in terms of π.

The number of marks is given in brackets [ ] at the end of each question or part question.
The total number of marks for this paper is **80**.

PARENT'S SIGNATURE	FOR EXAMINER'S USE		
	Units		/ 80
	Statements/Accuracy		
	Poor Presentation		

Setter: Ms Er Ying Ying Vetter: Ms Winnifred Lim

This question paper consists of 21 printed pages including this page.

[Turn Over

Mathematical Formulae

Compound interest

$$\text{Total amount} = P\left(1+\frac{r}{100}\right)^n$$

Mensuration

Curved surface area of a cone $= \pi rl$

Surface area of a sphere $= 4\pi r^2$

$$\text{Volume of a cone} = \frac{1}{3}\pi r^2 h$$

$$\text{Volume of a sphere} = \frac{4}{3}\pi r^3$$

$$\text{Area of triangle } ABC = \frac{1}{2}ab\sin C$$

Arc length $= r\theta$, where θ is in radians

$$\text{Sector area} = \frac{1}{2}r^2\theta, \text{ where } \theta \text{ is in radians}$$

Trigonometry

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$a^2 = b^2 + c^2 - 2bc\cos A$$

Statistics

$$\text{Mean} = \frac{\Sigma fx}{\Sigma f}$$

$$\text{Standard deviation} = \sqrt{\frac{\Sigma fx^2}{\Sigma f} - \left(\frac{\Sigma fx}{\Sigma f}\right)^2}$$

Answer **all** the questions.

1 **(a)** Simplify $2y-3(x-y+1)$.

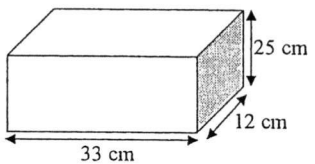
Answer (a) .....[1]

(b) Factorise $3p^2 - pq - 4q^2$.

Answer (b) .....[1]

---

2 The diagram below shows a wooden block measuring 33 cm by 12 cm by 25 cm.
An engineer wants to build cubes using these wooden blocks.

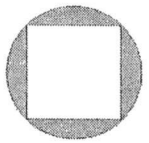


Find the smallest possible length of side of a cube that can be built.

Answer .....cm [2]

---

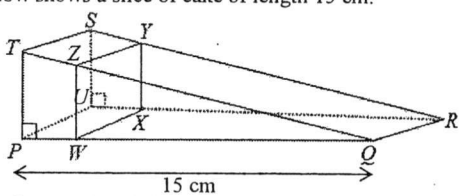
3 The diagram shows a square inscribed in a circle. The area of the circle is 1386 cm^2.
Find the area of the square. (Take $\pi = \frac{22}{7}$)



Answer .....cm^2 [2]

---

4 The diagram below shows a slice of cake of length 15 cm.

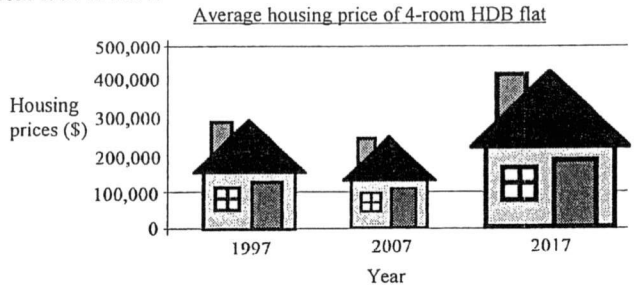


The slice of cake is cut at *WXYZ* into two pieces such that the ratio of area of the two slices of cake *WXYZ* to *PSTU* is 2 : 3. Calculate the length of the smaller piece of cake, *WQ*.

Answer .....cm [2]

---

5 The graph below shows the changes in average housing price of a 4-room HDB flat from 1997 to 2017.



State one aspect of the graph that may be misleading and explain how it may lead to a misinterpretation of the graph.

Answer

.....

.....

..... [2]

---

6 On a particular day, the temperature ranged between −11°C and 7 °C.

(a) Find the difference between the highest and lowest temperature.

Answer (a) .....°C [1]

(b) The temperature at 6 am was −11°C and the temperature at 2 pm was 7 °C. Given that the temperature rises at a constant rate, find the time when the temperature was 1.375 °C.

Answer (b) .....[2]

---

7 Write $\frac{2m-1}{21-3m}+\frac{3m+5}{5m-35}$ as a fraction in its simplest form.

Answer .....[3]

---

8 Kelvin wishes to loan $125000 to pay off the remaining cost of his HDB flat. A bank charges an interest rate of 2.35% per annum which is compounded half-yearly.

(a) Calculate the total amount he has to pay back in 7 years if he loans from the bank.

Answer (a) $.....[2]

(b) Calculate the monthly installment he has to pay.

Answer (b) $.....[1]

---

9 Given that $\varepsilon = \{x : x \text{ is a factor of } 72\}$

$A = \{x : x \text{ is a multiple of } 4\}$

$B = \{x : x \text{ is divisible by } 3\}$

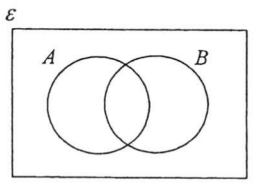
(a) List the elements in ε.

Answer (a) .....[1]

(b) Find $A \cap B'$.

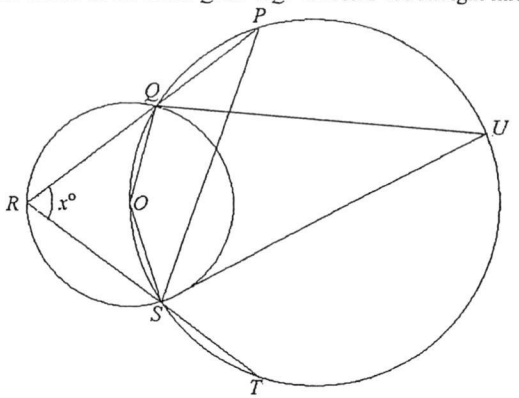
Answer (b) .....[1]

(c) In the Venn diagram below, shade $A' \cup B'$



[1]

10 In the diagram, circle QRS and circle PUT meet at points Q and S. O lies on circle PUT and is the centre of the circle QRS. RQP and RST are straight lines.



Given that $\angle QRS = x°$, showing all reasons clearly,

(a) find angle QUS in terms of x,

Answer (a) .....° [2]

(b) determine if ΔRPS is an isosceles triangle.

Answer (b)

.....

.....

.....[3]

11 **(a)** Express 630 and 495 each as a product of its prime factors.

Answer (a) 630 = ...

495 = ...[2]

(b) Find the highest common factor of 630 and 495.

Answer (b) .....[1]

(c) Find the smallest positive integer k, such that $630k$ is a perfect cube.

Answer (c) k = ...[1]

12 An estate of area 2.25 km^2 is represented on a map by an area of 36 cm^2.

(a) Express the scale of the map in the form 1: n.

Answer (a) .....[2]

(b) Find the area of the same piece of land, in cm^2, on another map of scale 5 : 900000.

Answer (b) .....cm^2 [2]

13 (a) The ratio of the distance from Town A to Town B and from Town B to Town C is 4 : 5. Sally drives at a constant speed of 65 km/h from Town A and she took 25 minutes to reach Town B. Find the distance between Town A and Town C.

Answer (a) ..... km [2]

(b) The price of petrol is \$1.35 per litre. Sally uses 0.27 l of petrol per 2.3 km of distance travelled. How much will it cost her to travel from Town A to Town C?

Answer (b) \$.....[2]

[Turn Over

14 $\overrightarrow{PQ} = \begin{pmatrix} 1 \\ 5 \end{pmatrix}, \overrightarrow{QR} = \begin{pmatrix} -6 \\ -7 \end{pmatrix}$ and $\overrightarrow{RS} = \begin{pmatrix} k^2 \\ -3 \end{pmatrix}$.

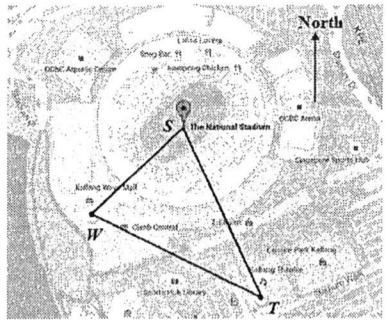
(a) Given that $\overrightarrow{QS}$ is parallel to $\overrightarrow{PQ}$, find the possible values of k.

Answer (a) k =.....[2]

(b) Hence, find $\left|\overrightarrow{PS}\right|$.

Answer (b) .....units [2]

15 The diagram below shows the map of the Singapore Sports Hub. S, T and W are positions of the National Stadium, Kallang Theatre and Kallang Wave Mall respectively. $SW = 550$ m, $WT = 675$ m, angle $SWT = 83°$ and the bearing of T from S is $161°$.



(a) Calculate the length of ST.

Answer (a) .....m [2]

(b) Calculate the bearing of W from T.

Answer (b) .....° [3]

16 A bag contains a total of 65 chocolates wrapped with silver, red and blue wrappers. There are 18 chocolates with red wrappers in the bag.

(a) If a chocolate is picked randomly, the probability of picking a chocolate with silver wrapper is $\frac{4}{13}$. How many chocolates with blue wrappers are there?

Answer (a) .....chocolates [1]

(b) If two chocolates are picked randomly, what is the probability of picking at least a chocolate with red wrapper?

Answer (b) .....[2]

(c) When x chocolates with blue wrappers are added to the 65 chocolates, the probability of picking a chocolate with silver wrapper becomes $\frac{5}{19}$. Find the value of x.

Answer (c) x =.....[2]

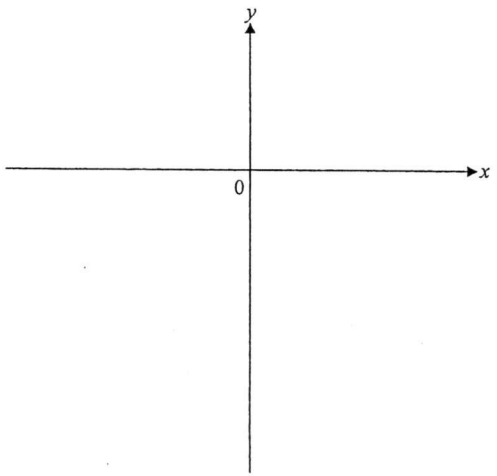
17 **(a)** Express $y = x^2 + 5x - 3$ in the form of $y = (x + h)^2 - k$.

Answer (a) .....[1]

(b) State the coordinates of the turning point.

Answer (b) (..... , ...) [1]

(c) Sketch the graph of $y = x^2 + 5x - 3$ on the axes provided, showing clearly the intercepts and turning point.



[2]

18 **(a)** Points A, B and C have coordinates $(0, 7)$, $(-3, k)$ and $(8, k)$ respectively. Given that the length of AB is $\sqrt{109}$ units and $k < 15$, find the value of k.

Answer (a) k = ...[2]

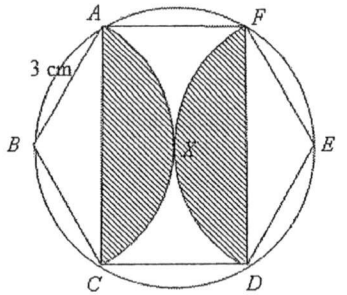
(b) Point D is a point such that $ABCD$ forms a trapezium with BC parallel to AD and the gradient of CD is $1\frac{3}{7}$. Find the coordinates of D.

Answer (b) D (..... , ...) [2]

(c) Calculate the area of trapezium $ABCD$.

Answer (c) .....square units [1]

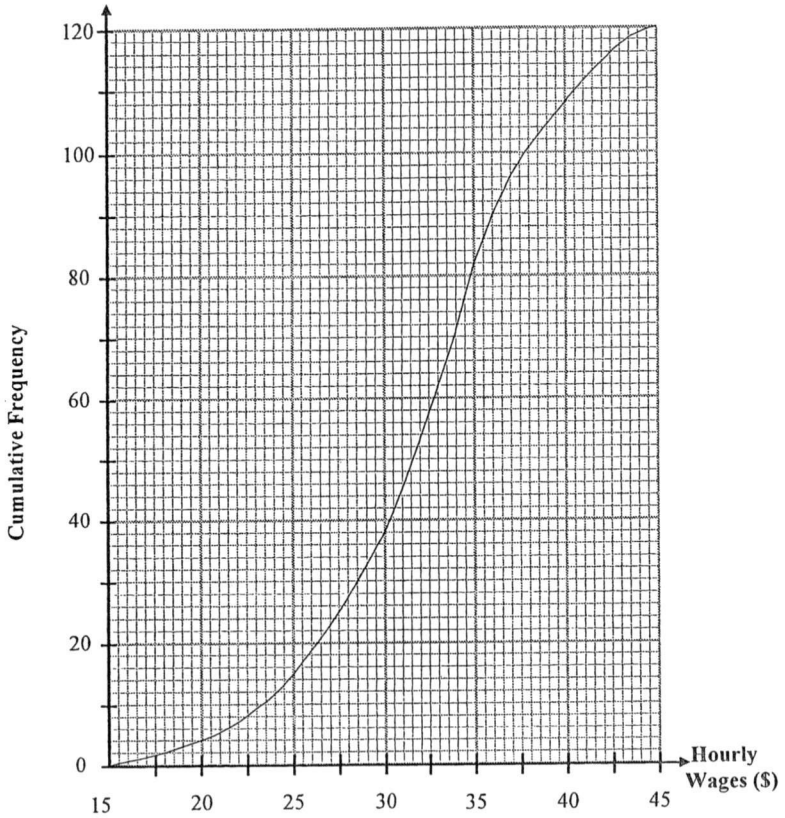
19 In the diagram below, *ABCDEF* is a regular hexagon inscribed in a circle with centre *X*. The length of each side of the hexagon is 3 cm, *AXC* and *DXF* are two identical circular arcs centered at *B* and *E* respectively and touch each other at the point *X*.



What percentage of the area of the hexagon is **not** shaded?

Answer .....% [5]

20 The cumulative frequency curve shows the distribution of the hourly wages of 120 workers in Sunshine Company.



(a) Use the graph to estimate the

(i) median hourly wage,

Answer (a)(i) $.....[1]

(ii) interquartile range.

Answer (a)(ii) $.....[1]

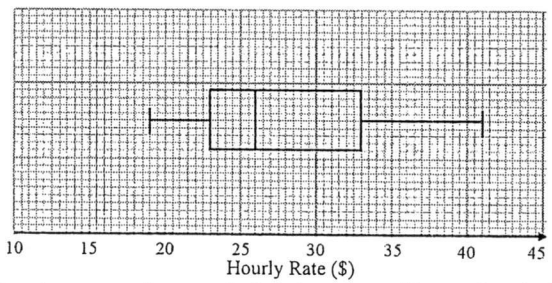
(b) Given that 10% of the workers hold managerial positions or higher, use the graph to estimate the minimum hourly wage of a middle manager.

Answer (b) $.....[1]

(c) Find the probability that a worker, chosen at random, is paid $30 or less per hour.

Answer (c) .....[1]

(d) The box-and-whisker plot below shows the hourly wages of 120 workers in Brightlight Company.



Write down two comparisons on the hourly wages paid to workers in Sunshine Company and Brightlight Company.

Answer (c)

.....

.....

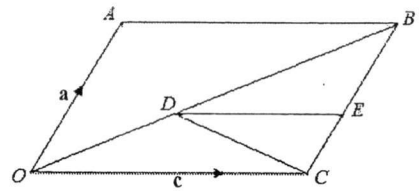
.....

.....

.....

.....[2]

21 In the diagram, $OABC$ is a parallelogram where $\overrightarrow{OA} = \mathbf{a}$ and $\overrightarrow{OC} = \mathbf{c}$. D is the point on OB such that $5\overrightarrow{OD} = 3\overrightarrow{DB}$.



(a) Express, as simply as possible, in terms of **a** and/or **c**,

(i) $\overrightarrow{OB}$,

Answer (a) (i).....[1]

(ii) $\overrightarrow{CD}$.

Answer (a) (ii).....[2]

(b) Given that $\frac{3}{8}BC = EC$, express $\overrightarrow{DE}$ in terms of **a** and/or **c**.

Answer (b) .....[2]

(c) Find, in the simplest form, the ratio of

(i) $\frac{\text{area of } \Delta BDE}{\text{area of } \Delta BOC}$,

Answer (c) (i)... .....[1]

(ii) $\frac{\text{area of } \Delta OCD}{\text{area of parallelogram } OABC}$.

Answer (c) (ii)... .....[1]

End of Paper

Q1a) $2y-3(x-y+1)$
$= 2y-3x+3y-3$
$= 5y-3x-3$ B1

1b) $3p^2-pq-4q^2$
$= (3p-4q)(p+q)$ B1

Q2)

3	33	12	25
2	11	4	25
2	11	2	25
5	11	1	25
5	11	1	5
11	11	1	1
	1	1	1

M1

LCM of 33, 12 and 25 $= 3\times2^2\times5^2\times11$
$= 3300$
∴ Smallest possible length $= 3300$cm A1

Q3) Let the radius of circle be r and length of square be l.

$\pi r^2 = 1386$
$r^2 = \frac{1386}{\frac{22}{7}}$ M1
$r = 21$ cm
$l^2 + l^2 = (2\times21)^2$
$2l^2 = 1764$
$l^2 = 882$
Area of square $= 882$ cm^2 A1

Q4) Let the length of WQ be x.

$\left(\frac{7}{15}\right)^3 = \frac{7}{5}$ M1
$x = 15\times\sqrt[3]{\frac{7}{5}}$
$= 11.052\,0945$ cm
≈ 11.1 cm (3 sf) A1

Q5) Readers may misinterpret the area of house as the average housing price of 4-room HDB flat B1
hence overestimating the average housing price of a 4-room HDB flat. B1

Q6a) Difference in temperature $= 7°C - (-11°C)$
$= 18°C$ B1

b) Rate of increase of temperature $= \frac{18°C}{8\text{ hours}}$
$= 2\frac{1}{4}°C/hr$

Duration taken for temperature to reach 1.375°C $= \frac{1.375°C-(-11°C)}{2\frac{1}{4}°C/hr}$ M1
$= 5\frac{1}{2}$ hr

Time when temperature was 1.375°C is 1130 am. A1

Q7) $\frac{2m-1}{21-3m} + \frac{3m+5}{5m-35}$
$= \frac{2m-1}{3(7-m)} + \frac{3m+5}{5(m-7)}$
$= \frac{2m-1}{3(7-m)} - \frac{3m+5}{5(7-m)}$ M1
$= \frac{5(2m-1)-3(3m+5)}{15(7-m)}$ M1
$= \frac{10m-5-9m-15}{15(7-m)}$
$= \frac{m-20}{15(7-m)}$ A1

OR

$\frac{2m-1}{21-3m} + \frac{3m+5}{5m-35}$
$= \frac{(2m-1)(5m-35)+(3m+5)(21-3m)}{(21-3m)(5m-35)}$ M1
$= \frac{10m^2-70m-5m+35+63m-9m^2+105-15m}{105m-735-15m^2+105m}$
$= \frac{m^2-27m+140}{-15m^2+210m-735}$
$= \frac{(m-20)(m-7)}{-15(m-7)^2}$ M1
$= \frac{m-20}{-15(m-7)}$
$= \frac{m-20}{15(7-m)}$ A1

Q8a) Total amount repayable $= \$125\,000\left(1+\frac{2.35}{100}\right)^{14}$ M1
$= \$147209.2147$
$\approx \$147209.21$ (nearest cents) A1

b) Monthly instalment $= \frac{\$147209.2147}{12\times7}$
$= \$1752.490651$
$\approx \$1752.49$ (nearest cents) B1

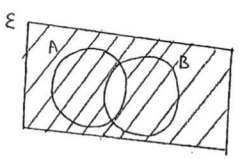
Q9a) $\xi = \{1, 2, 3, 4, 6, 8, 9, 12, 18, 24, 36, 72\}$ B1

b) $A = \{4, 8, 12, 24, 36, 72\}$

$B = \{3, 6, 9, 12, 18, 24, 36, 72\}$

$A \cap B = \{4, 8\}$ B1

c)



B1

Q10a) $\angle QOS = 2 \times \angle QRS$ ($\angle$ at centre = 2$\angle$ at circumference)

$= 2x°$ M1

$\angle QUS = 180° - \angle QOS$ ($\angle$s in opp. segment)

$= 180° - 2x°$ A1

b) $\angle RPS = \angle QUS$

$= 180° - 2x°$ ($\angle$s in same segment) M1

$\angle PSR = 180° - x° - (180° - 2x°)$ ($\angle$ sum of $\triangle$)

$= x°$ M1

$\because \angle PSR = \angle RPS = x°$, $\triangle RPS$ is an isosceles triangle. A1

Q11a) $630 = 2 \times 3^2 \times 5 \times 7$ B1

$495 = 3^2 \times 5 \times 11$ B1

b) HCF $= 3^2 \times 5$

$= 45$ B1

c) Smallest positive $k = 2^2 \times 3 \times 5^2 \times 7^2$

$= 14700$ B1

Q12a) $36\text{ cm}^2 : 2.25\text{ km}^2$

$6\text{ cm} : 1.5\text{ km}$

$1\text{ cm} : \frac{1.5 \times 1000 \times 100}{6}\text{ cm}$ M1

$1 : 25000$ A1

b) $5 : 900\,000$

$5\text{ cm} : 9\text{ km}$

$1\text{ cm} : 1.8\text{ km}$

$1\text{ cm}^2 : 3.24\text{ km}^2$

Area of land on map $= \frac{2.25}{3.24}$ M1

$= \frac{25}{36}\text{ cm}^2$ A1

Q13a) Town A to Town B : Town B to Town C

$4 : 5$

Distance from Town A to Town B $= 65 \times \frac{25}{60}$ M1

$= 27\frac{1}{12}\text{ km}$

Distance from Town A to Town C $= \frac{27\frac{1}{12}\text{ km}}{4} \times 9$

$= 60\frac{15}{16}\text{ km}$ A1

b) Total cost $= \frac{60\frac{15}{16}\text{ km}}{2.3\text{ km}} \times 0.27 \times \$1.35$ M1

$= \$9.657269022$

$\simeq \$9.66$ (nearest cent) A1

Q14a) $\vec{QS} = \vec{QR} + \vec{RS}$

$= \begin{pmatrix} -6 \\ -7 \end{pmatrix} + \begin{pmatrix} k^2 \\ -3 \end{pmatrix}$

$= \begin{pmatrix} -6+k^2 \\ -10 \end{pmatrix}$

$\because \vec{QS} \parallel \vec{PQ}$, $\vec{QS} = m\vec{PQ}$ where m is a constant

$\begin{pmatrix} -6+k^2 \\ -10 \end{pmatrix} = m\begin{pmatrix} 1 \\ 5 \end{pmatrix}$ M1

Comparing, $-10 = 5m$

$m = -2$

Sub $m = -2$, $-6 + k^2 = -2(1)$

$-6 + k^2 = -2$

$k^2 = 4$

$k = \pm 2$ A1

14b) $\vec{PS} = \vec{PQ} + \vec{QR} + \vec{RS}$

$= \begin{pmatrix}1\\5\end{pmatrix} + \begin{pmatrix}-6\\-7\end{pmatrix} + \begin{pmatrix}4\\-3\end{pmatrix}$

$= \begin{pmatrix}-1\\-5\end{pmatrix}$ M1

$|\vec{PS}| = \sqrt{(-1)^2+(-5)^2}$ units

$= \sqrt{26}$ units

$= 5.099019514$ units

$\simeq 5.10$ units (3sf) A1

Q15a) $S_T = \sqrt{550^2+675^2-2(550)(675)\cos 83°}$ M1

$= 817.0905779$ m

$\simeq 817$ m (3sf) A1

b) $\frac{\sin \angle SWT}{817.0905779} = \frac{\sin \angle WTS}{550}$ M1

$\sin \angle WTS = \frac{550 \times \sin 83°}{817.0905779}$

$\angle WTS = 41.92079507°$

Bearing of W from T $= 360° - (180° - 161°) - (41.92079507°)$ M1

$= 299.0792049°$ OR $180° + 161° - 41.92079507°$

$\simeq 299.1°$ (1dp) A1 $= 299.1°$ (1dp)

Q16a) P(silver wrapper) $= \frac{4}{13}$

$= \frac{20}{65}$

Number of chocolates with blue wrapper $= 65 - 18 - 20$

$= 27$ B1

b) P(picking at least 1 red wrapper) $= 1 - P(\text{both blue}) - P(\text{both silver}) - P(\text{silver, blue}) - P(\text{blue, silver})$

$= 1 - \frac{27}{65}\times\frac{26}{64} - \frac{20}{65}\times\frac{19}{64} - \frac{20}{65}\times\frac{27}{64} - \frac{27}{65}\times\frac{20}{64}$ M1

$= \frac{999}{2080}$ A1 OR P(red,red) + P(blue,red) + P(silver,red)

$= \frac{18}{65} + \frac{4}{13}\times\frac{18}{64} + \frac{27}{65}\times\frac{18}{64}$

c) P(silver wrapper) $= \frac{5}{19}$

$\frac{20}{65+x} = \frac{5}{19}$ M1

$380 = 325 + 5x$

$5x = 55$

$x = 11$ A1

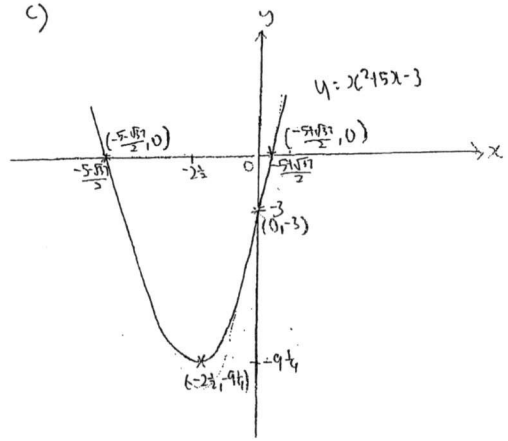
Q17a) x^2+5x-3

$= (x+\frac{5}{2})^2 - (\frac{5}{2})^2 - 3$

$= (x+\frac{5}{2})^2 - 9\frac{1}{4}$ B1

b) Turning point $= (-2\frac{1}{2}, -9\frac{1}{4})$ B1

c)



When $x=0$, $y=-3$

When $y=0$, $x^2+5x-3=0$

$x = \frac{-5\pm\sqrt{(5)^2-4(1)(-3)}}{2(1)}$

$= \frac{-5\pm\sqrt{37}}{2}$

$\simeq 0.541$ (3sf) or -5.54 (3sf)

B1 - shape, axis

B1 - Intercepts, turning point

Q18a) Length of AB $= \sqrt{109}$

$\sqrt{(0-(-3))^2+(7-k)^2} = \sqrt{109}$

$9 + (7-k)^2 = 109$

$9 + 49 - 14k + k^2 = 109$

$k^2 - 14k - 51 = 0$ M1

$(k-17)(k+3) = 0$

$k = 17$ or $k = -3$ (rejected)

$\therefore k = -3$ A1

18b) $\because BC \parallel AD$, gradient of AD = gradient of BC

$= 0$

$\therefore$ D must lie on $y = 7$

Let coordinates of D be $(x, 7)$

gradient of CD $= \frac{3}{7}$

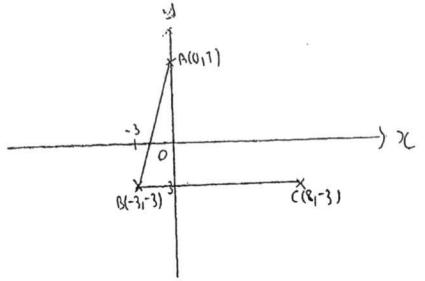
$\frac{7-(-3)}{x-8} = \frac{10}{7}$ M1

$\frac{10}{x-8} = \frac{10}{7}$

$x - 8 = \frac{7 \times 10}{10}$

$x = 15$

$\therefore D(15, 7)$ A1



c) Area of trapezium $= \frac{1}{2} \times (15+11) \times 10$

$= 130 \text{ unit}^2$ B1

Q19) Interior angle of hexagon $= \frac{180^\circ(6-2)}{6}$

$= 120^\circ$

$= \frac{2\pi}{3}$ rad M1

Area of segment $= \frac{1}{2} \times 3^2 \times \frac{2\pi}{3} - \frac{1}{2} \times 3^2 \times \sin\frac{2\pi}{3}$ M1

$= 5.527663644 \text{ units}^2$

$\angle ABX = \frac{120^\circ}{2}$

$= 60^\circ$

$\therefore \angle BAX = 180^\circ - 2(60^\circ)$ ($\angle$ sum of isos $\triangle$)

$= 60^\circ$

Area of hexagon $= 6 \times \frac{1}{2} \times 3^2 \times \sin\frac{\pi}{3}$ M1

$= 23.3826859 \text{ units}^2$

Percentage of hexagon that is unshaded $= \frac{23.3826859 - 2(5.527663644)}{23.3826859} \times 100\%$ M1

$= 52.72002825\%$

$\simeq 52.7\%$ (3sf) A1

Q20ai) Median hourly wage $\simeq \$32.75$ B1

ii) Interquartile range $\simeq \$36.00 - \$28.50$

$= \$7.50$ B1

b) Number of workers with managerial positions or higher $= \frac{10}{100} \times 120$

$= 12$

Minimum hourly wage of middle manager $\simeq \$40.00$ B1

c) $P(\leq \$30/\text{hr}) = \frac{38}{120}$

$= \frac{19}{60}$ B1

d) The hourly wages of workers in Sunshine Company is generally higher than the hourly wages of workers in Brightlight Company as the median hourly wage is higher for Sunshine company ($32.75) as compared to Brightlight Company ($26.00). B1

The hourly wages of workers in Sunshine Company is generally more consistent than that of Brightlight Company as the interquartile range is lower for Sunshine Company ($7.50) as compared to Brightlight Company ($10.00). B1

Q21ai) $\vec{OB} = \vec{OA} + \vec{AB}$

$= \vec{OA} + \vec{OC}$ ($\because$ OABC is a parallelogram)

$= \underset{\sim}{a} + \underset{\sim}{c}$ B1

ii) $\because 5\vec{OD} = 3\vec{DB}$

$\vec{OD} = \frac{3}{5}\vec{DB}$

$= \frac{3}{8}\vec{OB}$

$= \frac{3}{8}(\underset{\sim}{a} + \underset{\sim}{c})$

$\vec{CD} = \vec{CO} + \vec{OD}$

$= -\vec{OC} + \vec{OD}$

$= -\underset{\sim}{c} + \frac{3}{8}(\underset{\sim}{a} + \underset{\sim}{c})$ M1

$= \frac{1}{8}(3\underset{\sim}{a} - 5\underset{\sim}{c})$ A1

21 b) $\frac{3}{8}BC = EC$

$\overrightarrow{DE} = \overrightarrow{DB} + \overrightarrow{BE}$

$= \frac{5}{8}\overrightarrow{DB} + \frac{5}{8}\overrightarrow{BC}$

$= \frac{5}{8}(\underset{\sim}{a}+\underset{\sim}{c}) + \frac{5}{8}(-\underset{\sim}{a})$ M1

$= \frac{5}{8}\underset{\sim}{c}$ A1

ci) $\frac{\text{Area of } \triangle BDE}{\text{Area of } \triangle BDC} = \left(\frac{BE}{BC}\right)^2$

$= \left(\frac{5}{8}\right)^2$

$= \frac{25}{64}$ B1

ii) $\triangle ODC$ and $\triangle OBC$ has a common height, h cm.

$\frac{\text{Area of } \triangle ODC}{\text{Area of } \triangle OBC} = \frac{\frac{1}{2} \times OD \times h}{\frac{1}{2} \times DB \times h}$

$= \frac{OD}{DB}$

$= \frac{3}{8}$

$\frac{\text{Area of } \triangle OCD}{\text{Area of parallelogram } OABC} = \frac{\text{Area of } \triangle OCD}{\text{Area of } \triangle DBC} \times \frac{\text{Area of } \triangle OBC}{\text{Area of parallelogram } OABC}$

$= \frac{3}{8} \times \frac{1}{2}$

$= \frac{3}{16}$ B1

1 (a) Make t the subject of the formula $r = u + \sqrt{\frac{su}{t} + s^2}$. [2]

(b) Given that $x - \frac{1}{x} = 5$, find the value of $x^2 + \frac{1}{x^2}$. [2]

(c) Solve these simultaneous equations.

$$\frac{x-3y}{2} = \frac{2x-3y}{5}$$

$$7x - 6y = 19$$

[3]

(d) Express $\left(\frac{b}{a} + \frac{a}{b} + 2\right) \div \left(\frac{1}{a^2} - \frac{1}{b^2}\right)$ as a single fraction in its simplest form. [3]

2 Tickets to a concert were sold and the seats were divided into 4 categories. The number of tickets sold for Saturday and Sunday are summarized in the table below.

Categories	CAT 1	CAT 2	CAT 3	CAT 4
Saturday	64	85	110	87
Sunday	50	65	128	90

(a) Write down a 2×4 matrix **T** to represent the number of tickets sold for both days. [1]

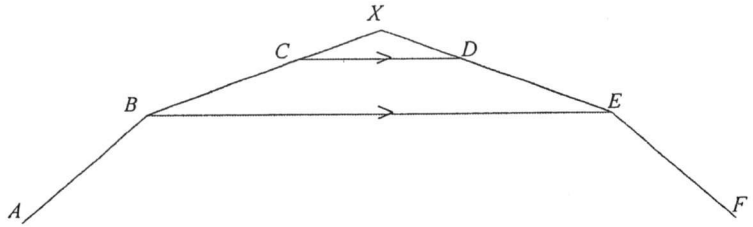
(b) The price per ticket is \$268 for Category 1, \$168 for Category 2, \$128 for Category 3 and \$78 for Category 4.
Represent the ticket prices in a column matrix **P**. [1]

(c) Evaluate the matrix **A** = **TP** [1]

(d) State what the elements of **A** represent. [1]

(e) A 10% discount was given for Matinee show tickets. 80 Category 1, 130 Category 2, 150 Category 3 and 185 Category 4 tickets were sold. Using matrix multiplication, calculate the total sales earned from the Matinee show. [3]

3



The diagram shows part of a regular n-sided polygon $ABCDEF\ldots$.
BCX and EDX are straight lines and CD is parallel to BE.

(a) Explain why triangles XCD and XBE are similar triangles. [2]

(b) It is given that $\angle BCD = 150°$. Find

(i) the value of n, [2]

(ii) angle CXD, [2]

(iii) angle ABE. [1]

4 The first 3 terms of a sequence of numbers T_1, T_2 and T_3 are given below:

$T_1 = 6(1)^2 + 30 = 36$
$T_2 = 6(2)^2 + 48 = 72$
$T_3 = 6(3)^2 + 66 = 120$

(a) (i) Find T_4. [1]

(ii) Find an expression, in terms of n, for T_n. Hence, or otherwise, explain why value of T_n must be even for all values of n. [3]

(iii) Evaluate T_{30}. [1]

(b) The first four terms in a different sequence are 4, 6, 8, 10.
Find an expression, in terms of n, for the nth term, P_n, of this sequence. [1]

(c) Find an expression, in terms of n, for the nth term, $\frac{T_n}{P_n}$. Hence, explain why the term $\frac{T_n}{P_n}$ is a multiple of 3. [3]

5 **Answer the whole of this question on a sheet of graph paper.**

The variables x and y are connected by the equation

$$y = \frac{1}{3}x(5 - x^2).$$

x	-3	-2	-1	0	1	2	3	4
y	4	-0.7	p	0	1.3	0.7	-4	-14.7

(a) Calculate the value of p. [1]

(b) Using a scale of 2 cm to 1 unit, draw a horizontal x-axis for $-3 \le x \le 5$.
Using a scale of 2 cm to 2 units, draw a vertical y-axis for $-16 \le y \le 4$.
On your axes, plot the points given in the table and join them with a smooth curve. [3]

(c) Determine the number of solutions to the equation $x(5 - x^2) = 3$.
Explain how this can be seen from your graph. [2]

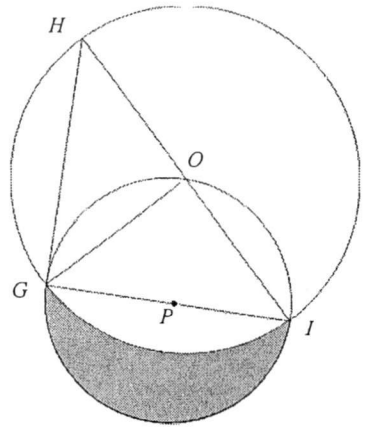
(d) By drawing a tangent, find the gradient of the curve at $(3,-4)$. [2]

(e) **(i)** On the same axes, draw a line with gradient 2 and passes through the point $(0, 2)$. [2]

(ii) Write down the x-coordinate(s) of the point where this line intersects the curve. [1]

(iii) Use your graph to find the range of values of x for which $x(5 - x^2) > 6x + 6$. [2]

6 The diagram shows two circles C_1 and C_2 with centre O and P respectively. HI and GI are diameters of C_1 and C_2 respectively.



(a) Show that triangles HGI and GOI are similar.
Give a reason for each statement you make. [2]

(b) Given that $HI = 10$ cm and $GI = 5\sqrt{2}$ cm.
Find the ratio area of triangle HGI : area of triangle GOI. [2]

(c) Find the shaded area. [4]

7 Queenie has allocated a budget of \$49 on buying ingredients for baking a cake. The budget is divided between flour, sugar and saffron in the ratio of 3 : 2 : 5.

(a) Find the amount Queenie spent on buying saffron. [2]

(b) **(i)** Given that the unit price of saffron is \$$x$ per gram, write down an expression for the amount of saffron bought. [1]

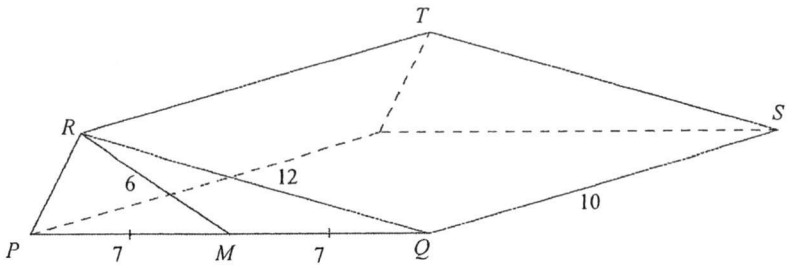
(ii) During a sale, the unit price of saffron is reduced by \$4. Write down an expression for the amount of saffron that can be bought for the same amount of money. [1]

(iii) If 4 more grams of saffron can be bought at the reduced price, form an equation in x and show that it reduces to $2x^2 - 8x - 49 = 0$. [3]

(iv) Solve the equation $2x^2 - 8x - 49 = 0$, giving your answers correct to 2 decimal places. [3]

(v) Janice who has \$50 wants to buy as much saffron as possible at the discounted price. Find the amount of saffron she can buy. [2]

8



The diagram shows a solid triangular prism with three rectangular faces.
$PM = QM = 7$ cm, $RM = 6$ cm, $QR = 12$ cm and $QS = 10$ cm.

(a) Show that $\angle MRQ = 24.53°$ corrected to 2 decimal places. [3]

(b) Calculate the vertical distance of R above PQ. [3]

(c) Given that $PR = \sqrt{26}$ cm, calculate the surface area of the prism. [3]

(d) Calculate the angle of elevation of T from Q. [3]

9 **(a)** The results of a Mathematics Test for a class of 20 students were recorded.
The results are shown in the stem-and-leaf diagram.

0	3 9
1	1 5
2	4 6 7
3	0 0 2 3 7 8
4	0 4 5 6 7 7 8

Key 1 | 3 means 13 marks

(i) Find the percentage of students who failed if the passing mark is 25. [1]

(ii) Find the mean mark. [1]

(iii) Find the standard deviation of the marks. [2]

(iv) It was discovered that the results had been recorded incorrectly.
The correct results were all 4 more than those recorded.

Explain how the median and standard deviation of the results have been affected by this error. [2]

(b) A bag contains 10 identical coloured balls. There are 4 pink and 6 blue balls.
Two balls are picked out at random, without replacement.

(i) Draw a tree diagram to show the probabilities of the possible outcomes. [2]

(ii) Find, as a fraction in its simplest form, the probability that one ball is pink and the other is blue. [2]

(iii) A third ball is now picked out at random. Calculate the probability that none of the three balls is blue. [2]

10 Sally is intending to set up a fruit punch stall during a charity bazaar to raise funds. Each cup will contain 200 ml of fruit punch and will be served in a plastic cup with a straw. Sally made a trip to the supermarket to find out the prices of the ingredients she needs. Information that Sally needs is provided in the tables below.

Sally's Fruit Punch Recipe

The recipe below makes 1 liter of punch:

1 lime

1 lemon

150ml pineapple juice

500ml lemonade

300ml orange juice

Ingredients Prices		
Item	**Description**	**Cost**
Lemon	Individual Pack of three	60¢ $1.50
Limes	Individual Pack of 5	75¢ $3.50
Lemonade	2L bottle	$1.85
Orange juice	1L carton	$1.25
Pineapple juice	500ml carton 2L carton	$2.50 $7.50
Plastic Cups	Pack of 20	$1.35
Straws	Pack of 100	$1.30

(a) Find the amount of orange juice needed to prepare 500ml of fruit punch. [1]

(b) Sally estimates that around 200 cups of fruit punch will be sold.

(i) What is the minimum number of bottles of lemonade that Sally should buy? [2]

Sally needs to decide how much she should charge for 1 cup of fruit punch. She must make sure that she covers all of her costs.

(ii) Suggest a sensible amount for her to charge 1 cup of fruit punch. Justify the decision you make and show your calculations clearly. [7]

- End of Paper -

4E5N Emath MYE 2017

Q1(a) $r = u + \sqrt{\frac{su}{t} + s^2}$

$r - u = \sqrt{\frac{su}{t} + s^2}$

$(r-u)^2 = \frac{su}{t} + s^2$ —— [M1]

$t = \frac{su}{(r-u)^2 - s^2}$ —— [A1]

(b) Given: $x - \frac{1}{x} = 5$

$(x - \frac{1}{x})^2 = 25$ —— [M1]

$x^2 - 2 + \frac{1}{x^2} = 25$

$x^2 + \frac{1}{x^2} = 27$ —— [A1]

(c) $\frac{x-3y}{2} = \frac{2x-3y}{5}$ —— ①

$7x - 6y = 19$ —— ②

From ②, $x = \frac{19+6y}{7}$; sub into ①

$\frac{\frac{19+6y}{7} - 3y}{2} = \frac{2(\frac{19+6y}{7}) - 3y}{5}$ —— [M1]

$\frac{5(19+6y)}{7} - 15y = 4(\frac{19+6y}{7}) - 6y$

$5(19+6y) - 105y = 4(19+6y) - 42y$

$51y = 19$

$y = \frac{19}{57} = \frac{1}{3}$ —— [A1]

$x = 3$ —— [A1]

$\therefore x = 3, \ y = \frac{1}{3}$

Q1(d) $\left(\frac{b}{a} + \frac{a}{b} + 2\right) \div \left(\frac{1}{a^2} - \frac{1}{b^2}\right)$

$= \left(\frac{b^2 + a^2 + 2ab}{ab}\right) \div \frac{b^2 - a^2}{(ab)^2}$ —— [M1]

$= \frac{(a+b)^2}{ab} \times \frac{(ab)^2}{(b-a)(b+a)}$ —— [M1]

$= \frac{ab(a+b)}{b-a}$ —— [A1]

Q2(a) $T = \begin{pmatrix} 64 & 85 & 110 & 87 \\ 50 & 65 & 128 & 90 \end{pmatrix}$ —— [B1]

(b) $P = \begin{pmatrix} 268 \\ 168 \\ 128 \\ 78 \end{pmatrix}$ —— [B1]

(c) $A = \begin{pmatrix} 64 & 85 & 110 & 87 \\ 50 & 65 & 128 & 90 \end{pmatrix} \begin{pmatrix} 268 \\ 168 \\ 128 \\ 78 \end{pmatrix}$

$= \begin{pmatrix} 52298 \\ 47724 \end{pmatrix}$ —— [B1]

(d) The elements of A represent the total sales of tickets on Saturday & Sunday respectively. —— [B1]

(e) Total sales $= 0.9 \begin{pmatrix} 80 & 130 & 150 & 185 \end{pmatrix} \begin{pmatrix} 268 \\ 168 \\ 128 \\ 78 \end{pmatrix}$ —— [M1]

$= \begin{pmatrix} 69219 \end{pmatrix}$ —— [M1]

∴ Total sales = $ 69219 —— [A1]

Q3 (a) $\angle BXE = \angle CXD$ (common $\angle$)

$\angle XCD = \angle XBE$ (corr. $\angle$s, $CD // BE$)

$\angle XDC = \angle XEB$ (corr. $\angle$s, $CD // BE$) } [M1]

∴ $\triangle XCD$ is similar to $\triangle XBE$ —— [A1]

(b)(i) $(n-2) \times 180 = 150 \times n$ —— [M1]

$n = 12$ —— [A1]

(ii) $\angle BCD = \frac{360}{12}$

$= 30°$ (ext. $\angle$ of polygon) —— [M1]

$\angle CXD = 180° - 30° - 30°$

$= 120°$ (base $\angle$ of isos $\triangle$) —— [A1]

(iii) $\angle ABE = 150° - 30°$

$= 120°$ —— [B1]

4 (a) (i) $T_4 = 6(4)^2 + 84$

$= 180$ —— [B1]

(ii) $T_n = 6n^2 + 18n + 12$ —— [M1]

$= 6(n+1)(n+2)$

$= 2[3(n+1)(n+2)]$ —— [A1]

Since T_n is divisible by 2, it must be even $\forall$ values of n. —— [A1]

(iii) $T_{30} = 6(30+1)(30+2)$

$= 5952$ —— [B1]

(b) $P_n = 2n + 2$ —— [B1]

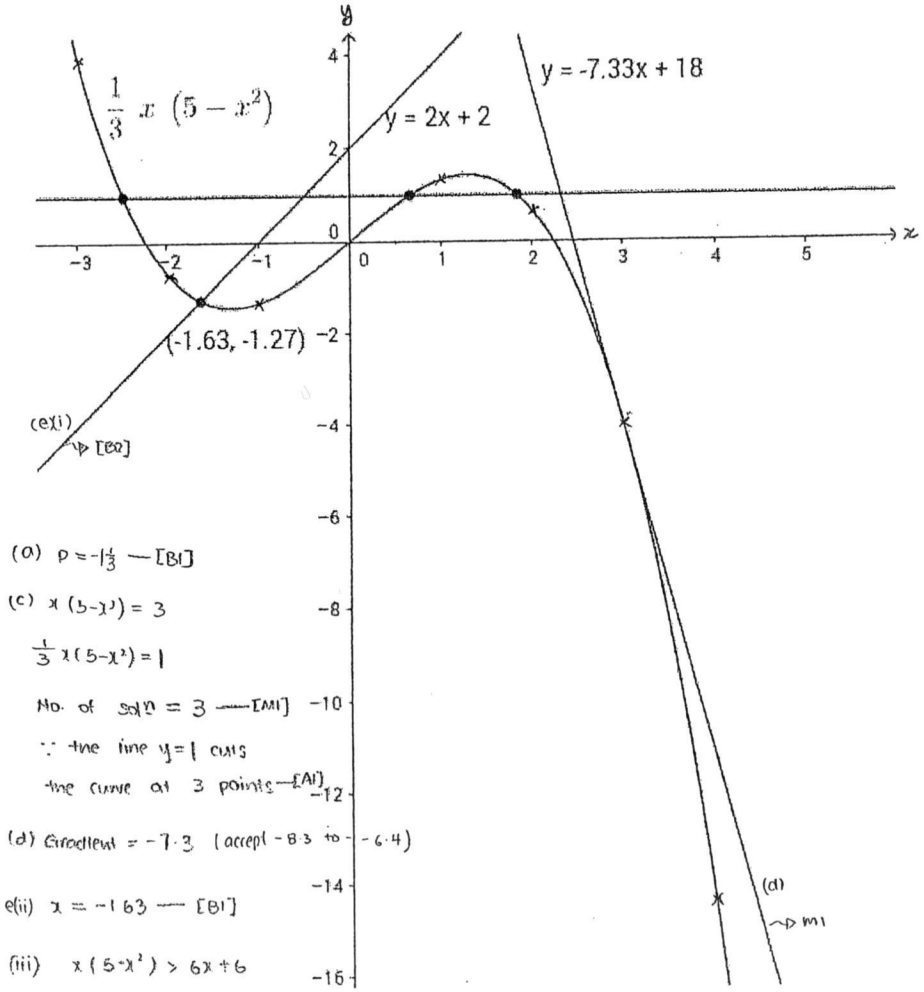
(c) $\frac{T_n}{P_n} = \frac{6(n+1)(n+2)}{2n+2}$ —— [M1]

$= \frac{6(n+1)(n+2)}{2(n+1)}$

$= 3(n+2)$ —— [A1]

Since $\frac{T_n}{P_n}$ is divisible by 3, it must be a multiple of 3. —— [A1]

Q5.



(a) $p = -1\frac{1}{3}$ —[B1]

(c) $x(5-x^2) = 3$

$\frac{1}{3}x(5-x^2) = 1$

No. of soln = 3 —[M1]

∵ the line $y = 1$ cuts the curve at 3 points —[A1]

(d) Gradient = -7.3 (accept -8.3 to -6.4)

e(ii) $x = -1.63$ — [B1]

(iii) $x(5-x^2) > 6x + 6$

$x(5-x^2) > 3(2x+2)$

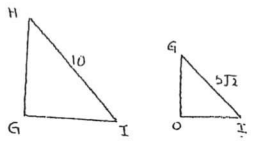
$\frac{1}{3}x(5-x^2) > 2x + 2$ —— [M1]

Range of x is $-3 \leq x < -1.63$

Q6 (a) $\angle HGI = \angle GOI = 90°$ ($\angle$s in semi circle)
$\angle HIG = \angle GIO$ (common $\angle$) } — [M1]

$\therefore \triangle HGI$ is similar to $\triangle GOI$ — [A1]

(b)



Area of $\triangle GHI$: Area of $\triangle GOI$

$= 10^2 : \left(5\sqrt{2}\right)^2$ — [M1]

$= 2 : 1$ — [A1]

(c) Area of quadrant GOI $= \frac{1}{4} \times \pi (5^2)$

$= \frac{25}{4}\pi$ — [M1]

Area of segment GPI $= \frac{25}{4}\pi - \frac{1}{2} \times 5 \times 5$

$= \frac{25}{4}\pi - \frac{25}{2}$ — [M1]

Area of shaded area $= \frac{1}{2} \times \pi \left(\frac{5\sqrt{5}}{2}\right)^2 - \left(\frac{25}{4}\pi - \frac{25}{2}\right)$ — [M1]

$= 12.5\ cm^2$ # — [A1]

Q7(a) Amount spent on saffron $= \frac{49}{10} \times 5$ — [M1]

$= \$24.50$ — [A1]

(b)(i) Amount of saffron bought $= \frac{24.5}{x}$ g — [B1]

(ii) Amount of saffron bought $= \frac{24.5}{x-4}$ g — [B1]

(iii) $\frac{24.5}{x-4} - \frac{24.5}{x} = 4$ — [M1]

$24.5x - 24.5x + 98 = 4x(x-4)$ — [M1]

$4x^2 - 16x - 98 = 0$

$2x^2 - 8x - 49 = 0$ — [A1]

(iv) $x = \frac{-(-8) \pm \sqrt{(-8)^2 - 4(2)(-49)}}{2(2)}$ — [M1]

$x = 7.3385\,39126$ or $x = -3.3385\,39126$

≈ 7.34 (2dp) # — [A1] ≈ -3.34 (2dp) # — [A1]

(v) Amount bought $= \frac{\$50}{7.3385\,39126 - 4}$ — [M1]

$= 14.97661046$

≈ 14.9 g (rounded down to 3sf) (accepted 15.0 g)

Q8(a) By Cosine Rule,

$6^2 + 12^2 - 2(6)(12)\cos \angle MRQ = 7^2$ —— [M1]

$\cos \angle MRQ = \frac{7^2 - 6^2 - 12^2}{-2(6)(12)}$ —— [M1]

$\angle MRQ = 24.533007^\circ$

$\approx 24.53^\circ$ (2 dp) —— [A1]

(b) Area of $\triangle RMQ = \frac{1}{2} \times 6 \times 12 \sin 24.533007^\circ$ —— [M1]

$= 14.94782593$

Vertical dist. of R above PQ $= \frac{14.94782593}{\frac{1}{2} \times 7}$ —— [M1]

$= 4.270807409$

≈ 4.27 cm (3sf) —— [A1]

(c) Surface area $= \left(\frac{1}{2} \times 14 \times 4.270807409\right) \times 2 + (\sqrt{26} + 14 + 12) \times 10$ —— [M2]

$= 370.7814989$

$\approx 371\ cm^2$ (3sf) —— [A1]

(d) By Pythagoras Theorem,

$TQ = \sqrt{12^2 + 10^2}$

$= \sqrt{244}$ —— [M1]

Let $\angle$ of elevation be θ

$\sin\theta = \frac{4.270807409}{\sqrt{244}}$ —— [M1]

$\theta = 15.86730841^\circ$

$\approx 15.9^\circ$ (1 dp) —— [A1]

Q9(a)(i) % of failures $= \frac{5}{20} \times 100\%$

$= 25\%$ —— [B1]

(ii) Mean $= \frac{632}{20}$

$= 31.6$ marks —— [B1]

(iii) Standard Deviation $= \sqrt{\frac{23542}{20} - 31.6^2}$ —— [M1]

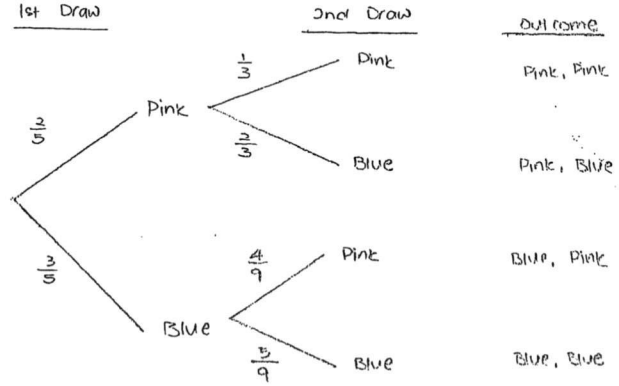
$= 13.36188609$

≈ 13.4 —— [A1]

(iv) Median would increase by 4 marks. —— [B1]

Standard deviation would remain the same —— [B1]

(b)(i)



1 branch 1 mark

(ii) $P(1\ pink, 1\ blue) = \left(\frac{3}{5}\right)\left(\frac{2}{3}\right) + \left(\frac{3}{5}\right)\left(\frac{4}{9}\right)$ —— [M1]

$= \frac{8}{15}$ —— [A1]

(iii) $P(\text{no blue balls}) = \left(\frac{2}{5}\right)\left(\frac{1}{3}\right)\left(\frac{3}{8}\right)$ —— [M1]

$= \frac{1}{30}$ —— [A1]

Q10 (a) Amount of orange juice $= \frac{1}{5} \times 300ml$

$= 150ml$ —— [A1]

(b) Total amount of fruit punch $= 200 \times 200ml$

$= 40\,000ml$

$= 40L$ —— [M1]

min. bottles of lemonade $= \frac{40 \times 0.5L}{2L}$

$= 10$ bottles —— [A1]

(c)

Amount of limes needed $= 40$

Amount of lemons needed $= 40$

Amount of pineapple juice $= 40 \times 0.15L$

$= 6L$

Amount of lemonade $= 40 \times 0.5L$

$= 20L$

Amount of orange juice $= 40 \times 0.3L$

$= 12L$

Accurate calculate of

lime + lemonade — M1

" " " lemon — M1

" " " pineapple + orange — M1

" " " plastic + straw — M1

cost of lime $= \frac{40}{5} \times \$3.50$

$= \$28$

cost of lemon $= \frac{39}{3} \times \$1.50 + 60¢$

$= \$20.10$

cost of pineapple juice $= \frac{6}{2} \times \$7.50$

$= \$22.50$

cost of lemonade $= 10 \times \$1.85$

$= \$18.50$

cost of orange juice $= 12 \times \$1.25$

$= \$15$

cost of plastic cups $= 10 \times \$1.35$

$= \$13.50$

cost of straws $= \$1.30 \times 2 = \$2.60$

total cost $= \$120.20$ —— [A1]

min cost $= \frac{\$120.20}{200}$ —— [M1]

$= \$0.601$

Since its a fund raising, Sally should consider her labour cost, rental of venue and also the amount she aims to raise for the event. ∴ sensible amount to charge will be more than $0.601 # —— [M1]

JUNYUAN SECONDARY SCHOOL
MID YEAR EXAMINATION 2017
SECONDARY FOUR EXPRESS / FIVE NORMAL (ACADEMIC)

CANDIDATE NAME

CLASS INDEX NUMBER

MATHEMATICS **4048/01**

Paper 1 3 May 2017

2 hours

Candidates answer on the Question Paper.

READ THESE INSTRUCTIONS FIRST

Write your name, class and index number on all the work you hand in.
Write in dark blue or black pen.
You may use an HB pencil for any diagrams or graphs.
Do not use paper clips, highlighters, glue or correction fluid.

Answer **all** questions.
If working is needed for any question it must be shown with the answer.
Omission of essential working will result in loss of marks.
The use of an approved scientific calculator is expected, where appropriate.
If the degree of accuracy is not specified in the question, and if the answer is not exact, give the answer to three significant figures. Give answers in degrees to one decimal place.
For π, use either your calculator value or 3.142, unless the question requires the answer in terms of π.

At the end of the examination, fasten all your work securely together.
The number of marks is given in brackets [ ] at the end of each question or part question.
The total of the marks for this paper is 80.

For Examiner's Use

Mathematical Formulae

Compound interest

$$\text{Total amount} = P\left(1+\frac{r}{100}\right)^n$$

Mensuration

Curved surface area of a cone $= \pi rl$

Surface area of a sphere $= 4\pi r^2$

Volume of a cone $= \frac{1}{3}\pi r^2 h$

Volume of a sphere $= \frac{4}{3}\pi r^3$

Area of triangle $ABC = \frac{1}{2}ab\sin C$

Arc length $= r\theta$, where θ is in radians

Sector area $= \frac{1}{2}r^2\theta$, where θ is in radians

Trigonometry

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$a^2 = b^2 + c^2 - 2bc\cos A$$

Statistics

$$\text{Mean} = \frac{\sum fx}{\sum f}$$

$$\text{Standard deviation} = \sqrt{\frac{\sum fx^2}{\sum f} - \left(\frac{\sum fx}{\sum f}\right)^2}$$

1 (a) Express 126 as the product of its prime factors.

Answer ..... [1]

(b) Using your calculator, find the value of $\pi^{3.14}$ correct to two decimal places.

Answer ..... [1]

2 Showing your working clearly, find the fraction exactly halfway between $\frac{7}{8}$ and $\frac{9}{10}$.

Answer ..... [2]

3 (a) Write down $\frac{4}{11}$ in recurring decimal form.

Answer ..... [1]

(b) Express $2\frac{3}{5}$ as a percentage.

Answer ..... % [1]

4 Given that $a : b = 5 : 4$, calculate the value of $\frac{2a}{7b}$.

Answer ..... [2]

5 Given that $-3 \le x \le 2$ and $-7 \le y \le 3$, calculate

(a) the largest possible value of $x - y$,

Answer ..... [1]

(b) the smallest possible value of x^2.

Answer ..... [1]

6 (a) A train left a station at 21:47 and arrived at its destination at 03:56 the following day.

How many minutes did the journey take?

Answer ..... minutes [1]

(b) Jenny runs 2.88 km in 30 minutes.

Calculate her speed in metres per second.

Answer ..... m/s [2]

7 The cost of a circular ornament, $\$C$, is directly proportional to the square of its radius, r metres.

(a) Given that $C = 24$ when $r = 0.2$, form an equation connecting C and r.

Answer ..... [2]

(b) Hence, calculate the cost of a circular ornament with $r = 0.7$.

Answer $ ..... [1]

8 For his holiday, Alex changed 2 400 Malaysian Ringgits (MYR) to Singapore Dollars (SGD) when the exchange rate was 1 SGD = 3.20 MYR.
At the end of his holiday, he had 25 SGD left.

(a) How much did he spend in Singapore Dollars?

Answer ..... SGD [2]

(b) He changed his remaining 25 SGD for 77 MYR at the end of his holiday.

What was the exchange rate of MYR to 1 SGD at the time of exchange?

Answer ..... MYR [1]

9 A scale model of a steam engine has a length of 60 cm.
The actual engine is 9.6 m long.

(a) Find the scale of the model engine to the actual engine in the form 1: n.

Answer 1 : ..... [1]

(b) Given that the name plate on the model measures 4 cm by 5 cm, calculate the actual area of the name plate on the engine in square metres.

Answer ..... m^2 [2]

10 **(a)** Find the gradient of the straight line L_1 which has the equation $2y - 5x = 4$.

Answer ..... [1]

(b) Another straight line L_2 has the equation $3y + 2x - 4 = 0$.
It passes through the point $(3k, k + 2)$.

Calculate the value of k.

Answer $k =$ ..... [2]

11 A metal beam structure weighs 13.2 megagrams and has a volume of $1.1\times10^5\ \text{cm}^3$.
(mega = 10^6)

(a) Express 13.2 megagrams in grams, giving your answer in standard form.

Answer ..... g [1]

(b) Hence find the density of the structure in g/cm^3, giving your answer in standard form.

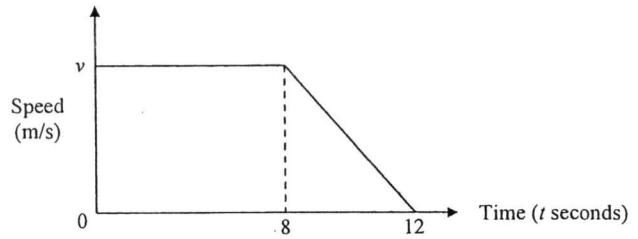
Answer ..... g/cm^3 [2]

---

12 Solve the inequalities $3x-2\le 5x+1<7-x$, and represent your solution on the number line below.

Answer ..... [3]

13 The diagram shows a speed-time graph for part of a car's journey.



The retardation of the car between $t=8$ and $t=12$ is $3.75\ \text{m/s}^2$.

(a) Find the value of v.

Answer $v=$ ..... [1]

(b) Calculate the total distance travelled by the car in the 12 seconds.

Answer ..... m [2]

---

14 Given that $6^x=9$, where x is a positive integer, find the value of

(a) 6^{-x},

Answer ..... [1]

(b) $6^{\frac{x}{2}}$,

Answer ..... [1]

(c) 6^{3x+1}.

Answer ..... [2]

15 Solve

(a) $\frac{5x+6}{4} = x$,

Answer $x =$ ..... [2]

(b) $3y^2 = 2y$.

Answer $y =$ ..... or ..... [2]

16 (a) Simplify $\frac{(2a^{-2}b)^3}{\sqrt{16a^2b^8}}$, giving your answer in positive index.

Answer ..... [2]

(b) Given that $8^h = \frac{16}{4^k}$, express h in terms of k.

Answer $h =$ ..... [2]

17 (a) $\xi = \{ x : x \text{ is an integer, where } 40 \le x \le 50 \}$

$P = \{ x : x \text{ is a prime number} \}$

$Q = \{ x : x \text{ is a multiple of } 6 \}$

(i) List the elements of Q.

Answer ..... [1]

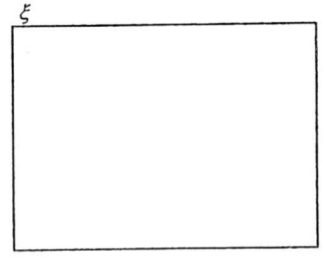
(ii) List the elements of $(P \cup Q)'$.

Answer ..... [1]

(b) On the Venn diagrams below, draw sets A and B when

(i) $A \cap B = \varnothing$,

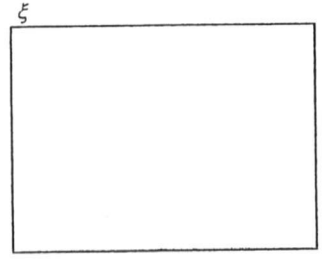
Answer (b)(i)



[1]

(ii) $A \subset B$.

Answer (b)(ii)



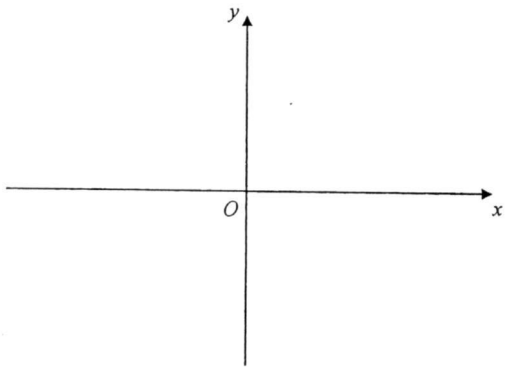
[1]

22 Calculate the exact principal amount if the difference between simple interest and compound interest compounded yearly for a period of 3 years at 5% per annum is \$305.

Answer \$ ..... [4]

23 **(a)** Sketch the graph of $y = -(x-3)^2 + 1$.

Answer (a)



[2]

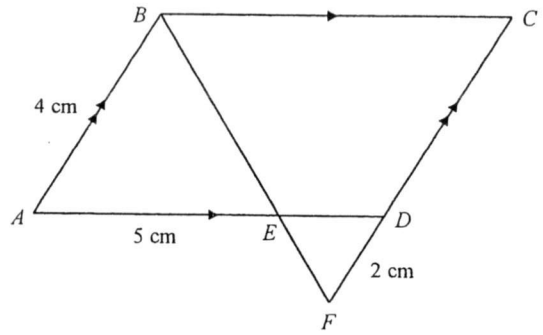
(b) State the coordinates of the turning point of the graph $y = -(x-3)^2 + 1$.

Answer (..... , ...) [1]

(c) Write down the equation of the line of symmetry.

Answer ..... [1]

24



ABCD is a parallelogram.
BEF and *CDF* are straight lines.
$AB = 4$ cm, $DF = 2$ cm and $AE = 5$ cm.

(a) By stating your reasons, show that triangle *ABE* is similar to triangle *CFB*.

Answer (a)

[2]

(b) Calculate *BC*.

Answer ..... cm [2]

(c) Triangle *DFE* is also similar to triangle *ABE*.

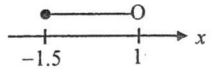
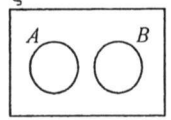
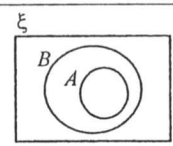
Given that the area of triangle *DFE* is 1.5 cm², find the area of triangle *ABE*.

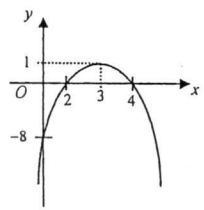
Answer ..... cm² [2]

End of Paper

Mid-Year Exam 2017 Secondary 4E/5N EM P1
(MARKING SCHEME)

1	(a)	$2\times3^2\times7$ **(Ans)**	B1
	(b)	36.40 **(Ans)**	B1
2		$\frac{1}{2}\left(\frac{7}{8}+\frac{9}{10}\right)$	M1
		$=\frac{1}{2}\left(\frac{35+36}{40}\right) \quad =\frac{1}{2}\left(\frac{71}{40}\right) \quad =\frac{71}{80}$ **(Ans)**	A1
3	(a)	$0.3636363636\ldots\ldots = 0.\dot{3}\dot{6}$ **(Ans)**	B1
	(b)	$\frac{13}{5}\times100\% \quad =260\%$ **(Ans)**	B1
4		$\frac{2}{7}\times\frac{a}{b} \quad =\frac{2}{7}\times\frac{5}{4}$	M1
		$=\frac{5}{14}$ **(Ans)**	A1
5	(a)	$(2)-(-7) \quad =9$ **(Ans)**	B1
	(b)	$(0)^2 \quad =0$ **(Ans)**	B1
6	(a)	21:47 to 03:47 = 6 hours = 360 mins, 03:47 to 03:56 = 9 mins. So total time taken is 369 mins **(Ans)**	B1
	(b)	$\frac{2.88\times1000\text{ m}}{30\times60\text{ s}}$	M1
		$=1.6$ m/s **(Ans)**	A1
7	(a)	$C=kr^2$, where k is a constant.	
		$24=k\times(0.2)^2 \quad \Rightarrow \quad k=600$	M1
		$\therefore\ C=600r^2$ **(Ans)**	A1
	(b)	$C=600\times(0.7)^2 \quad \Rightarrow \quad C=\$294$ **(Ans)**	B1√
8	(a)	$\$\frac{2400}{3.20} \quad =\$750$	M1
		$\therefore\ \$750-\$25 \quad =\$725$ **(Ans)**	A1
	(b)	$\frac{77}{25}$ MYR $\quad =3.08$ MYR **(Ans)**	B1
9	(a)	60 cm : 9.6 m $\Rightarrow$ 1 cm : 0.16 m $\Rightarrow$ 1 cm : 16 cm $\therefore\ n=16$ **(Ans)**	B1
	(b)	Area of model = 20 cm^2	
		Area scale = (1 cm)^2 : (0.16m)^2 $\Rightarrow$ 1 cm^2 : 0.0256 m^2	M1
		$\therefore$ 20 cm^2 represents $20\times0.0256 \quad =0.512$ m^2 **(Ans)**	A1
10	(a)	$2y=5x+4 \quad \Rightarrow \quad y=\frac{5}{2}x+2 \quad \Rightarrow$ gradient $=\frac{5}{2}$ **(Ans)**	B1
	(b)	$3(k+2)+2(3k)=4 \quad \Rightarrow \quad 3k+6+6k=4$	M1
		$\Rightarrow \quad 9k=-2 \quad \Rightarrow \quad k=-\frac{2}{9}$ **(Ans)**	A1
11	(a)	13.2×10^6 g $=$ 1.32×10^7 g **(Ans)**	B1
	(b)	Density $=\frac{1.32\times10^7\text{ g}}{1.1\times10^5\text{ cm}^3}$	M1
		$=1.2\times10^2$ g/cm^3 **(Ans)** *(accept if 1.20×10^2 is written)*	A1

12		$3x-2\le5x+1$ and $5x+1<7-x$ 	M1
		$-2x\le3 \quad 6x<6$	B1√
		$x\ge-1.5 \quad x<1$	A1
		$\therefore\ -1.5\le x<1$ **(Ans)**	
13	(a)	$\frac{v}{4}=3.75 \quad \Rightarrow \quad v=15$ m/s **(Ans)**	B1
	(b)	Total dist. traveled = Area under graph $=\frac{1}{2}\times(8+12)\times15$	M1√
		$=150$ m **(Ans)**	A1
14	(a)	$\frac{1}{6^x} \quad =\frac{1}{9}$ **(Ans)**	B1
	(b)	$\left(6^x\right)^{\frac{1}{2}} \quad =(9)^{\frac{1}{2}} \quad =3$ **(Ans)**	B1
	(c)	$6^{3x}\times6^1 \quad =\left(6^x\right)^3\times6$	M1
		$=(9)^3\times6 \quad =4\,374$ **(Ans)**	A1
15	(a)	$5x+6=4x$	M1
		$\therefore\ x=-6$ **(Ans)**	A1
	(b)	$3y^2-2y=0$	M1
		$y(3y-2)=0$	
		$\therefore\ y=0$ or $y=\frac{2}{3}$ **(both Ans)**	A1
16	(a)	$\frac{8a^{-6}b^3}{4ab^4} \quad =2a^{-7}b^{-1}$ *(for removing root and bringing in power)*	M1
		$=\frac{2}{a^7b}$ **(Ans)**	A1
	(b)	$2^{3h}=\frac{2^4}{2^{2k}} \quad \Rightarrow \quad 2^{3h}=2^{4-2k}$	M1
		$\Rightarrow \quad 3h=4-2k \quad \Rightarrow \quad \therefore\ h=\frac{4-2k}{3}$ **(Ans)**	A1
17	(a)(i)	$Q=\{42, 48\}$ **(Ans)** *(1 mark for both elements)*	B1
	(a)(ii)	$(P\cup Q)'=\{40, 44, 45, 46, 49, 50\}$ **(Ans)** *(1 mark for all 6 elements)*	B1
	(b)(i)		B1
	(b)(ii)		B1
18	(a)	$(0\times2)+(1\times5)+(2\times1)+(3\times4)+(4\times6)+(5\times2) \quad =53$ **(Ans)**	B1
	(b)	Mean $=(53\div20) \quad =2.65$ **(Ans)**	B1
	(c)	Modal number means 'mode' $=4$ **(Ans)**	B1
	(d)	Position $=\frac{20+1}{2}=10.5 \quad \Rightarrow$ 10th and 11th $\Rightarrow$ median $=3$ **(Ans)**	B1

19	(a)	$55+75+23(n-2)=360$ $130+23n-46=360 \Rightarrow 23n=276 \Rightarrow n=12$ **(Ans)**	M1 A1
	(b)	$\angle DCB=105° \Rightarrow 2x+110+75+105=540$ $\Rightarrow 2x=250 \Rightarrow x=125°$ **(Ans)**	M1 A1
20	(a)	$80\%\times 200=160$ From graph, $x=31.6$ mins **(Ans)**	M1 A1
	(b)	From graph, when $x=39$ mins, the cumulative frequency $y=182$. Thus, no. of people who spends more than 39 mins is $200-182=18$. $\therefore$ percentage of people $=\frac{18}{200}\times 100\%$ $=9\%$ **(Ans)**	M1 A1
21		Let x be the number of days Jim can repair a car alone. Jim alone takes: x days = 1 car $\Rightarrow$ 1 day $=\frac{1}{x}$ car Max alone takes: $(x+6)$ days = 1 car $\Rightarrow$ 1 day $=\frac{1}{x+6}$ car Jim and Max both takes: 4 days = 1 car $\Rightarrow$ 1 day $=\frac{1}{4}$ car Thus, $\frac{1}{x}+\frac{1}{x+6}=\frac{1}{4}$ $\Rightarrow \frac{(x+6)+x}{x(x+6)}=\frac{1}{4} \Rightarrow x^2+6x=4(2x+6)$ which reduces to $x^2-2x-24=0$ $(x-6)(x+4)=0$ $\therefore x=6$ or $x=-4$ (reject) Hence, Jim takes 6 days to repair a car alone, and Max takes 12 days to repair a car alone. **(both Ans)** *(1 mark can be respectively awarded for equivalent method)*	M1 M1 M1 A1
22		Let the principal amount be $\$x$. Compound Interest – Simple Interest $= x\left(1+\frac{5}{100}\right)^3-\left(x+\frac{x\times 5\times 3}{100}\right)$ $305 = x(1.05)^3-\left(\frac{100x+15x}{100}\right)$ $305 = 1.157625x-1.15x$ $305 = 0.007625x$ $\therefore x = 40\,000$ **(Ans)**	M1 M1 M1 A1
23	(a)	 • correct symmetrical shape = 1 mark • correct intersection of x and y-axes = 1 mark	B1 B1
	(b)	From sketch graph, max. point = (3, 1) **(Ans)**	B1
	(c)	$x=3$ **(Ans)**	B1

24	(a)	$\angle ABE=\angle CFB$ (alt. $\angle$s, AB is parallel FC) $\angle BAE=\angle FCB$ (opp. $\angle$s of parallelogram) $\therefore$ triangle ABE is similar to triangle CFB (AA property)	B1 B1
	(b)	$\frac{AB}{CF}=\frac{AE}{CB} \Rightarrow \frac{4}{6}=\frac{5}{CB} \Rightarrow 4\times CB=30$ $\Rightarrow \therefore BC=7.5$ cm **(Ans)**	M1 A1
	(c)	$\left(\frac{DF}{AB}\right)^2=\frac{1.5}{A_2} \Rightarrow \left(\frac{2}{4}\right)^2=\frac{1.5}{A_2}$ $\Rightarrow A_2=6$ cm^2 **(Ans)**	M1 A1

--- END OF MARKING SCHEME ---

1 **(a)** Expand and simplify $-(3p-2q)(2q+5p)$. [1]

(b) Given $4r^2 - h = 2f - ghr^2$, express r in terms of f, g and h. [2]

(c) Express $\dfrac{3x^2+x-10}{2x^2-8}$ as a single fraction in its simplest form. [2]

(d) Solve $\dfrac{1}{y-1} - \dfrac{2}{5} = \dfrac{7}{4y}$. [3]

(e) Cupcakes are sold at 3 for \$5.
Louisa bought m cupcakes and was given a 5% discount.

Write an expression, in terms of m, for the amount she paid in dollars. [2]

2 Jeria trekked 10 kilometres to a waterfall at an average speed of x kilometres per hour.

(a) Write down an expression, in terms of x, for the time taken in hours. [1]

(b) For her return journey from the waterfall, Jeria trekked the same 10 kilometres at an average speed of $(x+1)$ kilometres per hour.
The time taken for her return journey is 30 minutes faster than the time taken for her journey to the waterfall.

Write down an equation in x and show that it simplifies to $x^2 + x - 20 = 0$. [4]

(c) Solve the equation $x^2 + x - 20 = 0$. [3]

(d) Marianne has 300 minutes to trek to the waterfall and back.
The ratio of Jeria's to Marianne's trekking time to the waterfall is 2 : 3.

Find the maximum time, in minutes, Marianne could take for her return journey. [2]

3 The first four terms in a sequence of numbers are given below.

$T_1 = 1 = 1$

$T_2 = 3 + 5 = 8$

$T_3 = 7 + 9 + 11 = 27$

$T_4 = 13 + 15 + 17 + 19 = 64$

(a) Find T_5 and T_6. [2]

(b) What is the name given to the numbers 1, 8, 27, 64, ...? [1]

(c) How many numbers are added up to give the value of T_n? [1]

(d) Find an expression, in terms of n, for the value of T_n. [1]

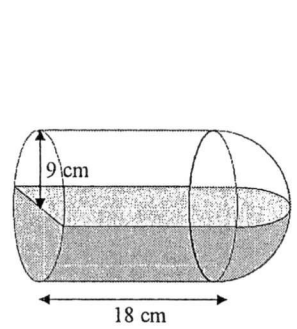
(e) Explain why the value of T_n is odd when n is odd and even when n is even. [2]

4 **(a)** **Diagram is not drawn to scale.**

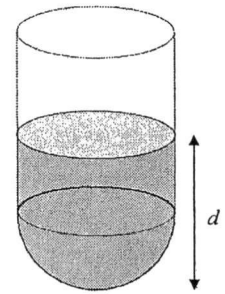
A closed container, made by joining a hemisphere of radius 9 cm and a cylinder of length 18 cm, is placed in two rest positions as shown below.

In Position A, the container is horizontal and exactly half filled with water.

In Position B, the half-filled container is held with its axis vertical and the hemisphere at the bottom.



Position A



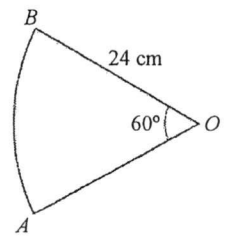
Position B

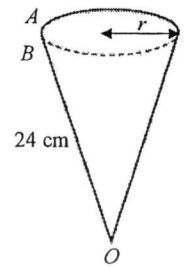
(i) Find the volume of water in the container, leaving your answer in π. [3]

(ii) Show that the depth of the water, d, in Position B is 15 cm. [3]

(iii) Find the total surface area of the container in contact with the water in Position B. [2]

(b) The wrapper of a giant ice-cream cone is formed from joining points A and B of the sector AOB.



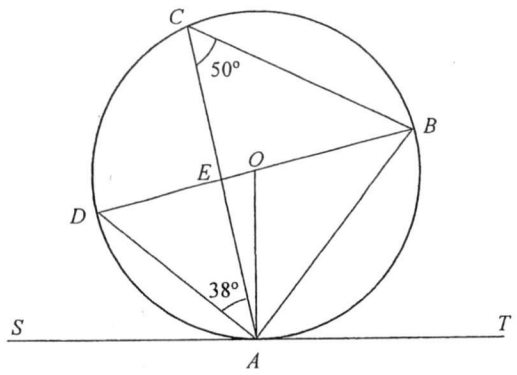


(i) Find the area of sector AOB in terms of π. [1]

(ii) Show that the radius, r, of the ice-cream cone wrapper is 4 cm. [2]

(iii) Find the volume of the ice-cream cone. [2]

5



The diagram shows a circle $ABCD$ with centre O.
SAT is a tangent to the circle at point A and the line AC intersects the line BD at E.
Angle $DAC = 38°$ and angle $ACB = 50°$.

(a) Find, giving reasons for each answer,

(i) angle OBA, [2]

(ii) angle OAC, [1]

(iii) angle SAD, [1]

(iv) angle CED. [2]

(b) Given that $AB = 15$ cm, find

(i) the diameter of the circle, [2]

(ii) the area of triangle AOB. [2]

6 **Answer the whole of this question on a sheet of graph paper.**

The variables x and y are connected by the equation

$$y = -\frac{x^2}{3} - \frac{11}{x} + 11.$$

Some corresponding values of x and y, correct to two decimal places, are given in the table.

x	1	1.5	2	3	4	5	6
y	− 0.33	2.92	q	4.33	2.92	0.47	− 2.83

(a) Find the value of q. [1]

(b) Using the scale of 2 cm to represent 1 unit on each axis, draw a horizontal x-axis for $0 \le x \le 6$ and a vertical y-axis for $-3 \le y \le 5$.

On your axes, plot the points given in the table and join them with a smooth curve. [3]

(c) By drawing a tangent, find the gradient of the curve at (1.5, 2.92). [2]

(d) By drawing a suitable straight line on the same axes, find the solutions to the equation $-\frac{x^2}{3} - \frac{11}{x} + 12 = x$. [2]

(e) Use your graph to find the coordinates of the maximum point of $y = -\frac{x^2}{3} - \frac{11}{x} + 11$ in the range of $1 \le x \le 6$. [2]

7 Tickets to a Korean-pop concert were sold online by an organiser.
The number of tickets sold for each category on Friday and Saturday respectively are summarised in the table below.

Day	Category 1	Category 2	Category 3	Category 4
Friday	135	120	100	105
Saturday	150	140	125	85

(a) Write down a 2×4 matrix **T** to represent the number of tickets sold by the organiser. [1]

(b) The price per ticket is \$228 for Category 1, \$168 for Category 2, \$128 for Category 3 and \$98 for Category 4.

Represent the ticket prices in a column matrix **S**. [1]

(c) Evaluate the matrix $\mathbf{K} = \mathbf{TS}$. [2]

(d) State what the elements of **K** represent. [1]

(e) The organiser decided to hold another concert on Sunday.
The prices per ticket for all categories were increased by 5%.

The number of tickets sold on Sunday is shown in the table below.

Day	Category 1	Category 2	Category 3	Category 4
Sunday	180	200	110	75

Using matrix multiplication, find the total sales revenue on Sunday. [3]

8 (a) In a team of software engineers, there are 12 females and 18 males.
Two engineers are selected at random, one after another, to attend a global conference.

(i) Draw a tree diagram to show the probabilities of the possible outcomes. [2]

(ii) Find, as a fraction in its simplest form, the probability that one female and male are selected. [2]

(iii) The manager decided to select a third engineer to attend the same conference.

Find the probability that all three engineers are females. [2]

(b) 15 fishermen from Town Koney were asked how many fishes they caught in an 8-hour work day.
The results are shown in the stem-and-leaf diagram.

Stem	Leaf
4	2
5	1 4 8 8 8
6	0 2 x
7	2 5 6 7
8	3
9	9

Key: 4 | 2 = 42 fishes

(i) The median number of fishes caught is 62.

Form an inequality to represent the range of possible values of x. [1]

(ii) Find the percentage of fishermen who caught at most 58 fishes a day. [1]

(iii) It is given that $x = 5$.

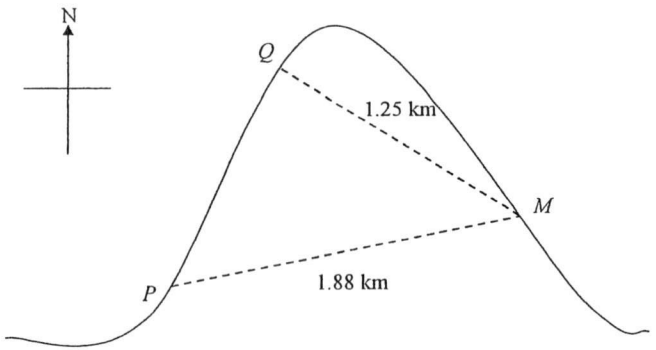
(a) Calculate the standard deviation of the number of fishes. [1]

(b) Determine and explain whether the mean is a good indicator of average in this distribution. [2]

(c) In Town Hona, the mean and standard deviation of the number of fishes are 68 and 13.2 respectively.

Use this information to comment on one difference between the two towns. [1]

9 Three jetties M, Q and P are situated around a coastal bay at the same ground level.
Jetty M is 1.88 km away at a bearing of 72° from Jetty P.
Jetty Q is 1.25 km away at a bearing of 300° from Jetty M.



(a) Find the straight line distance between Jetty Q and P. [2]

(b) Find the bearing of Jetty Q from Jetty P. [2]

(c) Ryan rides a water jet ski from Jetty Q towards PM.
He wishes to reach PM in the shortest time possible.

Find the distance of the route which allows him to reach in the shortest time. [2]

(d) A helicopter flies at a constant altitude of 950 m directly above PM.

(i) Find the angle of depression of the helicopter to Jetty Q when it is nearest to Q. [2]

(ii) A skydiver jumps off the helicopter when the helicopter is nearest to Q.

Determine whether the angle of elevation of the skydiver is smaller at point P or point M.
Show your workings clearly. [2]

10 Jerel will commence his Bachelor's Degree in Computing at the National University of Singapore (NUS) this August 2017.

Information that Jerel needs is on the opposite page.

Jerel is planning ahead for his annual cost of living.
He decides to stay on-campus to save the daily transportation time of 2 hours every day.

(a) Calculate his maximum estimated annual cost of living, excluding the vacation period. [1]

(b) Jerel receives $800 allowance from his parents every month.
To cover all his expenses. he works as a part-time tutor in a learning centre.
He is paid $20 per hour.

On average, find the minimum number of hours he will need to work monthly to cover his cost of living.
Give your answer in whole number. [3]

(c) Jerel has to complete four years of education in NUS for his degree.
He plans to take a POSB bank loan for his tuition fee payment.

Suggest whether it is more practical for him to take a 5 or 10 year bank loan.
Justify the decision you make and show your calculations clearly. [6]

COST OF LIVING
Full-Time Undergraduate Studies (2017)

The annual costs below have been derived based on a conservative estimate for a reasonably comfortable lifestyle, excluding the vacation period.

Item	Annual Estimated Cost
On-Campus accommodation (single/double occupancy)	$2 625 to $7 000
Meals	$2 600
Personal expenses	$2 200
Transportation within Singapore	$800
Average cost of books/supplies	$400

Notes:
- All amounts stated are in Singapore dollars.
- The costs are calculated on an annual basis, excluding the vacation period.

ANNUAL TUITION FEES
For New Students enrolled in Academic Year 2017/2018

National University of Singapore Fees payable by students per academic year				
Course	Arts & Social Sciences	Business / Accountancy	Engineering / Computing	Law
Annual Tuition Fees	$8 050	$9 450	$8 050	$12 500

POSB BANK LOAN
Education Loan Exclusive Promotion!

Key Features
- Lower interest rates of 4.6% per annum with 2% processing fee
- Up to 10 years repayment period which starts after course completion

END OF PAPER

JYSS 4E5N Mid Year 2017 Paper 2

No.		Answer	Workings	Marks	*Remarks
1	(a)	$4pq-15p^2+4q^2$	$-(3p-2q)(2q+5p)$ $=-(6pq+15p^2-4q^2-10pq)$ $=4pq-15p^2+4q^2$	B1	
	(b)	$r=\pm\sqrt{\frac{2f+h}{4+gh}}$	$4r^2-h=2f-ghr^2$ $4r^2+ghr^2=2f+h$ $r^2(4+gh)=2f+h$ $r^2=\frac{2f+h}{4+gh}$ $r=\pm\sqrt{\frac{2f+h}{4+gh}}$	M1 A1	
	(c)	$\frac{(3x-5)}{2(x-2)}$	$\frac{3x^2+x-10}{2x^2-8}$ $=\frac{(3x-5)(x+2)}{2(x^2-4)}$ $=\frac{(3x-5)(x+2)}{2(x+2)(x-2)}$ $=\frac{(3x-5)}{2(x-2)}$	M1 A1	*1m for correct expansion of either numerator or denominator.
	(d)	$y=1.70$ or -2.57	$\frac{1}{y-1}-\frac{2}{5}=\frac{7}{4y}$ $\frac{5-2(y-1)}{5(y-1)}=\frac{7}{4y}$ $\frac{7-2y}{5y-5}=\frac{7}{4y}$ $35y-35=28y-8y^2$ $8y^2+7y-35=0$ $y=\frac{-7\pm\sqrt{7^2-4(8)(-35)}}{2(8)}$ $y=\frac{-7\pm\sqrt{1169}}{16}$ $y=1.70$ or $y=-2.57$	M1 M1 A1	
	(e)	$\$\frac{19}{12}m$	m cupcakes cost $\$\frac{5}{3}m$ Price after discount $=\frac{95}{100}\times\frac{5}{3}m$ $=\$\frac{19}{12}m$	M1 A1	Accept $\$1\frac{7}{12}m$

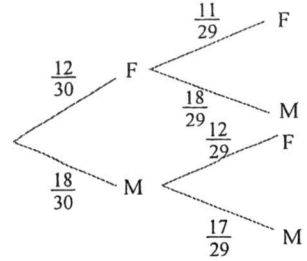
No.		Answer	Workings	Marks	*Remarks
2	(a)	$\frac{10}{x}$h	$\frac{10}{x}$h	B1	
	(b)	$x^2+x-20=0$ (shown)	Time taken for return journey $=\frac{10}{x+1}$h $\frac{10}{x}-\frac{10}{x+1}=\frac{1}{2}$ $\frac{10x+10-10x}{x(x+1)}=\frac{1}{2}$ $x^2+x=20$ $x^2+x-20=0$ (shown)	M1 M1 M1 A1	
	(c)	$x=4$ or $x=-5$	$x^2+x-20=0$ $(x-4)(x+5)=0$ $x=4$ or $x=-5$	M1 A2	*Deduct 1m if student rejects $x=-5$.
	(d)	75 min	Time taken for Jeria's journey to waterfall $=\frac{10}{4}=2.5$ h Time taken for Marianne's journey to waterfall $=\frac{2.5}{2}\times 3$ $=3.75$ h $=225$ min Maximum time for return journey $=300-225=75$ min	M1 A1	
3	(a)	$T_5=21+23+25+27+29=125$ $T_6=31+33+35+37+39+41=216$		B1 B1	
	(b)	Perfect Cubes		B1	
	(c)	n		B1	
	(d)	n^3		B1	
	(e)	When n is odd, n^2 will be the product of two odd numbers, which will be odd. $n^3=n^2\times n$ = odd x odd = odd. When n is even, n^2 will be the product of two even numbers, which will be even. $n^3=n^2\times n$ = even x even = even.		B1 B1	Or equivalent reasonings.

No.		Answer	Workings	Marks	*Remarks
4	(a) (i)	972π cm^3	Volume of cylinder $= \pi(9)^2(18)$ $= 1458\pi$ Volume of hemisphere $= \frac{2}{3}\pi(9)^3$ $= 486\pi$ Volume of water $= \frac{1}{2}(1458\pi + 486\pi)$ $= 972\pi$ cm^3	M1 M1 A1	
	(a) (ii)	$d = 6$ cm (shown)	Volume of cylinder $= 972\pi - 486\pi$ $= 486\pi$ cm^3 $\pi(9)^2 h = 486\pi$ $h = 6$ cm $d = 6 + 9 = 15$ cm (shown)	M1 M1 A1	
	(a) (iii)	848 cm^2	Total surface area in contact with water $= 2\pi(9)^2 + 2\pi(9)(6)$ $= 848.23$ $= 848$ cm^2	M1 A1	*1m for either curved surface area of cylinder or hemisphere
	(b) (i)	96π cm^2	area of sector AOB $= \frac{1}{2}(24)^2(\frac{\pi}{3})$ $= 96\pi$ cm^2	B1	*In degree: $\pi(24)^2(\frac{60^\circ}{360^\circ})$ $= 96\pi$ cm^2
	(b) (ii)	4 cm (shown)	Circumference = arc length of sector $= 24(\frac{\pi}{3})$ [or in degree] $= 8\pi$ cm $2\pi r = 8\pi$ $r = 4$ cm	M1 A1	Alternatively, $\pi r l = 96\pi$ $\pi(r)(24) = 96\pi$ $r = 4$ cm
	(b) (iii)	586 cm^3	Height of cone $= \sqrt{24^2 - 4^2}$ $= \sqrt{560}$ cm Volume of cone $= \frac{1}{3}\pi(\sqrt{560})(4)^2$ $= 396$ cm^3	M1 A1	*Accept 397 when using $\pi = 3.142$.

No.		Answer	Workings	Marks	*Remarks
5	(a) (i)	40°	$\angle AOB = 50^\circ \times 2 = 100^\circ$ ($\angle$ at centre $= 2\angle$ at circumference) $\angle ABO = (180^\circ - 100^\circ) \div 2 = 40^\circ$ (base $\angle$ of isos triangle)	M1 A1	
	(a) (ii)	12°	$\angle OAC = 90^\circ - 38^\circ - 40^\circ = 12^\circ$ ($\angle$ in a semicircle)	B1	Accept alternative method
	(a) (iii)	40°	$\angle SAD = 90^\circ - 38^\circ - 12^\circ = 40^\circ$ (tan $\perp$ rad)	B1	Accept alternative method
	(a) (iv)	88°	$\angle CBE = 38^\circ$ ($\angle$ in same segment) $\angle CED = 38^\circ + 50^\circ = 88^\circ$ (ext angle of triangle)	M1 A1	Accept alternative method
	(b) (i)	19.6 cm	$\cos 40^\circ = \frac{15}{DB}$ $DB = 19.581$ cm $= 19.6$ cm	M1 A1	
	(b) (ii)	47.2 cm^2	Radius $= 19.581 \div 2 = 9.7905$ Area of AOB $= \frac{1}{2} \times (9.7905)^2 \times \sin 100^\circ$ $= 47.2$ cm^2	M1 A1	

No.		Answer	Workings	Marks	*Remarks
6	(a)	$q = 4.17$		B1	
	(b)	(as attached) 1m – correct plotting of point 1m – correct scale 1m – smooth curve		B3	
	(c)	Gradient $= \frac{4.65 - 1.2}{2 - 1}$ $= 3.9 \pm 0.5$ (Accept values from 3 to 4) *sample a few scripts before confirming the acceptable range.*		M1 A1	*1m – drawing of line *1m – correct value
	(d)	$-\frac{x^2}{3} - \frac{11}{x} + 12 = x$ $y + 1 = x$ Draw $y = x - 1$ Solutions: $x = 1.05$ or $x = 3.95$ (Accept values ± 0.1)		M1 A1	*M1 – drawing of line $y = x - 1$ (not accepted if equation is found but line is not drawn)
	(e)	(2.55, 4.5) (Accept values ± 0.1)		B2	*1m each for x and y coordinates *deduct 1m for non coordinates form answer

No.		Answer	Workings	Marks	*Remarks
7	(a)	$\mathbf{T} = \begin{pmatrix} 135 & 120 & 100 & 105 \\ 150 & 140 & 125 & 85 \end{pmatrix}$		B1	
	(b)	$\mathbf{S} = \begin{pmatrix} 228 \\ 168 \\ 128 \\ 98 \end{pmatrix}$		B1	
	(c)	$\mathbf{K} = \begin{pmatrix} 74030 \\ 82050 \end{pmatrix}$	$\mathbf{K} = \mathbf{TS}$ $= \begin{pmatrix} 135 & 120 & 100 & 105 \\ 150 & 140 & 125 & 85 \end{pmatrix} \begin{pmatrix} 228 \\ 168 \\ 128 \\ 98 \end{pmatrix}$ $= \begin{pmatrix} 74030 \\ 82050 \end{pmatrix}$	B2	B1 – 74030 B1 – 82050
	(d)	**K** represents the <u>total ticket sales</u> for all categories on Friday and Saturday <u>respectively</u>.		B1	
	(e)	\$100873.50	Prices of tickets of each category on Sunday $= 1.05 \begin{pmatrix} 228 \\ 168 \\ 128 \\ 98 \end{pmatrix}$ $= \begin{pmatrix} 239.40 \\ 176.40 \\ 134.40 \\ 102.90 \end{pmatrix}$ Total sales revenue $= \begin{pmatrix} 180 & 200 & 110 & 75 \end{pmatrix} \begin{pmatrix} 239.40 \\ 176.40 \\ 134.40 \\ 102.90 \end{pmatrix}$ $= (100873.50)$ Total sales revenue = \$100873.50	M1 M1^ A1	*0m if no matrix multiplication is used. *M2 can be awarded if at least 1 matrix multiplication is used. *Not to penalize for 1d.p. within matrix ^can accept: $1.05\begin{pmatrix} 228 & 168 & 128 & 98 \end{pmatrix}\begin{pmatrix} 180 \\ 200 \\ 110 \\ 75 \end{pmatrix}$ or $1.05\begin{pmatrix} 180 & 200 & 110 & 75 \end{pmatrix}\begin{pmatrix} 228 \\ 168 \\ 128 \\ 98 \end{pmatrix}$ or find $\begin{pmatrix} 180 & 200 & 110 & 75 \end{pmatrix}\begin{pmatrix} 228 \\ 168 \\ 128 \\ 98 \end{pmatrix}$ then $\times 1.05$ etc.

No.		Answer	Workings	Marks	*Remarks
8	(a) (i)			B2	Deduct 1m for each wrong probability. Do not penalize marks if fractions are not reduced to simplest form.
	(a) (ii)	$\frac{72}{145}$	P(one female one male) = P(F, M) + P(M, F) $= \frac{2}{5} \times \frac{18}{29} + \frac{3}{5} \times \frac{12}{29}$ $= \frac{72}{145}$	M1 A1	
	(a) (iii)	$\frac{11}{203}$	P(F, F, F) $= \frac{2}{5} \times \frac{11}{29} \times \frac{10}{28}$ $= \frac{11}{203}$	M1 A1	
	(b) (i)	$62 \le x \le 69$		B1	
	(b) (ii)	40%	$\frac{6}{15} \times 100 = 40\%$	B1	
	(b) (iii) (a)	13.9	SD $= \sqrt{\frac{\sum fx^2}{\sum f} - \left(\frac{\sum fx}{\sum f}\right)^2}$ $= \sqrt{\frac{68246}{15} - \left(\frac{990}{15}\right)^2}$ = 13.9 fishes (3 sf)	B1	
	(b) (iii) (b)	There is an <u>outlier in the data</u> (99 fish). Hence the mean is <u>not a good indicator</u> of average in this distribution.		M1 A1	1m – yes / no 1m – reason *0 m given if no / wrong reason is given.
	(b) (iii) (c)	The <u>mean is higher</u> in Town Hona, hence the fishermen in Town Hona caught <u>more fishes</u> in a day. or The <u>standard deviation is lower</u> in Town Hona, hence the number of fishes caught by the fishermen in Town Hona is <u>more consistent</u>.		B1 √	Follow through their SD in part (b)(iii)(a)

No.		Answer	Workings	Marks	*Remarks
9	(a)	1.40 km	$\angle QMP = 48^\circ$ By Cosine Rule $QP = \sqrt{1.25^2 + 1.88^2 - 2\times1.25\times1.88\times\cos 48^\circ}$ $= 1.39713$ km $= 1.40$ km	M1 A1	
	(b)	030.3°	By Sine Rule $\frac{1.25}{\sin QPM} = \frac{1.39713}{\sin 48^\circ}$ $\angle QPM = 41.6735^\circ = 41.7^\circ$ Bearing of Q from P $= 72^\circ - 41.6735^\circ$ $= 30.3265^\circ$ $= 030.3^\circ$	M1 A1	
	(c)	0.929 km	$\sin 48^\circ = \frac{QX}{1.25}$ Shortest distance QX $= 0.92893$ km $= 0.929$ km	M1 A1	
	(d) (i)	45.6°	$\tan\theta = \frac{0.95}{0.92893}$ $\theta = 45.6^\circ$	M1 A1	
	(d) (ii)		*X is the point of shortest distance from Q to PM.* Distance of PX (1.044 km) is longer than the distance of XM (0.833 km). Hence, angle of elevation from Point P to the skydiver is smaller.	M1 A1	1m – reason 1m – Point P *Award 0 m if only calculations given without any explanations or justifications

No.		Answer	Workings	Marks	*Remarks
10	(a)	$13000	Maximum estimated annual cost of living $7000 + $2600 + $2200 + $400 + $800 = $13000	B1	
	(b)	15 hours	Annual expenses that he has to pay on his own = $13000 – $800 × 12 = $3400 Per month = $3400 ÷ 12 = $283.333 Maximum number of hours = $283.333 ÷ 20 = 14.167 ≈ 15 hours	M1 M1 A1	*Accept reasonable calculations continued from part (a)
	(c)		Total Tuition fee (4 years) = $8050 × 4 = $32200 Including 2% processing fee = $32200 × $\frac{102}{100}$ = $32844 **5 year loan** Interest incurred $I = \frac{PRT}{100} = \frac{32844\times4.6\times5}{100} = \$7554.12$ Total amount he has to pay = $7554.12 + $32844 = $40398.12 Amount per month = $40398.12 ÷ 5 ÷ 12 = $673.302 } Not needed if reasoning is (1) **10 year loan** Interest incurred $I = \frac{PRT}{100} = \frac{32844\times4.6\times10}{100} = \$15108.24$ Total amount he has to pay = $15108.24 + $32844 = $47952.24 Amount per month = $47952.24 ÷ 10 ÷ 12 = $399.602 } Not needed if reasoning is (1) **Conclusion:** (1) Jerel will pay lesser per month if he goes for a 10-year loan. However, the total interest for a 5-year loan is $7554.12 lower than that of a 10-year loan. It will be more practical for him to go for a 5-year loan. **Also accept:** (2) Although the total interest for a 10-year loan is $7554.12 higher than that of a 5-year loan, he gets to pay a comfortable amount of $400 a month for 10 years. It will be more practical for him to go for a 10-year loan.	M1 M1 M1 M1 M1 A1	o.e.

Name:	Class	Index Number

Jurong West Secondary School

Mid-Year Examinations 2017

80

MATHEMATICS 4048/01

Secondary Four Express/ Five Normal Academic
Paper 1

4 May 2017
0800 – 1000
2 hours

Candidates answer on the Question Paper.

READ THESE INSTRUCTIONS FIRST

Write your name, class and index number on all the work you hand in.
Write in dark blue or black pen.
You may use an HB pencil for any diagrams or graphs.
Do not use staples, paper clips, glue or correction fluid.

Answer **all** questions.
If working is needed for any question it must be shown with the answer.
Omission of essential working will result in loss of marks.
The use of an approved scientific calculator is expected, where appropriate.
If the degree of accuracy is not specified in the question, and if the answer is not exact, give the answer to three significant figures. Give answers in degrees to one decimal place.
For π, use either your calculator value or 3.142, unless the question requires the answer in terms of π.

At the end of the examination, fasten all your work securely together.
The number of marks is given in brackets [ ] at the end of each question or part question.
The total of the marks for this paper is 80.

After checking of answer script		
Checked by Student	**Signature**	**Date**

This document consists of 23 printed pages.

Mathematical Formulae

Compound interest

$$\text{Total amount} = P\left(1+\frac{r}{100}\right)^n$$

Mensuration

Curved surface area of a cone $= \pi rl$

Surface area of a sphere $= 4\pi r^2$

$$\text{Volume of a cone} = \frac{1}{3}\pi r^2 h$$

$$\text{Volume of a sphere} = \frac{4}{3}\pi r^3$$

$$\text{Area of triangle } ABC = \frac{1}{2}ab\sin C$$

Arc length $= r\theta$, where θ is in radians

Sector area $= \frac{1}{2}r^2\theta$, where θ is in radians

Trigonometry

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$a^2 = b^2 + c^2 - 2bc\cos A$$

Statistics

$$\text{Mean} = \frac{\sum fx}{\sum f}$$

$$\text{Standard deviation} = \sqrt{\frac{\sum fx^2}{\sum f} - \left(\frac{\sum fx}{\sum f}\right)^2}$$

Answer **all** questions.

1. Factorise completely $4ax - 3ay - 16x + 12y$.

Answer... ... ... [2]

---

2. Solve $(3x-4)(x-2)=1$.

Answer x = ... or ..... [2]

---

3. Mr Heng bought a painting for \$3 500. Some years later he sold it for a profit of 150% on the price he paid. Find the selling price.

Answer \$ ..... [2]

---

4. The temperature of a pie was $-4^{\circ}C$ when taken out of the freezer. The pie was immediately heated up, and after 8 minutes its temperature was $16^{\circ}C$.

(a) By how much did the temperature increase during the 8 minutes?

Answer (a) ..... $^{\circ}C$ [1]

(b) Given that the temperature of the pie increased at a constant rate, calculate the number of minutes it has been warmed when its temperature reached $2^{\circ}C$?

Answer (b) ..... minutes [1]

---

5. Miss Gan invested \$7 500 in a saving account for 6 years. The rate of compound interest was fixed at 3% per annum compounded half yearly. Calculate the amount of interest Miss Gan earned at the end of 6 years.

Answer \$ ..... [2]

---

6. Highlighters are sold in packs of three at a cost of x cents per pack. Find an expression for the number of highlighters that can be bought for y dollars.

Answer..... [2]

7. Simplify

(a) $5(2x+y)-4(3x-2y)$,

Answer (a) ..... [1]

(b) $\left(\frac{x^{12}}{81}\right)^{-\frac{1}{4}}$

Answer (b) ..... [2]

8. **(a)** Express $x^2-6x+13$ in the form $(x-p)^2+q$.

Answer (a) ..... [2]

(b) Write down the coordinates of the minimum point of the graph $y=x^2-6x+13$.

Answer (b) (..... , ...) [1]

9. **(a)** Solve the inequality $-6<2x+7\le 7$.

Answer (a) ..... [2]

(b) Hence write down smallest integer value of x which satisfies $-6<2x+7\le 7$.

Answer (b) ..... [1]

10. 30 students took a Mathematics test. The mean mark was 13.2 and the median was 14.

An extra student took the test late. The new mean was 13.

(a) What mark did the extra student get?

Answer (a) ..... [2]

(b) Complete the sentence below with the correct phrase from the list.

The new median ...

is definitely smaller than 14.
is definitely bigger than 14.
might be smaller than 14 but might still be 14.
might be bigger than 14 but might still be 14.
is still 14.

Answer (b) The new median .....

..... [1]

11. P is the point $(3, 4)$. Q is the point $(-5, 1)$.

(a) Write down the column vector $\overrightarrow{PQ}$.

Answer (a) ..... [1]

(b) Find $\left|\overrightarrow{PQ}\right|$.

Answer (b) ..... [2]

12. $\xi = \{\text{integers } x: 3 \leq x \leq 11\}$

$A = \{\text{factors of } 24\}$

$B = \{\text{prime numbers}\}$

(a) Draw a Venn diagram to illustrate this information. [2]

Answer (a)

(b) Write down $n(A \cup B)$.

Answer (b) ..... [1]

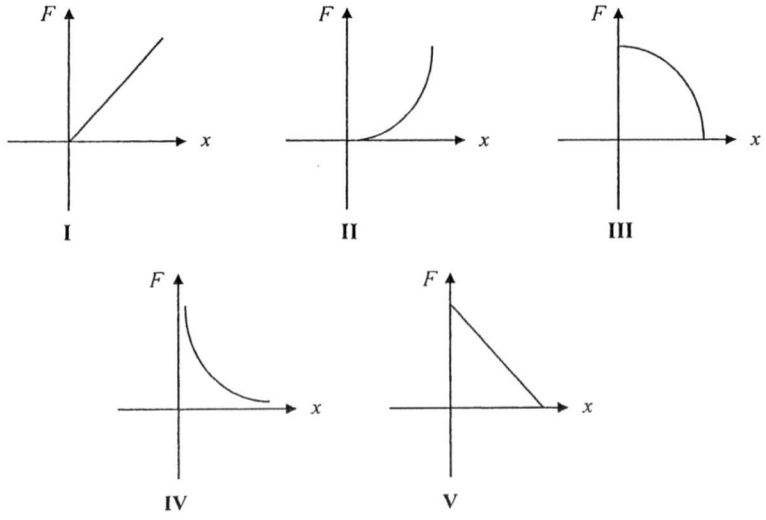
(c) List the elements contained in the set $A' \cap B'$.

Answer (c) ..... [1]

---

13. The force of attraction, F newtons, between two magnets is inversely proportional to the square of the distance, x centimetres, between them.

(a) Which one of the following sketch graphs below represents the relationship between the force of attraction and the distance?



Answer (a) Graph ..... [1]

(b) When the magnets are 4 cm apart, the force is 3 newtons. Find the equation connecting F and x.

Answer (b) ..... [2]

(c) Find the distance between the two magnets when the force is 0.75 newtons.

Answer (c) ..... cm [1]

---

14. The actual perimeter of a reservoir is 37 km. The perimeter of the reservoir is represented by a length of 185 cm on the map.

(a) Write down the scale of the map in the form of $1 : n$.

Answer (a) 1 : ..... [1]

(b) A plantation has an actual area of 3.2 km^2. Find the area, in square centimeters, of the plantation on the map.

Answer (b) ..... cm^2 [2]

---

15. When written as product of their prime factors,

$$P = 2^3 \times 3^6$$

$$Q = 2^3 \times 3 \times 5,$$

$$R = 2^2 \times 3^2 \times 5.$$

(a) Find the value of the cube root of P.

Answer (a) ..... [1]

(b) Given that the HCF and LCM of Q, R and S is 60 and 2520 respectively, find the smallest possible value of S.

Answer (b) ..... [2]

16. Two containers are geometrically similar. The larger container has a capacity of 1.08 litres and the smaller container has a capacity of 0.32 litres. The height of the larger container is 18 cm.

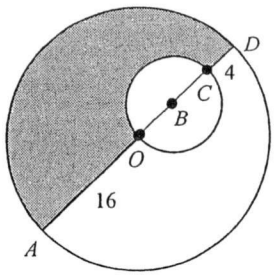
(a) Calculate the height of the smaller container.

Answer (a) ..... cm [2]

(b) The ratio surface area of larger container : surface area of smaller container can be written in the form k : 1. Find the value of k.

Answer (b) k =..... [2]

17.



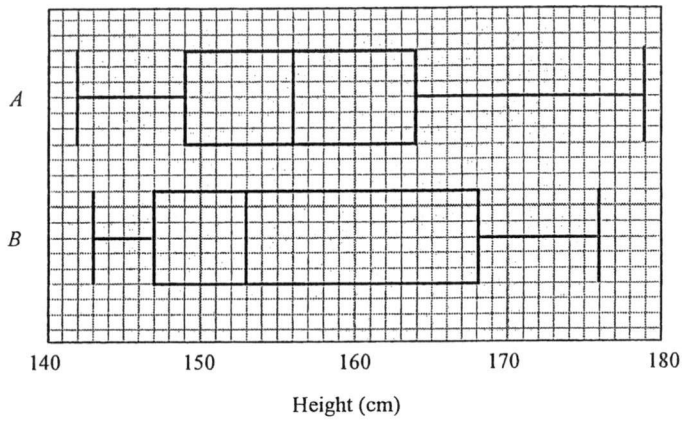
(a) In the diagram, the big circle, centre O, has a radius of 16 cm. The small circle, centre B, has OC as diameter. $AOBCD$ is a straight line with $CD = 4$ cm. Find the perimeter of the shaded region, leaving your answer in terms of π.

Answer (a) ..... cm [2]

(b) A point is chosen, at random within the big circle. Find the probability that this point is in the shaded region.

Answer (b) ..... [2]

18. These box plots show the distributions of the heights of students in two schools A and B.



(a) Find the median for school A.

Answer (a) ..... cm [1]

(b) Find the interquartile range for school B.

Answer (b) ..... cm [1]

(c) Here are two statements comparing the heights for the two schools.
For each one, write whether you agree or disagree. Give a reason for each answer.

(i) On average, students from school A is taller than students from school B.

Answer (c)(i) ..... because .....

.....

..... [1]

(ii) A greater proportion of the students has height of more than 165 cm in school *A* than in school *B*.

Answer (c)(ii)..... because .....

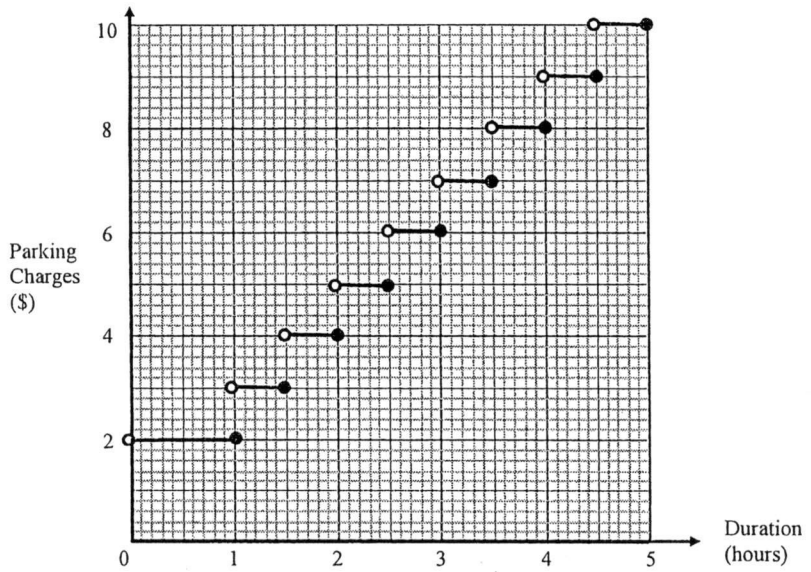
.....

..... [1]

---

19. The step-function graph below shows the parking charges for the first 5 hours at Carpark *P*.

Answer (b)(i)



(a) Find the parking charges for a car that is parked at Carpark *P* for

(i) $1\frac{1}{2}$ hours,

Answer (a)(i) $ ..... [1]

(ii) 3 hours 10 minutes.

Answer (a)(ii) $ ..... [1]

(b) Another nearby carpark, Carpark *Q* offers the parking charges as shown below.

$2.60 upon entry.
2 cents per minute thereafter
(Maximum charges of $8)

Mr Chan wishes to park his car at one of these carparks for $2\frac{1}{2}$ hours.

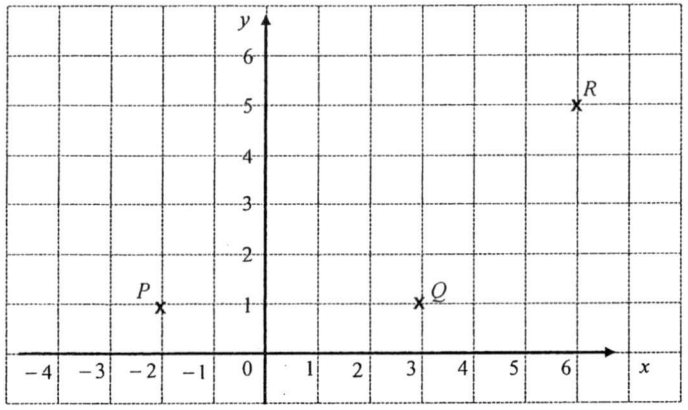
(i) On the same axes, draw the graph of parking charges at Carpark *Q*. [1]

(ii) State the carpark Mr Chan should choose to park his car.

Answer (b)(ii) ..... [1]

---

20. The diagram shows three points $P(-2, 1)$, $Q(3, 1)$ and $R(6, 5)$.



(a) Find the gradient of PR.

Answer (a) ..... [1]

(b) Find the area of triangle PQR.

Answer (b) ..... [1]

(c) Find the value of $\tan \angle QPR$.

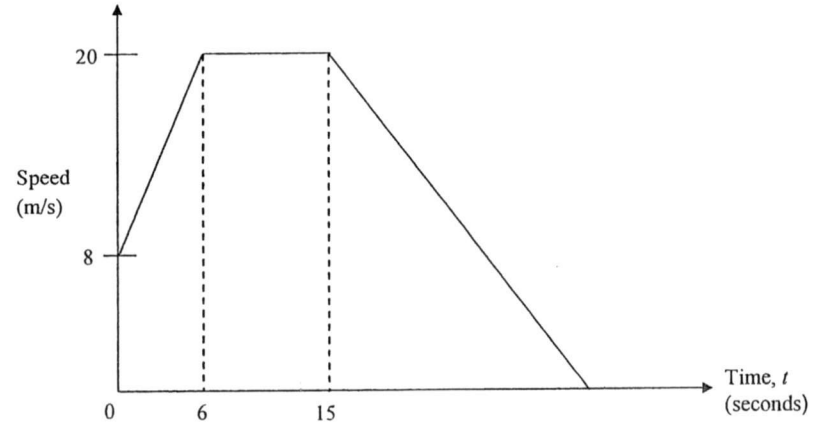
Answer (c) ..... [1]

(d) $PQRS$ is a trapezium with PQ parallel to SR. The area of the trapezium is 28 units^2. Find the coordinates of S.

Answer (d) S (..... , ...) [2]

---

21. The diagram represents the speed-time graph of a moving object.



(a) Calculate the speed of the object when $t = 4$.

Answer (a) ..... m/s [2]

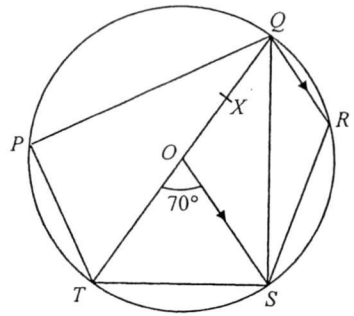
(b) Calculate the distance travelled in the first 15 seconds.

Answer (b) ..... m [2]

(c) Given that the rate at which the object slows down after $t = 15$ is equal to half the rate at which it accelerates during the first 6 seconds, calculate the time at which it stops.

Answer (c) ..... s [2]

22. A circle, centre O, passes through the points P, Q, R, S and T. QT is the diameter of the circle, angle $SOT = 70°$ and QR is parallel to OS.



(a) Find

(i) angle OQS,

Answer (a)(i) angle OQS =..... [1]

(ii) angle OTS,

Answer (a)(ii) angle OTS =..... [1]

(iii) angle QRS,

Answer (a)(iii) angle QRS =..... [1]

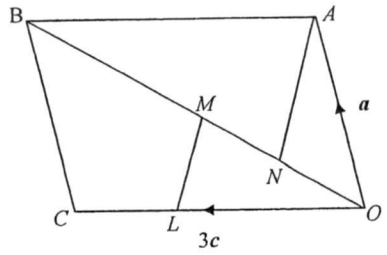
(iv) angle QSR,

Answer (a)(ii) angle QSR =..... [1]

(b) X is the point on QT such that $QX = \frac{1}{4}QT$. Given that the area of triangle PQT is 90 cm^2, calculate the area of PXT.

Answer (b) ..... cm^2 [2]

23.



$OABC$ is a parallelogram. M is the midpoint of OB, N is the point on OM such that $OM = 2ON$ and L is the point on OC such that $OL = 2LC$. $\overrightarrow{OA} = \boldsymbol{a}$, $\overrightarrow{OC} = 3\boldsymbol{c}$ and $\overrightarrow{ML} = -\frac{1}{2}\boldsymbol{a} + \frac{1}{2}\boldsymbol{c}$.

(a) Write each of the following in terms of $\boldsymbol{a}$ and $\boldsymbol{c}$. Give your answers in the simplest form.

(i) $\overrightarrow{OB}$

Answer (a)(i) ..... [1]

(ii) $\overrightarrow{ON}$

Answer (a)(ii) ..... [1]

(iii) $\overrightarrow{AN}$

Answer (a)(iii) ..... [2]

(b) Explain why ML is parallel to AN.

Answer (b) .....

.....

..... [1]

(c) Find the ratio $AN : ML$.

Answer (c) ..... [1]

End of Paper

Answer **all** questions.

1. Factorise completely $4ax - 3ay - 16x + 12y$.

$4ax - 3ay - 16x + 12y = a(4x - 3y) - 4(4x - 3y)$ M1

$= (a - 4)(4x - 3y)$ A1

Answer..... [2]

---

2. Solve $(3x - 4)(x - 2) = 1$.

$(3x - 4)(x - 2) = 1$

$3x^2 - 6x - 4x + 8 = 1$ M1

$3x^2 - 10x + 7 = 0$

$(3x - 7)(x - 1) = 0$

either $3x - 7 = 0$ or $x - 1 = 0$

$x = 2\frac{1}{3}$ $x = 1$ A1

Answer $x =$ ..... or ..... [2]

---

3. Mr Heng bought a painting for \$3 500. Some years later he sold it for a profit of 150% on the price he paid. Find the selling price.

Selling price $= \frac{250 \times 3500}{100}$ M1

$= \$8750$ A1

Answer \$ ..... [2]

---

4. The temperature of a pie was $-4^{\circ}C$ when taken out of the freezer. The pie was immediately heated up, and after 8 minutes its temperature was $16^{\circ}C$.

(a) By how much did the temperature increase during the 8 minutes?

Increase in temperature $= 16 - (-4) = 20^{\circ}C$ B1

Answer (a) ..... $^{\circ}C$ [1]

(b) Given that the temperature of the pie increased at a constant rate, calculate the number of minutes it has been warmed when its temperature reached $2^{\circ}C$?

$20^{\circ}C$ take 8 minutes

$6^{\circ}C$ take $\frac{6 \times 8}{20} = 2.4$ minutes B1

Answer (b) ..... minutes [1]

---

5. Miss Gan invested \$7 500 in a saving account for 6 years. The rate of compound interest was fixed at 3% per annum compounded half yearly. Calculate the amount of interest Miss Gan earned at the end of 6 years.

Total amount $= 7500\left(1 + \frac{1.5}{100}\right)^{12}$ M1

$\approx \$8967.14$

Interest $= 8967.136286 - 7500$

$\approx \$1467.14$ A1

Answer \$ ..... [2]

---

6. Highlighters are sold in packs of three at a cost of x cents per pack. Find an expression for the number of highlighters that can be bought for y dollars.

3 highlighters cost x cents

1 highlighter costs $\frac{x}{3}$ cents

Number of highlighters that can be bought $= 100y \div \frac{x}{3}$ } M1 for either method seen

$= \frac{300y}{x}$ A1

Answer..... [2]

---

7. Simplify

(a) $5(2x+y)-4(3x-2y)$,

$5(2x+y)-4(3x-2y) = 10x+5y-12x+8y$

$= -2x+13y$ B1

Answer (a) ..... [1]

(b) $\left(\frac{x^{12}}{81}\right)^{-\frac{1}{4}}$

$\left(\frac{x^{12}}{81}\right)^{-\frac{1}{4}} = \left(\frac{81}{x^{12}}\right)^{\frac{1}{4}}$ M1 or $\left(\frac{x^{12}}{81}\right)^{-\frac{1}{4}} = \frac{x^{-3}}{\frac{1}{3}}$ M1

$= \frac{3}{x^3}$ A1 $= \frac{3}{x^3}$ A1

Answer (b) ..... [2]

---

8. **(a)** Express $x^2-6x+13$ in the form $(x-p)^2+q$.

$x^2-6x+13 = x^2-6x+\left(-\frac{6}{2}\right)^2-\left(-\frac{6}{2}\right)^2+13$

$= (x-3)^2+4$ B1 B1 for correct p and q

Answer (a) ..... [2]

(b) Write down the coordinates of the minimum point of the graph $y = x^2-6x+13$.

Coordinates of minimum point $= (3, 4)$ B1

Answer (b) (..... , ...) [1]

---

9. **(a)** Solve the inequality $-6 < 2x+7 \le 7$.

$-6 < 2x+7$ and $2x+7 \le 7$

$2x > -13$ $2x \le 0$

$x > -6\frac{1}{2}$ $x \le 0$ B1 for either $x > -6\frac{1}{2}$ or $x \le 0$ seen

$\therefore \ -6\frac{1}{2} < x \le 0$ B1

Answer (a) ..... [2]

(b) Hence write down smallest integer value of x which satisfies $-6 < 2x+7 \le 7$.

$x = -6$ B1

Answer (b) ..... [1]

---

10. 30 students took a Mathematics test. The mean mark was 13.2 and the median was 14.

An extra student took the test late. The new mean was 13.

(a) What mark did the extra student get?

Mark $= 31 \times 13 - 30 \times 13.2$ M1

$= 403 - 396$

$= 7$ A1

Answer (a) ..... [2]

(b) Complete the sentence below with the correct phrase from the list.

The new median ...

is definitely smaller than 14.
is definitely bigger than 14.
might be smaller than 14 but might still be 14.
might be bigger than 14 but might still be 14.
is still 14.

Answer (b) The new median might be smaller than 14 but might still be 14. B1

11. P is the point $(3, 4)$. Q is the point $(-5, 1)$.

(a) Write down the column vector $\overrightarrow{PQ}$.

$\overrightarrow{PQ} = \overrightarrow{OQ} - \overrightarrow{OP}$ or $\overrightarrow{PQ} = \overrightarrow{PO} + \overrightarrow{OQ}$

$= \begin{pmatrix} -5 \\ 1 \end{pmatrix} - \begin{pmatrix} 3 \\ 4 \end{pmatrix}$ $= \begin{pmatrix} -3 \\ -4 \end{pmatrix} + \begin{pmatrix} -5 \\ 1 \end{pmatrix}$

$= \begin{pmatrix} -8 \\ -3 \end{pmatrix}$ B1 $= \begin{pmatrix} -8 \\ -3 \end{pmatrix}$ B1

Answer (a) ..... [1]

(b) Find $\left|\overrightarrow{PQ}\right|$.

$\left|\overrightarrow{PQ}\right| = \sqrt{(-8)^2 + (-3)^2}$ M1

$= \sqrt{73}$

≈ 8.54 units A1

Answer (b) ..... [2]

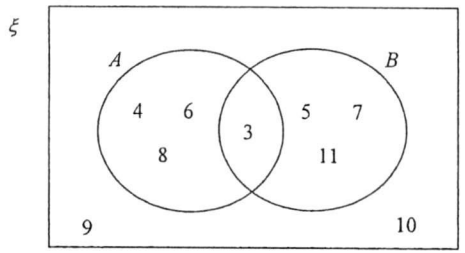
12. $\xi = \{\text{integers } x: 3 \le x \le 11\}$

$A = \{\text{factors of } 24\}$

$B = \{\text{prime numbers}\}$

(a) Draw a Venn diagram to illustrate this information. [2]

Answer (a)



B1 B1 for any two out of four groups of numbers correct

(b) Write down $\text{n}(A \cup B)$.

$\text{n}(A \cup B) = 7$ B1

Answer (b) ..... [1]

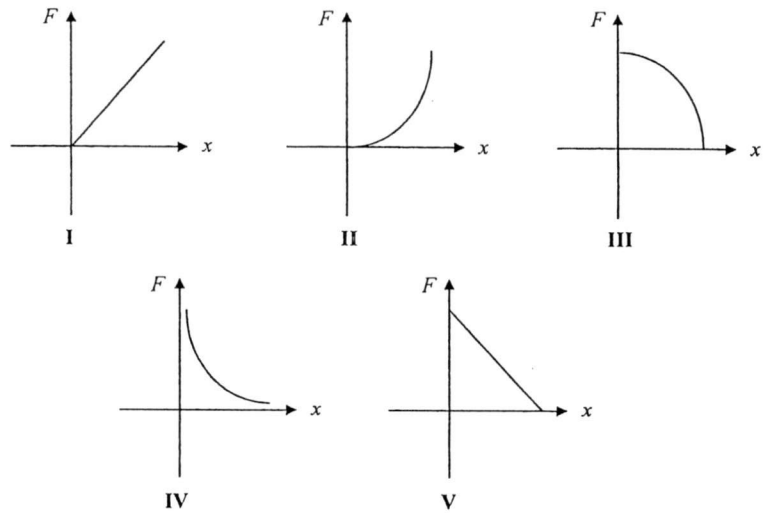
(c) List the elements contained in the set $A' \cap B'$.

$A' \cap B' = \{9, 10\}$ B1

Answer (c) ..... [1]

13. The force of attraction, F newtons, between two magnets is inversely proportional to the square of the distance, x centimetres, between them.

(a) Which one of the following sketch graphs below represents the relationship between the force of attraction and the distance?



Graph IV B1

Answer (a) Graph ..... [1]

(b) When the magnets are 4 cm apart, the force is 3 newtons. Find the equation connecting F and x.

$F = \frac{k}{x^2}$ M1

$3 = \frac{k}{4^2}$

$k = 48$

$F = \frac{48}{x^2}$ A1

Answer (b) ..... [2]

(c) Find the distance between the two magnets when the force is 0.75 newtons.

$F = \frac{48}{x^2}$

$0.75 = \frac{48}{x^2}$

$x^2 = \frac{48}{0.75}$

$x = 8$ cm or $x = -8$ (N.A.) B1 for $x = 8$

Answer (c) ..... cm [1]

---

14. The actual perimeter of a reservoir is 37 km. The perimeter of the reservoir is represented by a length of 185 cm on the map.

(a) Write down the scale of the map in the form of $1 : n$.

185 cm : 37 km

185 cm : 3700000 cm

1 : 20000 B1

Answer (a) 1 : ..... [1]

(b) A plantation has an actual area of 3.2 km^2. Find the area, in square centimeters, of the plantation on the map.

185 cm : 37 km

1 cm : 0.2 km

1 cm^2 : 0.04 km^2 M1

Area on the map $= \frac{3.2}{0.04}$

$= 80$ cm^2 A1

Answer (b) ..... cm^2 [2]

---

15. When written as product of their prime factors,

$P = 2^3 \times 3^6$

$Q = 2^3 \times 3 \times 5$,

$R = 2^2 \times 3^2 \times 5$.

(a) Find the value of the cube root of P.

$P = 2^3 \times 3^6$

$= (2 \times 3^2)^3$

$\sqrt[3]{P} = \sqrt[3]{(2 \times 3^2)^3}$

$= 18$ B1

Answer (a) ..... [1]

(b) Given that the HCF and LCM of Q, R and S is 60 and 2520 respectively, find the smallest possible value of S.

HCF $= 60 = 2^2 \times 3 \times 5$

LCM $= 2520 = 2^3 \times 3^2 \times 5 \times 7$ M1 for either $60 = 2^2 \times 3 \times 5$ or $2520 = 2^3 \times 3^2 \times 5 \times 7$

$S = 2^2 \times 3 \times 5 \times 7$

$= 420$ A1

Answer (b) ..... [2]

---

16. Two containers are geometrically similar. The larger container has a capacity of 1.08 litres and the smaller container has a capacity of 0.32 litres. The height of the larger container is 18 cm.

(a) Calculate the height of the smaller container.

$\left(\frac{h}{18}\right)^3 = \frac{0.32}{1.08}$ M1

$\left(\frac{h}{18}\right)^3 = \frac{8}{27}$

$\frac{h}{18} = \frac{2}{3}$

$h = 12$ cm A1

Answer (a) ..... cm [2]

(b) The ratio surface area of larger container : surface area of smaller container can be written in the form $k : 1$. Find the value of k.

$\left(\frac{3}{2}\right)^2 = \frac{9}{4}$ M1

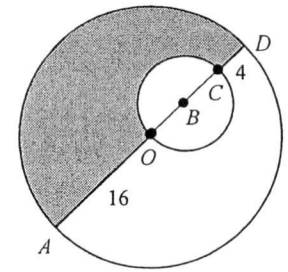
$= \frac{9}{\frac{4}{1}}$

$\therefore$ Ratio $= 2.25 : 1$

$\therefore$ $k = 2.25$ A1

Answer (b) $k =$..... [2]

17.



(a) In the diagram, the big circle, centre O, has a radius of 16 cm. The small circle, centre B, has OC as diameter. $AOBCD$ is a straight line with $CD = 4$ cm. Find the perimeter of the shaded region, leaving your answer in terms of π.

Perimeter of shaded region

$= \frac{1}{2}(2)(\pi)(16) + \frac{1}{2}(2)(\pi)(6) + 16 + 4$ M1

$= 16\pi + 6\pi + 20$

$= 22\pi + 20$ cm A1

Answer (a) ..... cm [2]

(b) A point is chosen, at random within the big circle. Find the probability that this point is in the shaded region.

P (point is in the shaded region)

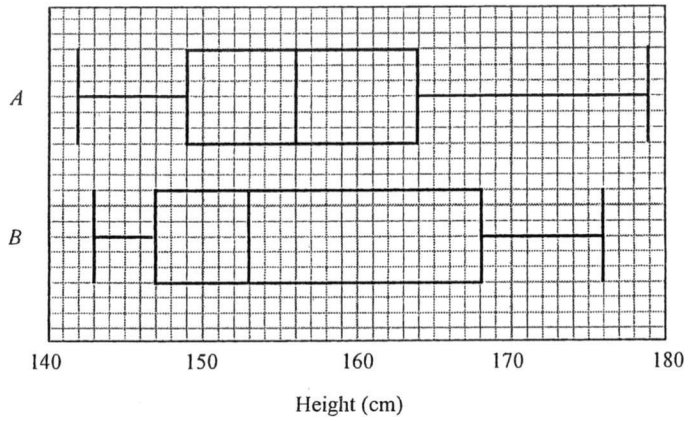
$= \frac{\frac{1}{2}(\pi)(16)^2 - \frac{1}{2}(\pi)(6)^2}{\pi(16)^2}$ M1

$= \frac{128\pi - 18\pi}{256\pi}$

$= \frac{55}{128}$ A1

Answer (b) ..... [2]

18. These box plots show the distributions of the heights of students in two schools A and B.



(a) Find the median for school A.

Median = 156 cm B1

Answer (a) ..... cm [1]

(b) Find the interquartile range for school B.

Interquartile range = 168 – 147

= 21 cm B1

Answer (b) ..... cm [1]

(c) Here are two statements comparing the heights for the two schools.

For each one, write whether you agree or disagree. Give a reason for each answer.

(i) On average, students from school A is taller than students from school B.

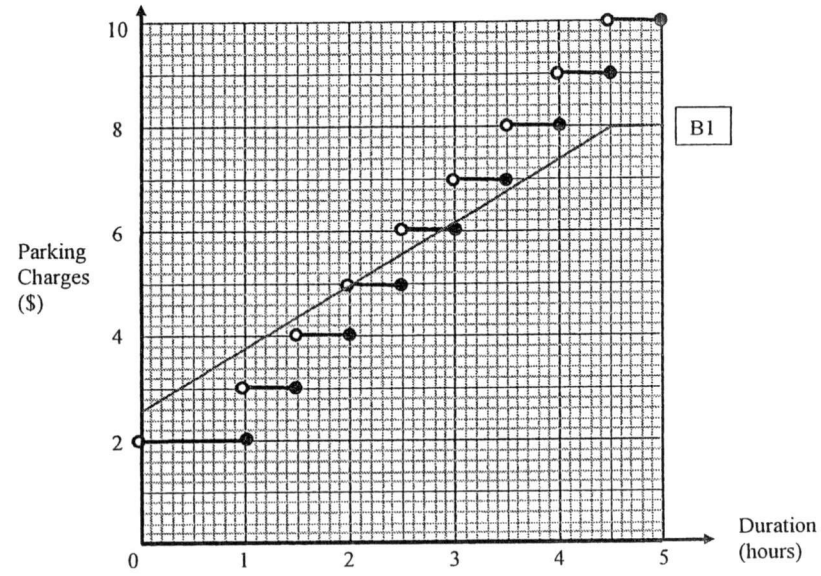
Answer (c)(i) Agree because the median height of the students from school A is higher as compared to school B. B1

(ii) A greater proportion of the students has a height of more than 165 cm in school A than in school B.

Answer (c)(ii) Disagree because less than 25% of the students has height of more than 165 cm in school A but more than 25% of the students has height of more than 165 cm in school B. B1

19. The step-function graph below shows the parking charges for the first 5 hours at Carpark P.

Answer (b)(i)



(a) Find the parking charges for a car that is parked at Carpark P for

(i) $1\frac{1}{2}$ hours,

$3 B1

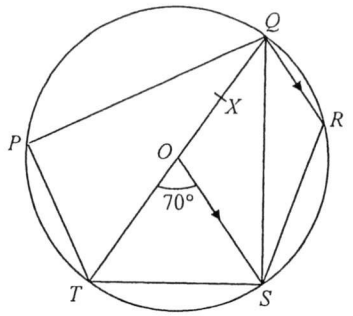
Answer (a)(i) $ ..... [1]

(ii) 3 hours 10 minutes.

$7 B1

Answer (a)(ii) $ ..... [1]

22. A circle, centre O, passes through the points P, Q, R, S and T. QT is the diameter of the circle, angle $SOT = 70°$ and QR is parallel to OS.



(a) Find

(i) angle OQS,

angle $OQS = 35°$ ($\angle$ at centre $= 2\angle$ at circumference) B1

Answer (a)(i) angle OQS =..... [1]

(ii) angle OTS,

angle $OTS = \dfrac{180° - 70°}{2}$ (base $\angle$s of isos. Δ)

$= 55°$ B1

Answer (a)(ii) angle OTS =..... [1]

(iii) angle QRS,

angle $QRS = 180° - 55°$ (opp. $\angle$s of cyclic quad)

$= 125°$ B1

Answer (a)(iii) angle QRS =..... [1]

(iv) angle QSR,

angle $OSQ = 35°$ (base $\angle$s of isos. Δ)

angle $SQR = 35°$ (alt. $\angle$s)

angle $QSR = 180° - 35° - 125°$ ($\angle$ sum of Δ)

$= 20°$ B1

Answer (a)(ii) angle QSR =..... [1]

(b) X is the point on QT such that $QX = \dfrac{1}{4}QT$. Given that the area of triangle PQT is 90 cm^2, calculate the area of PXT.

$$\frac{\text{Area of } \Delta PXT}{\text{Area of } \Delta PQT} = \frac{\frac{1}{2} \times XT \times h}{\frac{1}{2} \times QT \times h}$$

$$\frac{\text{Area of } \Delta PXT}{90} = \frac{XT}{QT}$$

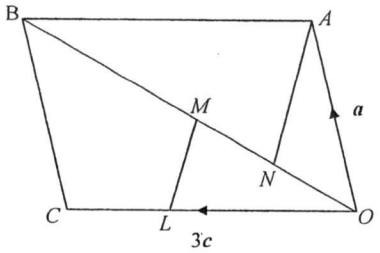
$$\frac{\text{Area of } \Delta PXT}{90} = \frac{3}{4}$$ M1

$$\text{Area of } \Delta PXT = \frac{3}{4} \times 90$$

$= 67.5$ cm^2 A1

Answer (b) ..... cm^2 [2]

23.



$OABC$ is a parallelogram. M is the midpoint of OB, N is the point on OM such that $OM = 2ON$ and L is the point on OC such that $OL = 2LC$. $\overrightarrow{OA} = \boldsymbol{a}$, $\overrightarrow{OC} = 3\boldsymbol{c}$ and $\overrightarrow{ML} = -\frac{1}{2}\boldsymbol{a} + \frac{1}{2}\boldsymbol{c}$.

(a) Write each of the following in terms of $\boldsymbol{a}$ and $\boldsymbol{c}$. Give your answers in the simplest form.

(i) $\overrightarrow{OB}$

$\overrightarrow{OB} = \overrightarrow{OA} + \overrightarrow{AB}$

$= \boldsymbol{a} + 3\boldsymbol{c}$ B1

Answer (a)(i) ..... [1]

(ii) $\overrightarrow{ON}$

$\overrightarrow{ON} = \frac{1}{4}\overrightarrow{OB}$

$= \frac{1}{4}(\boldsymbol{a} + 3\boldsymbol{c})$ B1

Answer (a)(ii) ..... [1]

(iii) $\overrightarrow{AN}$

$\overrightarrow{AN} = \overrightarrow{AO} + \overrightarrow{ON}$

$= -\boldsymbol{a} + \frac{1}{4}(\boldsymbol{a} + 3\boldsymbol{c})$ M1

$= -\frac{3}{4}\boldsymbol{a} + \frac{3}{4}\boldsymbol{c}$ A1

Answer (a)(iii) ..... [2]

(b) Explain why ML is parallel to AN.

$\overrightarrow{AN} = -\frac{3}{4}\boldsymbol{a} + \frac{3}{4}\boldsymbol{c}$ and $\overrightarrow{ML} = \frac{1}{2}(-\boldsymbol{a} + \boldsymbol{c})$

$= \frac{3}{4}(-\boldsymbol{a} + \boldsymbol{c})$

$\overrightarrow{ML} = \frac{2}{3}\overrightarrow{AN}$

Answer (b) Since $\overrightarrow{ML} = \frac{2}{3}\overrightarrow{AN}$, ML is parallel to AN. B1

Answer (b) .....

..... [1]

(c) Find the ratio $AN : ML$.

$AN : ML = 3 : 2$ B1

Answer (c) ..... [1]

End of Paper

Answer **all** questions.

1 **(a)** Simplify $\frac{5p^2-20}{3p^2+6p}$. [3]

(b) Express as a single fraction in its simplest form.

(i) $\frac{3xy^2}{16} \div \frac{15y}{4x}$ [2]

(ii) $\frac{7}{2x+1} - \frac{1}{x-2}$ [2]

(c) Solve the equation $\frac{8x-1}{x-2} - 3 = 2x$. [3]

(d) It is given that $ab = cd + 2bc^2$.

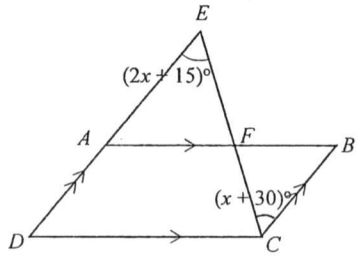
(i) Find a when $b = 0.8, c = 2$ and $d = -1$. [1]

(ii) Express b in terms of a, c and d. [2]

---

2 **(a)** The interior angle of a regular polygon is 160°.
Calculate the number of sides that the polygon has. [2]

(b) The diagram shows a parallelogram $ABCD$ with DA produced to E. F is the point of intersection of AB and CE.
Angle $AEF = (2x + 15)°$ and angle $ECB = (x + 30)°$.



(i) Find the value of x. [2]
(ii) Show that triangles AEF and DEC are similar. [2]
(iii) Given that the length of $AF = 3$ cm, $CD = 5$cm and $AE = 4$ cm, find the length of AD. [2]

3 Mr Tan plans to organize a family outing to Singapore Zoo.
The cost of a ticket to Singapore Zoo is \$22 for a child, \$33 for an adult and \$15 for a senior citizen if the ticket is bought at the entrance of the zoo.
If the ticket is bought online, there would be a discount and the ticket would cost \$16.50 for a child, \$24.75 for an adult and \$11.25 for a senior citizen.

The information can be represented by the matrix $\mathbf{T} = \begin{matrix} \text{Child} & \text{Adult} & \text{Senior Citizen} \\ \begin{pmatrix} 22 & 33 & 15 \\ 16.5 & 24.75 & 11.25 \end{pmatrix} & & \begin{matrix} \text{Entrance} \\ \text{Online} \end{matrix} \end{matrix}$

(a) Mr Tan's family consists of 5 children, 6 adults and 2 senior citizens.
Represent the number of people in Mr Tan's family in a 3×1 column matrix **P**. [1]

(b) Evaluate the matrix **R = TP**. [2]

(c) State what the elements of **R** represent. [1]

(d) Evaluate the matrix $\mathbf{Q} = (1 \quad -1)\,\mathbf{R}$. [1]

(e) State what the elements of **Q** represent. [1]

---

4 The diagram shows part of a number grid.

2	4	6	8	10	12
14	16	18	20	22	24
26	28	30	32	34	36
38	40	42	44	46	48

A square outlining nine numbers, as shown, can be placed anywhere on the grid. n represents the number in the top left corner of the square.

(a) Write down an expression, in terms of n, for the number in the bottom right corner of the square. [1]

(b) Find the value of n given that the sum of the four numbers in all the corners of the square is 928. [2]

(c) Show that the difference between the products of the numbers in the diagonally opposite corners of the square is always 96. [2]

---

5 **Answer the whole of this question on a sheet of graph paper.**

The variables x and y are connected by the equation

$$y = \frac{x^2}{4} + \frac{1}{x} - 6.$$

Some corresponding values of x and y, correct to two decimal places, are given in the table below.

x	0.5	1	2	3	4	5	5.5
y	−3.94	−4.75	−4.50	−3.42	p	0.45	1.74

(a) Find the value of p. [1]

(b) Using a scale of 2 cm to represent 1 unit on each axis, draw a horizontal x-axis for $0 \leq x \leq 6$ and a vertical y-axis for $-5 \leq y \leq 2$.
On your axes, plot the points given in the table and join them with a smooth curve. [3]

(c) Use your graph to find the solutions of $\frac{x^2}{4} + \frac{1}{x} = 1\frac{1}{2}$ [2]

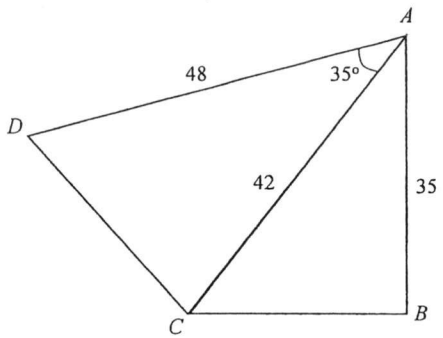
(d) By drawing a tangent, find the gradient of the curve at the point $(1, -4.75)$. [2]

(e) **(i)** On the same axes, draw the line $y = \frac{1}{2}x - 4$ for $0 \leq x \leq 6$. [2]

(ii) Write down the x-coordinate of the point where the line intersects the curve for $0.5 \leq x \leq 6$. [1]

(iii) This value of x is a solution of the equation $x^3 + Ax^2 + Bx + 4 = 0$.
Find the value of A and the value of B. [2]

6



The diagram shows a field $ABCD$ on horizontal ground, crossed by a path AC.
B is due south of A and C is due west of B.
$AC = 42$ m, $AD = 48$ m and $AB = 35$ m.
Angle $DAC = 35°$.

(a) Calculate

(i) BC, [1]

(ii) CD, [3]

(iii) angle ADC, [2]

(iv) the area of the field, [2]

(v) the bearing of D from A. [2]

(b) A bird is at E, which is vertically above C.
The angle of elevation of E from A is 35°.
Calculate the angle of depression of D from E. [3]

7 (a) A group of 80 students took a Mathematics examination. The highest mark for the test is 100. The table below shows a summary of their marks.

Marks	Frequency
$0 < x \leq 20$	2
$20 < x \leq 40$	10
$40 < x \leq 60$	23
$60 < x \leq 80$	42
$80 < x \leq 100$	3

(i) Calculate an estimate of
(a) the mean mark, [2]
(b) the standard deviation of the marks. [2]

(ii) Explain why the mean and the standard deviation calculated is only an estimation. [1]

(iii) The same group of students took a Science Examination and their results are summarized as follows.

Mean mark	47
Standard deviation	8.3

Make two comparisons between the performances of the students in the two examinations. [2]

(iv) A student is chosen at random from the group. Find the probability that the student scored more than 60 marks in the Mathematics Examination. [1]

(v) Two students are chosen at random from the group. Find the probability that both of them scored more than 20 marks but less than or equals to 60 marks in the Mathematics Examination. [2]

(b) The probability of passing an English Examination is 0.7 and the probability of passing a Mother Tongue Examination is 0.55.
A student is chosen at random from the same group of students.
Find the probability that the student will pass at least one examination. [2]

8 A watch dealer bought w watches, each at the same price, for a total cost of \$4 725.

(a) Find an expression, in terms of w, for the cost, in dollars, of each watch. [1]

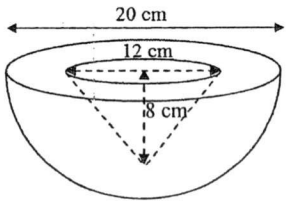
(b) The watch dealer kept 2 watches for himself and sold the rest at a profit of \$150 per watch. Write down an expression, in terms of w, for the total amount he received for all the watches sold. [1]

(c) Given that the watch dealer made a profit of \$1320 altogether, form an equation in w and show that it reduces to $5w^2 - 54w - 315 = 0$. [3]

(d) Solve the equation $5w^2 - 54w - 315 = 0$. [3]

(e) Find the selling price of each watch. [2]

9 (a) The solid in the figure below is made up by extracting a circular cone of base diameter 12 cm and height 8 cm from a metal hemisphere of diameter 20 cm.



Calculate
(i) the volume of the figure, [3]
(ii) the total surface area of the figure. [4]

(b) The solid is melted and all of the metal was used to make some cubes.
What is the maximum number of cubes that can be made if the length of each cube is 3 cm? [2]

10 AirSinga is a budget airline that charges passengers for checked baggage and excess baggage. The information on the cost of checked baggage and excess baggage for International Flights and Domestic Flights is given in the table below.

Cost of checked baggage for a round-trip ticket during the initial booking of International Flights	
Weight of baggage	**Price**
Up to 20 kg	$35
Up to 40 kg	$112

Cost of checked baggage for a round-trip ticket during the initial booking of Domestic Flights	
Weight of baggage	**Price**
Up to 15 kg	$13
Up to 30 kg	$30

Excess baggage for any flights after initial booking has been made
$22 per kg or part thereof

(a) Miss Lynn and Miss Sonia bought a round-trip ticket with AirSinga from Bangkok to Phuket. Each of them paid for checked baggage up to 15 kg for their initial booking of the domestic flight. When they left Bangkok, the weight of their baggage was 11 kg each. On their return trip, Miss Lynn's baggage weighed 8.5 kg more than the initial weight of her baggage and Miss Sonia's baggage weighed 2 kg more than the initial weight of her baggage. If Miss Lynn and Miss Sonia combined the weight of their baggage on the return trip, how much would Miss Lynn have to pay for her excess baggage? [2]

(b) Mr Willy bought a round-trip ticket from Singapore to Taiwan. He paid for checked baggage up to 20 kg during his initial booking for the International Flight. The weight of his baggage was 15 kg when he left Singapore. On his return trip, his baggage weighed 24 kg and hence, he was charged for the excess baggage. Should Mr Willy pay for checked baggage up to 20 kg or 40 kg during his initial booking to save money? Justify your decision with calculations, indicating the amount of money saved. [4]

(c) Mr Hassan wants to travel to Hong Kong and did a research on the cost of the air tickets of two airline companies, Air Singa and Singapura Airline. Some of the information is shown in the tables below.
Singapura Airline does not charge passengers for checked baggage, up to 30 kg.
The time indicated in the tables is the time of departure.

Cost of Air Singa's air tickets from Singapore to Hong Kong, including all taxes

Time \ Date	1 June	2 June	3 June	4 June	5 June	6 June	7 June
6 am	$150	$150	$180	$180	$140	$140	$150
4 pm	$160	$165	$180	$180	$135	$135	$135

Cost of Air Singa's air tickets from Hong Kong to Singapore, including all taxes

Time \ Date	1 June	2 June	3 June	4 June	5 June	6 June	7 June
11 am	$145	$160	$200	$200	$150	$150	$160
9 pm	$130	$180	$200	$200	$170	$180	$190

Cost of Singapura Airline's air tickets from Singapore to Hong Kong, including all taxes

Time \ Date	1 June	2 June	3 June	4 June	5 June	6 June	7 June
5 am	$200	$190	$275	$300	$200	$200	$220
3 pm	$180	$225	$275	$275	$180	$180	$200

Cost of Singapura Airline's air tickets from Hong Kong to Singapore, including all taxes

Time \ Date	1 June	2 June	3 June	4 June	5 June	6 June	7 June
10 am	$200	$220	$260	$280	$220	$230	$230
10 pm	$190	$220	$260	$260	$200	$220	$220

Mr Hassan would like to stay in Hong Kong for 6 days 5 nights and he has the flexibility to travel between 1 June and 7 June. He would like to depart from Singapore in the morning and depart from Hong Kong at night. He also wants to travel with the same airline for both trips.

Which airline should Mr Hassan choose to get the cheapest option given that his checked baggage for both trips was 24 kg? Justify your decision with calculations, indicating the date and time of the flights that he should choose. [5]

Marking Scheme

1. (a) $\frac{5p^2-20}{3p^2+6p} = \frac{5(p^2-4)}{3p(p+2)}$ [M1: $5(p^2-4)$ or $3p(p+2)$]

$= \frac{5(p+2)(p-2)}{3p(p+2)}$ [M1: $(p+2)(p-2)$]

$= \frac{5(p-2)}{3p}$ [A1: Accept $\frac{5p-10}{3p}$]

b(i) $\frac{3xy^2}{16} \div \frac{15y}{4x}$

$= \frac{3xy^2}{16} \times \frac{4x}{15y}$ [M1]

$= \frac{x^2y}{20}$ [A1]

b(ii) $\frac{7}{2x+1} - \frac{1}{x-2}$

$= \frac{7(x-2)-1(2x+1)}{(2x+1)(x-2)}$ [M1: Attempt to make common denominator]

$= \frac{5x-15}{(2x+1)(x-2)}$ [A1: Accept $\frac{5x-15}{2x^2-3x-2}$]

(c) $\frac{8x-1}{x-2} = 2x+3$

$8x-1 = (2x+3)(x-2)$

$8x-1 = 2x^2-x-6$ [M1: Expanded correctly]

$2x^2-9x-5 = 0$ [M1]

$(2x+1)(x-5) = 0$

$x = -0.5/-\frac{1}{2}$ or $x = 5$ [A1: both values of x are correct]

d(i) $a = 5.5 \ / \ 5\frac{1}{2} \ / \ \frac{11}{5}$ [B1]

d(ii) $ab = cd + 2bc^2$

$ab - 2bc^2 = cd$

$b(a-2c^2) = cd$ [M1: took out common factor b]

$b = \frac{cd}{a-2c^2}$ [A1]

2 (a) Method 1:

Exterior angle = 20° [M1]

Number of sides = $\frac{360}{20}$

= 18 [A1]

Method 2:

$(n-2)(180) = 160n$ [M1]

$180n - 360 = 160n$

$20n = 360$

$n = 18$ [A1]

(i) $2x + 15 = x + 30$ [M1]

$x = 15$

Angle $AEF = 45°$ [A1]

(ii) Angle AEF = Angle DEC (common angle)

Angle EAF = Angle EDC (Corresponding angle) [M1: one correct statement with reason]

∴ Triangles AEF and DEC are similar (AA Similarity test) [A1]

(iii) Using similar triangles:

$\frac{DE}{4} = \frac{5}{3}$ [M1]

$DE = \frac{20}{3}$ cm

$\therefore AD = \frac{20}{3} - 4$

$= 2\frac{2}{3} / \frac{8}{3} /$ 2.67 cm (3 s.f.) [A1]

3 (a) $\mathbf{P} = \begin{pmatrix} 5 \\ 6 \\ 2 \end{pmatrix}$ [B1]

(b) $\mathbf{R} = \begin{pmatrix} 22 & 33 & 15 \\ 16.5 & 24.75 & 11.25 \end{pmatrix} \begin{pmatrix} 5 \\ 6 \\ 2 \end{pmatrix}$

$= \begin{pmatrix} 338 \\ 253.5 \end{pmatrix}$ [B1: 338] [B1: 253.5]

(c) The elements of **R** represent the total amount that Mr Tan has to pay for the tickets for his family if the tickets are bought at the entrance and online respectively. [B1]

(d) $\mathbf{Q} = (1 \quad -1) \begin{pmatrix} 338 \\ 253.5 \end{pmatrix}$

= (84.5) [B1: Allow ecf from (b)]

(e) **Q** represents the total amount that Mr Tan would save if he has bought the tickets online. [B1]

4 (a) $n + 28$ [B1]

(b) $n + (n+4) + (n+24) + (n+28) = 928$ [M1]

$4n + 56 = 928$

$n = 218$ [A1]

(c) $(n+4)(n+24) - n(n+28)$ [M1: accept $n(n+28) - (n+4)(n+24)$]

$= (n^2 + 28n + 96) - n^2 + 28n$

$= 96$ [A1: accept -96]

5(a) B1: $p = -1.75$

(b) P1: Plot 5 points correctly
P1: Plot 2 other points correctly with correct scale
C1: Smooth curve

(c) M1: Ruled straight line through $(2, -4.5)$ or $y = -4.5$ seen.
A1 or B1: $x = 2$ or $x = 0.75(\pm 0.1)$

(d) M1: Attempt to draw a tangent at $(1, -4.75)$.
A1: Gradient $= -0.5(\pm 0.1)$

(e)i B1: Attempted to draw and labelled $y = \frac{1}{2}x - 4$ or table of values seen

B1 or B2: Drew $y = \frac{1}{2}x - 4$ correctly

(e)ii B1: $x = 3.8(\pm 0.1)$

(e)iii $\frac{x^2}{4} + \frac{1}{x} - 6 = \frac{1}{2}x - 4$ [M1]
$x^3 - 2x^2 - 8x + 4 = 0$
$\therefore A = -2, B = -8$ [A1]

6(a)(i) $BC = \sqrt{42^2 - 35^2}$
$= \sqrt{539}$
$= 23.2$ m (3 s.f.) [B1]

(a)(ii) Using the cosine rule,
$(CD)^2 = 48^2 + 42^2 - 2(48)(42)\cos 35$ [M1: Substituted correct values into cosine rule]
$= \sqrt{765.1789574}$
$CD = 27.66186829$ [M1]
$= 27.7$ m (3s.f.)
[A1]

(a)(iii) Using the sine rule,
$\frac{\sin \angle ADC}{42} = \frac{\sin 35}{27.66186829}$ [M1: Applied sine rule correctly; Allow ECF from (ii)]

$\angle ADC = 60.56124173$
$= 60.6°$ (1 d.p.) [A1]

(a)(iv) Area of the field
$= \frac{1}{2}(48)(42)\sin 35 + \frac{1}{2}(\sqrt{539})(35)$ [M1: Either $\frac{1}{2}(48)(42)\sin 35$ OR $\frac{1}{2}(\sqrt{539})(35)$]
$= 984.4515847$
$= 984$ m^2 (3 s.f.)
[A1]

(a)(v) $\cos \angle CAB = \frac{35}{42}$ [M1: Attempted to use trigo ratios to find $\angle CAB$]
$\angle CAB = 33.55730976°$
Bearing of D from A
$= 180 + 33.55730976 + 35$
$= 248.6°$ (1 d.p.)
[A1]

(b) $\tan 35 = \frac{EC}{42}$ [M1]
$EC = 42\tan 35$
$\tan \angle EDC = \frac{42\tan 35}{27.66186829}$ [M1: allow ecf for value of CD from a(ii)]

$\angle EDC = 46.8°$ (1 d.p)
$\therefore$ Angle of depression $= 46.8°$ [A1]

7(a)(i)(a) Mean $= \frac{2(10)+10(30)+23(50)+42(70)+3(90)}{80}$ [M1]
$= 58.5 / 58\frac{1}{2}$ [A1]
[Note: Award only B1 if no working is shown]

(a)(ii)(b) Standard deviation of the marks
$= \sqrt{\frac{296800}{80} - \left(58\frac{1}{2}\right)^2}$ [M1: Correct substitution]
$= 17.0$ (3 s.f.) [A1]
[Note: Award only B1 if no working is shown]

(a)(ii) The exact mark of each student is not known. [B1]

(a)(iii) The students did better in the Math exam as the mean mark of the Math exam is higher. [B1]
However, the result of the Science exam is more consistent as the standard deviation of the marks of the Science exam is lower. [B1]

(a)(iv) $\frac{45}{80} = \frac{9}{16} / 0.5625$ [B1]

(a)(v) $\frac{33}{80} \times \frac{32}{79}$ [M1]
$= \frac{66}{395} / 0.167$ (3s.f.) [A1]

(b) $(0.55 \times 0.7) + (0.55 \times 0.3) + (0.45 \times 0.7)$ [M1: Accept $1 - 0.3(0.45)$]
$= \frac{173}{200} / 0.865$ [A1]

8 (a) $\$\frac{4725}{w}$ [B1]

(b) $\$(w-2)(\frac{4725}{w} + 150)$ [B1]

(c) $(w-2)\left(\frac{4725}{w} + 150\right) = 4725 + 1320$ [M1: Formed the correct equation]

$4725 + 150w - \frac{9450}{w} - 300 = 6045$ [M1: Correct expansion]

$150w^2 - 1620w - 9450 = 0$
$5w^2 - 54w - 315 = 0$ (shown) [A1]

(d) $5w^2 - 54w - 315 = 0$

Using the general formulae,

$w = \frac{-(-54) \pm \sqrt{(-54)^2 - 4(5)(-315)}}{2(5)}$ OR $(w - 15)(5w + 21) = 0$ [M1: Correct substitution OR factorized correctly]

$w = 15$ or $w = -4.2 / -4\frac{1}{5} / -\frac{21}{5}$ [A2: minus 1 mark if students reject any of the answers]

*Award only B2 if students got both values of w correct without showing working

(e) Selling price $= \frac{4725}{15} + 150$
$= \$465$ [M1: $\frac{4725}{15}$ (allow ecf for M1)]
[A1]

9(a)(i) Volume of the figure
$= \frac{2}{3}\pi(10)^3 - \frac{1}{3}\pi(6)^2(8)$
$= 1792.802208 \text{ cm}^3$
$= 1790 \text{ cm}^3$ (3 s.f.)
[M1: attempted to find the vol. of hemisphere or cone]
[M1: attempted to subtract the vol. of cone from vol. of hemisphere]
[A1]

(a)(ii) Slant height of the cone
$= \sqrt{6^2 + 8^2}$
$= 10$ cm
Total surface area of the cone
$= 2\pi(10)^2 + \pi(6)(10) + [\pi(10)^2 - \pi(6)^2]$
$= 1020 \text{ cm}^2$ (3 s.f.)
[M1: Attempted to find slant height of the cone]
[M1: Found 1 of the 3 S.A. correctly]
[M1: Found the other 2 S.A. correctly]
[A1]

(b) Max number of cubes
$= \frac{1792.802208}{3^3}$
$= 66$
[M1: Allow ECF from (i)
[A1]

10(a) Excess baggage weight
$= 8.5 - 4 - 2$ [M1]
$= 2.5$ kg
Amount paid for excess baggage
$= 3 \times 22$
$= \$66$ [A1]

(b) Amount that he has to pay for the excess baggage [M1: Identified that excess baggage = 4kg]
$= 4 \times 22$
$= \$88$
Total amount spent with checked baggage up to 20kg [M1: Attempted to find total amount spent with checked baggage up to 20kg]
$= \$88 + \$35$
$= \$123$
Total amount saved with checked baggage up to 40kg [M1: Attempted to find the amount saved]
$= 123 - 112$
$= \$11$ [A1: Indicated the decision clearly stating the amount of money saved]
∴ Mr Willy should pay for checked baggage up to 40kg during his initial booking as he would save \$11.

(c) To travel for 6D5N, there are only 2 options:
1 – 6 June or 2 – 7 June

Air ticket for Air Singa for 1 June, 6am to 6 June, 9pm + checked baggage up to 40 kg
$= \$150 + \$180 + \$112$ [M1]
$= \$442$ [A1]

Air ticket for Singapura Airline for 2 June, 5am to 7 June, 10pm
$= \$190 + 220$ [M1]
$= \$410$ [A1]

Therefore, Mr Willy should buy the air ticket from Singapura Airline, choosing the departure flight on 2 June at 5am and the arrival flight on 7 June at 10pm. [A1]

Name : _____

Class	Index Number

METHODIST GIRLS' SCHOOL

Founded in 1887

MID-YEAR EXAMINATION 2017
Secondary 4

Monday
15 May 2017

MATHEMATICS
Paper 1

4048/01
2 h

READ THESE INSTRUCTIONS FIRST

Write your name, class and index number on the work you hand in.
Write in dark blue or black pen on both sides of the paper.
You may use an HB pencil for any diagrams or graphs.
Do not use staples, paper clips, highlighters, glue or correction fluid.

Answer **all** questions.
If working is needed for any question, it must be shown with the answer.
Omission of essential working will result in loss of marks.
The use of an approved scientific calculator is expected, where appropriate.
If the degree of accuracy is not specified in the question, and if the answer is not exact, give the answer to three significant figures. Give answers in degrees to one decimal place.
For π, use either your calculator value or 3.142, unless the question requires the answer in terms of π.

The number of marks is given in brackets [ ] at the end of each question or part question.
The total of the marks for this paper is 80.

80

This question paper consists of 19 printed pages.

Mathematical Formulae

For Examiner's Use

Compound Interest

$$\text{Total amount} = P\left(1+\frac{r}{100}\right)^n$$

Mensuration

Curved surface area of a cone $= \pi rl$

Surface area of a sphere $= 4\pi r^2$

Volume of a cone $= \frac{1}{3}\pi r^2 h$

Volume of a sphere $= \frac{4}{3}\pi r^3$

Area of a triangle $= \frac{1}{2}ab\sin C$

Arc length $= r\theta$, where θ is in radians

Sector area $= \frac{1}{2}r^2\theta$, where θ is in radians

Trigonometry

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$a^2 = b^2 + c^2 - 2bc\cos A$$

Statistics

$$\text{Mean} = \frac{\sum fx}{\sum f}$$

$$\text{Standard deviation} = \sqrt{\frac{\sum fx^2}{\sum f} - \left(\frac{\sum fx}{\sum f}\right)^2}$$

Answer all the questions

For Examiner's Use

1 **(a)** Calculate $\dfrac{23-\left(-4\frac{3}{5}\right)^3}{\sqrt[3]{(7.2)^2+\sqrt{6.81}}}$.

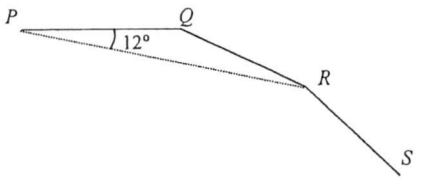
Write down the first five digits on your calculator display.

Answer (a) ..... [1]

(b) Write your answer to part **(a)** correct to 1 significant figure.

Answer (b) ..... [1]

2



PQ, QR and RS are adjacent sides of a regular polygon.

Given that $\angle QPR = 12°$, calculate

(a) the exterior angle of the polygon,

(b) the number of sides of the polygon,

(c) $\angle PRS$.

Answer (a) ..... ° [1]

(b) ..... [1]

(c) $\angle PRS$ = ... ° [1]

For Examiner's Use

3 The masses of 20 apples were measured, to the nearest grams.

The results are shown in the stem and leaf diagram below.

6	0 5
7	4 4 6 8 x 9
8	1 1 2 5 5
9	4 4 4 5 6
10	1 7

Key 7 | 4 means 74 grams

(a) Given that the percentage of apples whose weights are less than 80 grams is 40%, state a possible value of x.

Answer (a) ..... [1]

(b) Find the median weight.

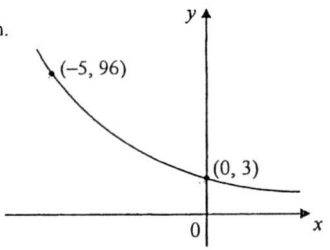
Answer (b) .....grams [1]

(c) The same information is to be shown on a pie chart.
Calculate the angle of the sector which would represent the number of apples whose weights are between 80 grams and 100 grams.

Answer (c) ..... ° [1]

4 The sketch shows the graph of $y = ka^x$.
The points (0, 3) and (–5, 96) lie on the graph.

Find the values of k and a.

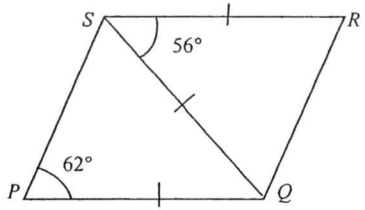


Answer k = ... [1]

a = ... [1]

5 In the diagram, PQS and RSQ are isosceles triangles in which $PQ = QS = SR$, $\angle SPQ = 62°$ and $\angle QSR = 56°$

For Examiner's Use



(a) Prove that ΔPQS and ΔRSQ are congruent.

(b) Write down the special name of quadrilateral $PQRS$.

Answer (a) .....

.....

.....

..... [2]

(b) ..... [1]

6 **(a)** Find the least values of x and y such that $2^3 \times 3^x \times 5^y$ is a multiple of 10.

Answer (a) x = ... [1]

y = ... [1]

(b) Written as a product of its prime factors, $201684 = 2^2 \times 3 \times 7^5$.
Find the smallest positive integer k such that $201684k$ is a cube number.

Answer (b) k = ... [1]

7 A map is drawn to the scale of 1 : 25 000.

For Examiner's Use

(a) The representation of a lake on the map has a perimeter of 8 cm.
How many kilometres will a girl cover if she walks around the lake?

Answer (a) ..... km [1]

(b) The area of a plantation is 75 000 m^2. Calculate the area of the plantation on the map in square centimetres.

Answer (b) ..... cm^2 [2]

8

	Singapore	Indonesia	China
Population	5.7×10^6	260×10^6	1.38×10^9
Area (km^2)	719.1	1.9×10^6	9.6×10^6

For Examiner's Use

Use as much information from the table as necessary to answer the following.

(a) Find the ratio of the population of Indonesia to the population of China.
Give your answer in the form $a : b$.

(b) How many times is the population of Indonesia that of Singapore?
Give your answer to the nearest whole number.

(c) Calculate the average number of people per square kilometre in China.
Give your answer to the nearest whole number.

Answer (a) ..... [1]

(b) ..... [1]

(c) ..... [1]

9 $\varepsilon = \{ x : x \text{ is an integer and } 4 \le x \le 25 \}$
$A = \{ x : x \text{ is divisible by } 5 \}$
$B = \{ x : x \text{ is a perfect square} \}$
$C = \{ x : x \text{ is a prime number} \}$

For Examiner's Use

(a) List the element/s contained in the set $A \cap B$,

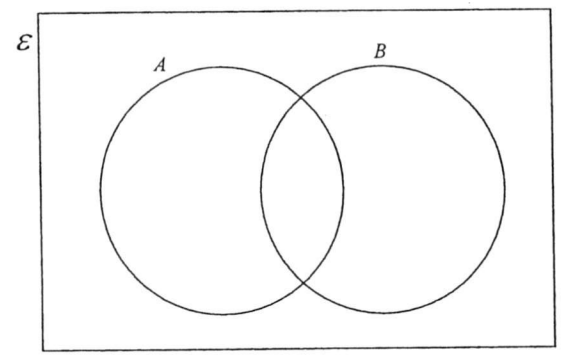
Answer (a) ..... [1]

(b) Write down in set notation an equation involving B and C.

Answer (b) ..... [1]

(c) In the Venn diagram below, shade the region representing $(A \cap B')'$.

Answer



[1]

10 Solve the equation $\frac{x}{x+1}-\frac{x+1}{3x-1}=\frac{1}{4}$.

For Examiner's Use

Answer $x =$ ....., ...[3]

11 **(a)** Express $x^2-6x+11$ in the form $(x-p)^2+q$.

Answer (a) ..... [2]

(b) **Hence**, write down the minimum value of $x^2-6x+11$.

Answer (b) ..... [1]

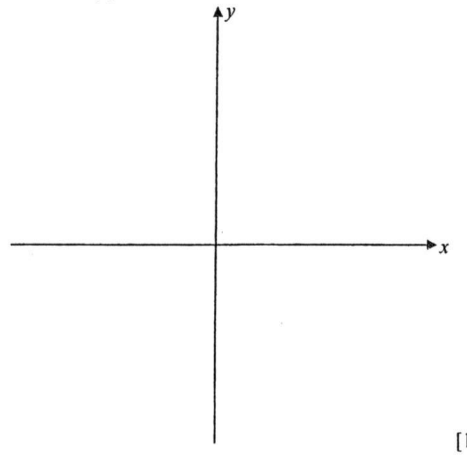
(c) Write down the equation of the line of symmetry of the graph of $y=x^2-6x+11$.

Answer (c) ..... [1]

12 On the axes given, sketch the graphs of

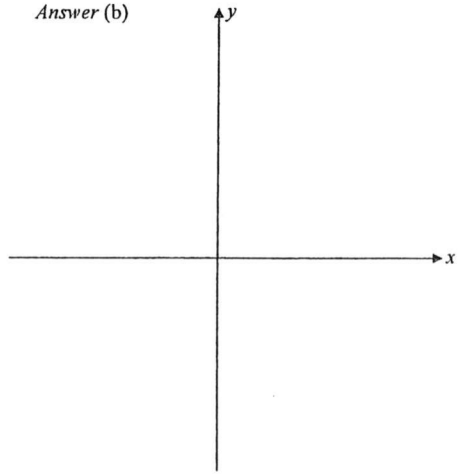
For Examiner's Use

(a) $y=2-x^3$, *Answer* (a)



[1]

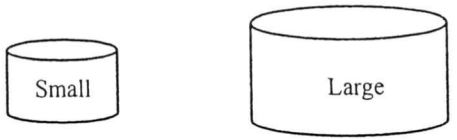
(b) $y=\frac{1}{x^2}$. *Answer* (b)



[1]

13 The base areas of two geometrically similar tins of sardines labelled Small and Large are 240 cm^2 and 540 cm^2 respectively.

For Examiner's Use



(a) The height of the Large tin is 18 cm. Find the height of the Small tin.

Answer (a) ..... cm [2]

(b) The price of the Small tin of sardines is \$3.20. A shopkeeper used the ratio of the base areas given above to price the Large tin of sardines.
Explain, with clear working, why the Large tin is better value for money.

Answer (b) .....
.....
.....
.....
.....
.....
..... [2]

14 Two outlets sold the following number of cups of drinks on a particular day.

For Examiner's Use

	Number of cups of drinks		
	Tea	Coffee	Milo
Outlet 1	32	67	56
Outlet 2	24	56	43

It costs \$0.50, \$0.80 and \$0.60 for the outlets to prepare a cup of tea, coffee and milo respectively. Each cup of tea, coffee and milo was sold for \$3.50, \$4.50 and \$4.00 respectively. It is given that

$$\mathbf{P} = \begin{pmatrix} 32 & 67 & 56 \\ 24 & 56 & 43 \end{pmatrix}, \quad \mathbf{C} = \begin{pmatrix} 0.50 \\ 0.80 \\ 0.60 \end{pmatrix} \text{ and } \mathbf{S} = \begin{pmatrix} 3.50 \\ 4.50 \\ 4.00 \end{pmatrix}$$

(a) Find $\mathbf{P(S - C)}$.

Answer (a) = ... [1]

(b) Explain what your answer to **(a)** represents.

Answer (b) .....
.....
.....
..... [1]

(c) Using matrix multiplication of a 1×2 matrix with the matrix obtained in part **(a)**, calculate the total profit made from the two outlets for the day.

Answer (c) \$..... [2]

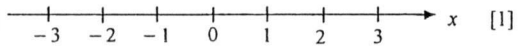
15 **(a)** Solve the inequalities $\frac{x-1}{4} < \frac{2x+1}{3} \leq 3x-2$.

For Examiner's Use

Answer (a) ..... [3]

(b) Represent solution set on the number line below.

Answer (b)



[1]

(c) Write down the smallest prime number which satisfies the above inequalities.

Answer (c) ..... [1]

16 **(a)** Factorise completely $4x+4xy-12y^2-12y$.

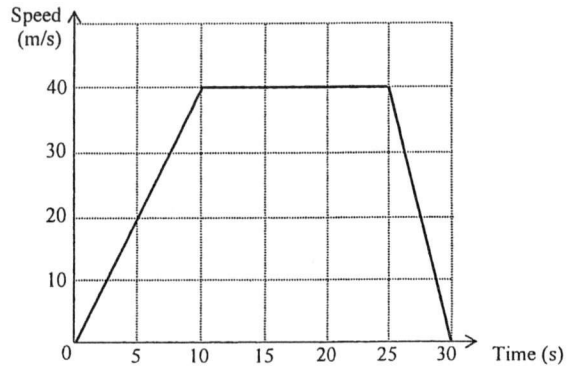
Answer (a) ..... [2]

(b) Simplify $\frac{a^4}{b} \times \left(\frac{b^6}{a^9}\right)^{-\frac{1}{3}}$, giving your answer in positive indices only.

Answer (b) ..... [2]

17 The diagram is the speed-time graph for the first 30 seconds of a car's journey.

For Examiner's Use



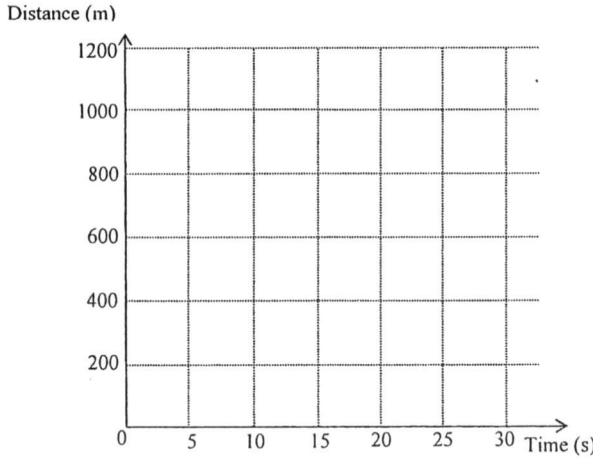
(a) Calculate the speed when the time is 7 seconds.

Answer (a) ..... m/s [1]

(b) Calculate the deceleration in the last 5 seconds.

Answer (b) ..... m/s^2 [1]

(c) On the axes below, sketch the distance-time graph for the journey.



[3]

18 A source of light is observed from a distance of d metres.
The amount of light received, L units, is inversely proportional to square of the distance from the source.

For Examiner's Use

(a) When the distance is 2 m apart, the amount of light received is 9 units.
Find the relationship between L and d.

Answer (a) ..... [2]

(b) Find the amount of light received when the distance is 5 m.

Answer (b) ..... units [1]

(c) When the source is at a certain distance, the amount of light received is p units.
Find the amount of light received, in terms of p, when the distance is doubled.

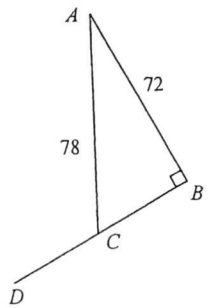
Answer (c) ..... units [1]

19 In the diagram, $\angle ABC = 90°$ and BCD is a straight line.
All measurements are in centimetres.

Without the use of calculators, find
(a) $\sin \angle BAC$,

(b) $\cos \angle ACD$.

Give both answers in the simplest form of $\frac{a}{b}$,
where a and b are integers.

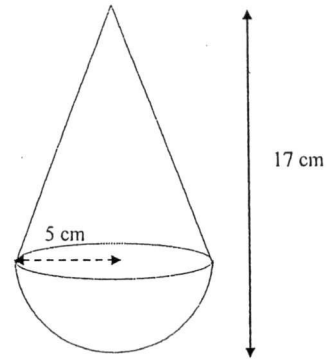


Answer (a) $\sin \angle BAC =$ ..... [2]

(b) $\cos \angle ACD =$ ..... [1]

20 The diagram shows a solid wooden toy made from a cone and a hemisphere of radius 5cm.
The total height of the toy is 17 cm.
The cost of painting this wooden toy is 0.5 cents per cm^2.

For Examiner's Use

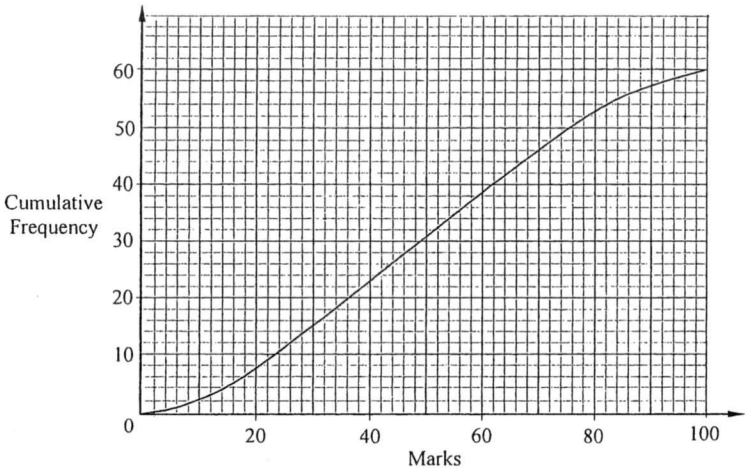


Find the total cost of painting the wooden toy in dollars.

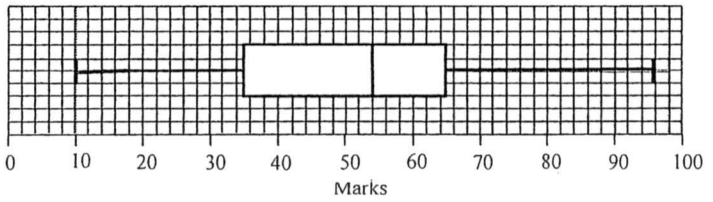
Answer $ ..... [3]

21 The cumulative frequency curve below shows the marks obtained, out of 100, by 60 students in an Additional Mathematics test.

For Examiner's Use



The same 60 students also set for the Physics paper.
The box-and-whisker diagram below illustrates the marks obtained.
The maximum mark was again 100.



Use the two diagrams to complete this table for the two tests.

Subject	Lower Quartile	Median	Upper Quartile	Interquartile Range
Add. Mathematics		49	68	
Physics	35		65	

[2]

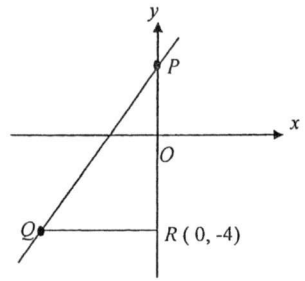
A student commented that the results for Additional Mathematics were better than for Physics.
Do you agree? Give a reason for your answer.

Answer .....

..... [1]

22 In the diagram below, R is the point $(0, -4)$ and P is a point on the y-axis. The line PQ meets the horizontal line through R at Q.

For Examiner's Use



(a) Write down the equation of line QR.

(b) Given that the equation of the line PQ is $3y - 7x = 9$, find the coordinates of Q.

(c) Find the area of triangle PQR.

(d) Calculate the perpendicular distance from R to PQ, giving your answer correct to two decimal places.

Answer (a) ..... [1]

(b) (,) [1]

(c) ..... units^2 [2]

(d) ..... units [2]

23 A playground is in the shape of a quadrilateral $ABCD$.
It is given that $AB = 90$ m, B is due East of A, $AD = 70$ m, $\angle BAD = 115°$, $\angle ADC = 85°$ and the bearing of C from $B = 010°$.

For Examiner's Use

(a) Complete the scale drawing of the playground, using a scale of 1cm to 10 m. [2]

(b) On your drawing, construct

(i) the perpendicular bisector of CD, [1]

(ii) the angle bisector of $\angle ABC$. [1]

(c) A statue , P, is to be built inside the playground such that it is nearer to D than C and equidistant from AB and BC.

Mark and label a possible position of the statue on your drawing. [1]

End of Paper

Answer all the questions

For Examiner's Use

1 **(a)** Calculate $\dfrac{23-\left(-4\frac{3}{5}\right)^3}{\sqrt[3]{(7.2)^2+\sqrt{6.81}}}$.

Write down the first five digits on your calculator display.

31.749

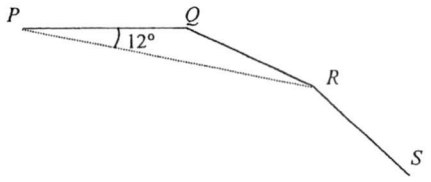
Answer (a) ..... [1]

(b) Write your answer to part **(a)** correct to 1 significant figure.

30

Answer (b) ..... [1]

2



PQ, QR and RS are adjacent sides of a regular polygon.

Given that $\angle QPR = 12°$, calculate

(a) the exterior angle of the polygon,

(b) the number of sides of the polygon,

(c) $\angle PRS$.

(a) an exterior angle of the polygon = 12° + 12° = 24°

(b) $\dfrac{360°}{24°} = 15$ (c) $\angle PRS$ = 156° - 12° =144°

Answer (a) 24 [1]

(b) 15 ° [1]

(c) $\angle PRS$ = 144 ° [1]

For Examiner's Use

3

The masses of 20 apples were measured, to the nearest grams.

The results are shown in the stem and leaf diagram below.

6	0 5
7	4 4 6 8 *x* 9
8	1 1 2 5 5
9	4 4 4 5 6
10	1 7

Key 7 | 4 means 74 grams

(a) Given that the percentage of apples whose weights are less than 80 grams is 40%, state a possible value of x.

8, 9

Answer (a) ..... [1]

(b) Find the median weight.

81.5

Answer (b) .....grams [1]

(c) The same information is to be shown on a pie chart.
Calculate the angle of the sector which would represent the number of apples whose weights are between 80 grams and 100 grams.

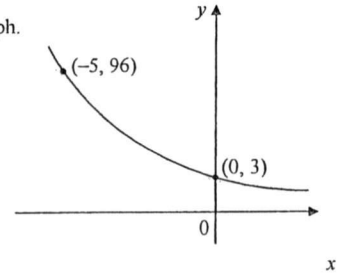
216 °

Answer (c) ..... [1]

4 The sketch shows the graph of $y = ka^x$.
The points (0, 3) and (–5, 96) lie on the graph.

Find the values of k and a.

When $x = 0$, $y = 3$, $3 = k\,a^0$,
$k = 3$



$y = 3a^x$
$96 = 3a^{-5}$
$a^5 = \dfrac{3}{96}$
$a = \dfrac{1}{2}$

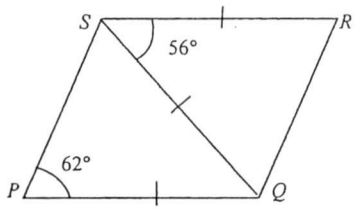
Answer k = 3 [1]

$\frac{1}{2}$

a = ... [1]

5 In the diagram, PQS and RSQ are isosceles triangles in which $PQ = QS = SR$ $\angle SPQ = 62°$ and $\angle QSR = 56°$

For Examiner's Use



(a) Prove that ΔPQS and ΔRSQ are congruent.

(b) What special type of quadrilateral is $PQRS$?

Answer (a) $\angle PQS = 180^0 - 62^0 - 62^0 = 56^0$ [$\angle$ sum of Δ]
$\therefore \angle PQS = \angle RSQ$
SQ is common
$PQ = RS$ [Given]
$\therefore \Delta PQS \equiv \Delta RSQ$ [SAS] [2]

(b) parallelogram [1]

6 **(a)** Find the least values of x and y such that $2^3 \times 3^x \times 5^y$ is a multiple of 10.

Answer (a) x = ...0..... [1]
y = ...1..... [1]

(b) Written as a product of its prime factors, $201684 = 2^2 \times 3 \times 7^5$.
Find the smallest positive integer k such that $201684k$ is a cube number

$k = 2 \times 3^2 \times 7$

Answer (b) k = ...126..... [1]

7 A map is drawn to the scale of 1 : 25 000.

For Examiner's Use

(a) The representation of a lake on the map has a perimeter of 8 cm.
How many kilometres will a girl cover if she walks around the lake?

1 cm : 25 000 cm
: 250 m
: 0.25 km
8 cm : 8 X 0.25 km
: 2 km

Answer (a) .....2..... km [1]

(b) The area of a plantation is 75 000 m^2. Calculate the area of the plantation on the map in square centimetres.

1 cm : 250 m
1cm^2 : 62500 m^2
1.2 cm^2 : 75000 m^2

Answer (b) .....1.2..... cm^2 [2]

8

	Singapore	Indonesia	China
Population	5.7×10^6	260×10^6	1.38×10^9
Area (km^2)	719.1	1.9×10^6	9.6×10^6

For Examiner's Use

Use as much information from the table as necessary to answer the following.

(a) Find the ratio of the population of Indonesia to the population of China.
Give your answer in the form $a : b$.

(b) How many times is the population of Indonesia that of Singapore?
Give your answer to the nearest whole number.

(c) Calculate the average number of people per square kilometre in China.
Give your answer to the nearest whole number.

(a) $260 \text{ X } 10^6 : 1.36 \text{ X } 10^9$

(b) $\frac{260 \times 10^6}{5.7 \times 10^6} = 45.6$

(c) $\frac{1.38 \times 10^9}{9.6 \times 10^6} = 1.4375 \times 10^2$

Answer (a) ..... 13 : 69 ..... [1]

(b) ..... 46 ..... [1]

(c) 144 [1]

9 $\varepsilon = \{ x : x \text{ is an integer and } 4 \le x \le 25 \}$
$A = \{ x : x \text{ is divisible by } 5 \}$
$B = \{ x : x \text{ is a perfect square} \}$
$C = \{ x : x \text{ is a prime number} \}$

For Examiner's Use

(a) List the element/s contained in the set $A \cap B$,

Answer (a) {25} [1]

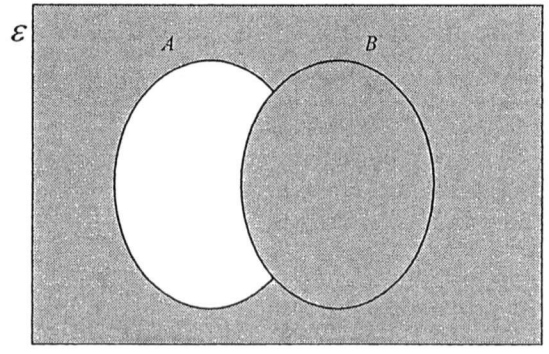
(b) Write down in set notation an equation involving B and C.

$B \cap C = \{\}$ or $B \cap C = \phi$

Answer (b) ..... [1]

(c) In the Venn diagram below, shade the region representing $(A \cap B')'$.

Answer



[1]

10 Solve the equation $\frac{x}{x+1}-\frac{x+1}{3x-1}=\frac{1}{4}$.

For Examiner's Use

$4x(3x-1)-4(x+1)^2=(x+1)(3x-1)$

$12x^2-4x-4(x^2+2x+1)=3x^2+2x-1$

$12x^2-4x-4x^2-8x-4-3x^2-2x+1=0$

$5x^2-14x-3=0$

$(5x+1)(x-3)=0$

$x=-\frac{1}{5}$ or $x=3$

Answer $x=-\frac{1}{5}$, 3 [3]

11 **(a)** Express $x^2-6x+11$ in the form $(x-p)^2-q$.

$x^2-6x+11$

$=(x-3)^2-9+11$

$=(x-3)^2+2$

Answer (a) $(x-3)^2+2$ [2]

(b) Hence write down the minimum value of $x^2-6x+11$.

Answer (b) 2 [1]

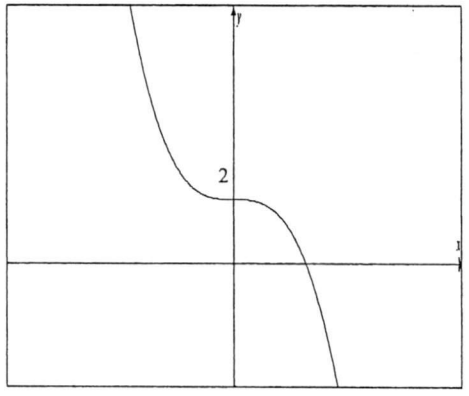
(c) Write down the equation of the line of symmetry of the graph of $y=x^2-6x+11$.

Answer (c) $x=3$ [1]

12 On the axes given, sketch the graphs of

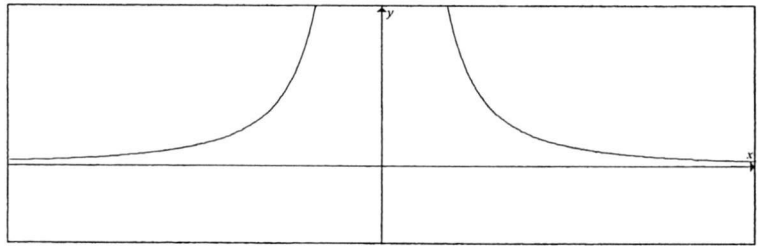
For Examiner's Use

(a) $y=2-x^3$ *Answer* (a)



[1]

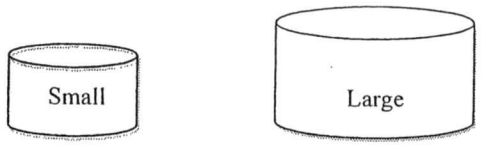
(b) $y=\frac{1}{x^2}$ *Answer* (b)



[1]

13 The base areas of two geometrically similar tin of sardines labelled Small and Large are 240 cm^2 and 540 cm^2 respectively.

For Examiner's Use



(a) The height of the Large tin is 18 cm. Find the height of the Small tin.

Let the height of the Small tin be x cm.

$$\left(\frac{x}{18}\right)^2 = \frac{240}{540}$$

$$\frac{x}{18} = \sqrt{\frac{4}{9}}$$

$$\frac{x}{18} = \frac{2}{3}$$

Answer (a) .....12..... cm [2]

(b) The price of the Small tin of sardines is \$3.20. A shopkeeper used the ratio of the base areas given above to price the Large tin of sardines. Explain , with clear working, why the Large tin is value for money.

Price of Large tin based on ratio of area $= \left(\frac{3}{2}\right)^2 \times \$3.20 = \$7.20$

Ratio of volume $= \left(\frac{2}{3}\right)^3 = \frac{8}{27}$

Price of Large tin based on volume $= \frac{27}{8} \times \$3.20 = \$10.80$

Answer (b) Based on the ratio of areas, the price paid for the Large tin is less.

$\therefore$ it is more value for money. [2]

14 Two outlets sold the following drinks on a particular day.

For Examiner's Use

	Tea	Coffee	Milo
Outlet 1	32	67	56
Outlet 2	24	56	43

It costs \$0.50, \$0.80 and \$0.60 for the outlets to prepare a cup of tea, coffee and milo respectively. Each cup of tea, coffee and milo was sold for \$3.50, \$4.50 and \$4.00 respectively. It is given that

$$\mathbf{P} = \begin{pmatrix} 32 & 67 & 56 \\ 24 & 56 & 43 \end{pmatrix}, \quad \mathbf{C} = \begin{pmatrix} 0.50 \\ 0.80 \\ 0.60 \end{pmatrix} \quad \text{and} \quad \mathbf{S} = \begin{pmatrix} 3.50 \\ 4.50 \\ 4.00 \end{pmatrix}$$

(a) Find **P(S – C)**.

$$\begin{pmatrix} 32 & 67 & 56 \\ 24 & 56 & 43 \end{pmatrix} \begin{pmatrix} 3.00 \\ 3.70 \\ 3.40 \end{pmatrix}$$

Answer (a) $= \begin{pmatrix} 534.30 \\ 425.40 \end{pmatrix}$ [1]

(b) Explain what your answer to **(a)** represents.

Answer (b) Profit from selling all 3types of drinks at each outlet respectively. [1]

(c) Using matrix multiplication of a 1 X 2 matrix with the matrix obtained in part **(a)**, calculate the total profit made from the two outlets for the day.

$$\begin{pmatrix} 1 & 1 \end{pmatrix} \begin{pmatrix} 534.30 \\ 425.40 \end{pmatrix} = (534.30 + 425.40) = (959.70)$$

Answer (c) \$959.70 [2]

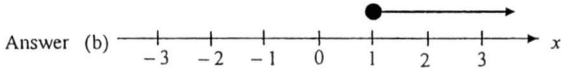
15 **(a)** Solve the inequalities $\frac{x-1}{4} < \frac{2x+1}{3} \leq 3x-2$.

$3(x-1) < 4(2x+1)$
$3x-3 < 8x+4$
$3x-8x < 7$
$-5x < 7$
$x > -\frac{7}{5}$

$2x+1 \leq 9x-6$
$2x-9x \leq -6-1$
$-7x \leq -7$
$x \geq 1$

Answer (a) $x \geq 1$ [3]

(b) Represent solution set on the number line below.

Answer (b) 

[1]

(c) Write down the smallest prime number which satisfies the above inequalities.

Answer (c) 2 [1]

16 **(a)** Factorise completely $4x+4xy-12y^2-12y$,

$4x(1+y)-12y(y+1)$
$=4(y+1)(x-3y)$

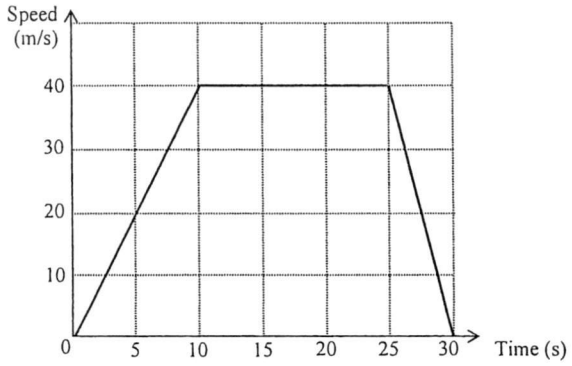
Answer (a) $4(y+1)(x-3y)$ [2]

(b) Simplify $\frac{a^4}{b} \times \left(\frac{b^6}{a^9}\right)^{-\frac{1}{3}}$, giving your answer in positive indices only.

$\frac{a^4}{b} \times \left(\frac{a^9}{b^6}\right)^{\frac{1}{3}}$
$= \frac{a^4}{b} \times \frac{a^3}{b^2}$

Answer (b) $\frac{a^7}{b^3}$ [2]

17 The diagram is the speed-time graph for the first 30 seconds of a car's journey.



(a) Calculate the speed when the time is 7 seconds.

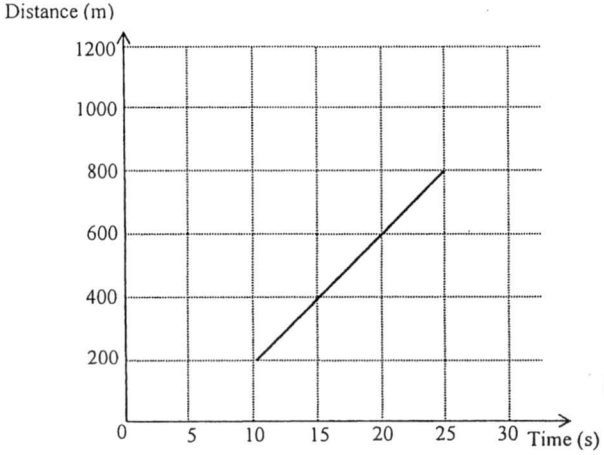
$\frac{v}{40} = \frac{7}{10}$ Answer (a) 28 m/s [1]

(b) Calculate the deceleration in the last 5 seconds.

$\frac{40}{5} = 8$

Answer (b) 8 m/s² [1]

(c) On the axes below, sketch the distance-time graph for the journey.



[3]

18 A source of light is observed from a distance of d metres.
The amount of light received, L units, is inversely proportional to square of the distance from the source.

For Examiner's Use

(a) When the distance is 2 m apart, the amount of light received is 9 units.
Find the relationship between L and d.

$$L = \frac{k}{d^2}$$

$$9 = \frac{k}{2^2}$$

$$k = 36$$

Answer (a) $L = \frac{36}{d^2}$ [2]

(b) Find the amount of light received when the distance is 5 m.

$$L = \frac{36}{5^2} = \frac{36}{25}$$

Answer (b) 1.44 units [1]

(c) When the source is at a certain distance, the amount of light received is p units.
Find the amount of light received, in terms of p, when the distance is doubled.

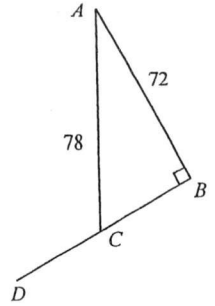
$$p = \frac{36}{d^2}$$

$$\frac{36}{(2d)^2} = \frac{36}{4d^2}$$

Answer (c) $\frac{1}{4}p$ units [1]

19 In the diagram, $\angle ABC = 90°$ and BCD is a straight line.
All measurements are in centimetres.

For Examiner's Use



Without the use of calculators, find
(a) $\sin \angle BAC$,

(b) $\cos \angle ACD$.

Give both answers in the simplest form of $\frac{a}{b}$,
where a and b are integers.

$BC = \sqrt{78^2 - 72^2} = 30$

(a) $\sin \angle BAC = \frac{30}{78} = \frac{5}{13}$

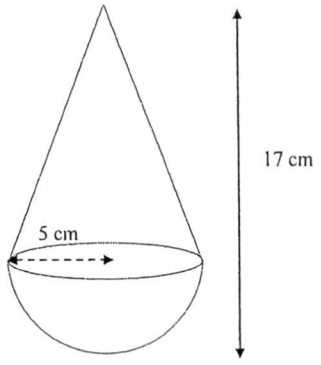
(b) $\cos \angle ACD = -\cos \angle ACB = -\frac{30}{78} = -\frac{5}{13}$

Answer (a) $\sin \angle BAC = \frac{5}{13}$ [2]

(b) $\cos \angle ACD = -\frac{5}{13}$ [1]

20 The diagram shows a solid wooden toy made from a cone and a hemisphere of radius 5cm.
The total height of the toy is 17 cm.
The cost of painting this wooden toy is 0.5 cents per cm^2.

For Examiner's Use



Find the total cost of painting the wooden toy in dollars.

Height of cone = 12

Curved surface of cone = $\pi(5)(13) = 65\pi$

Curved surface area of hemisphere = $2\pi(5)^2 = 50\pi$

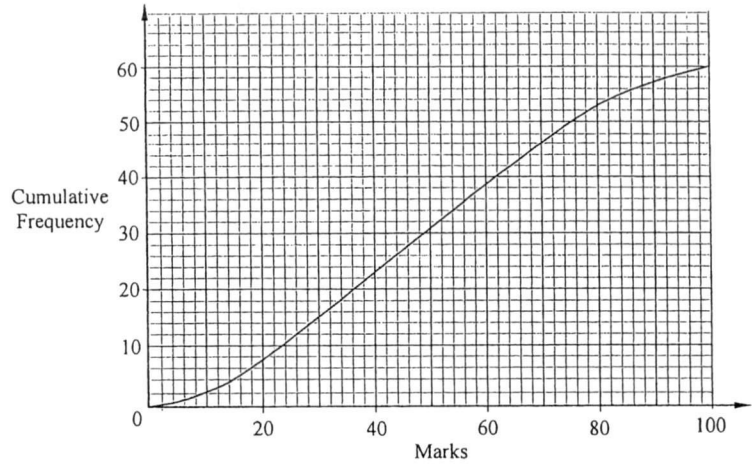
Total Surface Area = 115π

Cost of painting = $115\pi \times 0.5$ cents
= 180.64 cents

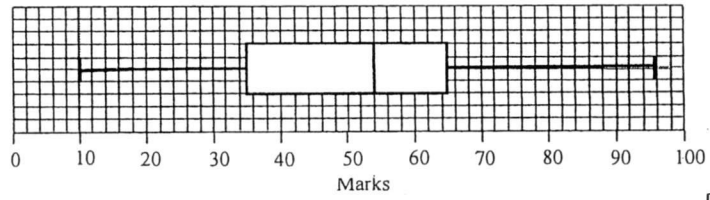
Answer $ 1.81 [3]

21 The cumulative frequency curve below shows the marks obtained, out of 100, by 60 students in an Additional Mathematics test.

For Examiner's Use



The same 60 students also set for the Physics paper.
The box-and-whisker diagram below illustrates the marks obtained.
The maximum mark was again 100.



[2]

Use the two diagrams to complete this table for the two tests.

Subject	Lower Quartile	Median	Upper Quartile	Interquartile Range
Add. Mathematics	30	49	68	38
Physics	35	54	65	30

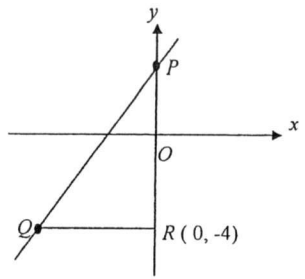
A student commented that the results for Additional Mathematics was better than for Physics.
Do you agree? Give a reason for your answer.

Answer (c) No, Physics has a higher median and a smaller interquartile range.

[1]

22 In the diagram below, R is the point $(0, -4)$ and P is a point on the y-axis. The line PQ meets the horizontal line through R at Q.

For Examiner's Use



(a) Write down the equation of line QR.

(b) Given that the equation of the line PQ is $3y - 7x = 9$, find the coordinates of Q.

(c) Find the area of triangle PQR.

(d) Calculate the perpendicular distance from R to PQ, giving your answer correct to two decimal places.

(b)

when $y = -4$, $\quad -4(3) - 7x = 9$

$$-7x = 21$$

$$x = -3$$

(c) when $x = 0$, $3y = 9$

$$y = 3$$

Coordinates of P = $(0, 3)$

Area of $\Delta PQR = \frac{1}{2} \times 3 \times 7 = 10.5$ units^2

Let d be the perpendicular distance from R to PQ

$PQ = \sqrt{3^2 + 7^2} = \sqrt{58}$

(d) $\frac{1}{2} \times \sqrt{58} \times d = 10.5$

$$d = \frac{21}{\sqrt{58}}$$

Answer (a) $y = -4$ [1]

(b) (-3 , -4) [1]

(c) 10.5 units^2 [2]

(d) 2.76 units [2]

23 A playground is in the shape of a quadrilateral $ABCD$.
It is given that $AB = 90$ m, B is due East of A, $AD = 70$ m, $\angle BAD = 115°$, $\angle ADC = 85°$ and the bearing of C from $B = 010°$.

For Examiner's Use

(a) Complete the scale drawing of the playground, using a scale of 1cm to 10 m. [2]

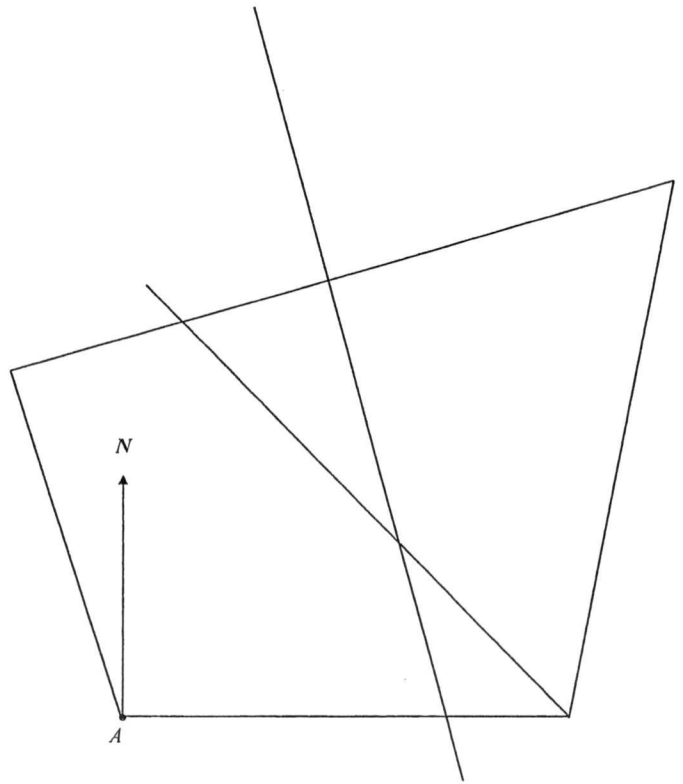
(b) On your drawing, construct

(i) the perpendicular bisector of CD, [1]

(ii) the angle bisector of $\angle ABC$. [1]

(c) A statue , P, is to be built inside the playground such that it is nearer to D than C and equidistant from AB and BC.

Mark and label a possible position of the statue on your drawing. [1]



End of Paper

Answer **all** the questions.

1 **(a)** **(i)** Factorise completely $2a^2 - 11a - 21$. [1]

(ii) **Hence**, factorise completely $2(2b+1)^2 - 22b - 32$. [2]

(b) Express $\frac{6t-8}{3t^2-5t+2} \div \frac{4-3t}{3t-3}$ as a single fraction in its simplest form. [3]

(c) It is given that $w = \frac{x+2t^2}{a}$.

(i) Find w when $x = -2$, $t = -3$ and $a = 19$. [1]

(ii) Express t in terms of w, x and a. [2]

2 The table below shows the distribution of the heights of class of 30 Primary One students.

Height x (cm)	Frequency
$100 \le x < 105$	2
$105 \le x < 110$	6
$110 \le x < 115$	15
$115 \le x < 120$	7

(a) State the modal class of the distribution. [1]

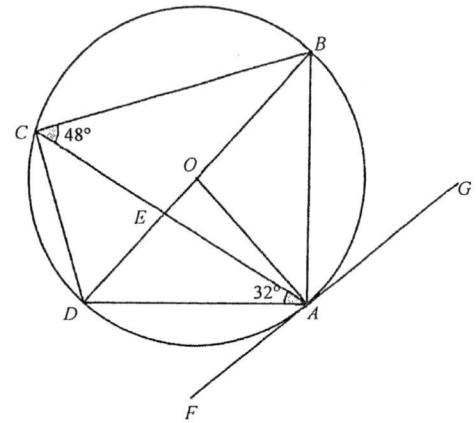
(b) Calculate an estimate of the mean height of the Primary One students. [2]

(c) Calculate an estimate of the standard deviation of their heights. [1]

(d) The standard deviation of the heights of a class of Primary Two students is 3.52. Use this information to comment on one difference between the two distributions. [1]

(e) It was found that the heights of all the Primary One students were measured incorrectly. The correct heights were all 3 cm more than those recorded. Explain how the estimated mean and standard deviation of the heights have been affected by this error. [2]

3 In the diagram, O is the centre of the circle through A, B, C and D. FG is the tangent to the circle at A. AC intersects BD at E. $\angle ACB = 48°$ and $\angle CAD = 32°$.



(a) Calculate the following angles, showing reasons,

(i) $\angle ABO$, [2]

(ii) $\angle BDA$, [1]

(iii) $\angle CDA$, [2]

(iv) $\angle GAB$. [2]

(b) Is BD parallel to GF? Justify your answer. [2]

4 **(a)** A sofa can be bought locally at \$1 200 before a discount of 5%. The same sofa can be purchased from China at RMB5000. However, this overseas purchase will incur a shipping charge of S\$50 and a 7% Goods and Service Tax (GST) based on total charges of the item. The exchange rate is RMB4.84 = S\$1.

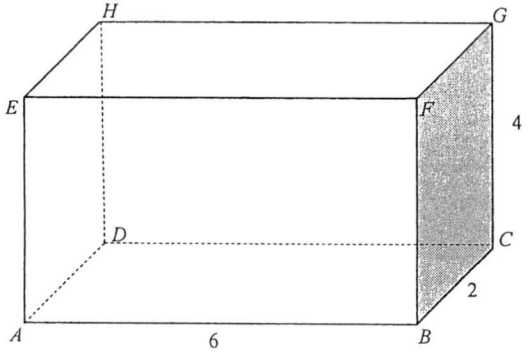
Determine, with clear working, the cheaper option to buy the sofa. [4]

(b) Mr Tan wants to deposit \$50 000 into Bank ABC. ABC offers two deposit plans.

Plan A	Plan B
Simple interest: 3% per annum	Half-yearly compounded interest: x% per annum

Mr Tan calculated that the interest yield from either plan is the same when he deposits the sum for 3 years. Calculate the value of x. [4]

5 The diagram shows a rectangular cuboid $ABCDEFGH$.
$AB = 6$ cm, $BC = 2$ cm and $CG = 4$ cm.



(a) Show that the angle $HBD = 32.3°$, correct to 1 decimal place. [2]

(b) Calculate angle AFC. [3]

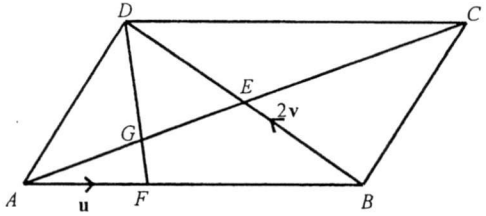
(c) Calculate the greatest angle of elevation of the point H when viewed from any point along AB. [1]

6 **(a)** J is the point $(-2, 4)$. The point K is the result of the translation of J by $\begin{pmatrix} 4 \\ -1 \end{pmatrix}$.

(i) Find the position vector of K. [1]

(ii) Find the equation of line JK. [2]

(b) In the diagram $ABCD$ is a parallelogram. The diagonals AC and BD intersect at E. F is a point on AB such that $AB = 3AF$. G is the midpoint of AE. $\overrightarrow{AF} = \mathbf{u}$ and $\overrightarrow{BD} = 2\mathbf{v}$.



(i) Express the following, as simply as possible, in terms of **u** and/or **v**,

(a) $\overrightarrow{FB}$, [1]

(b) $\overrightarrow{AG}$, [1]

(c) $\overrightarrow{AD}$, [1]

(d) $\overrightarrow{DF}$, [1]

(e) $\overrightarrow{DG}$. [2]

(ii) State two facts about the points D, G and F. [2]

(iii) Calculate the values of

(a) $\dfrac{Area\ of\ \Delta ADF}{Area\ of\ \Delta ADG}$, [1]

(b) $\dfrac{Area\ of\ \Delta ADG}{Area\ of\ ABCD}$. [2]

7 **(a)** The diagram shows part of a number grid.

1	2	3	4	5	6	7	8	9
10	11	12	13	14	15	16	17	18
19	20	21	22	23	24	25	26	27
28	29	30	31	32	33	34	35	36

A rectangle outlining 6 numbers, as shown, can be placed anywhere on the grid.

(i) If n represents the number in the top left corner of the rectangle, write down an expression, in terms of n, for the number in the bottom right of the rectangle. [1]

(ii) Show that the difference in the squares of any two numbers in the same column is always a multiple of 9. [2]

(iii) Find the number in the top right corner of the rectangle given that the sum of the six numbers in the rectangle is 777. [3]

(b) A bag contains 5 red and 7 green balls. Joan removes a ball from the bag. Paul then removes another ball from the bag.

(i) Draw a tree diagram to show the possible outcomes. [2]

(ii) Find, as a fraction in its simplest form, the probability that

(a) both Joan and Paul pick red balls, [1]

(b) Paul picks a green ball, [2]

(c) both Joan and Paul pick different coloured balls. [2]

8 **Answer the whole of this question on a sheet of graph paper.**

The variables x and y are connected by the equation $y = x + \frac{4}{x}$.

Some corresponding values of x and y, correct to one decimal place, are given in the following table.

x	0.5	1	2	3	4	5	6	7	8
y	8.5	5	4	4.3	5	a	6.7	7.6	8.5

(a) Find the value of a. [1]

(b) Using a scale of 2 cm to 1 unit, draw a horizontal x-axis for $0 \le x \le 8$.

Using a scale of 2 cm to 1 unit, draw a vertical y-axis for $0 \le y \le 9$.

On your axes, plot the points given in the table and join them with a smooth curve. [3]

(c) Use your graph to solve $5 - x = \frac{4}{x}$. [2]

(d) By drawing a tangent, find the gradient of the curve at the point $(1, 5)$. [2]

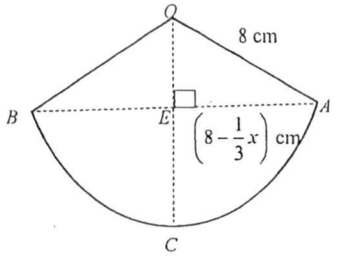
(e) **(i)** On the same axes, draw the graph of $y = -x + 7$. [1]

(ii) Write down the x-coordinate of the points at which the two graphs intersect. [1]

(iii) These values are solutions of the equation $2x^2 + ax + b = 0$.

Find the values of a and b. [2]

9 OAB is a sector of a circle, with centre O and radius 8 cm.
OC cuts AB at E. It is given that $AE = (8 - \frac{1}{3}x)$ cm and $\angle OEA = 90°$.



(a) Given that $EC = x$ cm, express OE in terms of x. [1]

(b) Hence, by making use of your answer in (a), write an equation in terms of x and show that it reduces to $5x^2 - 96x + 288 = 0$. [3]

(c) Solve the equation $5x^2 - 96x + 288 = 0$, giving your answers correct to two decimal places. [3]

(d) Calculate the length of AB. [1]

(e) Calculate angle AOB in terms of radians. [1]

(f) A cone is made from using the sector by joining OA to OB. Calculate the radius of the circular base of this cone. [3]

10 Mr Loh bought a new car.

(a) Mr Loh made a downpayment of $60 000 and had to pay 48 months of instalments of $1 200 each for the car.
Find the original price of the car given that the simple interest of the loan amount was 2.5% per annum, giving your answer to the nearest dollar. [3]

The following shows some information he found from the internet about his new car's fuel capacity and fuel efficiency.

Fuel Tank Capacity: 15 gallons

Fuel Efficiency:

City Driving: 22 miles per gallon
Expressway Driving: 26 miles per gallon

(b) In Singapore, fuel is sold by the litre and distance is measured by kilometre. Convert the above information for the fuel tank capacity to litres and fuel efficiency for both the city and expressway driving to kilometres per litre given that 1 gallon = 3.7854 litres and 1 mile = 1.60934 kilometres. [3]

(c) Mr Loh is deciding between a self-drive trip to Melaka in Malaysia or going by takin the bus coach for his family of 5 people.
He calculated that the driving distance from Singapore to Melaka is about 240km, of which 210km is via expressway driving. Petrol is priced at $1.50 per litre. Toll fee is about $50 per way. Coach fare per person per way is $35.
Determine, with clear working, which mode of travelling should Mr Loh decide to save money. [4]

End of Paper 2

1(a)(i) $2a^2 - 11a - 21$

$= (2a + 3)(a - 7)$ [B1]

(ii) $2(2b + 1)^2 - 22b - 32$

$= [2(2b + 1) + 3][2b + 1 - 7]$ [M1]

$= (4b + 5)(2b - 6)$

$= 2(4b + 5)(b - 3)$ [A1]

(b) $\dfrac{6t - 8}{3t^2 - 5t + 2} \div \dfrac{4 - 3t}{3t - 3}$

$= \dfrac{2(3t - 4)}{(3t - 2)(t - 1)} \times \dfrac{3(t - 1)}{4 - 3t}$ [factorise first denominator M1]

$= \dfrac{2(3t - 4)}{(3t - 2)(t - 1)} \times \dfrac{3(t - 1)}{-(3t - 4)}$ [change sign M1]

$= \dfrac{2(3)}{-(3t - 2)}$

$= \dfrac{6}{2 - 3t}$ [A1]

(c)(i) $w = \dfrac{x + 2t^2}{a}$

$w = \dfrac{-2 + 2(-3)^2}{19}$

$w = 0.842 (3sf)$ *or* $\dfrac{16}{19}$ [A1]

(ii) $w = \dfrac{x + 2t^2}{a}$

$wa = x + 2t^2$

$2t^2 = wa - x$ [M1]

$t = \pm\sqrt{\dfrac{wa - x}{2}}$ [A1]

2(a) $110 \leq x \leq 115$ [B1]

(b) Estimated mean $= \dfrac{102.5(2) + 107.5(6) + 112.5(15) + 117.5(7)}{30}$ [M1]

$= 112$ cm [A1]

(c) Standard deviation = 4.15 [B1]

(d) The heights of the Primary 2 students are more consistent [smaller spread] as the standard deviation is smaller. [B1]

(e) Estimated Mean will remain the same. [A1]

Standard deviation will reduce to 4.08. [A1]

3(a)(i) $\angle BOA = 48^\circ \times 2$ (angle at centre = 2 angle at circumference)

$= 96^\circ$ [M1]

$\angle ABO = (180° - 96°) \div 2$ (base angles of isos triangle)

$= 42°$ [A1]

Or $\angle ADB = 48^\circ$ (angles in same segment)

$\angle BAD = 90^\circ$ (right angle in semi-circle) [M1]

$\angle ABO = 180° - 48° - 90°$ (angles sum of triangle)

$= 42°$ [A1]

(ii) $\angle BDA = 48^\circ$ (angles in same segment) [A1]

(iii) $\angle DCA = 42^\circ$ (angles in same segment) [M1]

$\angle CDA = 180° - 42° - 32°$ (angles sum of triangle)

$= 106°$ [A1]

Or $\angle CBD = 32^\circ$ (angles in same segment) [M1]

$\angle CDA = 180° - 42° - 32°$ (angles in opposite segment)

$= 106°$ [A1]

(iv) $\angle GAB = 48°$ (alt segment theorem) [B2]

Or $\angle OAG = 90°$ (tan ⊥ rad) [M1]

$\angle OAB = 42°$ (base angle of isoc triangle)

$\angle GAB = 90° - 42° = 48°$ [A1]

(b) No. $\angle GAB = 48°$ $\angle DBA = 42°$ [M1]

Since they are not equal, they cannot be alternate angles of a set of parallel lines, hence, BD is not parallel to GF. [A1]

4(a) Cost of sofa when bought locally = $\$1200 \times \frac{95}{100} = \$1140$ [M1]

Cost of sofa when bought in China = $\left(\frac{5000}{484} + 50\right) \times 1.07 = \$1158.87 (2dp)$ [M2]

Cheaper option is to buy locally as the cost is lower. [A1]

3(b) Interest from Plan A = $\frac{3}{100} \times \$50000 \times 3$

$= \$4500$ [M1]

$4500 + 50000 = 50000(1 + \frac{x}{2 \times 100})^6$ [M1]

$\frac{109}{100} = (1 + \frac{x}{200})^6$

$1 + \frac{x}{200} = \sqrt[6]{\frac{109}{100}}$ [M1]

$\frac{x}{200} = \sqrt[6]{\frac{109}{100}} - 1$

$x = 2.89$ $(3sf)$ [A1]

5(a) $BD = \sqrt{(6)^2 + (2)^2} = \sqrt{40}$ [M1]

$\tan \angle HBD = \frac{4}{\sqrt{40}}$ [M1]

$\angle HBD = 32.3°$ (1dp)

(b) $AC = BD = \sqrt{40}$

$AF = \sqrt{6^2 + 4^2} = \sqrt{52}$

$FC = \sqrt{2^2 + 4^2} = \sqrt{20}$ [Any 2, M1]

$\cos \angle AFC = \frac{40 - 52 - 20}{-2\sqrt{52}\sqrt{20}}$ [M1]

$\angle AFC = 60.3°$ (1dp) [A1]

c) Greatest angle of elevation = $\tan^{-1}\left(\frac{4}{2}\right) = 63.4°$ (1dp) [A1]

6(a) (i) $\overrightarrow{JO} + \overrightarrow{OK} = \overrightarrow{JK}$

$\begin{pmatrix} 2 \\ -4 \end{pmatrix} + \overrightarrow{OK} = \begin{pmatrix} 4 \\ -1 \end{pmatrix}$

$\overrightarrow{OK} = \begin{pmatrix} 2 \\ 3 \end{pmatrix}$ [A1]

(ii) Gradient of $JK = -\frac{1}{4}$ [M1]

$y - 4 = -\frac{1}{4}(x + 2)$

$y = -\frac{1}{4}x + \frac{7}{2}$ [A1]

(b)(i)(a) $\overrightarrow{FB} = 2\mathbf{u}$ [B1]

(b) $\overrightarrow{AG} = \frac{1}{2}\overrightarrow{AE} = \frac{1}{2}(3\mathbf{u} + \mathbf{v})$ [A1]

(c) $\overrightarrow{AD} = 3\mathbf{u} + 2\mathbf{v}$ [B1]

(d) $\overrightarrow{DF} = -2(\mathbf{u} + \mathbf{v})$ [B1]

(e) $\overrightarrow{DG} = -3\mathbf{u} - 2\mathbf{v} + \frac{1}{2}(3\mathbf{u} + \mathbf{v})$ [M1]

$= -\frac{3}{2}\mathbf{u} - \frac{3}{2}\mathbf{v}$

$= -\frac{3}{2}(\mathbf{u} + \mathbf{v})$ [A1]

(ii) $\overrightarrow{DG} = \frac{3}{4}\overrightarrow{DF}$

D, F and G are collinear. [A1]

$DG = \frac{3}{4}DF$ [A1]

(iii)(a) $\frac{\text{Area of } \Delta ADF}{\text{Area of } \Delta ADG} = \frac{4}{3}$ [B1]

(b) $\frac{\text{Area of } \Delta ADG}{\text{Area of } \Delta DE} = \frac{1}{2}$

$\frac{\text{Area of } \Delta ADG}{\text{Area of ABCD}} = \frac{1}{2 \times 4}$ [M1]

$= \frac{1}{8}$ [A1]

7 (a)(i) $n+11$ [B1]

(ii) $(n+9)^2 - n^2$ [M1]

$= n^2 + 18n + 91 - n^2$

$= 18n + 81$

$= 9(2n+9)$ [M1]

It is a multiple of 9.

(iii) Sum of numbers $= n+n+1+n+2+n+9+n+10+n+11$

$= 6n+33$ [M1]

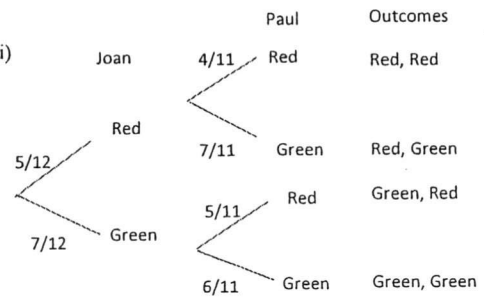
$6n+33 = 777$

$6n = 744$

$n = 124$ [M1]

Number in top right corner of rectangle = 126 [A1]

(b) (i)



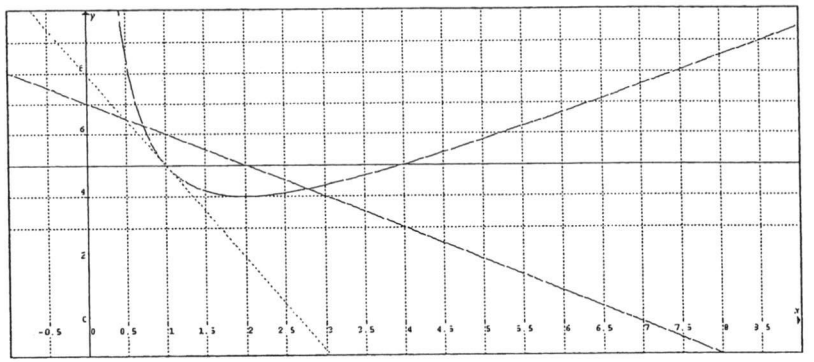
A1 for fractions, A1 for outcomes

(ii)(a) P(both red balls) $= \frac{5}{12} \times \frac{4}{11} = \frac{5}{33}$ [A1]

(b) P(Paul picks green ball) $= \frac{5}{12} \times \frac{7}{11} + \frac{7}{12} \times \frac{6}{11}$ [M1]

$= \frac{7}{12}$ [A1]

(c) P(both different coloured balls) $= \frac{5}{12} \times \frac{7}{11} + \frac{7}{12} \times \frac{5}{11}$ [M1] $= \frac{35}{66}$ [A1]



Q9 (a). $a = 5.8$ [B1]

(c) insert y= 5, therefore x = 1 and 4 [A1 each]

(d) Gradient $= -3$ Accept range from -3.938 to -2.3057 [A1] , tangent line [M1]

(e)(ii) x= 0.7 and x =2.8

(iii) $x + \frac{4}{x} = -x + 7$

$2x + \frac{4}{x} - 7 = 0$

$2x^2 - 7x + 4 = 0$ [M1]

a = -7, b =4 [A1]

9(a) $OE = (8- x)$cm [B1]

(b) $8^2 = (8-x)^2 + (8-\frac{1}{3}x)^2$ [M1]

$64 = 64 - 16x + x^2 + 64 - \frac{16}{3}x + \frac{1}{9}x^2$ [M1[

$10x^2 - 192x + 576 = 0$ [M1]

$5x^2 - 96x + 288 = 0$ (shown)

(c) $x = \frac{-(-96) \pm \sqrt{(-96)^2 - 4(5)(288)}}{2(50}$ [M1]

= 15.48 or 3.72 (2dp) [A1, A1]

(d) AB = $2\left[8 - \frac{1}{3}(3.72122\right] = 13.5cm(3sf)$ [A1]

(e) $\sin \angle EOA = \frac{8 - \frac{1}{3}(3.72122)}{8}$ [M1]

$\angle EOA = \sin^{-1}\left[\frac{8 - \frac{1}{3}(3.72122)}{8}\right.$

$\angle AOB = 2\angle EOA = 2.01rad$ [A1]

(f) arc AB =circumference of base of cone = 8 (2.0143) cm

Let radius be r.

$8(2.0143) = 2\pi r$

r =2.56 cm (3sf)

10(a) Total instalment paid = $48 \times 1200 = \$57600$ [M1]

Total Interest paid $= \frac{57600}{110} \times 10$ [M1]

=\$5236.3636..

Original car price = \$60000 +\$57600-\$5236.36363

=\$112364 (nearest dollar) [A1]

(b) Fuel capacity = 15×3.7854 =56.781 litres [B1]

City driving fuel efficiency= $\frac{22 \times 1.60934}{3.7854} = 9.35316.. = 9.35$ km/l [A1]

Expressway driving fuel efficiency = $\frac{26 \times 1.60934}{3.7854} = 11.0537.. = 11.1$ km/l [A1]

(c) Cost of coach fare = $5 \times 35 \times 2 = \$350$ [M1]

Cost of self-drive $= \left[\frac{210}{11.0537} + \frac{30}{9.35316}\right] \times 1.5 \times 2 + 100]$

= \$166 (nearest dollar) [M1]

Should self-drive as it is cheaper. [A1]

NORTH VISTA SECONDARY SCHOOL

MID-YEAR EXAMINATION 2017

80

NAME: _____ ( ) CLASS : _____

SUBJECT: MATHEMATICS (PAPER 1) DATE : 2 MAY 2017

LEVEL/STREAM :SECONDARY 4 EXPRESS/ SECONDARY 5 NORMAL ACADEMIC TIME : 2 HOURS

CODE : 4016/01

READ THESE INSTRUCTIONS FIRST

Write your name, register number and class on all the work you hand in.
Write in dark blue or black pen.
You may use a pencil for any diagrams or graphs.
Do not use staples, paper clips, highlighters, glue or correction fluid.

Answer all questions.
If working is needed for any question it must be shown with the question.
Omission of essential working will result in loss of marks.

The use of an approved scientific calculator is expected, where appropriate.
If the degree of accuracy is not specified in the question, and if the answer is not exact, give the answer to three significant figures. Give answers in degrees to one decimal place.
For π, use either your calculator value or 3.142, unless the question requires the answer in terms of π.

The number of marks is given in brackets [ ] at the end of each question or part question.
The total of the marks for this paper is 80.

For Examiner's Use	
Category	**Question No.**
Accuracy	
Brackets	
Fractions	
Units	
Others	
Marks Deducted	

This paper consists of **18** printed pages.

[Turn Over

Mathematical Formulae

Compound Interest

Total amount = $P\left(1+\frac{r}{100}\right)^n$

Mensuration

Curved surface area of a cone = πrl

Surface area of a sphere = $4\pi r^2$

Volume of a cone = $\frac{1}{3}\pi r^2 h$

Volume of a sphere = $\frac{4}{3}\pi r^3$

Area of triangle $ABC = \frac{1}{2}ab\sin C$

Arc length = $r\theta$, where θ is in radians

Sector area = $\frac{1}{2}r^2\theta$, where θ is in radians

Trigonometry

$$\frac{a}{\sin A}=\frac{b}{\sin B}=\frac{c}{\sin C}$$

$$a^2 = b^2 + c^2 - 2bc\cos A$$

Statistics

$$\text{Mean} = \frac{\sum fx}{\sum f}$$

$$\text{Standard deviation} = \sqrt{\frac{\sum fx^2}{\sum f} - \left(\frac{\sum fx}{\sum f}\right)^2}$$

Answer **all** the questions.

1 **(a)** Simplify $10-2(x-3)+x$.

Answer ..... [1]

(b) 150 g of beans contains x g of proteins. Find an expression for the amount of proteins, in grams, in terms of x and z, for z kg of beans.

Answer .....g [1]

---

2 Factorise completely $3(2a-1)-4a^2+2a$.

Answer ..... [2]

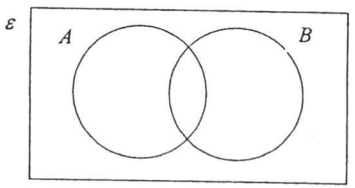
---

3 **(a)** $\varepsilon = \{x : x \text{ is a positive integer and } x < 15\}$
$A = \{x : x \text{ is a factor of } 12\}$
$B = \{x : 15-2x > 1\}$

Find $n(A' \cap B)$.

Answer ..... [1]

(b) On the Venn diagram, shade the region which represents $(A \cap B) \cup A'$. [1]



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4 A worker needs to cover a wall completely with square tiles of the same size. The dimension of the wall is 245 cm by 280 cm.

(a) Find the largest possible length of the side of a square tile.

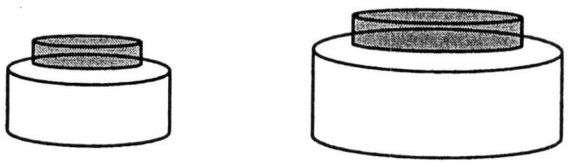
Answer .....cm [1]

(b) Find the total number of tiles that are needed to cover the wall completely.

Answer ..... [1]

---

5 A shop sells bottles of chilli sauce in two sizes as shown below.
The two bottles are geometrically similar. The height of the larger bottle is 15 cm and the height of the smaller bottle is 12 cm.



The smaller bottle costs \$9.60. The cost of the bottles are proportional to their capacities. Calculate the cost of the larger bottle, given that a discount of 20% is given for buying the larger bottle.

Answer \$..... [2]

---

6 Write as a single fraction in its simplest form $\frac{2}{9x^2-1}+\frac{1}{1-3x}$.

Answer ..... [2]

7 The mean mass of a golf ball and a tennis ball is 2.5×10^{-2} kg.
The mass of the tennis ball is 2×10^{-3} kg.

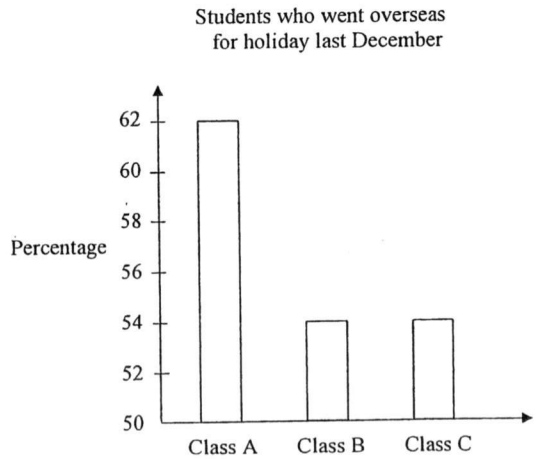
Find the ratio of the mass of the golf ball to the mass of the tennis ball.

Answer ..... [2]

8 A company invested \$625000 into an investment account which pays 2% per annum compound interest compounded monthly. Find the total amount in the account after 2 years, giving your answer correct to the nearest 10 cents.

Answer \$..... [2]

9 Jordan draws this graph to show the percentage of students from class A, B and C who went overseas for holiday last December.



Show one aspect of the graph that is misleading and explain how this may lead to a misinterpretation of the graph.

Answer .....

.....

..... [2]

10 **(a)** Simplify $\left(\frac{x^6}{64}\right)^{-\frac{1}{3}}$.

Answer ..... [1]

(b) Given that $a^5 \times a^{-2} \div \sqrt[3]{a^2} = a^k$, find the value of k.

Answer $k =$ ..... [1]

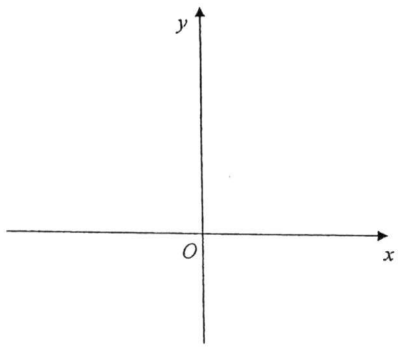
---

11 **(a)** Express $14 - 6x + x^2$ in the form $(x-h)^2 + k$.

Answer ..... [1]

(b) On the axes provided, sketch the graph of $y = 14 - 6x + x^2$. [2]

Answer



(c) Write down the equation of the line of symmetry for the graph $y = 14 - 6x + x^2$.

Answer ..... [1]

---

12 A model of a space shuttle is made using a scale of 1 : 150.

(a) The actual length of the space shuttle is 35.5 m long.
Find the length, in centimetres, of the model.

Answer .....cm [1]

(b) On the model, the area of the tail section painted red is 60 cm^2.
Find the actual area of the tail section that is painted red, giving your answer in square metres.

Answer .....m^2 [2]

---

13 The initial temperature of two substances X and Y at 08 00 were 28°C and −60°C respectively.
Substance X was cooled at a steady rate. After 5 minutes, its temperature was −12°C.
Substance Y was heated at a steady rate of 3°C per minute.

(a) Find the time, when the temperature of both substances will be the same.

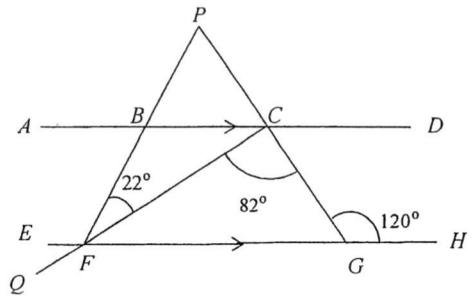
Answer ..... [2]

(b) Hence, find the temperature at this time.

Answer .....°C [1]

---

14 In the diagram, $ABCD$ and $EFGH$ are parallel and QFC, PBF and PCG are straight lines.
Angle $PFC = 22°$, angle $FCG = 82°$ and angle $CGH = 120°$.



(a) Calculate angle EFQ.

Answer ..... [1]

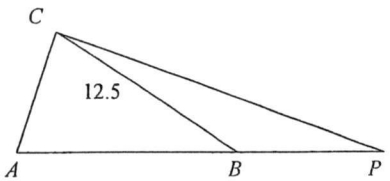
(b) Explain why $PB = PC$.

Answer .....

.....

..... [3]

15 In the diagram, ABC is an isosceles triangle, $BA = BC = 12.5$ cm, $\sin\angle PBC = \frac{7}{25}$ and AB is produced to P.



(a) Calculate the area of triangle ABC.

Answer .....cm^2 [2]

(b) Calculate the length from B where C is the nearest to the line AB.

Answer .....cm [2]

16 **(a)** The line $ax + by + 1 = 0$ is parallel to the x-axis and passes through the point $(-5, 4)$.
Find the values of a and b.

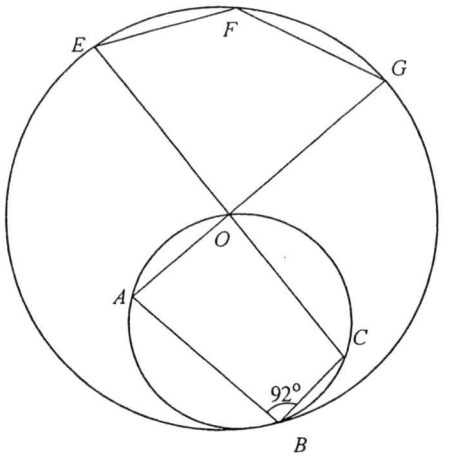
Answer $a =$ .....

$b =$ ..... [2]

(b) A is a point $(2, -1)$ and B is the point $(0, 5)$.
The line AB meets the straight line $x = 3$ at C.
Find the coordinates of C.

Answer (.....,.....) [2]

17 The figure shows two circles. In $ABCO$, $\angle ABC = 92^\circ$. Circle $BEFG$ has centre O.
EOC and AOG are straight lines.



(a) Calculate the angle OEG, giving reasons for your answer.

Answer .....

.....

..... [2]

(b) Explain whether it is possible for a circle to be drawn passing through the points E, F, G and O.

Answer .....

.....

..... [3]

18 When written as the product of their prime factors,

$$p = 3^2 \times 5, \quad q = 3^2 \times 7, \quad r = 3^m \times 5^n \times 7$$

(a) Find the LCM of p and q, giving your answer as the product of its prime factors.

Answer ..... [1]

(b) Given that $\frac{pq}{k}$ is a perfect cube, find the smallest possible integer value of k.

Answer $k =$ ..... [1]

(c) Find the greatest number that will divide p and q exactly.

Answer ..... [1]

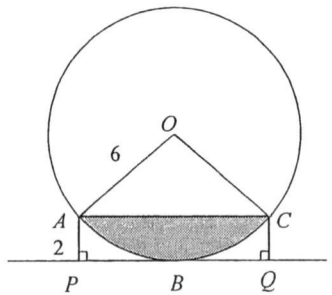
(d) State, with reason(s) if the product of p, q and r is an odd or even number.

Answer .....

..... [1]

---

19 The diagram shows a circle with centre O and radius 6 cm passing through the points A, B and C. $AP = CQ = 2$ cm, $\angle APB = \angle BQC = 90^\circ$ and the circle touches the line PQ at the point B. Calculate the area of the shaded region.



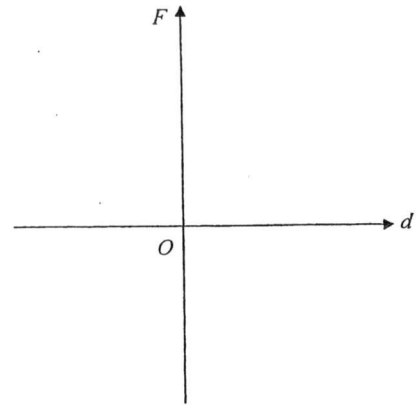
Answer .....cm^2 [5]

---

20 The force, F newtons, between 2 particles is inversely proportional to the square of the distance, d metres, between them.

(a) Sketch a graph to represent the relation between the force and the distance between the particles. [1]

Answer



The force is 3 newtons when the particles are 6 m apart.

(b) Find the equation for F in terms of d.

Answer ..... [2]

(c) Calculate the distance between the particles when the force is 12 newtons.

Answer .....m [1]

---

21 It is given that A is the point $(2,-7)$, $\overrightarrow{BA} = \begin{pmatrix} 12 \\ -16 \end{pmatrix}$ and $\overrightarrow{CD} = \begin{pmatrix} -3 \\ m \end{pmatrix}$.

Find

(a) $\left|\overrightarrow{BA}\right|$,

Answer ..... [1]

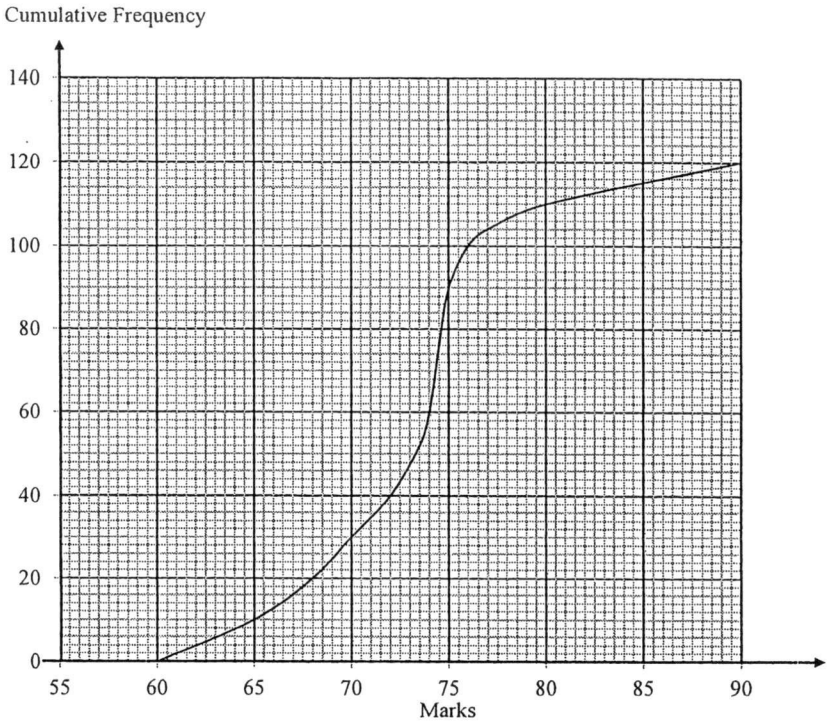
(b) the coordinates of B,

Answer ..... [1]

(c) the value of m if CD is parallel to BA.

Answer $m =$ ..... [2]

22 The cumulative frequency curve below illustrates the marks obtained by 120 students in an English test.



(a) Use your graph to find the interquartile range.

Answer ..... [2]

(b) If only 15% of the students scored a distinction, find the least marks for a student to obtain a distinction.

Answer ..... [2]

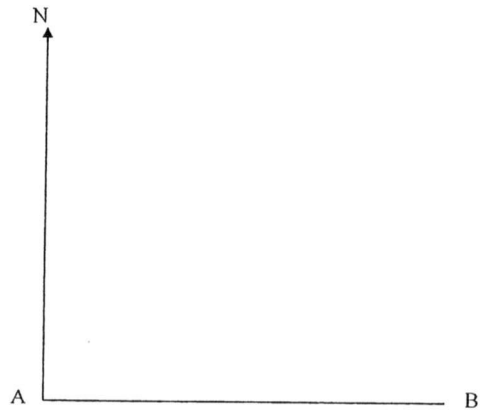
(c) Two students are chosen at random. Find the probability that one student scored above 80 marks and the other student scored below 65 marks.

Answer ..... [2]

23 The scale drawing shows the positions of two buildings A and B.
Building B is due east of building A.

(a) Building C is due north of building B and on a bearing of 055° from building A. [1]
Mark and label the position of building C.

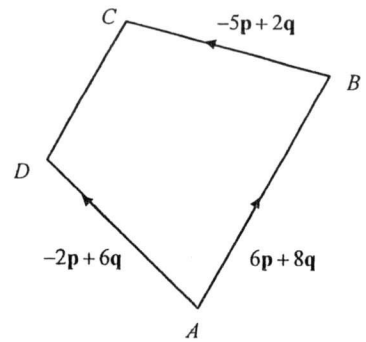
(b) Building D is equidistant from the lines AB and AC and equidistant from the points B and C. By constructing suitable line(s), find and label the position of building D. [2]



(c) Given that the scale of the diagram is 1 cm to 100 m, calculate the actual distance of AD in metres.

Answer .....m [1]

24 In the diagram, $\overrightarrow{AB} = 6\mathbf{p} + 8\mathbf{q}$, $\overrightarrow{BC} = -5\mathbf{p} + 2\mathbf{q}$ and $\overrightarrow{AD} = -2\mathbf{p} + 6\mathbf{q}$.



(a) Express $\overrightarrow{DC}$ in terms of **p** and **q**.

Answer ..... [1]

(b) Explain why *ABCD* is a trapezium.

Answer .....

..... [2]

(c) Find the ratio $\left|\overrightarrow{DC}\right| : \left|\overrightarrow{AB}\right|$.

Answer ..... [1]

(d) Find the numerical value of

(i) $\dfrac{\text{area of triangle } ADC}{\text{area of triangle } ABC}$,

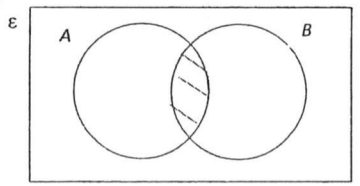
Answer ..... [1]

(ii) $\dfrac{\text{area of triangle } ADC}{\text{area of trapezium } ABCD}$.

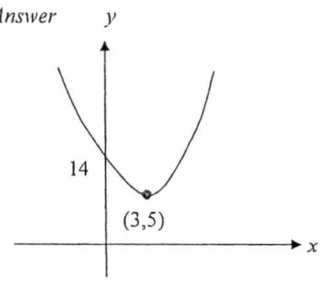
Answer ..... [1]

End of Paper

Mid-year Exam 2017- E Math Paper 1 (Marking Scheme)

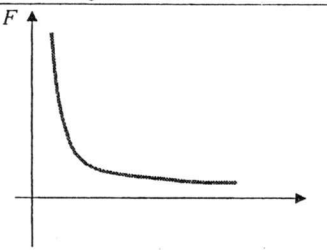
Qn.	Solutions	Marks
1(a)	$x-2(3-x)+10$ $=x-6+2x+10$ $=3x+4$	 B1
1(b)	y hours $-$ 5 km $60y$ mins -5 km 1 min $-\dfrac{5}{60y}$ km Distance $=\dfrac{5x}{60y}=\dfrac{x}{12y}$ km	 B1
2(a)	a^3+a^2-a-1 $=a^2(a+1)-(a+1)$ $=(a^2-1)(a+1)=(a-1)(a+1)^2$	 M1 A1
3(a)	$\varepsilon=\{1,2,3,4,5....,14\}$ $A=\{1,2,3,4,6,12\}$ $A'=\{5,7,8,9,10,11,13,14\}$ $B=\{1,2,3,4,5,6\}$ $A'\cap B=\{5\}$	 B1
3(b)		B1
4(a)	HCF 245 and 280 = 5×7 Largest possible length = 35 cm	 B1
4(b)	Total number of tiles = $8\times7=56$	B1
5	Cost of the bigger bottle $=\left(\dfrac{15}{12}\right)^3\times9.60$ $=\$18.75$ (exact) After discount $=\dfrac{18.75}{100}\times80$ $=\$15$ (exact)	M1 A1

6	$\dfrac{2}{9x^2-1}+\dfrac{1}{1-3x}$ $=\dfrac{2}{(3x+1)(3x-1)}-\dfrac{1}{3x-1}$ $=\dfrac{2-(3x+1)}{(3x+1)(3x-1)}$ $=\dfrac{2-3x-1}{(3x+1)(3x-1)}=\dfrac{1-3x}{(3x+1)(3x-1)}$ $=\dfrac{-(3x-1)}{(3x+1)(3x-1)}=\dfrac{-1}{(3x+1)}$	 M1 *Common denominator* A1
7	Total mass $=2.5\times10^{-2}\times2=5\times10^{-2}$ Mass of golf ball $=5\times10^{-2}-2\times10^{-3}=4.8\times10^{-2}$ $\dfrac{4.8\times10^{-2}}{2\times10^{-3}}=24$	 M1 A1
8	Total amount $=625000\left(1+\dfrac{\frac{1}{6}}{100}\right)^{24}$ $=650485.0747\approx\$650485.10$	M1 A1
9	Misleading feature –The baseline is missing in this graph and it starts at some arbitrary number (that is 50 in this case instead of zero. Accept – The vertical scale did not start from zero Effect- It looks like the percentage of students who went overseas from Class A is **2 times more** than that for Class B & C.	B1 B1
10 (a)	$\left(\dfrac{125}{a^6}\right)^{-\frac{1}{3}}=\left(\dfrac{a^6}{125}\right)^{\frac{1}{3}}=\dfrac{a^2}{5}$	B1
10 (b)	$\dfrac{a^4\times a^{-1}}{\sqrt[3]{a^2}}=a^k$ $a^{4-1-\frac{2}{3}}=a^k$ $a^k=a^{\frac{7}{3}}\Rightarrow k=\dfrac{7}{3}$ or $2\dfrac{1}{3}$	 B1
11 (a)	$14-6x+x^2=(x-3)^2+5$	B1

11 (b)	*Answer*  	y-intercept & shape correct B1 Min point correctly indicated B1
11 (c)	$x = 3$	B1
12 (a)	$\frac{35.5}{150} = 23\frac{2}{3} cm$	B1
12 (b)	$1\ \text{cm}^2 - 2.25\text{m}^2$ $2.25 \times 60 = 135\ \text{cm}^2$	M1,A1
13 (a)	$28^0C \Rightarrow -12^0C$ Every 1 min, temperature will drop 8^0C Let t be the time when the temperature is the same for X and Y $28 - 8t = -60 + 3t$ $88 = 11t$ $t = 8$ mins Time is 08 08	M1 –use of 8^0C A1
13 (b)	$-60 + 3(8) = -36^0C$	B1
14 (a)	$\angle CFG = 120^o - 82^o = 38^o$ (ext angle of a triangle) $\angle EFQ = 38^o$ (vert. opp angles)	B1

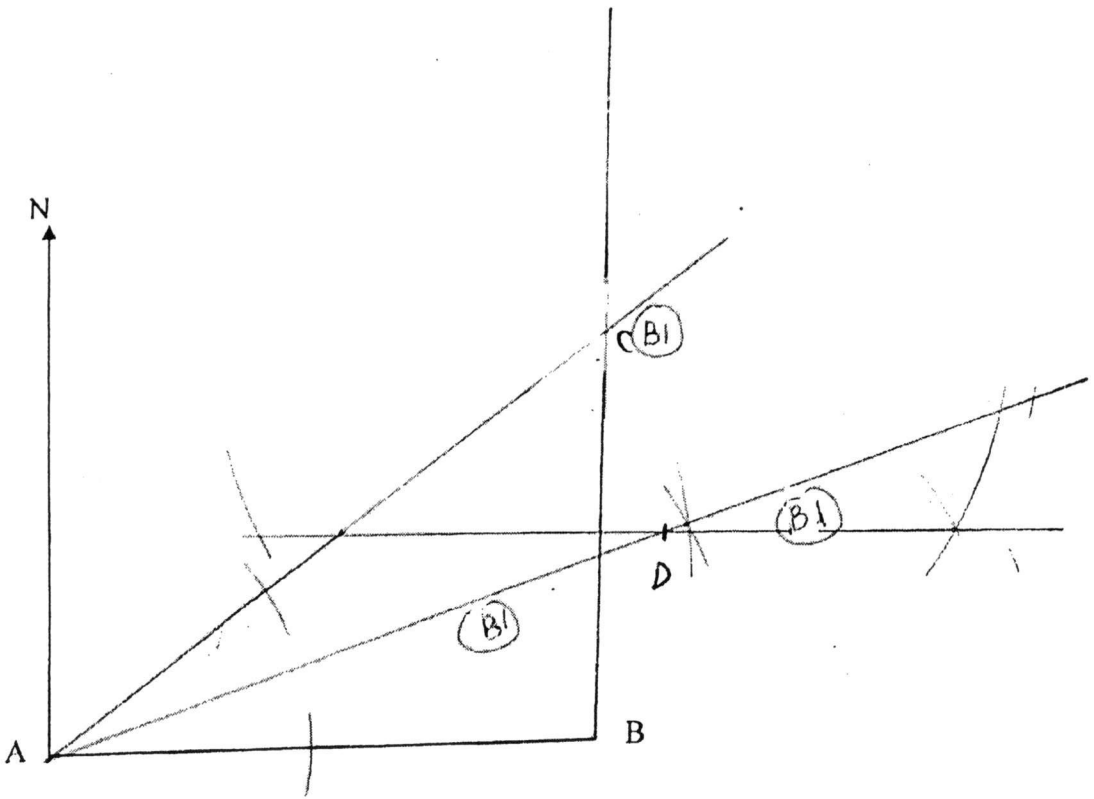
14 (b)	$\angle PBC = \angle BFC = 22^o + 38^o = 60^o$ (corr angles) $\angle PCB = \angle PGF$ (corr angles) $\angle PGF = 180^o - 120^o$ (angles on str line) $\therefore \angle PCB = 60^o$ Since $\angle PBC = \angle PCB = 60^o$, ΔPBC is a isos/equilateral triangle $\Rightarrow PB = PC$	B1 B1 B1 – mention isos/equilateral triangle & $PB = PC$
15 (a)	Area of triangle ABC $= \frac{1}{2}(12.5)^2 \sin\angle ABC$ $= 27\frac{7}{8}\text{cm}^2$ M1 - use $\sin\angle ABC = \sin\angle PBC$	 M1 A1
15 (b)	Let x be the perpendicular height $\frac{1}{2} \times AB \times x = 27\frac{7}{8} \Rightarrow x = 3.5\text{cm}$ $d^2 + 3.5^2 = 12.5^2$ $d = 12cm$	 M1- for x A1
16 (a)	$ax + by + 1 = 0$ $y = -\frac{a}{b}x - \frac{1}{b}$ *Parallel* to x-axis $\Rightarrow$ gradient =0 $y = -\frac{1}{b}$ $-\frac{1}{b} = 4 \Rightarrow b = -\frac{1}{4}$ and $a = 0$	 B1, B1
16 (b)	$Gradient = \frac{5+1}{0-2} = -3$ $y = -3x + 5$ *Sub* $x = 3$ $y = -3(3) + 5 = -4$ $C(3, -4)$	 M1 A1

17	$\angle ADC = 180^o - 92^o = 88^o$ (angles in opp segment) $\angle EDG = 88^o$ (*vert* opp angles) $\angle DEG = \frac{180^o - 88^o}{2} = 46^o$ (*isos* triangle)	B1 – any 2 correct reasons B1 - answer
17 (b)	Reflex $\angle EDG = 360^o - 88^o = 272^o$ (angle at a point) $\angle EFG = \frac{272^o}{2} = 136^o$ (angle at the centre = twice angle at circumference) $\angle EFG + \angle EDG = 136^o + 88^o$ $= 224^o \neq 180^o$ angle in opp segment does not apply $\Rightarrow$ Not possible for a circle to pass through E, F, G and D	B1 B1 – Use angles in the opp segment B1
18 (a)	$p = 3^2 \times 5,\ q = 3^2 \times 7$ $LCM = 3^2 \times 5 \times 7$	B1
18 (b)	$\frac{pq}{k} = 3^4 \times 5 \times 7 \times \frac{1}{k}$ is a perfect cube $k = 3 \times 5 \times 7 = 105$	B1
18 (c)	$p = 3^2 \times 5,\ q = 3^2 \times 7$ $HCF = 3^2 = 9$	B1
18 (d)	$pqr = 3^{4+m} \times 5^{1+n} \times 7^2$ p, q and r are expressed as a product of prime factors that are odd numbers, thus p, q and r are odd numbers. The product of 3 odd numbers is still an odd number	B1

19	$\angle AOB = \cos^{-1}\frac{4}{6}$ $\angle AOC = \cos^{-1}\frac{4}{6} \times 2 = 96.379$ or $\angle AOC = 83.621$ Area of sector $= \frac{96.3793}{360} \times \pi \times 6^2 = 30.278$ or Area of sector $= 26.270$ Area of triangle $= \frac{1}{2}6^2 \sin 96.3793 = 17.888$ or Area of triangle $= 4\sqrt{20}$ / Area of segment $= 30.278 - 17.888 = 12.4cm^2$ *or* Area of segment $= 8.38cm^2$	B1 B1 B1 B1 B1	Many left the question blank. For students who assume angle AOC as 90^0 or 60^0 no marks are awarded. However, for wrong value of angle AOC, ecf is awarded for correct use of formula for area of triangle and area of sector
20(a)		B1	A few students obtained the correct answer.
20(b)	$3 = \frac{k}{6^2} \Rightarrow k = 108$ $F = \frac{108}{d^2}$	M1 A1	Common error $3 = \frac{k}{6} \Rightarrow k = 18$ $F = \frac{18}{d}$ Weaker students left the question blank
20(c)	$F = \frac{108}{d^2}$ $12 = \frac{108}{d^2}$ $d^2 = \frac{108}{12} \Rightarrow d = \pm 3,\ d = 3$	B1	For those who did not obtain correct answer for (b) also did not obtain the correct answer for (c) Weaker students left the question blank
21(a)	$\sqrt{12^2 + (-16)^2} = 20units$	B1	Majority obtain the correct answer

21(b)	$\overrightarrow{OB} = \overrightarrow{OA} - \overrightarrow{AB}$ $= \begin{pmatrix} 2 \\ -7 \end{pmatrix} - \begin{pmatrix} 12 \\ -16 \end{pmatrix} = \begin{pmatrix} -10 \\ 9 \end{pmatrix}$ $B(-10,9)$	B1	Majority obtain the correct answer. Few students did not put bracket. No marks awarded There are some who obtain answer $\begin{pmatrix} 12 \\ -16 \end{pmatrix} - \begin{pmatrix} 2 \\ -7 \end{pmatrix} = \begin{pmatrix} 10 \\ -9 \end{pmatrix}$ $B(10,-9)$
21(c)	$\overrightarrow{CD} = k\overrightarrow{BA}$ $\begin{pmatrix} -3 \\ m \end{pmatrix} = k \begin{pmatrix} 12 \\ -16 \end{pmatrix}$ $12k = -3 \Rightarrow k = -\frac{1}{4}$ $m = -16k \Rightarrow m = -16(-\frac{1}{4}) = 4$	M1 A1	Well answered. Except for a few students in weaker classes.
22(a)	$75 - 70 = 5$	B1	Well answered. Common wrong answer $90 - 30 = 60$
22(b)	$\frac{80}{100} \times 120 = 96$ From graph, least marks = 75.5	B1	A number of students gave 76. No marks award as the marking is clear at 75.5
22(c)	$\frac{10}{120} \times \frac{10}{119} \times 2 = \frac{5}{357}$	M1 for first pair A1	Badly done
23	See the attached 950 m	B4	Badly done. Many students did not understand the meaning of bearing of C from A.
24(a)	$\overrightarrow{DC} = \overrightarrow{DA} + \overrightarrow{AB} + \overrightarrow{BC}$ $\overrightarrow{DC} = 3p + 4q$	 B1	
24(b)	$\overrightarrow{AB} = 2(3p + 4q)$ $\overrightarrow{AB} = 2\overrightarrow{DC} \Rightarrow \overrightarrow{AB} // \overrightarrow{DC}$ DA is not parallel to BC Since it has one pair of parallel sides $\Rightarrow ABCD$ is a trapezium	 B1-ratio +parallel B1	$AB = 2DC$ must mention parallel if not 1m will not be awarded
24(c)	$\overrightarrow{AB} = 2\overrightarrow{DC}$ $\lvert\overrightarrow{DC}\rvert = \frac{1}{2}\lvert\overrightarrow{AB}\rvert$ *Ratio* =1:2	 B1	Dependent if they have the correct answer in (b) Accepted 0.5 as answer
24(d)	(i) $\frac{\text{area of triangle } ADC}{\text{area of triangle } ABC} = \frac{DC}{AB} = \frac{1}{2}$	B1	Badly done
24(d)	(ii) $\frac{\text{area of triangle } ADC}{\text{area of triangle } ABCD} = \frac{1}{3}$	B1	Badly done

23)



NORTH VISTA SECONDARY SCHOOL

MID-YEAR EXAMINATION 2017

100

NAME: _____( ) CLASS: _____

SUBJECT : MATHEMATICS (PAPER 2) DATE : 3 MAY 2017

LEVEL/STREAM : SECONDARY 4 EXPRESS/ SECONDARY 5 NORMAL(ACADEMIC) TIME : 2 HOURS 30 MINUTES

CODE : 4048/02

READ THESE INSTRUCTIONS FIRST

Write your register number and name on all the work you hand in.
Write in dark blue or black pen on both sides of the paper.
You may use a pencil for any diagrams or graphs.
Do not use staples, paper clips, highlighters, glue or correction fluid.

Answer all questions.
If working is needed for any question it must be shown with the answer.
Omission of essential working will result in loss of marks.
The use of an approved scientific calculator is expected, where appropriate.
If the degree of accuracy is not specified in the question, and if the answer is not exact, give the answer to three significant figures. Give answers in degrees to one decimal place.
For π, use either your calculator value or 3.142, unless the question requires the answer in terms of π.

At the end of the examination, fasten all your work securely together.
The number of marks is given in brackets [ ] at the end of each question or part question.
The total of the marks for this paper is 100.

Mathematical Formulae

Compound Interest

$$\text{Total amount} = P\left(1+\frac{r}{100}\right)^n$$

Mensuration

Curved surface area of a cone $= \pi rl$

Surface area of a sphere $= 4\pi r^2$

Volume of a cone $= \frac{1}{3}\pi r^2 h$

Volume of a sphere $= \frac{4}{3}\pi r^3$

Area of triangle $ABC = \frac{1}{2}ab\sin C$

Arc length $= r\theta$, where θ is in radians

Sector area $= \frac{1}{2}r^2\theta$, where θ is in radians

Trigonometry

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$a^2 = b^2 + c^2 - 2bc\cos A$$

Statistics

$$\text{Mean} = \frac{\Sigma fx}{\Sigma f}$$

$$\text{Standard Deviation} = \sqrt{\frac{\Sigma fx^2}{\Sigma f} - \left(\frac{\Sigma fx}{\Sigma f}\right)^2}$$

Answer **all** the questions.

1 **(a)** It is given that $a = \dfrac{5b + c^2}{b - 2c}$.

(i) Find a when $b = 6$ and $c = -2$. [1]

(ii) Express b in terms of a and c. [2]

(b) Solve the equation $\dfrac{3x+4}{5} - \dfrac{x}{7} = 4$. [2]

(c) Solve the simultaneous equations.

$$3x + 2y = 4$$
$$x - y = -7$$ [3]

(d) Simplify $\dfrac{2x^2 + x - 3}{4x^2 - 9}$. [3]

---

2 The table below shows the number of customers for different types of treatments offered by three salons, A, B and C on average for a one-week period.

	Straightening	Curling	Colouring
Salon A	6	4	9
Salon B	4	7	5
Salon C	8	2	3

(a) Write down a 3×3 matrix **P** to represent the number of customers in salon A, B and C on average in a week. [1]

(b) One month is defined as four weeks.

Evaluate the matrix $\mathbf{M} = 4\mathbf{P}$. [1]

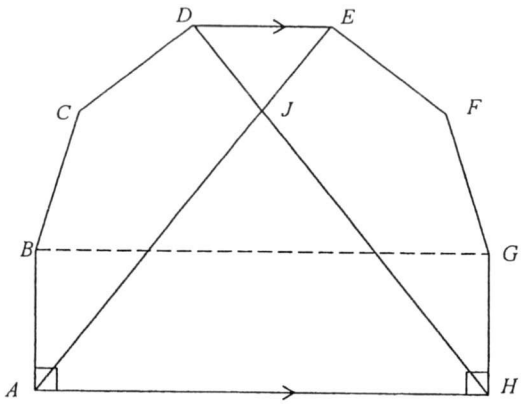
(c) Evaluate the matrix $\mathbf{T} = (1 \quad 1 \quad 1)\,\mathbf{M}$. Explain what the elements of **T** represent. [2]

(d) The cost of straightening, curling and colouring is \$150, \$120 and \$55 respectively. It can be represented as a 3×1 column matrix **Q**.

Evaluate the matrix **MQ**. [2]

(e) State what the elements of **MQ** represent. [1]

3



The diagram shows an octagon, *ABCDEFGH*.

BCDEFG is part of a regular decagon and *ABGH* is a rectangle.

It is given that *AH* is parallel to *DE*.

(a) Find

(i) angle *CDE*, [2]

(ii) angle *ABC*. [2]

(b) Explain why triangles *EFG* and *DCB* are congruent. [2]

(c) It is given that $DJ = JE$ and that angle $EDJ = 55°$.

Jason claims that points *A*, *H* and *J* lies on the circumference of a circle, and that *AH* is the diameter of the circle. Do you agree? Explain your answer. [2]

---

4 The diagram shows a sequence of figures made up of dots and lines.

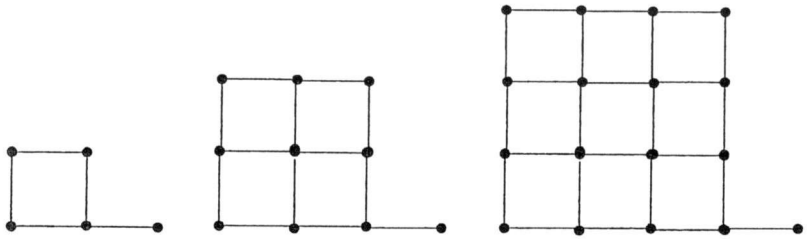


Figure 1 Figure 2 Figure 3

The number of dots and lines can be written as sequences as shown in the table below.

Figure	Number of dots (D_n)	Number of lines (T_n)	$T_n - D_n$
1	5	$1+2\left(1+1^2\right)=5$	0
2	10	$1+2\left(2+2^2\right)=13$	3
3	17	$1+2\left(3+3^2\right)=25$	8
4	***a***	***b***	***c***
...	...	...	...
n			

(a) Find the values of a, b and c. [3]

(b) Find an expression, in terms of n, for the nth term of the sequence T_n. [1]

(c) Explain why the number of lines must be odd for all values of n. [1]

(d) **(i)** Find an expression, in terms of n, for the nth term of the sequence D_n, and show that $D_n = n^2 + 2n + 2$. [1]

(ii) Hence, show that $T_n - D_n = (n+1)(n-1)$. [2]

5 **Answer the whole of this question on a sheet of graph paper.**

The variables x and y are connected by the equation

$$y = \frac{3}{x^2} + x.$$

Some of the corresponding values of x and y are given in the table below.

x	0.5	1	2	3	4	5	6
y	12.5	4	p	3.33	4.19	5.12	6.08

(a) Find the value of p. [1]

(b) Using a scale of 2 cm to represent 1 unit, draw a horizontal x-axis for $0 \le x \le 6$.
Using a scale of 1 cm to represent 1 unit, draw a vertical y-axis for $-1 \le y \le 13$. [3]

(c) Use your graph to find the solutions to the equation, $\frac{3}{x^2} + x = 3$ in the range $0 \le x \le 6$. [1]

(d) By drawing a tangent, find the gradient of the curve at (1, 4). [2]

(e) **(i)** On the same axes, draw the line $y = -\frac{1}{2}x + 7$ for $0 \le x \le 6$. [2]

(ii) Write down the x-coordinate(s) of the point(s) for which the line intersects the curve. [1]

(iii) The value of x is a solution of the equation $3x^3 + Ax^2 + Bx + 6 = 0$.
Find the value of A and the value of B. [2]

6 A light aircraft flew from Singapore to Ho Chi Minh and returned to Singapore.
The distance between Singapore and Ho Chi Minh is 1080 km.

(a) On the outward flight, the average speed of the aircraft was x kilometers per hour. Write down an expression, in terms of x, for the time taken in hours. [1]

(b) On the return flight, the average speed was 30 km/h greater than the average speed on the outward flight. Write down an expression, in terms of x, for the time taken, in hours, on the return flight. [1]

(c) The time taken on the return flight was 30 minutes less than the time taken on the outward flight. Form an equation in x and show that it reduces to

$$x^2 + 30x - 64800 = 0 .$$ [3]

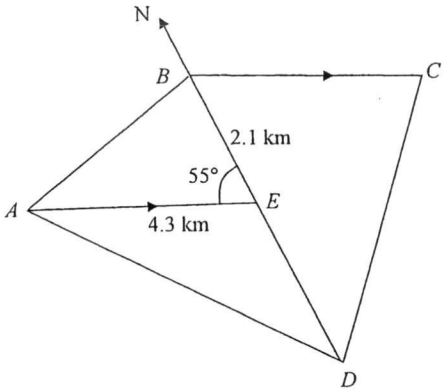
(d) Solve the equation $x^2 + 30x - 64800 = 0$. [3]

(e) Calculate,

(i) the time taken, in hours, on the outward flight, [2]

(ii) the average speed for the whole flight from Singapore to Ho Chi Minh and back to Singapore, assuming that there is no rest time. [2]

7



The diagram shows five points A, B, C, D and E on an empty piece of land where BED is a straight line, AE is parallel to BC and D is south of B. $AE = 4.3$ km, $BE = 2.1$ km and $\angle AEB = 55°$.

(a) Given that the bearing of D from C is 200°, find

(i) the bearing of C from B, [1]

(ii) the bearing of C from D. [1]

(b) Calculate the length of AB. [3]

(c) If the area of ΔAED is 7 km^2, find the length of ED. [2]

A man walked from A, along AE, until he reached a point X where BX is the shortest distance from B to AE.

(d) Calculate the distance the man walked. [2]

(e) A vertical tower TB stands at B such that T is the top of the tower. Given that the angle of elevation of T from E is 19°, calculate the angle of depression of D from T. [3]

---

8 (a) The time taken, in minutes, by 20 pupils to complete puzzle A was recorded below.

Time (x min)	$15 < x \leq 20$	$20 < x \leq 25$	$25 < x \leq 30$	$30 < x \leq 35$
Frequency	8	7	3	2

(i) Hence, calculate the mean and standard deviation. [2]

The time taken by the same 20 pupils to complete puzzle B, was recorded. The mean and standard deviation of the time taken to complete puzzle B are 26.4 min and 4.89 min respectively.

(ii) Which puzzle was easier? Explain. [2]

(iii) Due to an error in the stopwatch used to record the time taken for the 20 pupils to complete puzzle B, the timing for each pupil should be 1 min more than the recorded timing. How will this affect the mean and standard deviation? [2]

(b) A box contains 6 red marbles and 9 blue marbles. Two marbles are drawn from the box one after another without replacement.

(i) Draw a tree diagram to show the probabilities of the possible outcomes. [2]

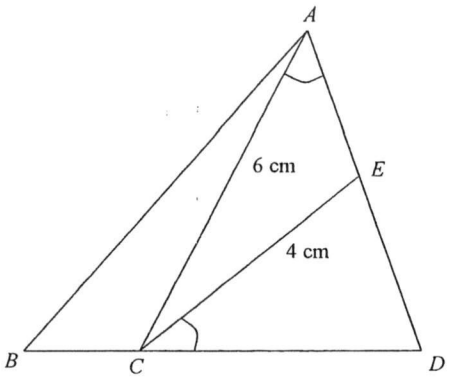
(ii) Find, as a fraction in its simplest form, the probability that

(a) both marbles drawn are of different colours, [1]

(b) the second marble drawn is red. [1]

(iii) A third marble is drawn. Find the probability that only the third marble drawn is red. [2]

9 In the diagram, E lies on AD and C lies on BD such that $BC : CD = 1 : 3$.
It is given that $AC = 6$ cm, $CE = 4$ cm and $\angle ECD = \angle CAD$.



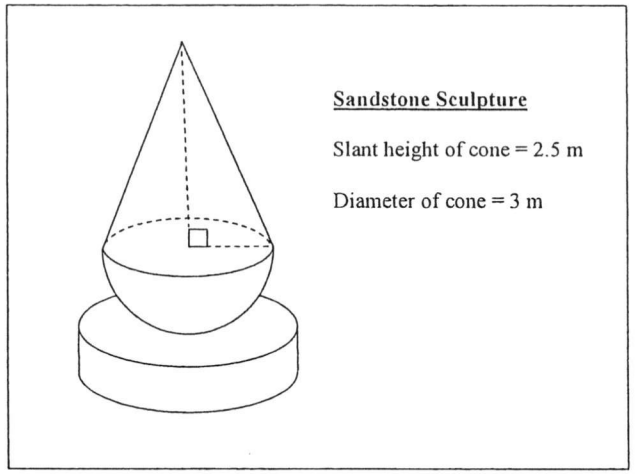
(a) Prove that ΔECD and ΔCAD are similar. [2]

(b) Write down the ratio of $CD : AD$. [1]

(c) Find the ratio of area of ΔECD : area of ΔCAD. [1]

(d) Given that the area of ΔACE is 8 cm^2, find the area of ΔBAD. [4]

10 The diagram shows a sandstone sculpture consisting of a cone attached to a hemisphere.



(a) Calculate the volume of the sculpture. [4]

(b) The sandstone sculpture is to be mounted on a cylindrical concrete platform.
The mass (kg) of the platform must be more than the mass (kg) of the sculpture so that it can provide sufficient support.

The recommended radius of the platform is 2 m.

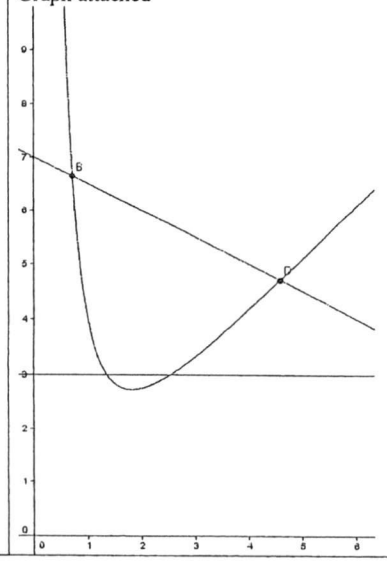
Recommend a suitable height for the platform. Justify your answer using mathematical reasoning. [6]

Useful Information

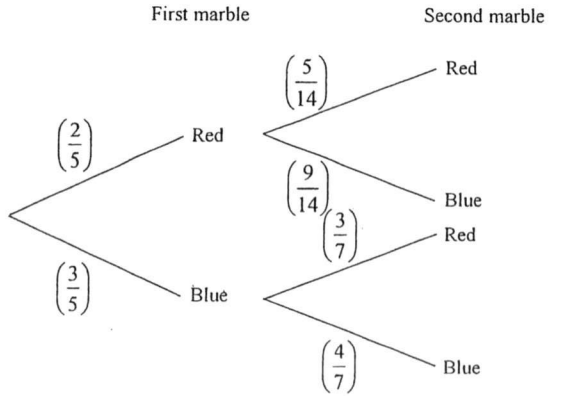
Density of concrete = 2400 kg/m^3
Density of sandstone = 1442 kg/m^3

End of Paper

Q	Answer	Marks / Remarks
1ai	$3\frac{2}{5}$ or 3.4	B1 (leave in mixed numbers)
1aii	$a=\frac{5b+c^2}{b-2c}$ $ab-2ac=5b+c^2$ $ab-5b=c^2+2ac$ $b(a-5)=c^2+2ac$ $b=\frac{c^2+2ac}{a-5}$ $=\frac{c(c+2a)}{a-5}$	M1 A1
1b	$\frac{3x+4}{5}-\frac{x}{7}=4$ $\frac{7(3x+4)-5x}{35}=\frac{4}{1}$ $21x+28-5x=140$ $x=7$	M1 (combine fractions) A1
1c	$x=-2$ $y=5$	M1 (substitution or elimination) B1 B1
1d	$\frac{2x^2+x-3}{4x^2-9}=\frac{(2x+3)(x-1)}{(2x+3)(2x-3)}$ $=\frac{x-1}{2x-3}$	M1 (factorise numerator) M1 (factorise denominator) A1
2a	$\mathbf{P}=\begin{pmatrix}6&4&9\\4&7&5\\8&2&3\end{pmatrix}$	B1
2b	$\mathbf{M}=\begin{pmatrix}24&16&36\\16&28&20\\32&8&12\end{pmatrix}$	B1
2c	$\mathbf{T}=(1\ \ 1\ \ 1)\begin{pmatrix}24&16&36\\16&28&20\\32&8&12\end{pmatrix}$ $=(72\ \ 52\ \ 68)$ T represents the total number of customers who went for straightening, curling and colouring in a month respectively.	M1 A1
2d	$\mathbf{Q}=\begin{pmatrix}150\\120\\55\end{pmatrix}$ $\mathbf{MQ}=\begin{pmatrix}24&16&36\\16&28&20\\32&8&12\end{pmatrix}\begin{pmatrix}150\\120\\55\end{pmatrix}$ $=\begin{pmatrix}7500\\6860\\6420\end{pmatrix}$	M1 A1
2e	**MQ** represents the amount of money received by shops A, B and C respectively in a month.	B1
3ai	Since *BCDEFG* is part of a regular decagon, Sum of interior angles of a decagon $=(10-2)\times180$ $=1440^\circ$ $\angle CDE=\frac{1440}{10}$ $=144^\circ$	M1 A1
3aii	Since ABCDEFGH is an octagon, Sum of interior angles of an octagon $=(8-2)\times180$ $=1080^\circ$ $\angle ABC=\frac{1080-2(90)-4(144)}{2}$ $=162^\circ$ Alternative: $\angle ABC=\frac{144}{2}+90=162^\circ$	M1 A1 M1,A1
3b	$BC = CG$ (sides of a regular polygon) $CD = EF$ (sides of a regular polygon)	

	Angle BCD = angle GFE (interior angle of a regular polygon) Therefore, by SAS, triangle EFG and DCB are congruent.	B1 B1
3c	$\angle DJE = 180 - 55 - 55$ $= 70^\circ$ (isos Δ) Since $\angle DJE = 70^\circ \neq 90^\circ$, by the property of angles in a semicircle, Jason's claim is untrue.	M1 A1
4a	$a = 26$ $b = 41$ $c = 15$	B1 B1 B1
4b	$T_n = 1 + 2\left(n + n^2\right)$ or $1 + 2n + 2n^2$	B1
4c	$2\left(n + n^2\right)$ will give an even number since any number multiplied by 2 will be an even number. By adding 1 to the product, it gives an odd number. Hence, the number of lines will always be an odd number.	B1
4e	$D_n = (n+1)^2 + 1$	B1
4f	$T_n - D_n = 1 + 2\left(n + n^2\right) - \left((n+1)^2 + 1\right)$ $= 1 + 2n + 2n^2 - n^2 - 2n - 2$ $= n^2 - 1$ $= (n+1)(n-1)$	M1 M1
5a	$p = 2.75$ (exact)	B1
5b	Graph attached 	Axes – B1 Points – B1 Smooth curve – B1

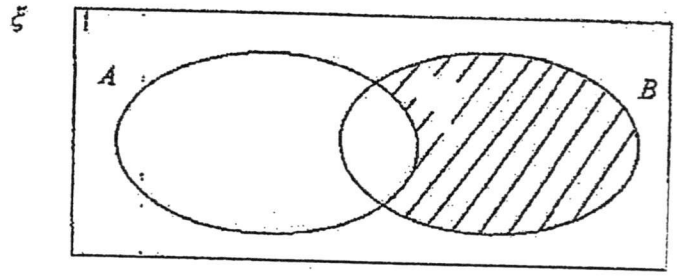
5c	$x = 1.35, 2.53 \pm 0.5$	B1
5d	Tangent drawn Gradient = -5 ± 1	M1 A1
5ei	Table of values Correct line with label	M1 A1
5eii	$x = 0.71, 4.57$	B1
5eiii	$\frac{3}{x^2} + x = -\frac{1}{2}x + 7$ $3 + x^2 = -\frac{1}{2}x^3 + 7x^2$ $3x^3 - 14x^2 + 6 = 0$ Therefore, A = –14 and B = 0	M1 (equate) A1 (values of A and B)
6a	$\frac{1080}{x}$ h	B1
6b	$\frac{1080}{x+30}$ h	B1
6c	$\frac{1080}{x} - \frac{1080}{x+30} = \frac{1}{2}$ $\frac{1080(x+30) - 1080x}{x(x+30)} = \frac{1}{2}$ $64800 = x^2 + 30x$ $x^2 + 30x - 64800 = 0$	M1 M1 M1
6d	$x = \frac{-30 \pm \sqrt{30^2 - 4(1)(-64800)}}{2(1)}$ $x = \frac{-30 \pm \sqrt{260100}}{2}$ $x = 240$ or -270	M1 M1 A1
6e(i)	Time $= \frac{1080}{240}$ Time $= 4\frac{1}{2}$ h	M1 A1
6e(ii)	Average speed $= \frac{1080 \times 2}{4\frac{1}{2} + 4} = 254\frac{2}{17}$ km/h	M1, A1
7a(i)	125°	B1
7a(ii)	020°	B1
7b	$AB^2 = 2.1^2 + 4.3^2 - 2(2.1)(4.3)\cos 55^\circ$ $AB^2 = 12.541$ $AB = 3.5414 = 3.54$ km (3s.f.)	M1 M1 A1
7c	$\angle AED = 125^\circ$ $\frac{1}{2}(4.3)(ED)\sin 125^\circ = 7$	M1

	$ED = 3.9746 = 3.97$ km (3s.f.)	A1
7d	$\cos 55° = \frac{XE}{2.1}$ $XE = 1.2045$ $AX = 4.3 - 1.2045$ $= 3.0955$ $= 3.10$ km (3s.f.)	M1 A1
7e	$\tan 19° = \frac{TB}{2.1}$ $TB = 0.72309$ km $\tan\theta = \frac{0.72309}{6.0746}$ $\theta = \tan^{-1}\left(\frac{0.72309}{6.0746}\right) = 6.8°$ (1 d.p.)	 M1 M1 A1
8a(i)	Mean $= \frac{8\times17.5+7\times22.5+3\times27.5+2\times32.5}{20} = 22.25$ min Standard deviation $= \sqrt{\frac{10375}{20} - 22.25^2} = 4.8669 = 4.87$ min	B1 B1
8a(ii)	Puzzle A was easier as the mean is lower.	B2, no marks if no reason given.
8a(iii)	The mean will increase by 1 min. The standard deviation will not change.	B1 B1
8b(i)		B2,1,0
8b(ii)a	$\left(\frac{2}{5}\right)\left(\frac{9}{14}\right)+\left(\frac{3}{5}\right)\left(\frac{3}{7}\right)=\frac{18}{35}$	B1
8b(ii)b	$\left(\frac{2}{5}\right)\left(\frac{5}{14}\right)+\left(\frac{3}{5}\right)\left(\frac{3}{7}\right)=\frac{2}{5}$	B1
8b(iii)	$\frac{9}{15}\times\frac{8}{14}\times\frac{6}{13}=\frac{72}{455}$	M1, A1

9a	$\angle ECD = \angle CAD$ (given) $\angle CDA$ is a common angle. Therefore, ΔECD and ΔCAD are similar. (AA)	M1 A1
9b	2:3	B1
9c	$\frac{\text{area of } \Delta ECD}{\text{area of } \Delta CAD} = \left(\frac{4}{6}\right)^2$ $= \frac{4}{9}$ area of ΔECD : area of $\Delta CAD = 4:9$	 B1
9d	$\frac{\text{area of } \Delta ACE}{\text{area of } \Delta CAD} = \frac{5}{9}$ $\frac{\text{area of } \Delta CAD}{\text{area of } \Delta BAD} = \frac{3}{4}$ $\frac{\text{area of } \Delta ACE}{\text{area of } \Delta BAD} = \frac{\text{area of } \Delta ACE}{\text{area of } \Delta CAD} \times \frac{\text{area of } \Delta CAD}{\text{area of } \Delta BAD}$ $= \frac{5}{9}\times\frac{3}{4}$ $\frac{8}{\text{area of } \Delta BAD} = \frac{5}{12}$ area of $\Delta BAD = \frac{8\times12}{5} = 19\frac{1}{5}$ cm² or 19.2 cm²	M1 M1 M1 A1
10a	Height of cone $= \sqrt{2.5^2 - 1.5^2} = 2$ m Volume of cone $= \frac{1}{3}\pi(1.5)^2(2) = 4.71238$ m³ Volume of hemisphere $= \frac{2}{3}\pi(1.5)^3 = 7.06858$ m³ Volume of sculpture $= 4.71238 + 7.06858$ $= 11.7809$ $= 11.8$ m³ (3 s.f.)	M1 M1 M1 A1
10b	Mass of sculpture $= 11.7809\times1442$ $= 16988$ kg Min volume of sculpture $= \frac{16988.0578}{2400} = 7.078357$ m³ $\pi(2)^2 h_{\min} = 7.078357$ $h_{\min} = 0.563$ m Recommended height = Any height above 0.563m Must be accompanied by reason for recommendation. E.g.: to accommodate unforeseen increase in the mass of the sculpture.	M1 M1 M1 M1 M1 A1

Answer all the questions.

1 Express in set notation, the set shaded in the following Venn diagram.



Answer ..... [1]

2 (a) Simplify $(3+2x)(1+x)$.

Answer ..... [1]

(b) Factorise completely $32a^2-18b^2$.

Answer ..... [2]

3 Factorise completely $12bx-6ay+8by-9ax$.

Answer ..... [2]

4 Write as a single fraction in its simplest form $\frac{5}{2+x}+\frac{6x}{x^2-4}$.

Answer ..... [2]

5 Show that for all p, where p is a positive integer, $(7p-3)^2-4p(p-3)+6$ is divisible by 15.

Answer

[2]

---

6 (a) Express $5-6x-x^2$ in the form $p-(x+q)^2$.

Answer ..... [2]

(b) Hence, sketch the graph of $y=5-6x-x^2$ indicating the y-intercept and the coordinates of the turning point on the graph.

Answer

[2]

---

7 A bicycle rental shop uses the formula $C = 5.5 + 3.5h$ to calculate charges for rental of bicycles, where C is the cost of rental and h is the number of hours of rental.

(a) State the basic charge to be paid regardless of the number of hours of rental.

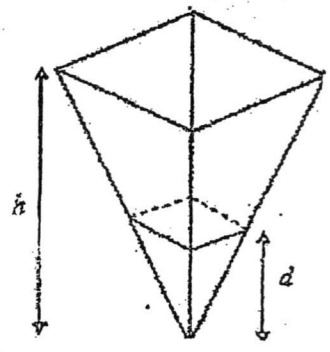
Answer $ ..... [1]

(b) Mathew and Ethan both rented a bicycle each for different number of hours.
The difference in the cost of rental between the two of them is $14.
Find the difference in the number of hours of rental between the two boys.

Answer ..... hours [2]

---

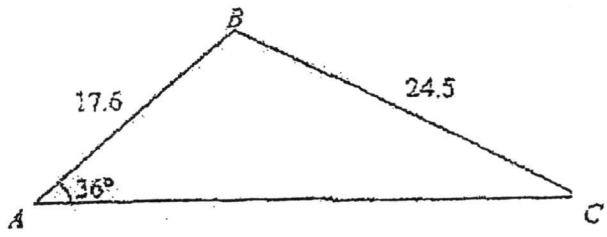
8 The diagram shows an inverted pyramid with a capacity of 800 cm³.



The depth of the liquid in the inverted pyramid, d, is one-third the height, h, of the pyramid.
Calculate the volume of the liquid.

Answer ..... cm³ [2]

9



ABC is a triangle, where $AB = 17.6$ cm, $BC = 24.5$ cm and angle $BAC = 36°$

Find angle ABC.

Answer angle ABC = ... [3]

10 Jane plans to travel back to Singapore from the United States.

In Singapore, the exchange rate is SGD \$1 = USD \$0.71.

In the United States, the exchange rate is USD \$100 = SGD \$153.

Jane wants to change USD \$1426 into Singapore dollars.

Which country should Jane change her money in order to get a better deal?

You must show your calculations.

Answer ..... [3]

11 Hector was arranging 315 one-centimetre cubes into a cuboid.

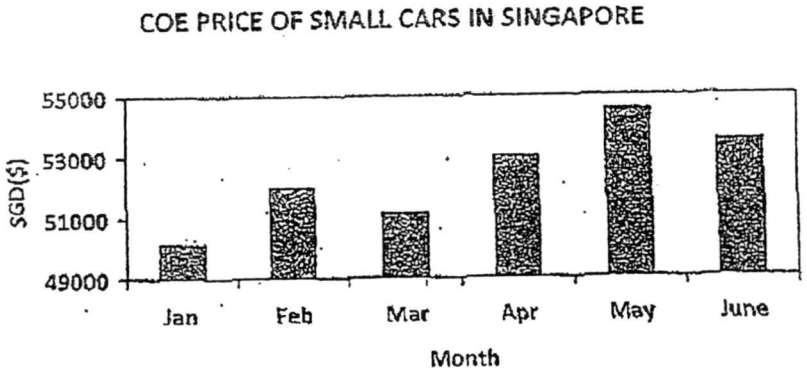
The perimeter of the base of the cuboid is 28 cm.

Each side of the cuboid has a length greater than 3 cm.

Find the height of the cuboid.

Answer .. cm [2]

12 The bar graph shows the COE price of small cars in Singapore over a period of 6 months.



State one aspect of the graph that may be misleading and explain how this may lead to a misinterpretation of the graph.

Answer .....

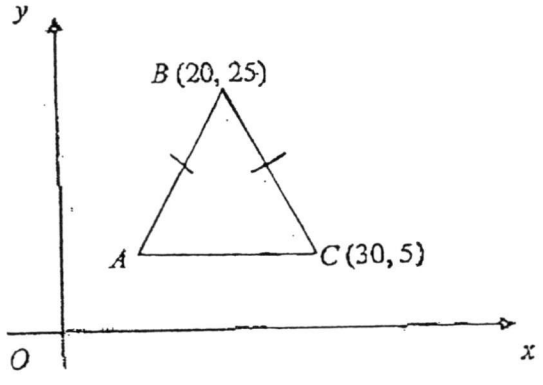
.....

..... [2]

13 The diagram shows an isosceles triangle.

AC is parallel to the x-axis.

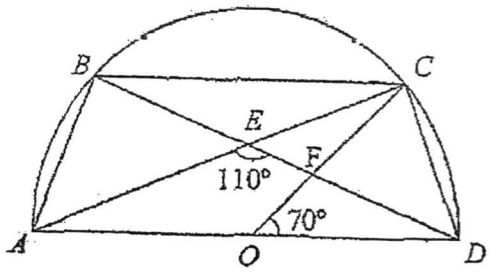
Point B has coordinates (20, 25) and C has coordinates (30, 5)



Find the coordinates of A.

Answer (....., ...) [1]

14



$ABCD$ is a semicircle with centre O.

BED and AEC are straight lines.

Angle $COD = 70°$ and angle $AED = 110°$.

(a) Stating your reasons clearly, calculate

(i) angle ACD,

Answer angle ACD = ...[1]

(ii) angle ADC,

Answer angle ADC = ...[1]

(iii) angle ABC,

Answer angle ABC = ...[1]

(iv) angle BFC.

Answer angle BFC = ...[3]

(b) Explain why BC is parallel to AD.

Answer .....

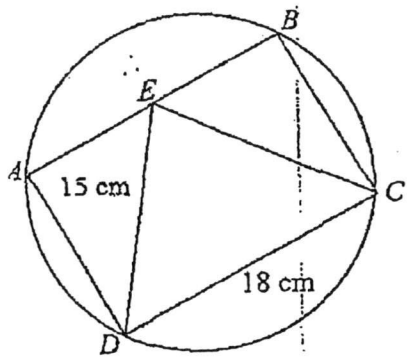
.....[1]

15 The diagram shows a circle $ABCD$.

E is the midpoint of the chord AB.

$ABCD$ is a rectangle.

$DE = 15$ cm and $DC = 18$ cm.



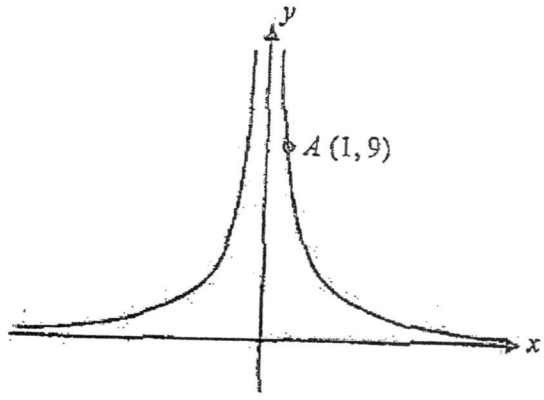
(a) Calculate the area of triangle ADE.

Answer ..... cm² [2]

(b) Calculate the circumference of the circle.

Answer ..... cm [2]

16 The sketch shows the graph of $y = 3^k \times x^{-n}$.
The graph passes through the point A (1, 9).



(a) (i) State a possible value of n.

Answer $n =$ ..... [1]

(ii) Find the value of k.

$k =$ ..... [1]

(b) Given that the coordinates of B is (−2, 2.25), find the length of the line segment AB.

Answer ..... [2]

---

17 (a) Express 3780 as the product of its prime factors.

Answer ..... [1]

(b) Using your answer to **part (a)**, explain why 3780 is **not** multiple of 49.

Answer .....

..... [1]

(c) c is a composite number and p is a prime number.
Find the values of p and c such that $3780 \times \frac{c}{p}$ is a perfect square and c has the least value.

Answer $p =$ .....

$c =$ ..... [2]

---

18 A map of Singapore is such that 9 cm^2 on the map represents the actual area of 36 km^2 on the land.

(a) Express the scale of the map in the form $1 : n$.

Answer 1 : ..... [2]

(b) The length of Bukit Timah Expressway on the map is 5 cm.
Calculate the actual distance, in kilometres, of the Bukit Timah Expressway.

Answer ..... km [1]

19 The table shows the prices of one litre of petrol and the discounts offered by leading petrol companies

Company	Petrol price per litre	Discount
A	\$1.723	18%
B	\$1.689	15%
C	\$1.702	12% discount plus \$3 off for every \$30 sale after discount

(a) Ronn wants to fill up his car with 55 litres of petrol at Company *C*.
Calculate the total amount Ronn paid for the petrol.

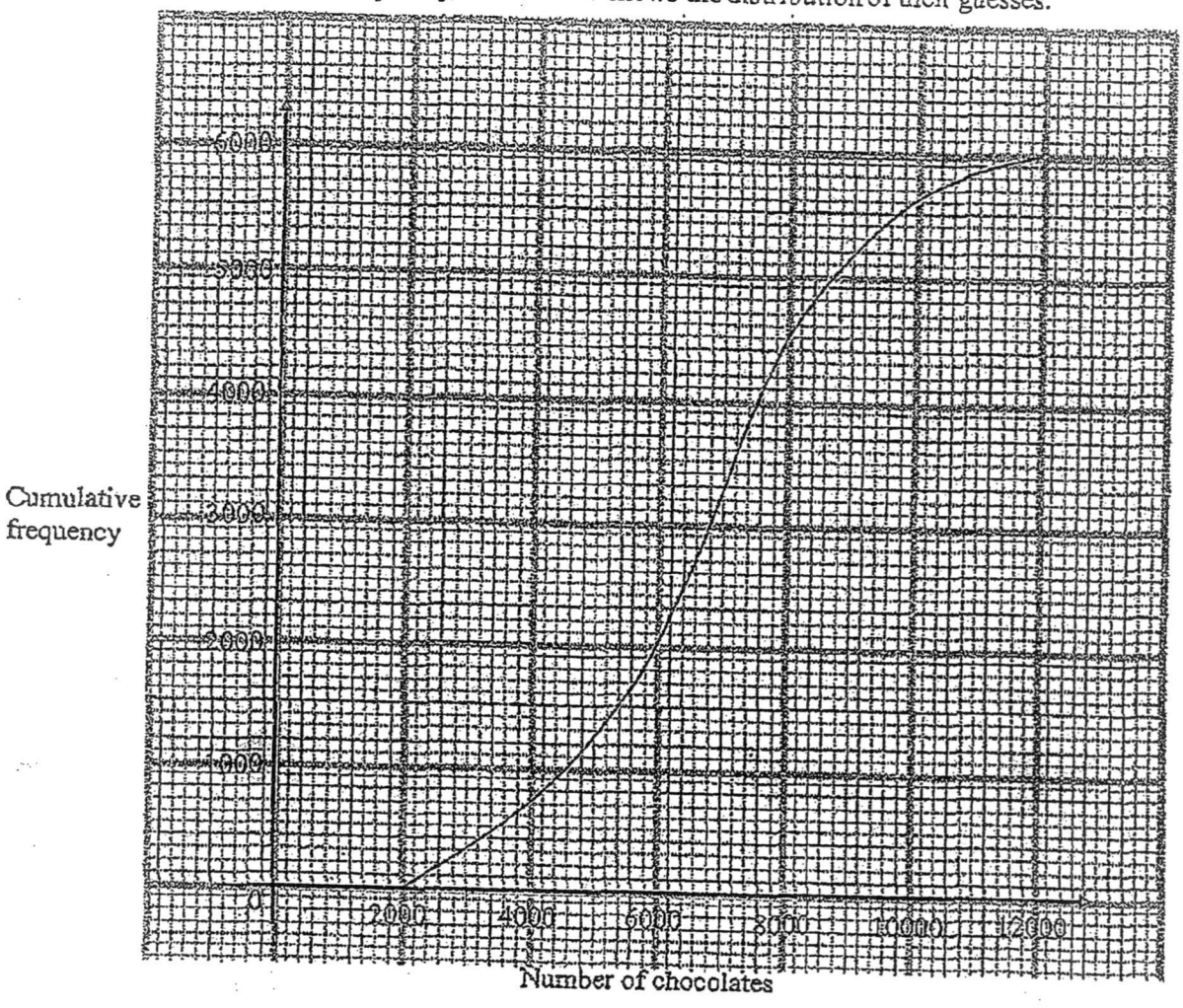
Answer \$ ..... [2]

(b) Comparing Company *A* and *B*, show clearly which company offers a better deal.

Answer ..... [2]

20 6000 customers participated in a contest where they have to guess the number of chocolates in a big glass container.

The cumulative frequency curve below shows the distribution of their guesses.



The actual number of chocolates is 6000.

(a) Find the median.

Answer ..... chocolates [1]

(b) Find the interquartile range.

Answer ..... chocolates [1]

(c) Find the probability that a customer, chosen at random, gave an estimate within 10% of the actual number of chocolates.

Answer ..... [3]

21 Gate B and Gate C are 400 m apart in a park. Gate A is such that angle $ACB = 105°$ and $AB = 550$ m.

(a) Using a scale of 1 cm to 50 m and the line BC is drawn for you, complete the scale drawing of triangle ABC. [1]

(b) A pavilion, inside the park, is located equidistant from the three gates.
By construction, find and label the position of the pavilion P. [2]

(c) Measure and calculate the actual distance between Gate A and the pavilion P.

Answer ..... m [1]

22 The position vectors of A and B are $\begin{pmatrix} 3 \\ 4 \end{pmatrix}$ and $\begin{pmatrix} -3 \\ 4 \end{pmatrix}$ respectively.

(a) Find the length of $\overrightarrow{OB}$.

Answer ..... [1]

(b) C is the point $(0, p)$ where $p > 0$.
$\overrightarrow{OC} = 4\overrightarrow{OA} + 4\overrightarrow{OB}$.
Find the value of p.

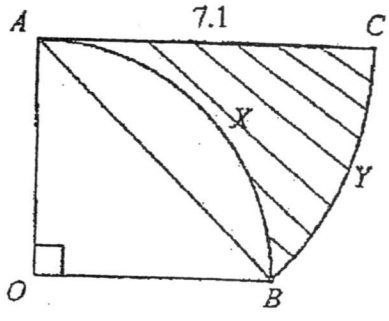
Answer $p =$ ..... [2]

(c) What type of quadrilateral is $OACB$?

Answer ..... [1]

---

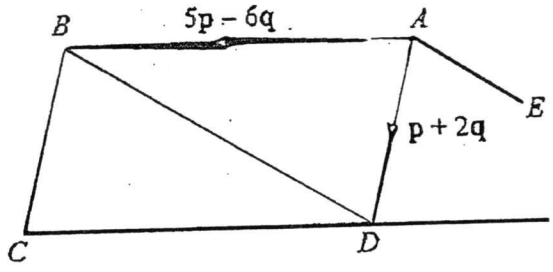
23 In the diagram, angle $AOB = 90°$, AC is parallel to OB and $AC = 7.1$ cm.
AXB is an arc of a circle with centre O and CYB is an arc of a circle with centre A.
Find the area of the shaded region.



Answer ..... cm^2 [5]

---

24 In the diagram, $ABCD$ is a parallelogram, $\overrightarrow{AD} = \mathbf{p} + 2\mathbf{q}$ and $\overrightarrow{AB} = 5\mathbf{p} - 6\mathbf{q}$.



(a) Express, as simply as possible, in terms of p and q,

(i) $\overrightarrow{CB}$,

Answer ..... [1]

(ii) $\overrightarrow{DB}$.

Answer ..... [2]

(b) E is a point such that $\overrightarrow{EA} = \mathbf{p} - 2\mathbf{q}$.

(i) Explain why $\overrightarrow{DB}$ is parallel to $\overrightarrow{EA}$.

Answer .....

..... [1]

(ii) Find the ratio of the area of triangle ADE to the area of triangle DBA.

Answer ..... : ..... [2]

End of Paper

MATHEMATICAL FORMULAE

Compound Interest

$$\text{Total amount} = P\left(1+\frac{r}{100}\right)^n$$

Mensuration

Curved surface area of cone = πrl

Surface area of a sphere = $4\pi r^2$

$$\text{Volume of a cone} = \frac{1}{3}\pi r^2 h$$

$$\text{Volume of sphere} = \frac{4}{3}\pi r^3$$

$$\text{Area of triangle ABC} = \frac{1}{2}ab\sin C$$

Arc length = $r\theta$, where θ is in radians

$$\text{Sector area} = \frac{1}{2}r^2\theta, \text{ where } \theta \text{ is in radians}$$

Trigonometry

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$a^2 = b^2 + c^2 - 2bc\cos A$$

Statistics

$$\text{Mean} = \frac{\sum fx}{\sum f}$$

$$\text{Standard Deviation} = \sqrt{\frac{\sum fx^2}{\sum f} - \left(\frac{\sum fx}{\sum f}\right)^2}$$

Answer all the questions.

1 (a) (i) Factorise $-3x^2-2x+5$. [1]

(ii) Simplify $\dfrac{6x+12}{3x^2-15x-42}$. [2]

(b) It is given that $d=\sqrt{\dfrac{5e-f}{ef}}$.

(i) Find d when $e=4$ and $f=2$. [1]

(ii) Express e in terms of d and f. [2]

(c) Solve the equation $\dfrac{3x+2}{5}-\dfrac{1}{2}=\dfrac{x}{2}$. [2]

(d) Solve these simultaneous equations.

$$7x+4y=-37$$
$$x-5y=17$$ [3]

2 In one small packet of gummies, there are both gummy bears and gummy snakes in two colours; red and green. In a large packet, there are 10 small packets.

The information can be represented by the matrix $A = \begin{matrix} \text{Green} & \text{Red} \\ \begin{pmatrix} 5 & 5 \\ 4 & 6 \end{pmatrix} & \begin{matrix} \text{Bear} \\ \text{Snake} \end{matrix} \end{matrix}$

(a) Evaluate the matrix $B = 10A$. [1]

(b) It costs \$0.10 and \$0.12 to produce 1 green and red gummy respectively.

Represent the cost of each colour of gummy in a 2×1 column matrix $\mathbf{C}$ in dollars. [1]

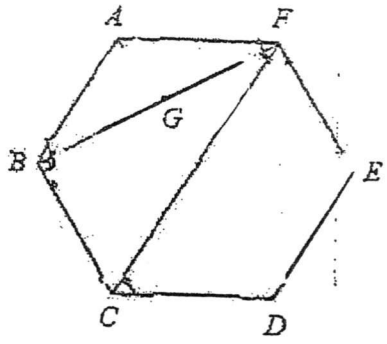
(c) Evaluate the matrix $D = BC$. [1]

(d) State what the elements of D represent. [1]

(e) Another gummy-making company, Company *Y*, packs 6 green gummy bears, 4 red gummy bears, 7 green gummy snakes and 3 red gummy snakes in one small packet. The costs to produce one green gummy and one red gummy remain the same. One large packet is also made up of 10 small packets.

Calculate the total cost for Company *Y* to produce one large packet. [3]

3 (a) The diagram shows a regular hexagon.



(i) Calculate the interior angle of a regular hexagon. [2]

(ii) It is given that $2AG = BC$. Find $\frac{\text{area of triangle } ABF}{\text{area of triangle } BFC}$. [2]

(b) (i) Simplify $\frac{(mn^2)^3}{p^5} \div \frac{n^5}{p^4}$. [2]

(ii) Given that $\frac{2^{q+5}}{4^{2q}} = \frac{1}{16}$, find the value of q. [3]

4 The first five terms in a sequence of numbers are given below.

0, 3, 8, 15, 24...

(a) Find the next two terms. [2]

(b) Find an expression, in terms of n, for the nth term, T_n, of the above sequence. [1]

(c) T_n and T_{n+1} are consecutive terms in the sequence.

Find and simplify an expression, in terms of n, for $T_{n+1} - T_n$. [3]

(d) Explain why two consecutive terms of the sequence cannot have a difference of 8. [2]

5 Answer the whole of this question on a sheet of graph paper.

The variables x and y are connected by the equation

$$y = x^3 - 4x^2 + \frac{5}{2}$$

Some corresponding values of x and y are given in the table below.

x	−1.5	−1	−0.5	0	0.5	1	1.5	2
y	−9.875	−2.5	1.375	2.5	p	−0.5	−3.125	−5.5

(a) Find the value of p. [1]

(b) Using a scale of 4 cm to represent 1 unit, draw a horizontal x-axis for $-1.5 \le x \le 2$.
Using a scale of 1 cm to represent 1 unit, draw a vertical y-axis for $-12 \le y \le 4$.
On your axes, plot the points given in the table and join them with a smooth curve. [3]

(c) Use your graph to find the coordinates of the maximum point of $y = x^3 - 4x^2 + \frac{5}{2}$, in the range of $-1.5 \le x \le 2$. [1]

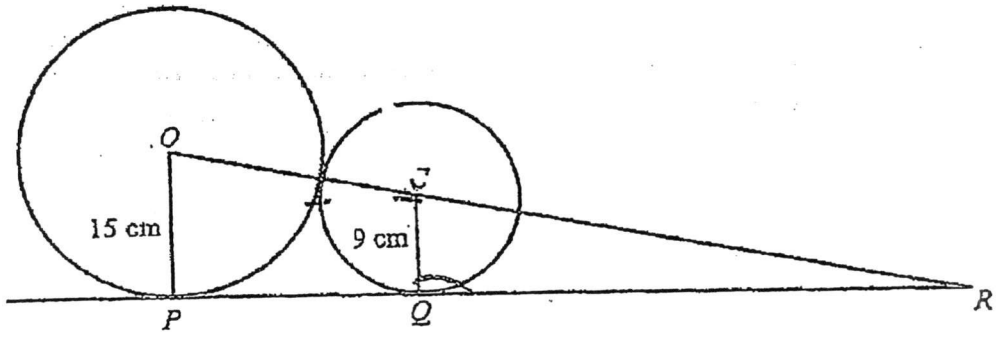
(d) Use your graph to find the solutions to the equation $x^3 - 4x^2 + 6 = 0$, in the range $-1.5 \le x \le 2$. [3]

(e) By drawing a tangent, find the gradient of the curve at $(-1, -2.5)$. [2]

(f) (i) On the same axes, draw the line $y = -3x - 4$ for $-1.5 \le x \le 2$. [1]

(ii) Write down the coordinates of the point where this line intersects the curve. [1]

6 The diagram shows a circle, centre O, with radius 15 cm touching another circle, centre C, with radius 9 cm.
OCR and PQR are straight lines and PQR is a tangent to both the circles at points P and Q.



(a) State the value of angle CQR and explain your answer. [2]

(b) Show that triangles OPR and CQR are similar.
Give a reason for each statement you make. [2]

(c) Find the value of $\dfrac{\text{area of triangle } CQR}{\text{area of trapezium } OCQP}$ [2]

(d) Find the difference in the areas of the two circles.
Leave your answer in terms of π. [2]

7 A company manufactures and sells posters for decoration and display.

(a) The posters manufactured by the company are sold in local shops and department stores. In a particular week, the number of posters available for sale in local shops and department stores are in the ratio 3 : 7.
Given that 160 more posters are available for sale in department stores, find the total number of posters available for sale in that week. [2]

(b) A shop owner bought x posters for \$60 from the company.

(i) Write down an expression, in terms of x, for the cost of each poster in dollars. [1]

The shop owner decides to sell the posters at a profit of \$1 each.

(ii) Write down an expression, in terms of x, for the selling price of each poster in dollars. [1]

The shop owner managed to sell 10 posters at the selling price in (ii).
He decided to sell the rest of the posters at \$5 each.

(iii) Write down an expression, in terms of x, for the total amount of money in dollars, that he collected from the sale of all posters. [1]

(iv) Given that the shop owner collected a total of \$130 from the sale of all posters, write down an equation in x to represent this information and show that it reduces to

$$x^2 - 34x + 120 = 0$$

[3]

(v) Solve the equation $x^2 - 34x + 120 = 0$. [3]

(vi) Find the cost price of each poster. [1]

8 The diagram shows a table used by an interior designer.

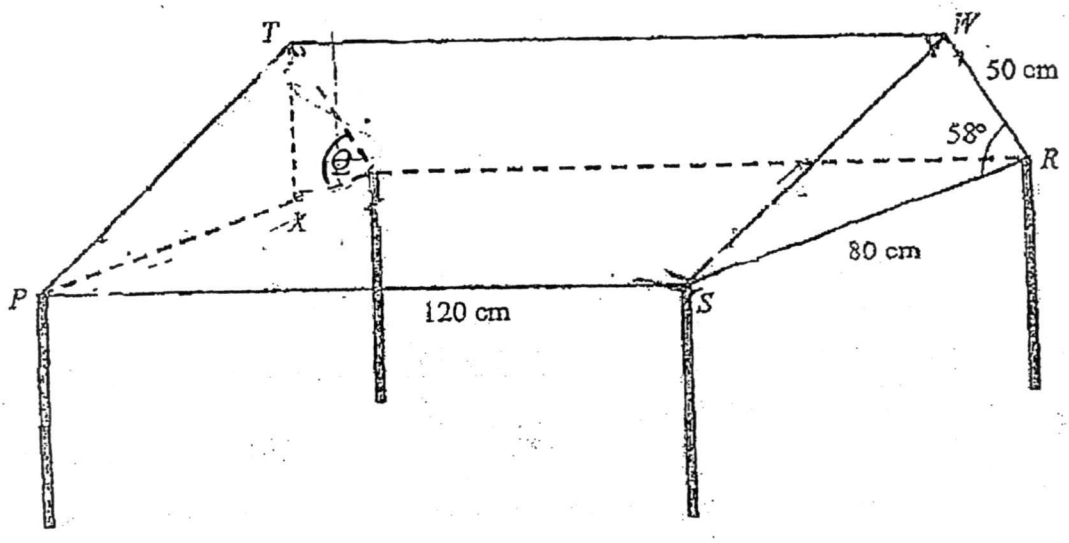
It is made up of a prism and 4 table legs for support.

The rectangle $PQRS$ lies on a horizontal plane.

T is vertically above X.

$PS = 120$ cm, $RS = 80$ cm and $WR = 50$ cm.

Angle $WRS = 58°$.



Calculate

(a) WS, [3]

(b) the volume of the prism, [3]

(c) TX, [2]

(d) XS, [4]

(e) the angle of elevation of T from S. [2]

9 (a) The amount of money, in dollars, spent by a group of 20 students (Group *A*) in the month of May is shown in the stem-and-leaf diagram below.

5	2	5			
6	2	3	7		
7	1	1	5	8	9
8	0	4	5	6	
9	2	3	8	9	
10	5	8			

Key 5|6 means $56

(i) Find the mean amount of money spent by the 20 students. [1]

(ii) Find the standard deviation of the amount of money spent by the 20 students. [1]

(iii) The mean and standard deviation of the amount of money spent by another group of 20 students (Group *B*) in May were $70 and $12 respectively.
Use the information to comment on two differences between the two distributions. [2]

(b) John plays a game at a carnival. In this game, he has to pick 2 coloured balls from two bags, *A* and *B*. He is only allowed to pick one ball from each bag. He has to pick one ball from Bag *A*, followed by another ball from Bag *B*.
Bag *A* contains 2 red balls, 3 blue balls and 6 yellow balls.
Bag *B* contains 4 red balls, 1 blue ball and 4 yellow balls.

(i) Draw a tree diagram to show the probabilities of the possible outcomes. [2]

(ii) John will win a large prize if he picks 2 balls that are blue, a small prize if he picks only one ball that is blue and goes home empty-handed otherwise.
Find, as a fraction in the simplest form, the probability that

(a) John will win a large prize, [1]

(b) John will win a small prize, [1]

(c) John will not win anything. [1]

10 A group of students are tasked to design, print and distribute brochures containing tips to save water to students in school, as part of the school's effort to raise awareness of the importance of saving water in school.

The students have been allocated a budget of $1200 to complete this task.

The students are required to print and distribute a copy of the brochure to each student and teacher in the school.

Each brochure is printed on both sides of 2 sheets of A4 size paper.

Students will be given brochures printed in black and white and teachers will be given brochures printed in colour. They will have to purchase the sheets of A4 size paper and toner cartridges from *ABC* bookstore, which will be delivered to school.

In addition, the students are also tasked to design and print 50 copies of A3 size coloured posters containing tips to save water, to be put up in all classrooms and various areas in the school. They have sourced for an external supplier, *XYZ* supplier, to print the posters. The posters will be delivered to school as well.

The information that the students require is found in Annex A, on the opposite page.

The students estimates that they have to distribute the brochures to 1360 students and 90 teachers.

(a) How many sheets of A4 size paper will the students require to purchase to print the brochures for all students and teachers? [1]

(b) How many toner cartridges will the students require to purchase to print the brochures for all students and teachers? [3]

(c) Given that one of the students in the group is a member of *ABC* bookstore and that the students aim to reduce the cost as far as possible, determine if the amount of budget allocated is sufficient to cover all costs.
Justify your answer with relevant mathematical working. [6]

Annex A

1) Cost of purchasing stationaries from *ABC* Bookshop:

Item	Description	Unit Cost (excluding GST)
A4 Paper	White paper	
	1 pack of 100 sheets	$2.00
	1 pack of 500 sheets	$5.00
	5 packs of 500 sheets each	$22.50
	10 packs of 500 sheets each	$42.00
Toner Cartridges	Black printing (each cartridge is able to print 1200 pages)	$136.00
	Colour printing (each cartridge is able to print 900 pages)	$140.00
The above prices are subjected to 7% Goods and Services Tax (GST). Member discount: 10% off total bill, after 7% GST Delivery cost: $30 per trip (not subjected to 7% GST) (Free delivery for minimum purchase of $200 in total bill, inclusive of 7% GST and after member discount.)		

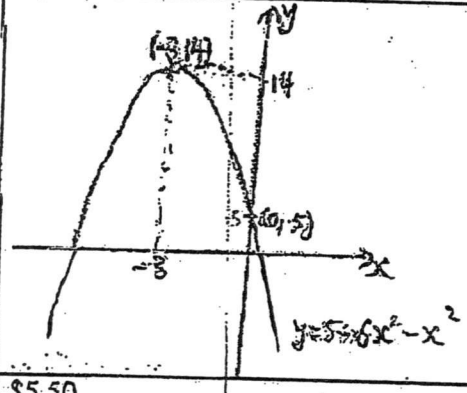
2) Cost of printing A3 size coloured posters

Supplier: *XYZ* Printing

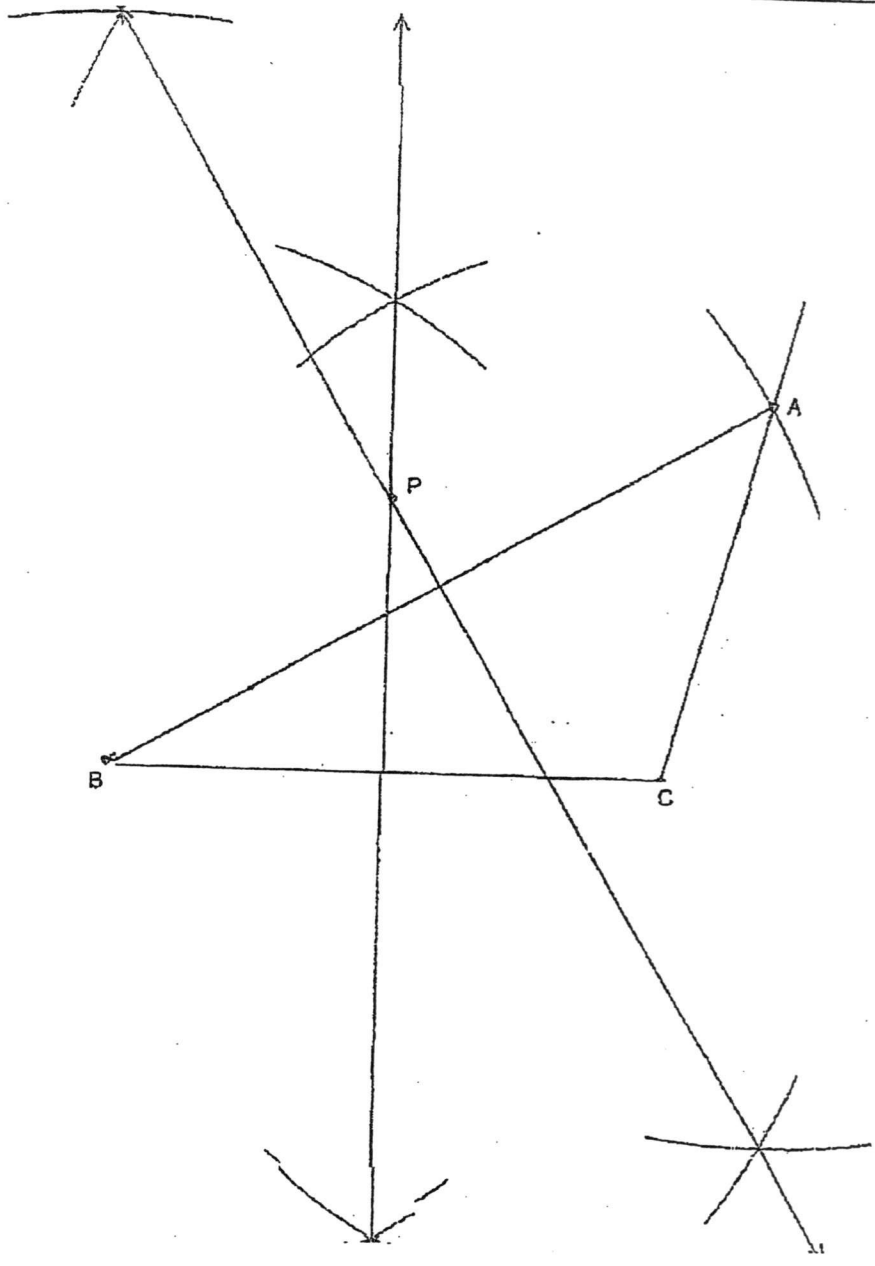
Item	Description	Unit Cost (excluding GST)
Black and White Posters	10 sheets	$25.00
	50 sheets	$120.00
Coloured Posters	10 sheets	$35.00
	50 sheets	$170.00
The above prices are subjected to 7% Goods and Services Tax (GST). Delivery cost: $20 per trip (not subjected to 7% GST) (Free delivery for minimum purchase of $200 in total bill, inclusive of 7% GST.)		

End of Paper

Pei Hwa Secondary School
Mid Year Examination 2017
Sec 4E & 5N Mathematics Paper 1
Answer Key

1(a)	$A' \cap B$
2(a)	$-2x^2+x+3$
2(b)	$2(4a+3b)(4a-3b)$
3	$(4b-3a)(3x+2y)$
4	$\dfrac{11x-10}{(x+2)(x-2)}$
5	$(7p-3)^2-4p(p-3)+6$ $=49p^2-42p+9-4p^2+12p+6$ $=45p^2-30p+15$ $=15(3p^2-2p+1)$ $\therefore$ for all p, $(7p-3)^2-4p(p-3)+6$ is divisible by 15. (Shown)
6(a)	$14-(x+3)^2$
6(b)	
7(a)	\$5.50
7(b)	4 hours
8	$29.6cm^3$ (3s.f.)
9	119.0° (1d.p.)
10	Amount of money Jane will get in Singapore $=\dfrac{1426}{0.71}$ $=SGD\$2008.45$ Amount of money Jane will get in the United States $=\dfrac{153}{100}\times 1426$ $=SGD\$2181.78$

	Jane will change her money in the United States as she will get back more Singapore dollars.
11	7 cm
12	In the graph, the data doesn't start at \$0, but somewhere around \$49000. This makes the differences appear much larger proportionally.
13	(10, 5)
14(a)(i)	90°
14(a)(ii)	55°
14(a)(iii)	125°
14(a)(iv)	75°
14(b)	Angle $BCE = 35°$ (Angles in the same segment) Since angle BCE = angle CAO (by property of alternate angles), BC is parallel to AD.
15(a)	$54cm^2$
15(b)	$68.0cm$
16(a)(i)	$n = -2$
16(a)(ii)	$9 = 3^k \times \left(\frac{1}{3}\right)^{-2}$ $k = 2$
16(b)	7.39 units
17(a)	$2^2 \times 3^3 \times 5 \times 7$
17(b)	Index of 7 is not at least 2
17(c)	$c = 15$ $p = 7$
18(a)	1 : 200000
18(b)	10 km
19(a)	\$76.38
19(b)	Company B offers a better deal.
20(a)	6800
20(b)	2600
20(c)	$\frac{1}{5}$

21a,b	
21(c)	Distance $= 5.7\ (\pm 0.1) \times 50$ $= 285\ (\pm 5)$ m
22(a)	5 units
22(b)	$p = 32$

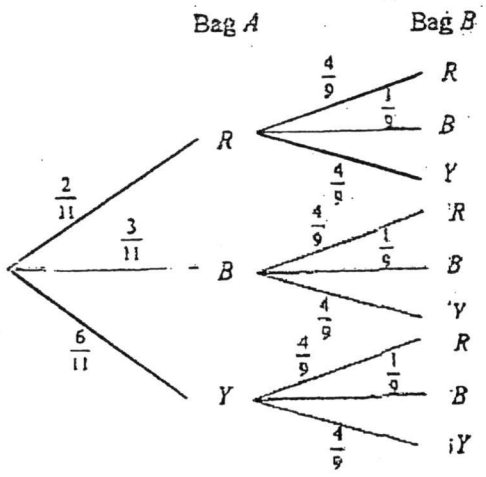
22(c)	Kite
23	12.6 cm^2
24(a)(i)	$-p-2q$
24(a)(ii)	$4p-8q$
24(b)(i)	$\overrightarrow{DB}$ $= 4(p-2q)$ $= 4\ \overrightarrow{EA}$
24(b)(ii)	$\frac{1}{4}$

PHSS 4E EM MYE Paper 2 2017 Answer Key

No.	Answer
1(a)(i)	$-3x^2-2x+5=(3x+5)(1-x)$
1(a)(ii)	$\frac{2}{x-7}$
1(b)(i)	$d=1.5$ or $d=1\frac{1}{2}$
1(b)(ii)	$e=\frac{f}{5-d^2f}$
1(c)	$x=1$
1(d)	$x=-3,\ y=-4$
2(a)	$\mathbf{B}=\begin{pmatrix}50 & 50\\ 40 & 60\end{pmatrix}$
2(b)	$\mathbf{C}=\begin{pmatrix}0.10\\ 0.12\end{pmatrix}$
2(c)	$\mathbf{D}=\begin{pmatrix}11\\ 11.2\end{pmatrix}$
2(d)	The elements of D represent the cost to produce all the gummy bears and gummy snakes in a large packet respectively.
2(e)	Total cost = \$10.80 + \$10.60 = \$21.40
3(a)(i)	120°
3(a)(ii)	$\frac{1}{2}$
3(b)(i)	$\frac{m^3n}{p}$
3(b)(ii)	$q=3$
4(a)	$T_6=35$ $T_7=48$
4(b)	$T_n=n^2-1$ or $(n+1)(n-1)$
4(c)	$T_{n+1}-T_n=n^2+2n-(n^2-1)$ $=2n+1$

No.	Answer
4(d)	$2n+1=8$ $n=3.5$ Assuming that the difference between two terms is 8, the first consecutive term is 3.5, which does not exist. Therefore, two consecutive terms cannot have a difference of 8. OR The difference $(2n+1)$ is an odd number. Therefore, two consecutive terms cannot have a difference of 8, which is an even number.
5(a)	$p=1.625$
5(b)	If all 8 points plotted correctly, otherwise, at least 6 points plotted correctly. Smooth curve
5(c)	Maximum point = (0, 2.5)
5(d)	From the graph, $x=-1.10\pm0.10$ and $x=1.55\pm0.10$
5(e)	Gradient = 8.67 ± 3
5(f)(i)	Correctly drawn line
5(f)(ii)	$(-0.85, -1.4)$
6(a)	$\angle CQR=90°$ tangent perpendicular to radius
6(b)	$\angle OPR=90°$ (tangent perpendicular to radius) $\angle OPR=\angle CQR$ $\angle PRO=\angle QRC$ (common angle) $\angle POR=\angle QCR$ (corresponding angles, $OP//CQ$) Hence, triangle OPR is similar to triangle CQR. (AA Similarity)
6(c)	$\frac{9}{16}$
6(d)	144π cm^2
7(a)	400
7(b)(i)	$\$\left(\frac{60}{x}\right)$
7(b)(ii)	$\$\left(\frac{60}{x}+1\right)$
7(b)(iii)	$\frac{600}{x}+5x-40$

7(b)(iv)	$\frac{600}{x}+10+5x-50=130$ $\frac{600}{x}+5x-170=0$ $600+5x^2-170x=0$ $5x^2-170x+600=0$ $x^2-34x+120=0$ (shown)
7(b)(v)	$x=30$ or $x=4$
7(b)(vi)	\$2
8(a)	68.3 cm
8(b)	204000 cm^3
8(c)	$TX=42.4$ cm
8(d)	$XS=131$ cm
8(e)	$\theta=17.9°$
9(a)(i)	\$80.15
9(a)(ii)	\$15.60
9(a)(iii)	1. The mean amount of money spent by students in Group *A* is higher than that of Group *B*. On average, students in Group *A* spent more money than students in Group *B*. 2. The standard deviation of the amount of money spent by students in Group *B* is lower than that of Group *A*. There is a smaller spread in the amount of money spent by students in Group *B*./ The amount of money spent by students in Group *B* is more consistent.

9(b)(i)	
9(b)(ii)(a)	$\frac{1}{33}$
9(b)(ii)(b)	$\frac{32}{99}$
9(b)(ii)(c)	$\frac{64}{99}$
10(a)	2900
10(b)	6
10(c)	Cost of purchase from *ABC* Bookstore Total cost with delivery cost, after member discount = \$816.1425 Cost of purchase from *XYZ* Printing Total cost with delivery = \$20 + \$181.90 = \$201.90 Grand total cost = \$816.1425 + \$201.90 = \$1018.04 The amount of budget of \$1200 is sufficient to cover all costs.

Name:_____( )	Class : Sec ____

TAMPINES SECONDARY SCHOOL

MID-YEAR EXAMINATION 2017

SECONDARY FOUR EXPRESS/4N1

SECONDARY FIVE NORMAL ACADEMIC

MATHEMATICS ***4048/1***
PAPER 1 ***2 hours***
3 May 2017

Candidates answer on the Question Paper.

80

READ THESE INSTRUCTIONS FIRST

Write your name, class and register number on all the work you hand in.
Write in dark blue or black pen.
You may use a pencil for any diagrams or graphs.
Do not use staples, paper clips, highlighters, glue or correction fluid.

Answer **all** questions.
If working is needed for any question it must be shown with the answer.
Omission of essential working will result in loss of marks.
You are expected to use a scientific calculator to evaluate explicit numerical expressions.
If the degree of accuracy is not specified in the question, and if the answer is not exact, give the answer to three significant figures. Give answers in degrees to one decimal place.
For π, use either your calculator value or 3.142, unless the question requires the answer in terms of π.

At the end of the examination, fasten all your work securely together.
The number of marks is given in brackets [ ] at the end of each question or part question.
The total marks for this paper is 80.

This paper consists of 20 printed pages

Mathematical Formulae

Compound Interest

$$\text{Total amount} = P\left(1+\frac{r}{100}\right)^n$$

Mensuration

Curved surface area of a cone = πrl

Surface area of a sphere = $4\pi r^2$

$$\text{Volume of a cone} = \frac{1}{3}\pi r^2 h$$

$$\text{Volume of a sphere} = \frac{4}{3}\pi r^3$$

$$\text{Area of a triangle } ABC = \frac{1}{2}ab\sin C$$

Arc length = $r\theta$, where θ is in radians

$$\text{Sector area} = \frac{1}{2}r^2\theta, \text{ where } \theta \text{ is in radians}$$

Trigonometry

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$a^2 = b^2 + c^2 - 2bc\cos A$$

Statistics

$$\text{Mean} = \frac{\Sigma fx}{\Sigma f}$$

$$\text{Standard deviation} = \sqrt{\frac{\Sigma fx^2}{\Sigma f} - \left(\frac{\Sigma fx}{\Sigma f}\right)^2}$$

Answer **all** the questions.

1. Find the values of x where $0° \leq x \leq 180°$ for $\sin x = 0.866$, giving your answers correct to the nearest degree.

Answer $x =$..... [1]

2. Given that $3^{2017} + 3^{2017} + 3^{2017} = 3^x$, state the value of x.

Answer $x =$..... [1]

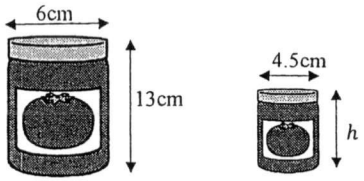
3. (a) Express 50 seconds as a percentage of 5 minutes.

Answer .....% [1]

(b) Express $62\frac{3}{5}\%$ as a decimal.

Answer ..... [1]

4.



The two bottles of pasta sauce in the above diagram are geometrically similar.
The larger bottle has a diameter of 6cm and height 13cm. The diameter of the smaller bottle is 4.5cm.

(a) Find the height, h, of the smaller bottle.

Answer (a) .....cm [1]

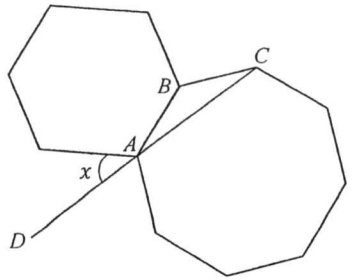
(b) The larger bottle costs \$6 and the smaller bottle costs \$2.50.
Which bottle is more value for money? Justify your answer with calculations.

Answer (b)..... [2]

5. Solve $x(2x + 7) = 4$.

Answer $x =$..... [3]

6.



A regular hexagon and regular octagon share a side AB as shown in the diagram above. DAC is a straight line. Calculate the size of angle x.

Answer $x =$..... ° [3]

7. The difference between the simple interest and the compound interest earned from a principal amount P, invested for a period of 3 years at 4% per annum is \$48.64. Find the principal amount P.

Answer \$..... [3]

8. Express the following as a single fraction in its simplest form

$$\frac{2}{(x+2)^2}+\frac{4}{2x+4}.$$

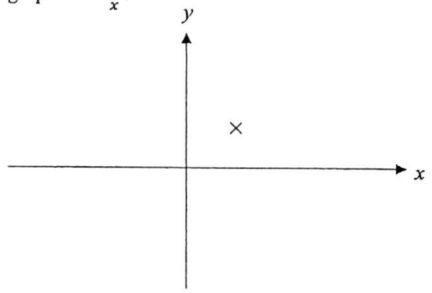
Answer ..... [3]

9. Expand and simplify

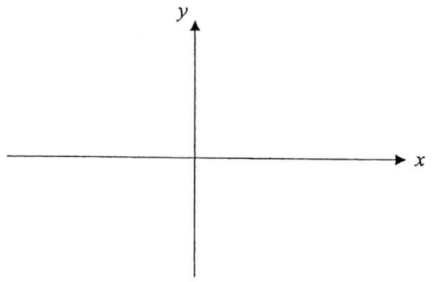
$$(x+1)^2 - x(x-2) + 1 .$$

Answer ..... [3]

10. (a) In the axes provided below, the point (1,1) has been marked.
Sketch the graph of $= \frac{1}{x}$. [1]



(b) Sketch the graph of $y = (x+3)(x-5)$ in the axes provided, indicating the x-intercept(s) and y-intercept(s) if any. [2]



11.

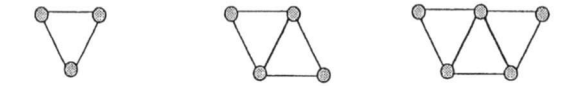


Diagram 1 Diagram 2 Diagram 3

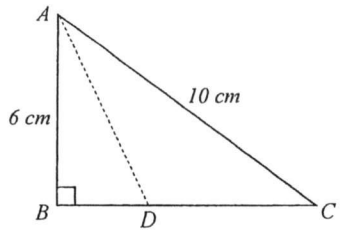
The figure above shows the number of sticks used to form each diagram.

(a) Draw diagram 4 in the space below. [1]

(b) Find an expression in terms of n, the number of sticks used to form Diagram n, reduced to its simplest form.

Answer (b)..... [2]

12.



In the diagram, $\angle ABC = 90°, AB = 6$ cm and $AC = 10$ cm.

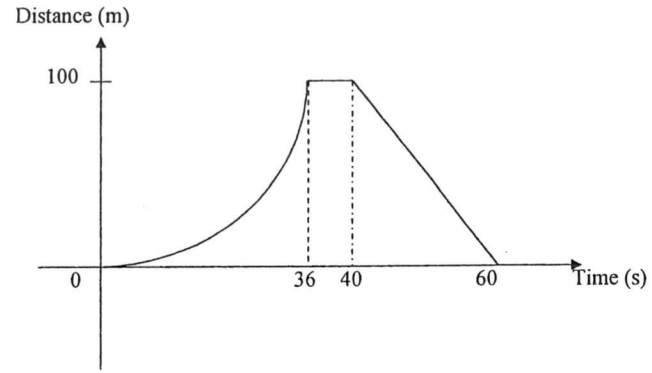
(a) Write down the exact value of $\sin \angle ACB$.

Answer $\sin \angle ACB =$..... [1]

(b) D is a point on BC such that $\angle BAD = \angle DAC$. Find the length of BD.

Answer .....cm [2]

13. The diagram shows the distance-time graph of a cyclist.



(a) Describe the motion of the cyclist for the first 40 seconds.

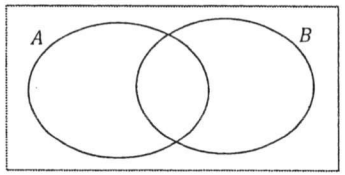
Answer (a).....

..... [2]

(b) Calculate the average speed of the cyclist for the entire 60 seconds.

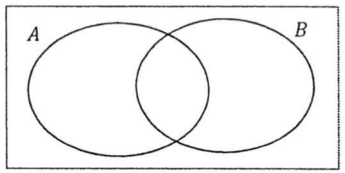
Answer (b).....m/s [1]

14. (a)



(i) On the Venn diagram above, shade the region that represents $A' \cap B$. [1]

(ii) On the Venn diagram below, draw the set K where $K \subset B$ and $A \cap K \neq \emptyset$. [1]



(c) Let $\varepsilon = \{x: x \text{ is an integer and } 2 < x \leq 10\}$
$A = \{x: x \text{ is an odd number}\}$
$B = \{x: x \text{ is a prime number}\}$.

List the elements of

(i) $A' \cup B$

Answer (b)(i)..... [1]

(ii) $(A \cup B)'$

Answer (b)(ii)..... [1]

15. The arrival frequency of trains at a train interchange is shown in the table below:

Train	Frequency
North Line	Every 3 minutes
South Line	Every 4 minutes
East Line	Every 6 minutes
West Line	Every 7 minutes

If all four trains arrived together at 8am,

(a) find the next time that all four trains will arrive at the same time at the interchange.

Answer (a)..... [2]

(b) Calculate the total number of times all four trains arrive at the interchange from 8am to 8pm inclusive on a particular day.

Answer (b)..... [2]

16. Given that $(x+y)^2 = 1$ and $4xy = 15$,
(a) find an expression for $x^2 + y^2$.

Answer (a)..... [1]

(b) Hence, find the value of $(2x+3y)^2 - 5y^2$.

Answer (b)..... [3]

17. (a) Express 1323 as a product of its prime factors.

Answer (a) ..... [1]

(b) Given that $1323k$ is both a perfect square and a perfect cube, write down the smallest integer value of k.

Answer (b)..... [1]

(c) The highest common factor of 1323 and another integer x is 9 and the lowest common multiple of both numbers is 2646. Find the value of x.

Answer (c)..... [2]

18. The variables x, y and z are related.
x is directly proportional to the cube of y and y is inversely proportional to the square root of z.
Given that when $x = 216, y = 4$ and $z = 4$,
(a) find an expression for
(i) z in terms of y,

Answer (a)(i)..... [2]

(ii) y in terms of x.

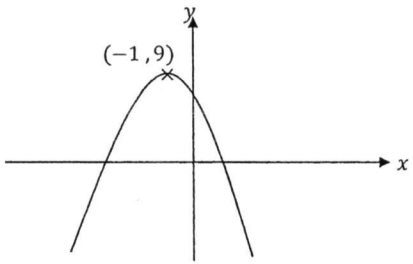
Answer (a)(ii)..... [2]

(b) Hence or otherwise, express z in terms of x and describe the relationship between x and z.

Answer (b)..... [2]

Relationship : ..... [1]

19.



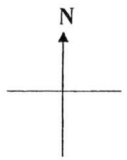
The diagram shows the graph of $y = -x^2 + bx + c$, where b and c are constants. Given that the turning point of the graph is $(-1, 9)$,
(a) state the equation of line of symmetry of the graph

Answer ..... [1]

(b) find the constants b and c.

Answer b =..... c =..... [4]

20. Building A is at a bearing 030° from Building C.
Building B is at a bearing 120° from Building C.
Given that $AC = 40$m and $BC = 100$m,
(a) Use a scale of 1cm to 10m, make an accurate scale drawing to create triangle ABC.
[2]



(b) From your drawing, find the actual distance, in metres, between buildings A and B.

Answer (b).....m [1]

(c) By drawing a perpendicular bisector and an angle bisector, identify and mark the point P that is equidistant from B and C, and from AC and BC. [2]

21. The table below shows the results of 3 students for their Elementary Mathematics Preliminary Examination.
The total possible mark for Paper 1 and 2 is 80 and 100 respectively.
Papers 1 and 2 have equal weightage.

Student	Paper 1	Paper 2
Ali	50	65
Adam	60	58
Alex	30	70

(a) Represent the data by a 3 x 2 matrix, A.

Answer (a)..... [1]

(b) Another matrix Y is represented by $\begin{pmatrix} 0.625 \\ 0.5 \end{pmatrix}$.

(i) Find AY.

Answer (b)(i)..... [2]

(ii) Explain how the two numbers in Y came about.

Answer (b)(ii).....

.....[2]

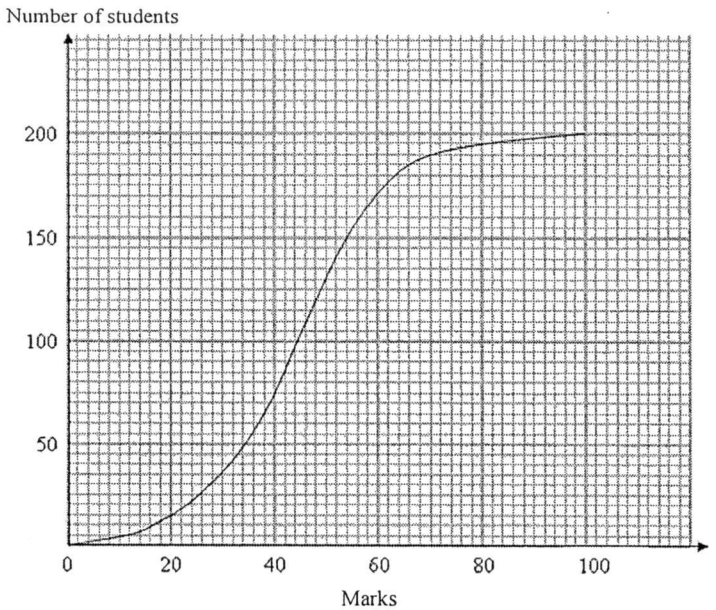
(iii) Describe what the entries in AY represent.

Answer (b)(iii).....

.....[1]

22. *Part (c) of this question is on the next page.*

The cumulative frequency curve below shows the marks obtained by 200 students in a Mathematics examination.



(a) Use the graph to find

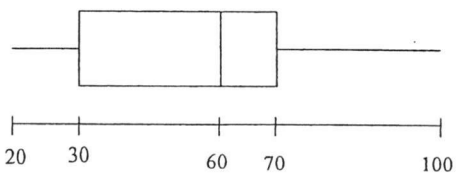
(i) the median mark

(ii) the interquartile range.

Answer (a)(i)..... [1]

(ii)..... [1]

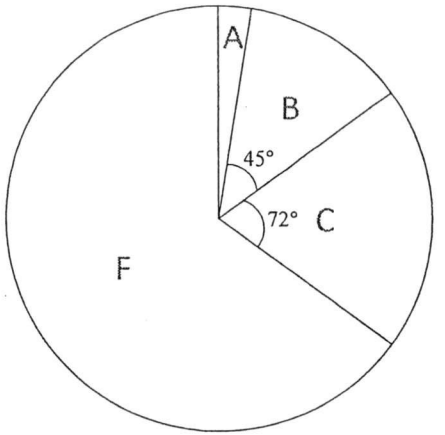
(b) The following box and whisker diagram illustrates the marks obtained when the same group of students sat for a Science examination.
Which subject did the students do better in? Explain why.



Answer (b) .....

..... [2]

(c) The mathematics examination data can also be represented by a pie chart.
Calculate the missing angles that represent the areas for A and F.



A: 80 marks and above

B: 60 to 79 marks

C: 50 to 59 marks

F: below 50 marks

Answer (c) *A*: .....°

F: .....° [3]

THE END

Answer **all** the questions.

For Examiner's Use

1. Find the values of x where $0° \leq x \leq 180°$ for $\sin x = 0.234$, giving your answers correct to the nearest degree.

0.866

Answer $x =$ 60° or 120° [1] B1 for both

2. Given that $3^{2017} + 3^{2017} + 3^{2017} = 3^x$, state the value of x.

$3(3^{2017}) = 3^x$

$3^{2018} = 3^x$

B1

Answer $x =$ 2018 [1]

3. (a) Express 50 seconds as a percentage of 5 minutes.

$\frac{50}{5 \times 60} \times 100\% = \frac{50}{3}$

B1

Answer $16\frac{2}{3}$ or 16.7 % [1]

(a) Express $62\frac{3}{5}\%$ as a decimal.

$\frac{62\frac{3}{5}}{100} =$

B1

Answer 0.626 [1]

Q4 a) $\frac{6}{4.5} = \frac{13}{h}$

$h = 9\frac{3}{4}$ or 9.75 B1

b) $\left(\frac{6}{4.5}\right)^3 = \frac{64}{27}$.

M1 find $\frac{V_1}{V_2}$ & use for comparison. → take $\frac{\text{vol}}{\text{cost}}$ or $\frac{\text{cost}}{\text{vol}}$.

Larger bottle

64 units → \$6

1 unit → \$0.09375

10.6667 unit ← \$1

Smaller bottle

27 units → \$2.50

1 unit → \$0.09259

10.8 unit ← \$1

Ans: smaller bottle. A1

4

5 Solve $x(2x + 7) = 4$.

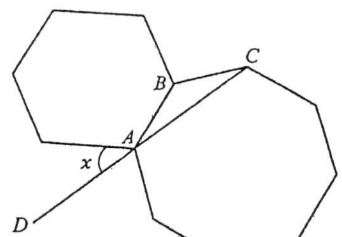
$2x^2 + 7x - 4 = 0$ M1 expand & make RHS = 0

$(2x - 1)(x + 4) = 0$ M1 factorisation or any relevant mtd.

$x = \frac{1}{2}$ or $x = -4$

A1 for both.

*8.6



A regular hexagon and regular octagon share a side AB as shown in the diagram above. DAC is a straight line. Calculate the size of angle x.

either 1 (M1) int ∡ of hexagon $= \frac{(6-2)\times 180^\circ}{6} = 120^\circ$

int ∡ of octagon $= \frac{(8-2)\times 180^\circ}{8} = 135^\circ$

(M1) $\angle BAC = \frac{180^\circ - 135^\circ}{2} = 22.5^\circ$

$x = 180^\circ - 120^\circ - 22.5^\circ = 37.5^\circ$ (A1)

Answer $x = 37.5^\circ$ [3]

Answer .... [ ]

*7. The difference between the simple interest and the compound interest earned from a principal amount P, invested for a period of 3 years at 4% per annum is \$48.64. Find the principal amount P.

simple interest : $I = \frac{P(4)(3)}{100}$

cpd interest : $I = P(1+\frac{4}{100})^3 - P$

either 1 (M1) find interest for either simple / cpd

(M1) $P(1+\frac{4}{100})^3 - P - \frac{12P}{100} = 48.64$

$\frac{76}{15625}P = 48.64$

$P = 10\,000$ (A1)

recognise that cpd interest is > simple interest ∴ cpd I − simple I = 48.64

Answer \$10 000 [3]

8. Express the following as a single fraction in its simplest form

$$\frac{2}{(x+2)^2} + \frac{4}{2x+4}.$$

$\frac{2}{(x+2)^2} + \frac{4}{2(x+2)}$

$= \frac{4}{2(x+2)^2} + \frac{4(x+2)}{2(x+2)^2}$ (M1) make denominator the same & multiply numerator by relevant factor

$= \frac{4+4x+8}{2(x+2)^2}$

(M1) $= \frac{4x+12}{2(x+2)^2} = \frac{2x+6}{(x+2)^2}$ (A1) combined into 1 fraction

Answer .... [3]

9. Expand and simplify $(x+1)^2 - x(x-2) + 1$.

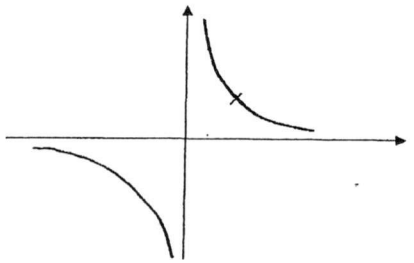
(M1) $= x^2+2x+1 - x^2+2x + 1$ (M1)

$= 4x + 2$ (A1)

accept $2(2x+1)$

Answer ..... [3]

10. (a) In the axes provided below, the point (1,1) has been marked.
Sketch the graph of $y = \frac{1}{x}$. [1]

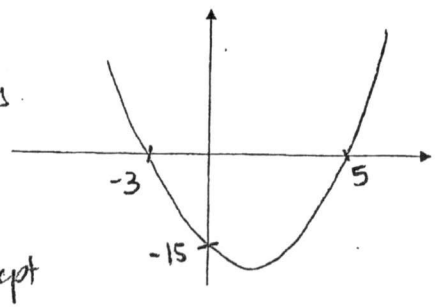


(B1) must pass through (1,1).

(b) Sketch the graph of $y = (x+3)(x-5)$ in the axes provided, indicating the ~~coordinates of the~~ x-intercept(s) and y-intercept(s) if any. [2]

(B1) shape

(B1) all intercepts labelled



when $y=0$, $x=-3$ or 5

when $x=0$, $y=-15$

* min pt cannot be at y-intercept (if not, -1)

11.

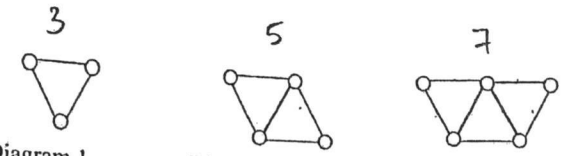
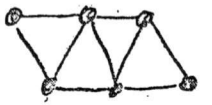


Diagram 1 Diagram 2 Diagram 3

The figure above shows the number of sticks used to form each diagram.

(a) Draw diagram 4 in the space below. [1]



(B1)

(b) Find an expression in terms of n, the number of sticks used to form Diagram n, reduced to its simplest form.

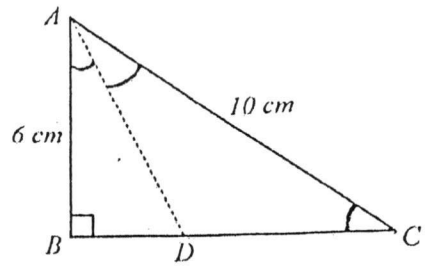
(M1) $3 + 2(n-1)$

$= 3 + 2n - 2$

$= 2n + 1$ (A1)

or (B2)

12.



In the diagram, $\angle ABC = 90°, AB = 6$ cm and $AC = 10$ cm.

(a) Write down the exact value of $\sin \angle ACB$.

$\sin \angle ACB = \frac{3}{5}$ or 0.6

Answer (exact) ..... [

* (b) D is a point on BC such that $\angle BAD = \angle DAC$. Find the length of BD.

$\cos \angle CAB = \frac{6}{10}$

$\angle CAB = 53.130°$ (M1) find $\angle CAB$.

$\tan \angle BAD = \frac{BD}{6}$

$BD = 6 \tan\left(\frac{53.130}{2}\right)°$

$= 3$ (A1) answer is exact (no rounding off)

accept 3.00

Answer .....cr

13.

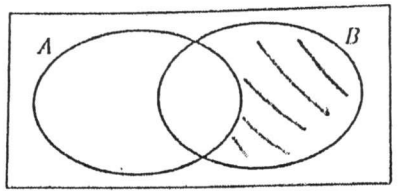
a) 0-36sec : move with increasing speed (B1)

36-40sec : stationary / at rest (B1)

(b) average speed $= \frac{200}{60}$

$= 3\frac{1}{3}$ m/s. (B1)

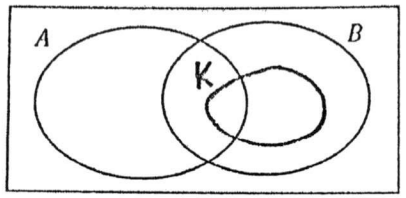
14. (a)



B1

(i) On the Venn diagram above, shade the region that represents $A' \cap B$. [1]

(ii) On the Venn diagram below, draw the set K where $K \subset B$ and $A \cap K \neq \emptyset$. [1]



B1

(b) Let $\varepsilon = \{x: x \text{ is an integer and } 2 < x \leq 10\}$
$A = \{x: x \text{ is an odd number}\}$
$B = \{x: x \text{ is a prime number}\}$.

List the elements of

(i) $A' \cup B$

B1

Answer (b)(i) 3,4,5,6,7,8,10 [1]

(ii) $(A \cup B)'$

B1

Answer (b)(ii) 4,6,8,10 [1]

15. The arrival frequency of trains at a train interchange is shown in the table below:

Train	Frequency
North Line	Every 3 minutes
South Line	Every 4 minutes
East Line	Every 6 minutes
West Line	Every 7 minutes

If all four trains arrived together at 8am,

(a) find the next time that all four trains will arrive at the same time at the interchange.

3 | 3,4,6,7
2 | 1,4,2,7
2 | 1,2,1,7
7 | 1,1,1,7
1,1,1,1

LCM = 84 min M1
= 1h 24 min

A1

Answer (a) 9.24 am [2]

(b) Calculate the total number of times all four trains arrive at the interchange from 8am to 8pm inclusive on a particular day.

12 hours = 720 min.

M1

$\frac{720}{84} = 8\frac{4}{7}$

no. of times = 8 + 1
= 9

A1

Answer (b) 9 [2]

ne: 1

16. Given that $(x+y)^2 = 1$ and $4xy = 15$,
(a) find an expression for $x^2 + y^2$.

For Examiner's Use

$x^2+2xy+y^2 = 1$

$x^2+y^2 = 1-2xy$ (B1)

Answer (a)..... [1]

(b) Hence, find the value of $(2x+3y)^2 - 5y^2$.

$= 4x^2+12xy+9y^2-5y^2$ (M1) expand correctly

$= 4x^2+12xy+4y^2$

$= 4(x^2+y^2)+12xy$ (M1) factorise

$= 4(1-2xy)+12xy$

$= 4-8xy+12xy$

$= 4+4xy$

$= 4+15$

$= 19$ (A1)

Answer (b)..... [3]

17. (a) Express 1323 as a product of its prime factors.

For Examiner's Use

3 | 1323
3 | 441
3 | 147
7 | 49
7

(B1)

Answer (a) $3^3 \times 7^2$ ..... [1]

(b) Given that $1323k$ is both a perfect square and a perfect cube, write down the smallest integer value of k.

(B1)

Answer (b) $3^3 \times 7^4$ or 64827 ..... [1]

(c) The highest common factor of 1323 and another integer x is 9 and the lowest common multiple of both numbers is ~~1323~~ 3646. Find the value of x.

$x = 2 \times 3^2$

$1323 = 3^3 \times 7^2$

$HCF = 3^2$

$LCM = 2 \times 3^3 \times 7^2$

(M1) comparing index notation

$x = 2 \times 9$

$= 18$ (A1)

18. The variables x, y and z are related. $x = ky$

x is directly proportional to the cube of y and y is inversely proportional to the square root of z. $y = \frac{k}{z}$

Given that when $x = 216$, $y = 4$ and $z = 4$,

(a) find an expression for

(i) z in terms of y,

$y = \frac{k}{\sqrt{z}}$

$4 = \frac{k}{\sqrt{4}}$

$k = 8$ (M1)

$\sqrt{z} = \frac{8}{y}$

$z = \frac{64}{y^2}$ (A1)

Answer (a)(i) ..... [1] (2m)

(ii) y in terms of x.

$y = k\sqrt[3]{x}$

$4 = k\sqrt[3]{216}$

$k = \frac{4}{6}$

$= \frac{2}{3}$ (M1)

or $x = ky^3$

$216 = k(9)^3$

$k = \frac{27}{8}$ (M1)

Answer (a)(ii) $y = \frac{2}{3}\sqrt[3]{x}$ (A1) (B1) (2m) [1]

(b) Hence or otherwise, express z in terms of x and describe the relationship between x and z.

$z = \frac{64}{\left(\frac{2}{3}\sqrt[3]{x}\right)^2}$ (M1) ecf.

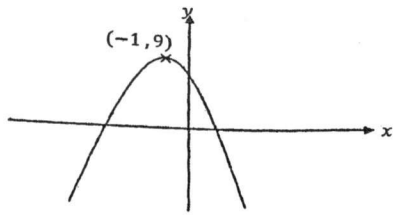
$= \frac{64}{\frac{4}{9}\sqrt[3]{x^2}}$

$= \frac{144}{\sqrt[3]{x^2}}$ (A1)

Answer (b) ..... [2]

Relationship : z is inversely proportional to cube root of x^2. (ecf) ..... [1]

19.



The diagram shows the graph of $y = -x^2 + bx + c$, where a and b are constants.
Given that the turning point of the graph is $(-1, 9)$,

(a) state the equation of line of symmetry of the graph

Answer $x = -1$ (B1) ..... [1]

(b) find the constants b and c.

$y = -(x-h)^2 + k$

$= -(x+1)^2 + 9$ (M1) sub. max pt into $y = -(x-h)^2 + k$

$= -(x^2 + 2x + 1) + 9$ (M1) expand

$= -x^2 - 2x - 1 + 9$

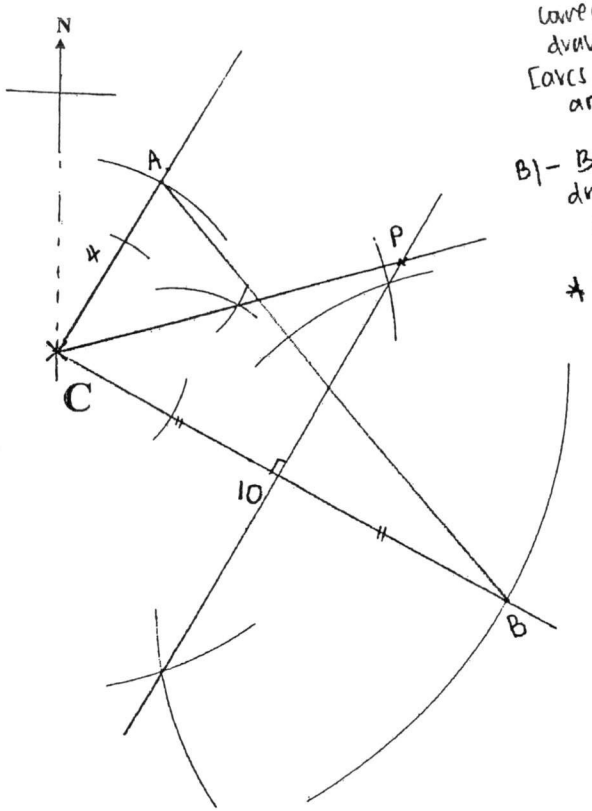
$= -x^2 - 2x + 8$

Answer $b = -2$ $c = 8$ ..... [4]

For Examiner's Use

(a) Use a scale of 1cm to 10m, make an accurate scale drawing to create triangle ABC.

[2]



B1 – AC correctly drawn to scale [arcs for length are optional]

B1 – BC correctly drawn to scale

* AB must be connected (-1 if AB not connected)

(b) From your drawing, find the actual distance, in metres, between buildings A and B.

AB = 10.8 cm ⟶ 108m
±0.1 ±1

(B1)

Answer (b) 108 m [1]

(c) By ~~drawing~~ constructing a perpendicular bisector (B1) of BC and an angle bisector (B1) of ∠ACB to identify and mark the point P that is equidistant from B and C, and from AC and BC. [2]

↑ -1 if not marked

21. The table below shows the results of 3 students for their Elementary Mathematics Preliminary Examination.
The total possible mark for Paper 1 and 2 is 80 and 100 respectively.
Papers 1 and 2 have equal weightage.

Student	Paper 1	Paper 2
Ali	50	65
Adam	60	58
Alex	30	70

(a) Represent the data by a 3 x 2 matrix, A.

Answer (a) $A = \begin{pmatrix} 50 & 65 \\ 60 & 58 \\ 30 & 70 \end{pmatrix}$ [1] (B1)

(b) Another matrix Y is represented by $\begin{pmatrix} 0.625 \\ 0.5 \end{pmatrix}$.

(i) Find AY.

$$\begin{pmatrix} 50 & 65 \\ 60 & 58 \\ 30 & 70 \end{pmatrix}\begin{pmatrix} 0.625 \\ 0.5 \end{pmatrix}$$

$$= \begin{pmatrix} 50(0.625)+65(0.5) \\ 60(0.625)+58(0.5) \\ 30(0.625)+70(0.5) \end{pmatrix} \text{(M1)}$$

$$= \begin{pmatrix} 63.75 \\ 66.5 \\ 53.75 \end{pmatrix} \text{(A1)}$$

Answer (b)(i) [2]

* (ii) Explain how the entries in Y came about.

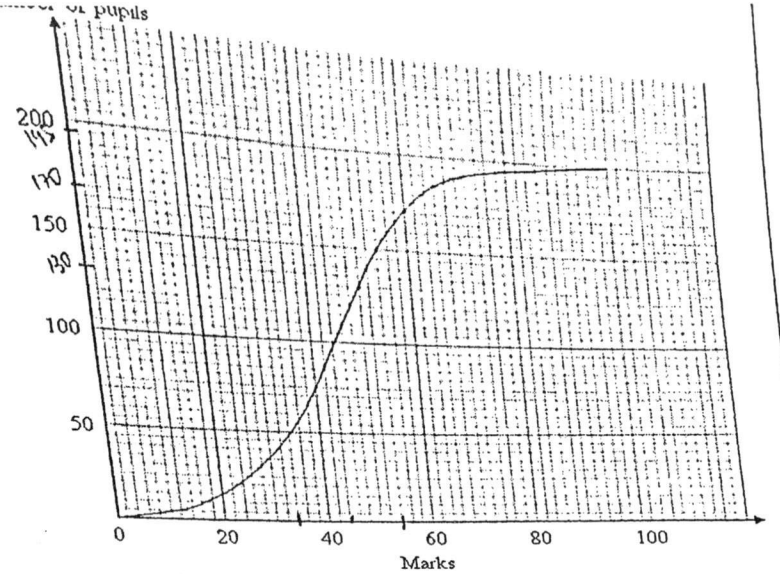
~~What do the 2 numbers in Y represent?~~ equal weightage

Answer (b)(ii) $0.625 = \frac{1}{80} \times 100 \times 50\%$ (B1)

$0.5 = \frac{1}{100} \times 100 \times 50\%$ (B1) [2]

* (iii) Describe what the entries in PY represent.

Answer (b)(iii) Total marks for the whole exam for each student respectively. (B1) [1]



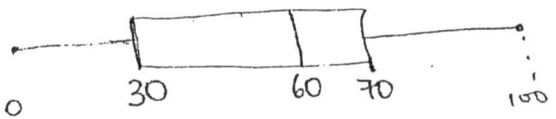
(a) Use the graph to find
(i) the median mark
(ii) the interquartile range.

54 - 34 (accept 45)

Answer (a)(i) 44 (B1) [1]

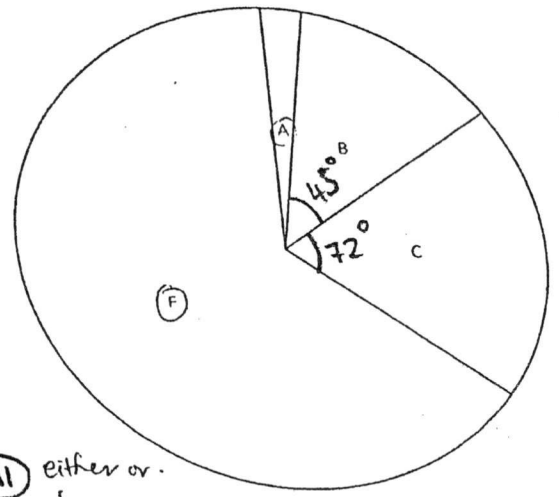
(ii) 20 (B1) [1]

(b) The following box and whisker diagram illustrates the marks obtained when the same group of students sat for a Science examination.
Which subject did the students do better in?
Explain your answer ~~in 2 ways~~. [2]



Science. median higher.
(B1) (B1)

(C) The mathematics examination data can also be represented by a pie chart. Calculate the missing angles that represent each area. [3]



A: 80 marks and above

B: 60 to ~~74~~ 79 ma

C: 50 to 59 ma

F: below 50 marks

(M1) either or.

$A: \frac{5}{200} \times 360^\circ = 9^\circ$ (A1)

$F: \frac{130}{200} \times 360^\circ = 234^\circ$ (A1)

Name:_____( )	Class : Sec _____

TAMPINES SECONDARY SCHOOL

MID-YEAR EXAMINATION 2017

SECONDARY FOUR EXPRESS/4N1

SECONDARY FIVE NORMAL (ACADEMIC)

MATHEMATICS ***4048/2***
PAPER 2 ***2 hours 30 minutes***
4 May 2017

Additional Materials:
Answer Paper
Graph Paper (1 sheet)

100

READ THESE INSTRUCTIONS FIRST

Write your name, class and register number on all the work you hand in.
Write in dark blue or black pen.
You may use a pencil for any diagrams or graphs.
Do not use staples, paper clips, highlighters, glue or correction fluid.

Answer **all** questions.
If working is needed for any question it must be shown with the answer.
Omission of essential working will result in loss of marks.
The use of an approved scientific calculator is expected, where appropriate.
If the degree of accuracy is not specified in the question, and if the answer is not exact, give the answer to three significant figures. Give answers in degrees to one decimal place.
For π, use either your calculator value or 3.142, unless the question requires the answer in terms of π.

At the end of the examination, fasten all your work securely together.
The number of marks is given in brackets [ ] at the end of each question or part question.

The total number of marks for this paper is 100.

This paper consists of 11 printed pages

Mathematical Formulae

Compound Interest

$$\text{Total amount} = P\left(1+\frac{r}{100}\right)^n$$

Mensuration

Curved surface area of a cone $= \pi rl$

Surface area of a sphere $= 4\pi r^2$

$$\text{Volume of a cone} = \frac{1}{3}\pi r^2 h$$

$$\text{Volume of a sphere} = \frac{4}{3}\pi r^3$$

$$\text{Area of a triangle } ABC = \frac{1}{2}ab\sin C$$

Arc length $= r\vartheta$, where ϑ is in radians

$$\text{Sector area} = \frac{1}{2}r^2\vartheta, \text{ where } \vartheta \text{ is in radians}$$

Trigonometry

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$a^2 = b^2 + c^2 - 2bc\cos A$$

Statistics

$$\text{Mean} = \frac{\Sigma fx}{\Sigma f}$$

$$\text{Standard deviation} = \sqrt{\frac{\Sigma fx^2}{\Sigma f} - \left(\frac{\Sigma fx}{\Sigma f}\right)^2}$$

Answer **all** the questions.

1 (a) Make z the subject of the formula [3]

$$v = \frac{2uz - 3u}{5z + 2}$$

(b) (i) Factorise $9(x-1)^2 - 64$ completely. [3]

(ii) Hence, find two factors of 18161, other than 1 and 18161. [2]

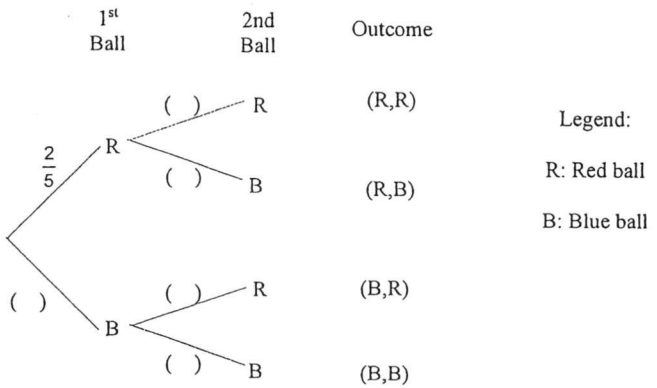
---

2 Sally had 4 red balls and 3 blue balls initially.
She bought more balls such that the number of blue balls bought is twice the number of red balls bought.
All the balls are placed in a box and one is randomly picked.

(a) Given that the probability that she picked a red ball is $\frac{2}{5}$, find the number of red and blue balls that Sally bought. [3]

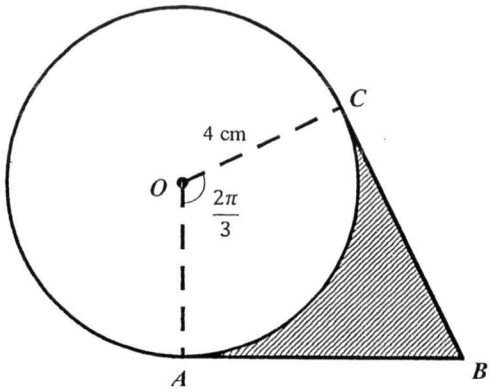
(b) Sally replaced the ball.
She then randomly picked two balls out of the box, one after another, with replacement.

(i) Copy and complete the following probability tree diagram. [2]



(ii) Find the probability that Sally picked two balls of the same colour. [2]

3(a)



The diagram above shows a circle with radius 4 cm.
AB and BC are tangents to the circle at A and C respectively.
Given that $\angle COA = \frac{2\pi}{3}$ radians, calculate

(i) the perimeter of the shaded region, [3]

(ii) the area of the shaded region. [4]

(b)

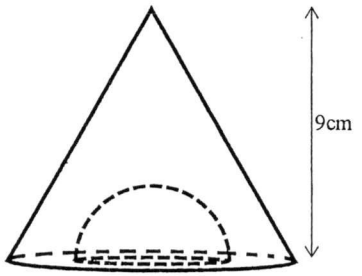


Figure 1

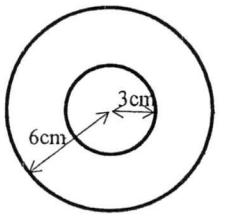
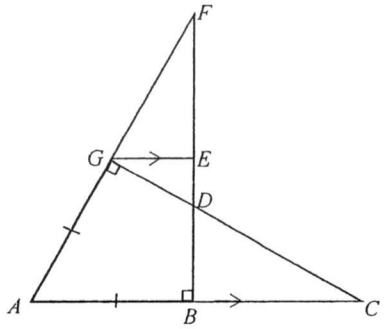


Figure 2

Figure 1 shows a solid ornament. Figure 2 shows the base of the ornament.
The ornament is a solid cone of height 9cm and radius 6cm with a hemispherical hole of radius 3cm.
Find the surface area of the ornament. [3]

4



In the diagram, $\angle ABF = \angle AGC = 90°$, GE is parallel to AC and $AG = AB$.
G is the midpoint of AF.

(a) Prove that $\Delta ABF \equiv \Delta AGC$. [2]

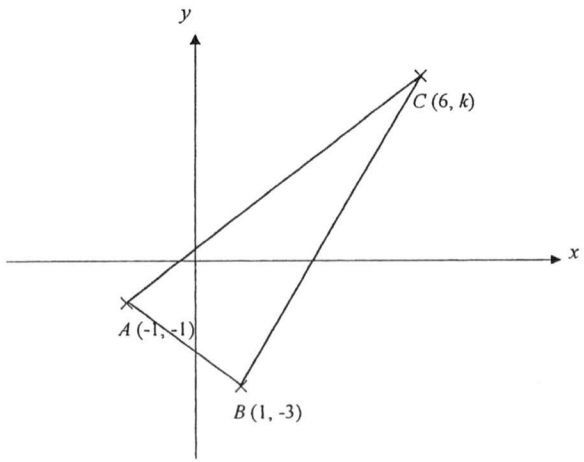
(b) Name two pairs of similar triangles. [2]

(c) Given that $\angle BFA = 30°$ and $AC = 26$ cm, find

(i) the length of BF, [2]

(ii) $\frac{\text{area of } \Delta GFE}{\text{area of } AGEB}$ [2]

5



The diagram shows a triangle ABC with $A(-1, -1), B(1, -3)$ and $C(6, k)$.
The gradient of AB is $-2n$ and the length of BC is 13 units. Find

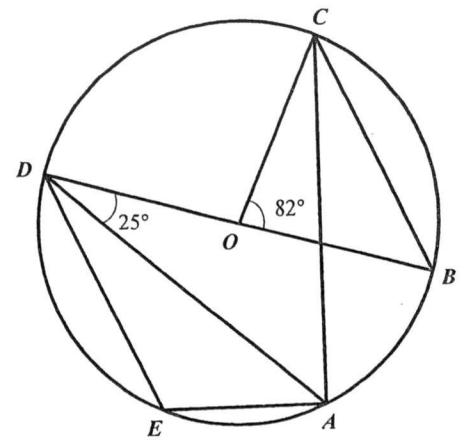
(a) the value of n, [2]

(b) the value of k [3]

(c) the equation of the line AB. [2]

(d) one possible coordinates of the point D if D is vertically above B and $ABCD$ is a trapezium. [4]

6



Points A, B, C, D and E lie on a circle with centre O.
BD is a diameter of the circle.
ED is parallel to BC, $\angle ADB = 25°$ and $\angle COB = 82°$.

(a) Giving your reasons, calculate

(i) $\angle CBO$ [1]

(ii) $\angle CAD$ [1]

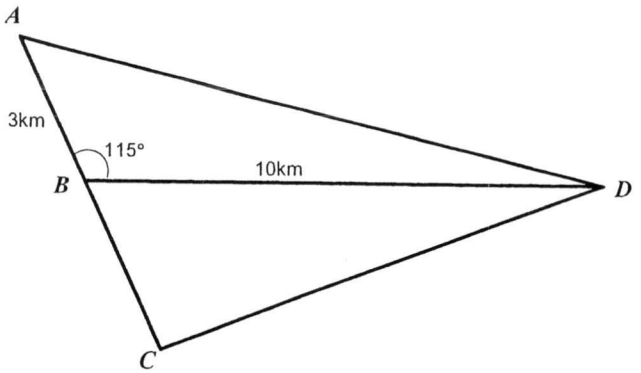
(iii) $\angle ADE$ [1]

(iv) $\angle AED$ [2]

(b) Prove that CO produced will pass through E. [3]

(c) Given that the radius of the circle is 3 cm, calculate the perpendicular distance between the chords DE and BC. [3]

7



The diagram shows four points A, B, C and D on a flat piece of land. ABC is a straight line.

Given that $AB = 3$ km, $BD = 10$ km and $\angle ABD = 115°$,

(a) calculate the length of AD. [3]

It is also known that the area of $\Delta BCD = 15.9$ km^2.

(b) Find the length of BC. [2]

A man walked along BD, from B until he reached a point E, where CE is the shortest distance from C to BD.

(c) Calculate the distance he walked. [3]

The same man flies a drone vertically above E such that the angle of elevation from C to the drone is 14°.

(d) Find the angle of depression from the drone to D. [3]

8 A plane flew a distance of 1700km from Singapore to Bali at an average speed of x km/h.

(a) Write down an expression in terms of x, for the time it took in hours to complete the journey. [1]

The same plane returned via the same route with an average speed that is 100km/h faster.

(b) Write down an expression in terms of x, for the time taken in hours to complete the return journey. [1]

Given that the return journey took 22 minutes shorter than the journey from Singapore to Bali,

(c) form an equation in x and show that it reduces to $11x^2 + 1100x - 5100000 = 0$. [3]

(d) Solve the equation $11x^2 + 1100x - 5100000 = 0$, leaving your answers correct to 1 decimal place. [3]

(e) If the plane departs from Singapore to Bali at 8am and stays in Bali for 3 hours for maintenance before returning back to Singapore, find the time that the plane will arrive back in Singapore. [4]

9 **Answer this question on a piece of graph paper.**

The variables x and y are connected by the equation $y = \frac{x^2}{8} + \frac{4}{x} - 4$.
The table below shows some values of x and the corresponding y values corrected to 2 decimal places.

x	1	2	3	4	5	6	7
y	0.13	-1.5	-1.54	-1	p	1.17	2.70

(a) Calculate the value of p. [1]

(b) Using a scale of 2cm to represent 1 unit on the horizontal x-axis and 4cm to represent 1 unit on the vertical y-axis, draw the graph of $y = \frac{x^2}{8} + \frac{4}{x} - 4$ for $1 \leq x \leq 7$. [3]

(c) By drawing another straight line, find the solution to $\frac{x^2}{8} + \frac{4}{x} - 6 = 0$. [2]

(d) Find the gradient of the curve at $x = 4$. [2]

(e) (i) On the same axes, draw the graph of $y = -x + 2$. [1]
(ii) Write down the x-coordinate of the point of intersection of the line and the curve. [1]
(iii) This value of x is a solution to the equation $ax^3 + bx^2 - 6x + 4 = 0$.
Find the values of a and b. [2]

10 Paul wants to travel from Tampines Secondary School to Marina Barrage at a certain time of day. He has decided to hire a private car to bring him to the location. His three choices of car companies are Comfy, Ubab and Grer.

The pricing information for each of his choices and other relevant information are given in the next page.

Assuming he wants to make the trip at 4pm on a Monday,
(a) calculate the amount he will need to pay if he chooses Grer. [1]
(b) Justify which company you would recommend to be the most economical for Paul to choose for this trip. Show your calculations clearly. [6]

If Paul wants to make the trip at 6.30pm on a Wednesday instead,
(c) would you recommend a different company to be more economical for Paul to choose? Why or why not? [3]

Distance between Tampines Secondary School and Marina Barrage		23km
Average time to travel from Tampines Secondary School to Marina Barrage	During non-peak period	25 minutes
	During peak period	30minutes

Comfy Taxi

Base Fare (1km or less)		$3.20
Every 400m thereafter or less, up to 10km		$0.22
Every 350m thereafter or less, after 10km		$0.22
Every 45 seconds of waiting time or less		$0.22
Peak period surcharge:	Monday to Friday, 6am – 9.29am Monday to Sunday, 6pm to 11.59pm	25% of meter fare
	Monday to Sunday, midnight – 5.59am	50% of meter fare

Ubab

Base Fare	$3
Per km	$0.45
Per minute	$0.20

Grer

Base Fare	$3
Per km	$0.80

End of Paper

1 (a)

$$V = \frac{2uz - 3u}{5z + 2}$$

$V(5z+2) = 2uz - 3u$ (M1)

$5vz + 2v = 2uz - 3u$

$5vz - 2uz = -2v - 3u$

$z(5v - 2u) = -2v - 3u$ (M1)

$z = \frac{-2v-3u}{5v-2u}$ (A1) or $\frac{2v+3u}{2u-5v}$ or $-\frac{2v+3u}{5v-2u}$

(b) (i) $9(x-1)^2 - 64$

$= [3(x-1)]^2 - 8^2$ (M1)

$= [3(x-1)+8][3(x-1)-8]$ (M1)

$= (3x+5)(3x-11)$ (A1)

or $9x^2 - 18x - 55$

$= (3x+5)(3x-11)$.

(ii) $9(x-1)^2 - 64 = 18161$

$(x-1)^2 = 2025$

$x - 1 = 45$

$x = 46$ (M1)

Ans: $3(46)+5$ and $3(46)-11$

$= 143$ and 127 (A1)

other acceptable answers

11, 1651,

13, 1397

2 (a) $P(\text{red ball}) = \frac{2}{5}$.

let x be the no. of red balls bought.

$\frac{4+x}{7+3x} = \frac{2}{5}$ (M1)

$5(4+x) = 2(7+3x)$

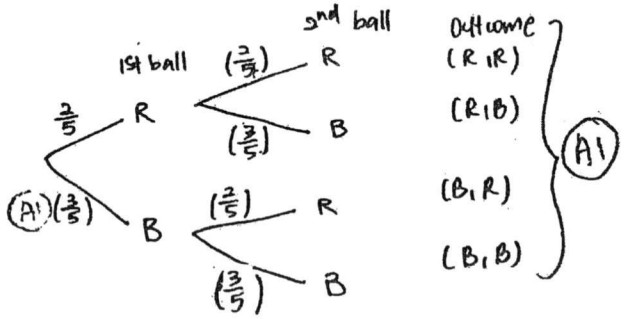
$20 + 5x = 14 + 6x$

$6 = x$

no. of red balls bought = 6 (A1)

no. of blue balls bought = 12 (A1)

(b) (i)



total no. of red = 10

total no. of blue = 15

(ii) $P(\text{both balls same colour}) = \frac{2}{5} \times \frac{2}{5} + \frac{3}{5} \times \frac{3}{5}$ (M1)

$= \frac{13}{25}$. (A1)

3 (a)(i) length of minor arc AC $= 4\left(\frac{2\pi}{3}\right)$ (M1)

$= 8.37758$

$\tan\left(\frac{\pi}{3}\right) = \frac{BC}{4}$

$BC = 4\tan\left(\frac{\pi}{3}\right)$ (M1)

$= 6.9282$

Perimeter of shaded region $= 8.37758 + 2\times 6.9282$

$= 22.23398$

≈ 22.2 cm (A1)

(a)(ii) area of half of minor sector COA $= \frac{1}{2}(4)^2\left(\frac{\pi}{3}\right)$ (M1)

$= \frac{8\pi}{3}$ 8.3776

or $\frac{1}{2}(4)^2\left(\frac{2\pi}{3}\right)$

$= 16.755$

area of Δ OAB $= \frac{1}{2}\times 4\times 4\tan\left(\frac{\pi}{3}\right)$ (M1) 27.713

$= 8\tan\left(\frac{\pi}{3}\right)$

area of shaded region $= \left[8\tan\left(\frac{\pi}{3}\right) - \frac{8\pi}{3}\right]\times 2$ (M1)

13.856 − 8.3776

$= 10.9577$ 5.4788 × 2

≈ 11.0 cm^2 (A1)

(b) surface area of ornament $= \pi(6)(\sqrt{117}) + \left[\pi(6)^2 - \pi(3)^2\right] + 2\pi(3)^2$ (M1) either one (M1)

203.88 84.83 56.549

$= 345.26$

≈ 345 cm^2 (A1)

4 (a) AG = AB (given)

$\angle ABF = \angle AGC = 90°$ (given)

$\angle GAC = \angle BAF$ (common) (M1)

$\therefore \Delta ABF \equiv \Delta AGC$ (AAS) (M1)

(b) ΔEGD and ΔBCD (B1) ΔGDE and ΔCDB

ΔFGE and ΔFAB (B1) ΔFGE and ΔGDE

etc.

(c)(i) BF = GC.

$\sin 30° = \frac{GC}{26}$ (M1)

$GC = 26\sin 30°$

BF = 13 (A1)

(ii) $\frac{\text{area of } \Delta GFE}{\text{area of } \Delta AFG} = \left(\frac{1}{2}\right)^2$ (M1)

$= \frac{1}{4}$

$\frac{\text{area of } \Delta GFE}{\text{area of AGEB}} = \frac{1}{3}$ (A1)

8

Q4c) (i) * Triangle should not exist — it does not follow trigo ratio.
Question should have been ∠BFA = 30°, not ∠BAF.
Because of this error, the following answers are all acceptable.

(1) using Pyth Thm:
$BF = \sqrt{26^2 - 13^2}$ (M1)
$= 22.5166605$
≈ 22.5 (A1)

↓ $BF^2 = 26^2 - 13^2$.

(2) using TOA CAH SOH
$\tan 30° = \frac{BF}{13}$ (M1)
$BF = 13\tan 30°$
≈ 7.51 (A1)

(3) using sine rule
$\frac{BF}{\sin 30°} = \frac{26}{\sin 90°}$ (M1)
$BF = 26\sin 30°$
$= 13.$ (A1)

(4) using cosine rule:
$BF^2 = 26^2 + 13^2 - 2(26)(13)\cos 30°$ (M1)
$BF^2 = 259.566827$
$BF \approx 16.1.$ (A1)

5 (a) gradient of AB $= \frac{-3+1}{1-(-1)}$ (M1)
$= \frac{-2}{2}$
$-2n = -1$
$n = \frac{1}{2}$ (A1)

(b) length of BC = 13
$\sqrt{(6-1)^2 + (k+3)^2} = 13$ (M1)
$\sqrt{25 + (k+3)^2} = 13$
$25 + (k+3)^2 = 169$
$(k+3)^2 = 144$
$k+3 = 12$ or -12
$k = 9$ or -15 (rejected)
(A1)

or $k^2 + 6k - 135 = 0$
$(k-9)(k+15) = 0$ (M1)

(c) equation of AB: $y = mx + c$
$y = -x + c$
Sub (-1, -1):
$-1 = -(-1) + c$
$c = -2$
∴ eqn of AB: $y = -x - 2$

(d) D(1, y) (M1)
grad of CD = grad of AB
$\frac{y-9}{1-6} = -1$ (M1)
$y - 9 = 5$
$y = 14$ (A1)
D(1, 14) (A1)

or grad of AD = grad of BC
$\frac{y+1}{1+1} = \frac{9+3}{6-1}$
$\frac{y+1}{2} = \frac{12}{5}$
$y = 3.8$
D(1, 3.8)

6 (a)(i) $\angle CBO = \frac{180° - 82°}{2}$ (base $\angle$ of isos $\triangle$)

$= 49°$ (B1)

(ii) $\angle CAD = 49°$ ($\angle$ in same seg.) (B1)

(iii) $\angle ADE = 49° - 25°$ (alt $\angle$, ED // BC)

$= 24°$ (B1)

(iv) $\angle BED = 90°$ ($\angle$ in semicircle)
$\angle BEA = 25°$ ($\angle$ in same seg.) } (M1)

$\angle AED = 90° + 25°$

$= 115°$ (A1)

(b) $\angle DAE = 180° - 115° - 24°$ ($\angle$ sum of $\triangle$) (M1)

$= 41°$

$\angle CAE = 41° + 49°$ (M1)

$= 90°$

since $\angle CAE = 90°$, CE is diameter (M1) ($\angle$ in semicircle)

$\Rightarrow$ CO produced will pass through E.

(c) Let x be the $\perp$ distance from BC to O.

$\sin 49° = \frac{x}{3}$ (M1)

$x = 3\sin 49°$

Perpendicular dist between DE and BC $= 2 \times 3\sin 49°$ (M1)

$= 4.53$ cm (to 3sf) (A1)

7 (a) $AD^2 = 3^2 + 10^2 - 2(3)(10)\cos 115°$ (M1) (M1) sqrt

$AD = 11.5913$

≈ 11.6 km (A1)

(b) $\frac{1}{2} \times BC \times 10 \sin 65° = 15.9$ (M1)

$BC = 3.50874$

≈ 3.51 km (A1)

(c) $\frac{1}{2} \times 10 \times CE = 15.9$

$CE = 3.18$ (M1)

$\tan 65° = \frac{3.18}{BE}$ (M1)

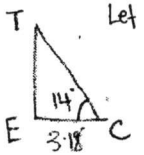
$BE = \frac{3.18}{\tan 65°}$

$= 1.48286$

≈ 1.48 km

He walked 1.48 km (A1)

(d) T



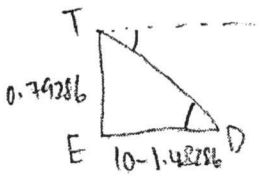
Let T be the position of the drone.

$\tan 14° = \frac{TE}{3.18}$ (M1)

$TE = 3.18 \tan 14°$

$= 0.79286$

11



$ED = 10 - 1.48286$ (M1)

$= 8.51714$

$\tan \angle TDE = \frac{0.79286}{8.51714}$

$\angle TDE = 5.3183$

$\approx 5.3°$

$\angle$ of depression from drone to D $= 5.3°$ (A1)

8 (a) $\frac{1700}{x}$ h (B1)

(b) $\frac{1700}{x+100}$ h (B1)

(c) $\frac{1700}{x} - \frac{1700}{x+100} = \frac{22}{60}$ (M1)

$\frac{1700(x+100) - 1700x}{x^2+100x} = \frac{22}{60}$ (M1)

$170000(60) = 22x^2 + 2200x$

$22x^2 + 2200x - 10200000 = 0$ (M1)

$11x^2 + 1100x - 5100000 = 0$ (shown)

(d) $11x^2 + 1100x - 5100000 = 0$

$x = \frac{-1100 \pm \sqrt{1100^2 - 4(11)(-5100000)}}{2(11)}$ (M1) show correct sub^n of values

$= 632.7$ or -732.7 (A1) (A1)

(e) SG to Bali : $\frac{1700}{632.7} = 2h\ 41min$ (M1)

Stop over : 3h

Depart Bali : 8am + 2h 41min + 3h
= 1.41 pm (M1)

Bali to SG : $\frac{1700}{732.7} = 2h\ 19min$ (M1)

Arrival Time : 1.41 pm + 2h 19min
= 4 pm. (A1)

9(b)

AXIS – 1
(Correct Scale)
points (all plotted – 1
(correctly)
smooth curve – 1

(a) $p = -0.08$ (B1)

(c) (B1) $y = 2$

(d) (B1)

(e)(i) (B1) $y = -x + 2$

(4, −1)

(2.7, −2)

(A1) (c) $x = 6.6 \pm 0.1$

(d) gradient
$= \frac{-1+2}{4-2.7}$
$= 0.769$ (A1) ± 0.1

(e)(ii) $x = 3.4$ (B1) ± 0.1

(iii) $\frac{x^2}{8} + \frac{4}{x} - 4 = -x + 2$ (M1)

(A1)

$a = \frac{1}{8}, b = 1$

$\frac{x^3}{8} + 4 - 4x = -x^2 + 2x$

$\frac{x^3}{8} + x^2 - 6x + 4 = 0$

Q9 e iii) question paper has a typo.

the equation is supposed to be $ax^3 + bx^2 - 6x + 4 = 0$

not $ax^3 + bx^2 - 8x + 4 = 0$.

(M1) was given to equate eq^n of curve to eq^n of line.

remaining (A1) would be given if student has tried to manipulate the equation and reduce it as much as possible.

---

a) amount to pay = $\$3 + \$0.80(23)$

$= \underline{\$21.40}$. (B1)

b) Ubab

amount to pay = $\$3 + \$0.45(23) + \$0.20(25)$

$= \underline{\$18.35}$ (B1).

Comfy.

base fare = $\$3.20$

first 10km : $\frac{10000}{400} = 25$

$25 \times \$0.22 = \underline{\$5.50}$ } (M1)

Next 13km : $\frac{13000}{350} = 37.14$

$38 \times \$0.22 = \underline{\$8.36}$ } (M1).

Waiting time : $\frac{25 \times 60}{45} = 33.33$

$\$0.22 \times 34 = \underline{\$7.48}$.

either (M1) for calculating base fare or waiting time

total cost = $\$3.20 + \$5.50 + \$8.36 + \$7.48$

$= \underline{\$24.54}$ (M1).

Conclusion : Ubab (A1).

10(c)

c) Comfy.

new waiting time $= \frac{30 \times 60}{45}$

$= 40$.

new waiting time charge $= 40 \times \$0.22$

$= \$8.80$.

meter fare $= \$3.20 + \$5.50 + \$8.36 + \$8.80$

$= \$25.86$.

price after surcharge $= \$25.86 \times 125\%$

$= \$32.33$.

M1 for showing working to calculate surcharge

Ubab.

amount to pay $= \$3 + \$0.45(23) + \$0.20(30)$

$= \$19.35$ (M1)

Conclusion: Stay with Ubab because still the cheapest. (A1).

Class	Register Number

Name ____________________

4048/01 **17/S4PR1/EM/1**

MATHEMATICS **PAPER 1**

Tuesday **2 May 2017** **2 hours**

VICTORIA SCHOOL

PRELIMINARY EXAMINATION ONE
SECONDARY FOUR

Candidates answer on the Question Paper.

READ THESE INSTRUCTIONS FIRST

Write your name, class and register number on all the work you hand in.
Write in dark blue or black pen.
You may use a pencil for any diagrams or graphs.
Do not use paper clips, highlighters, glue or correction fluid.

Answer **all** the questions.
If working is needed for any question it must be shown with the answer.
Omission of essential working will result in loss of marks.
You are expected to use a scientific calculator to evaluate explicit numerical expressions.
If the degree of accuracy is not specified in the question, and if the answer is not exact, give the answer to three significant figures. Give answers in degrees to one decimal place.
For π, use either your calculator value or 3.142, unless the question requires the answer in terms of π.

The number of marks is given in brackets [ ] at the end of each question or part question.
The total number of marks for this paper is 80.

This paper consists of **16** printed pages, including the cover page.

[Turn over

Mathematical Formulae

Compound interest

$$\text{Total amount} = P\left(1+\frac{r}{100}\right)^n$$

Mensuration

$$\text{Curved surface area of a cone} = \pi r l$$

$$\text{Surface area of a sphere} = 4\pi r^2$$

$$\text{Volume of a cone} = \frac{1}{3}\pi r^2 h$$

$$\text{Volume of a sphere} = \frac{4}{3}\pi r^3$$

$$\text{Area of triangle } ABC = \frac{1}{2}ab\sin C$$

$$\text{Arc length} = r\theta, \text{ where } \theta \text{ is in radians}$$

$$\text{Sector area} = \frac{1}{2}r^2\theta, \text{ where } \theta \text{ is in radians}$$

Trigonometry

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$a^2 = b^2 + c^2 - 2bc\cos A$$

Statistics

$$\text{Mean} = \frac{\Sigma fx}{\Sigma f}$$

$$\text{Standard deviation} = \sqrt{\frac{\Sigma fx^2}{\Sigma f} - \left(\frac{\Sigma fx}{\Sigma f}\right)^2}$$

1 Calculate $\sqrt[3]{\dfrac{45^2-63.2}{0.896-1}}$, giving your answer correct to 5 significant figures.

Answer ..... [1]

---

2 4.854×10^{-4} metres can be written as k micrometres. Find k.

Answer $k =$ ..... [1]

---

3 When written as the product of their prime factors,

p is $2\times3^2\times5^3$,
q is $2^3\times3\times7^3$,
r is $2^4\times3^2\times7^2$.

Find the

(a) value of the square root of r,

Answer (*a*) ..... [1]

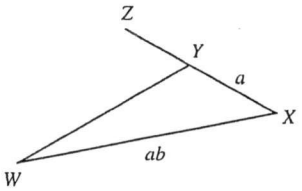
(b) greatest number that will divide p, q and r exactly,

Answer (*b*) ..... [1]

(c) LCM of p, q and r, giving your answer as the product of its prime factors.

Answer (*c*) ..... [1]

4 In the triangle WXY, $XY = a$ cm, $WX = ab$ cm and $WY = 2XY$. XY is produced to Z. Find $\cos \angle WYZ$ in terms of a and/or b.



Answer $\cos \angle WYZ =$ ..... [2]

---

5 The length of a rectangular block is 30 cm and its breadth is 20 cm, both correct to the nearest centimetre.

(a) Find the least possible base area of the block in m^2.

Answer (*a*) ..... m^2 [1]

(b) The volume of the block is 0.03 m^3, correct to the nearest 0.01 cubic metres. Find the greatest possible height of the block in metres.

Answer (*b*) ..... m [2]

6 **(a)** Express $-x^2-2x+1$ in the form $k-(x+h)^2$.

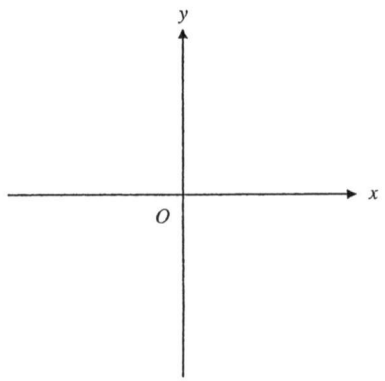
Answer (a) ..... [2]

(b) Hence, solve $x^2+2x=1$, giving your answers to 2 decimal places.

Answer (b) $x =$ ..... or ..... [2]

7 Sketch the graph of $y=\frac{1}{x}-1$.

Answer

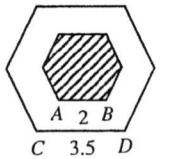


[2]

8 Evaluate $\frac{2^{1004}-15\left(2^{1000}\right)+1}{2^{1001}+2}$.

Answer ..... [2]

9 In a recent concert held at the Singapore Indoor Stadium, the hexagonal central stage is divided into 2 sections. The shaded area is a raised stage, similar to the central stage, while the remaining area is a water stage.



(a) Given that $AB=2$ m and $CD=3.5$ m, find the value of $\frac{\text{area of raised stage}}{\text{area of water stage}}$.

Answer (a) ..... [2]

(b) The area of the raised stage is 4 m^2. Find the volume of water required to fill the water stage given that the depth is 5 cm.

Answer (b) ..... m^3 [2]

10 **(a)** It is given that,
$\xi = \{\text{real numbers}\}$,
$N = \{\text{natural numbers}\}$,
$W = \{\text{whole numbers}\}$.

State if each of the following statements is True or False.

(i) $W \subset N$,

Answer (a)(i) ..... [1]

(ii) $-2 \in W$,

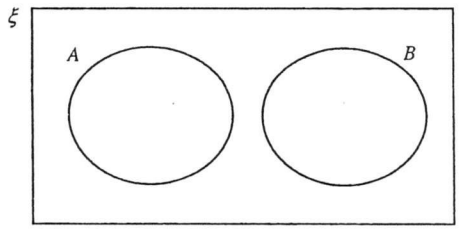
Answer (a)(ii) ..... [1]

(iii) $\mathrm{n}(N \cap W) = \mathrm{n}(N)$.

Answer (a)(iii) ..... [1]

(b) On the Venn Diagram shown in the answer space, shade the set $(A \cap B) \cup B'$.

Answer (b)



[1]

---

11 Simplify $\left(\frac{b^4}{2a}\right)^{-3} \div \frac{a^{\frac{1}{2}}(3b)^0}{\sqrt[3]{64b^{12}}}$.

Answer ..... [3]

12 It is given that $32^y \div 4^x = 1$.

(a) Show that $5y = 2x$.

Answer (a) .....

.....

.....

..... [1]

(b) Hence or otherwise, solve the simultaneous equations,

$$32^y \div 4^x = 1,$$
$$4x - (y + x) = 13.$$

Answer $x =$ ....., $y =$ ..... [3]

---

13 Simplify $\frac{6m^2 - mn - 2n^2}{10m + 5n + 2mn + n^2}$.

Answer ..... [4]

14 **(a)** Solve the inequality $\frac{4x-7}{3} < \frac{1}{2}(x+3)+1 \leq \frac{3}{4}x+8$.

Answer (a) ..... [3]

(b) Represent your answer to part **(a)** on the number line below.

Answer (b)

[1]

---

15 A map has a scale of $1 : n$.
A conservation enclosure has an area of 33.8 km^2. It is represented by an area of 80 cm^2 on the map.

(a) Find the value of n.

Answer (a) $n =$ ..... [2]

(b) The perimeter of the conservation enclosure on the map is 12 cm. Find the actual perimeter of the enclosure in kilometres.

Answer (b) ..... km [2]

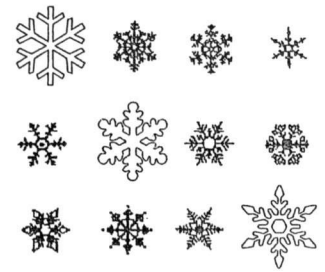
16 Blind boxes are sealed boxes containing collectibles from a specific series. Consumers do not know which specific design from the series they will receive from the purchased blind box. The probability of getting each design in each series is printed on the box.

Series I

COLLECT THEM ALL!

A	E	I	M
x	1/36	x	1/24
B	**F**	**J**	**N**
x	1/24	x	1/24
C	**G**	**K**	**O**
1/6	1/72	x	1/36
D	**H**	**L**	**P**
x	1/72	x	1/24

Series II



1/12 CHANCE OF EACH!

(a) Using the information from **Series I**,

(i) calculate the value of x,

Answer (a)(i) $x =$ ..... [1]

(ii) from designs A to P, state which is the most common design.

Answer (a)(ii) ..... [1]

(b) In **Series II**, consumers will receive either a small black snowflake or a large white snowflake. A carton of **Series II** contains 12 boxes, each with a different design. Two boxes from this carton were purchased. Find the probability of receiving

(i) 2 small black snowflakes,

Answer (b)(i) ..... [1]

(ii) at least 1 large white snowflake.

Answer (b)(ii) ..... [1]

17 Singapore will see her first water price revision in 17 years on 1 July 2017. The price will be increased in 2 steps, on 1 July 2017 and on 1 July 2018.

Water Price Revision 2017

		Current		From 1 July 2017		From 1 July 2018	
		Water Price ($/m³)		Water Price ($/m³)		Water Price ($/m³)	
		0 - 40m³	> 40m³	0 - 40m³	> 40m³	0 - 40m³	> 40m³
Potable Water	Tariff	$1.17	$1.40	$1.19	$1.46	$1.21	$1.52
	Water Conservation Tax (% of water tariff)	$0.35 (30% of $1.17)	$0.63 (45% of $1.40)	$0.42 (35% of $1.19)	$0.73 (50% of $1.46)	$0.61 (50% of $1.21)	$0.99 (65% of $1.52)
# Used Water	Waterborne Fee	$0.28	$0.28	$0.78	$1.02	$0.92	$1.18
	Sanitary Appliance Fee	$2.80 per fitting*		Combined into Waterborne Fee		Combined into Waterborne Fee	
Total Price		$2.10	$2.61	$2.39	$3.41	$2.74	$3.69

Refers to the water consumed by the household. Waterborne Fee and Sanitary Appliance Fee go towards meeting the cost of treating used water and maintaining the used water network.

The table above is taken from Singapore's National Water Agency, Public Utilities Board's website. It was also stated that key revision to the water price includes "a 30% increase in water price, phased over 2 years, starting from 1 July 2017".

(a) Determine how the 30% increase in water price is calculated. Show your working clearly.

Answer (a) .....

.....

.....

..... [2]

(b) Does this 30% increase in water price apply to all households regardless of water usage? Justify your answer.

Answer (b) .....

.....

..... [2]

18 A multiple choice test has a total of 50 questions. All questions must be attempted. There are 2 sections to the paper, Section *A* and Section *B*. The matrix **Q** shows the number of questions attempted by Victoria.

$$\mathbf{Q} = \begin{matrix} \text{Correct} & \text{Incorrect} \\ \end{matrix} \begin{pmatrix} 18 & 2 \\ x & 5 \end{pmatrix} \begin{matrix} \text{Section } A \\ \text{Section } B \end{matrix}$$

(a) State the value of x.

Answer (a) $x =$ ..... [1]

(b) 2 marks are awarded for each correct answer and 1 mark is deducted for each incorrect answer. Represent the mark allocation in a column matrix **M**.

Answer (b) **M** = ... [1]

(c) Evaluate the matrix **QM**.

Answer (c) **QM** = ... [1]

(d) State what the elements in **QM** represent.

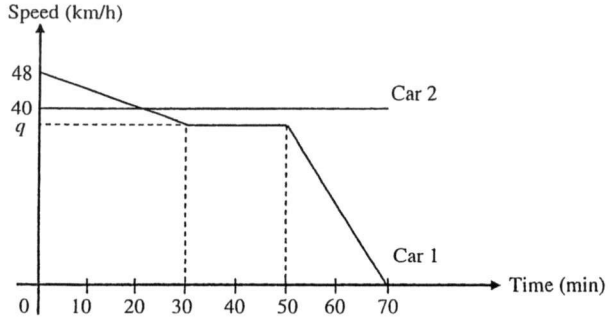
Answer (d) .....

..... [1]

(e) By matrix multiplication, find the total marks scored by Victoria in the test.

Answer (e) ..... marks [1]

19 The diagram shows the speed-time graphs of 2 cars travelling from Town A to Town B.



(a) Car 1 travelled at a constant deceleration of 24 km/h^2 for the first 30 minutes of the journey. Calculate the value of q.

Answer (a) $q =$ ..... [1]

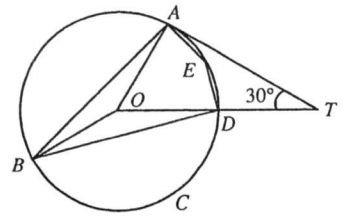
(b) Convert q km/h into m/s.

Answer (b) ..... m/s [1]

(c) Find the distance from Town A when the 2 cars meet.

Answer (c) ..... km [3]

20



O is the centre of the circle passing through A, B, C, D and E.
TA is a tangent to the circle.
Angle $ATO = 30°$.

(a) Find, giving reasons for each answer,

(i) angle ABD,

Answer (a)(i) Angle $ABD =$ ..... [3]

(ii) angle AED.

Answer (a)(ii) Angle $AED =$ ..... [1]

(b) Given that the radius of the circle is 2 cm and angle BOD = angle AED, show that the area of segment BCD is $\left(\frac{5}{3}\pi - 1\right)$ square centimetres.

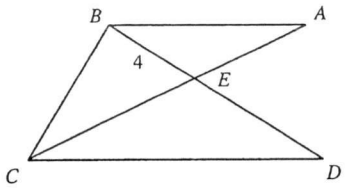
Answer (b) .....

.....

.....

..... [2]

21 Lines AC and BD intersect at E. It is given that $BE = 4$ cm, $BD = 10$ cm and $3AE = 2EC$.



(a) Prove a pair of similar triangles.

Answer (a) .....

.....

.....

.....

..... [3]

(b) Prove that AB is parallel to CD.

Answer (b) .....

..... [1]

(c) Find the value of

(i) $\dfrac{\text{area of triangle } AEB}{\text{area of triangle } CED}$,

Answer (c)(i) ..... [1]

(ii) $\dfrac{\text{area of triangle } AEB}{\text{area of triangle } BCD}$.

Answer (c)(ii) ..... [2]

22 The two towers within the One Raffles Place Complex are amongst the most iconic buildings in the Central Business District in Singapore.

Despite being constructed in the 1980s, Tower 1 (**on the left**) remains as one of the tallest buildings on the island. Given that the tower has a triangular cross-sectional area such that $AB = BC = 30$ m and $AC = 40$ m.

Using ruler and compass,

(a) construct the cross-sectional of the tower using a scale of 1 cm to 5 m, [1]

(b) construct the perpendicular bisector of BC and the angle bisector of angle ABC. [2]

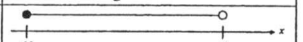
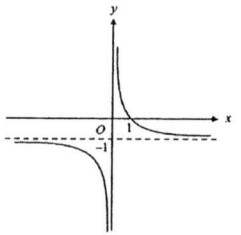
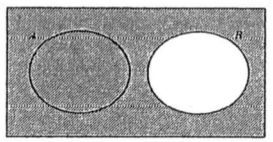
Answer (a), (b) and (c)

An ideal spot to view the sunset from the rooftop of the tower is equidistant from the edges AB and BC and nearer to point B.

(c) Mark and label a possible spot, S, that is ideal to view the sunset. [1]

End of Paper

Answer Key

1		-26.620		14	(a)	$-22 \le x < 5\frac{4}{5}$		
2		485.4			(b)			
3	(a)	84	(b) 6	15	(a)	65 000	(b)	7.8 km
	(c)	$2^4 \times 3^2 \times 5^3 \times 7^3$		16	(ai)	$\frac{1}{12}$	(aii)	C
4		$\frac{b^2-5}{4}$			(bi)	$\frac{6}{11}$	(bii)	$\frac{5}{11}$
5	(a)	$\frac{2301}{40000}$ m^2 or 0.057525 m^2		17	(a)	$\frac{2.74-2.10}{2.10} \times 100\% \approx 30\%$		
	(b)	$\frac{1400}{2301}$ m			(b)	For usage > 40 m^3, % increase $= 41\frac{7}{13}\%$ No, usage> 40 m^3, % inc $> 30\%$		
6	(a)	$-(x+1)^2+2$		18	(a)	25		
	(b)	$x = -2.41$ or $x = 0.41$			(b)	$\begin{pmatrix} 2 \\ -1 \end{pmatrix}$	(c)	$\begin{pmatrix} 34 \\ 45 \end{pmatrix}$
7					(d)	Marks obtained for Section A and Section B respectively.		
					(e)	$(1 \quad 1)\begin{pmatrix} 34 \\ 45 \end{pmatrix} = (79)$, 79 marks		
8		$\frac{1}{2}$		19	(a)	36		
9	(a)	$\frac{16}{33}$			(b)	10 m/s		
	(b)	$\frac{33}{80}$ m^3 or 0.4125 m^3			(c)	30 km		
10	(a)	False, False, True		20	(ai)	30°		
	(b)				(aii)	150°		
11		$\frac{32a^{\frac{5}{2}}}{b^8}$			(b)	$\frac{150°}{360°} \times \pi(2^2) - \frac{1}{2}(2)(2)\sin 150°$		
12	(a)	$2^{5y} \div 2^{2x} = 2^0$		21	(a)	$\angle AEB = \angle CED$ (vert. opp. $\angle$s) $\frac{AE}{CE} = \frac{2}{3}$ (given $3AE = 2EC$) $\frac{BE}{DE} = \frac{4}{10-4} = \frac{2}{3}$ $\therefore \frac{AE}{CE} = \frac{BE}{DE}$ ΔAEB & ΔCED are similar.		
	(b)	$x = 5, \quad y = 2$			(b)	$\angle BAE = \angle DCE$ (similar Δ) Angle properties of alternate angles, AB is parallel to CD.		
13		$\frac{(3m-2n)}{(5+n)}$			(ci)	$\frac{4}{9}$	(cii)	$\frac{4}{15}$

1 Calculate $\sqrt[3]{\frac{45^2-63.2}{0.896-1}}$, giving your answer correct to 5 significant figures.

Most students do get the correct answer.

$\sqrt[3]{\frac{45^2-63.2}{0.896-1}} = -26.6199$ (6sf)

$= -26.620$ (5sf) **B1**

Answer ..... [1]

---

2 4.854×10^{-4} metres can be written as k micrometres. Find k.

4.854×10^{-4} metres

$= 4.854 \times 10^2 \times 10^{-6}$ metres

$= 485.4$ micrometers

Some students took micrometers as 10^{-9} m. Some students leave the answer in standard form.

$k = 485.4$ **B1**

Answer $k =$ ..... [1]

---

3 When written as the product of their prime factors,

p is $2 \times 3^2 \times 5^3$,

q is $2^3 \times 3 \times 7^3$,

r is $2^4 \times 3^2 \times 7^2$.

Find the

This question was well attempted.

(a) value of the square root of r,

square root of $r = 2^2 \times 3 \times 7$

$= 84$ **A1**

Answer (*a*) ..... [1]

(b) greatest number that will divide p, q and r exactly,

HCF $= 2 \times 3$

$= 6$ **B1**

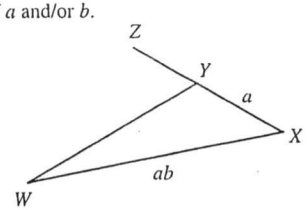
Answer (*b*) ..... [1]

(c) LCM of p, q and r, giving your answer as the product of its prime factors.

LMC $= 2^4 \times 3^2 \times 5^3 \times 7^3$ **B1**

Answer (*c*) ..... [1]

4 In the triangle WXY, $XY = a$ cm, $WX = ab$ cm and $WY = 2XY$. XY is produced to Z. Find $\cos \angle WYZ$ in terms of a and/or b.



$(ab)^2 = (2a)^2 + a^2 - 2(2a)(a)\cos \angle WYX$ **B1**

$4a^2 \cos \angle WYX = 5a^2 - a^2b^2$

$\cos \angle WYX = \dfrac{a^2(5-b^2)}{4a^2}$

$= \dfrac{5-b^2}{4}$

$\cos \angle WYZ = \dfrac{b^2-5}{4}$ **A1**

Many students apply the cos rule with respect $\cos \angle WYZ$ without considering the correct angle.

Answer $\cos \angle WYZ =$ ..... [2]

---

5 The length of a rectangular block is 30 cm and its breadth is 20 cm, both correct to the nearest centimetre.

(a) Find the least possible base area of the block in m^2.

Least possible area $= 29.5 \text{ cm} \times 19.5 \text{ cm}$

$= 0.295 \text{ m} \times 0.195 \text{ m}$

$= \dfrac{2301}{40000} \text{ m}^2$ or 0.057525 m^2 **A1**

Most students did well for this qn.

Answer (a) ..... m^2 [1]

(b) The volume of the block is 0.03 m^3, correct to the nearest 0.01 cubic metres. Find the greatest possible height of the block in metres.

Greatest possible height $= \dfrac{0.035}{0.057525}$ **M1**

$= \dfrac{1400}{2301}$ m **A1** (0.608 acceptable)

Most students use max volume as 0.03 m^3

Answer (b) ..... m [2]

6 **(a)** Express $-x^2 - 2x + 1$ in the form $k - (x+h)^2$.

$-x^2 - 2x + 1 = -(x^2 + 2x) + 1$ **M1**

$= -[x^2 + 2x + 1^2 - 1^2] + 1$

$= -[(x+1)^2 - 1] + 1$

$= -(x+1)^2 + 2$ **A1**

Quite a number of students attempt the qn by equating to zero.

Answer (a) ..... [2]

(b) Hence, solve $x^2 + 2x = 1$, giving your answers to 2 decimal places.

$x^2 + 2x = 1$

$-x^2 - 2x + 1 = 0$

$-(x+1)^2 + 2 = 0$ } **M1**

$(x+1)^2 = 2$

$x + 1 = \pm\sqrt{2}$

$x = \pm\sqrt{2} - 1$

$x = -2.414$ or $x = 0.414$ (3dp)

$x = -2.41$ or $x = 0.41$ (2dp) **A1** (both)

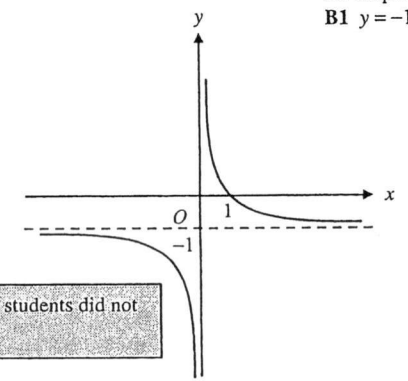
Answer (b) $x =$ ..... or ..... [2]

---

7 Sketch the graph of $y = \dfrac{1}{x} - 1$.

Answer

B1 shape & asymptote
B1 $y = -1$, $x = 1$



Question was badly done. Many students did not draw the asymptote line.

[2]

8 Evaluate $\dfrac{2^{1004}-15\left(2^{1000}\right)+1}{2^{1001}+2}$.

$\dfrac{2^{1004}-15\left(2^{1000}\right)+1}{2^{1001}+2}=\dfrac{2^4\left(2^{1000}\right)-15\left(2^{1000}\right)+1}{2\left(2^{1000}\right)+2}$ **B1** (same index 2^{1000})

$=\dfrac{2^{1000}+1}{2\left(2^{1000}+1\right)}$

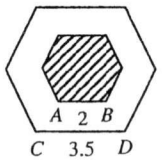
$=\dfrac{1}{2}$ **A1**

Question was badly done. Some students write NA as they compute using calculator.

Answer ..... [2]

---

9 In a recent concert held at the Singapore Indoor Stadium, the hexagonal central stage is divided into 2 sections. The shaded area is a raised stage, similar to the central stage, while the remaining area is a water stage.



(a) Given that $AB = 2$ m and $CD = 3.5$ m, find the value of $\dfrac{\text{area of raised stage}}{\text{area of water stage}}$.

Since the stages are similar,

$\dfrac{\text{area of raised stage}}{\text{area of whole stage}}=\left(\dfrac{2}{3.5}\right)^2=\dfrac{16}{49}$ **B1** $\left(\dfrac{2}{3.5}\right)^2$

$\dfrac{\text{area of raised stage}}{\text{area of water stage}}=\dfrac{16}{49-16}$

$=\dfrac{16}{33}$ **A1**

Question was quite well attempted.

Answer (a) ..... [2]

(b) The area of the raised stage is 4 m^2. Find the volume of water required to fill the water stage given that the depth is 5 cm.

Area of water stage $=\dfrac{4}{16}\times 33$

$=8.25$ m^2

Volume of water $=8.25\times 0.05$ **M1**

$=\dfrac{33}{80}$ m^3 or 0.4125 m^3 **A1**

Question was quite well attempted.

Answer (b) ..... m^3 [2]

10 **(a)** It is given that,
$\xi = \{\text{real numbers}\}$,
$N = \{\text{natural numbers}\}$,
$W = \{\text{whole numbers}\}$.

Questions were badly done.

State if each of the following statements is True or False.

(i) $W \subset N$,

Answer (a)(i) ..... False **B1** ..... [1]

(ii) $-2 \in W$,

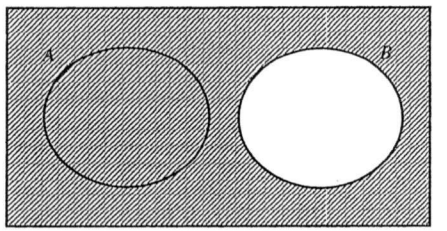
Answer (a)(ii) ..... False **B1** ..... [1]

(iii) $\text{n}(N \cap W) = \text{n}(N)$.

Answer (a)(iii) ..... True **B1** ..... [1]

(b) On the Venn Diagram shown in the answer space, shade the set $(A \cap B) \cup B'$.

Answer (b)



B1 [1]

---

11 Simplify $\left(\dfrac{b^4}{2a}\right)^{-3} \div \dfrac{a^{\frac{1}{2}}(3b)^0}{\sqrt[3]{64b^{12}}}$.

$\left(\dfrac{b^4}{2a}\right)^{-3} \div \dfrac{a^{\frac{1}{2}}(3b)^0}{\sqrt[3]{64b^{12}}}=\left(\dfrac{2a}{b^4}\right)^3 \times \dfrac{4b^4}{a^{\frac{1}{2}}}$

$=\dfrac{8a^3}{b^{12}} \times \dfrac{4b^4}{a^{\frac{1}{2}}}$

$=\dfrac{32a^{\frac{5}{2}}}{b^8}$ **A3** $\left(32, a^{\frac{5}{2}}, \dfrac{1}{b^8}\right.$ o.e$)$

Most students have no difficulty for this question except that some made careless mistakes.

Answer ..... [3]

12 It is given that $32^y \div 4^x = 1$.

(a) Show that $5y = 2x$.

Answer (a)

Questions was well attempted.

$32^y \div 4^x = 1$

$2^{5y} \div 2^{2x} = 2^0$

$5y - 2x = 0$ } **A1**

$5y = 2x$

[1]

(b) Hence or otherwise, solve the simultaneous equations,

$$32^y \div 4^x = 1,$$
$$4x - (y + x) = 13.$$

$5y = 2x \quad \therefore y = \frac{2}{5}x$

Sub $y = \frac{2}{5}x$, $4x - \left(\frac{2}{5}x + x\right) = 13$ **M1**

$\frac{13}{5}x = 13$

$x = 5$ **A1**

$x = 5, \quad y = \frac{2}{5}(5)$

$= 2$ **A1**

Answer $x =$ ....., $y =$ ..... [3]

---

13 Simplify $\frac{6m^2 - mn - 2n^2}{10m + 5n + 2mn + n^2}$.

Some students wrote the denominator as $5(2m+n)n(2m+n)$

$\frac{6m^2 - mn - 2n^2}{10m + 5n + 2mn + n^2} = \frac{(3m-2n)(2m+n)}{5(2m+n) + n(2m+n)}$ **B1**

M1

$= \frac{(3m-2n)(2m+n)}{(5+n)(2m+n)}$ **A1**

$= \frac{(3m-2n)}{(5+n)}$ **A1**

Answer ..... [4]

14 (a) Solve the inequality $\frac{4x-7}{3} < \frac{1}{2}(x+3)+1 \leq \frac{3}{4}x+8$.

$\frac{4x-7}{3} < \frac{1}{2}(x+3)+1$ and $\frac{1}{2}(x+3)+1 \leq \frac{3}{4}x+8$ **B1** (split, "and")

$2(4x-7) < 3(x+3)+6$ $\quad 2(x+3)+4 \leq 3x+32$

$8x - 14 < 3x + 9 + 6$ $\quad 2x + 6 + 4 \leq 3x + 32$

$5x < 29$ $\quad -22 \leq x$

$x < 5\frac{4}{5}$ $\quad x \geq -22$ **A1** (both)

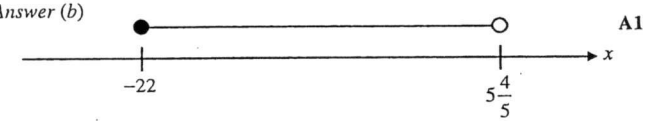
$\therefore -22 \leq x < 5\frac{4}{5}$ **A1**

Some students did not write "and".

Answer (a) ..... [3]

(b) Represent your answer to part (a) on the number line below.

Answer (b)



A1

[1]

---

15 A map has a scale of 1 : n.
A conservation enclosure has an area of 33.8 km^2. It is represented by an area of 80 cm^2 on the map.

(a) Find the value of n.

Area Scale = 80 cm^2 : 33.8 km^2 $\quad$ Linear Scale = $\sqrt{400}$ cm : $\sqrt{169}$ km **M1**

= 400 cm^2 : 169 km^2 $\quad$ = 20 cm : 13 km

= 1 cm : 0.65 km

= 1 : 65 000

$n = 65\,000$ **A1**

Poor presentation from majority of the students. Prone to wrong conversions when trying to convert 33.8 km^2 to cm^2.

Answer (a) $n =$ ..... [2]

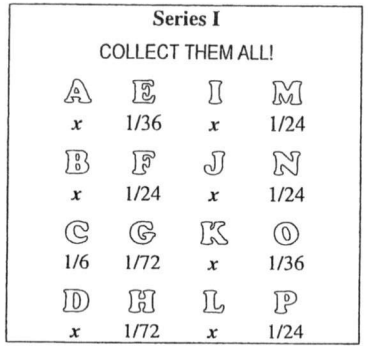
(b) The perimeter of the conservation enclosure on the map is 12 cm. Find the actual perimeter of the enclosure in kilometres.

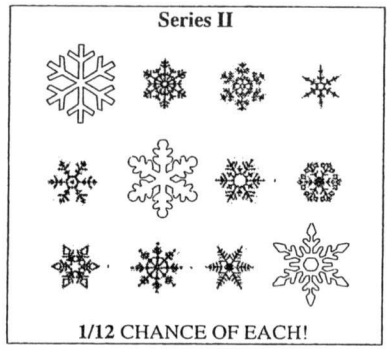
Perimeter = 12 × 0.65 **M1**

= 7.8 km **A1**

Answer (b) ..... km [2]

16 Blind boxes are sealed boxes containing collectibles from a specific series. Consumers do not know which specific design from the series they will receive from the purchased blind box. The probability of getting each design in each series is printed on the box.





(a) Using the information from **Series I**,

(i) calculate the value of x,

(a) Well attempted.

$$7x+\frac{1}{6}+2\left(\frac{1}{36}\right)+4\left(\frac{1}{24}\right)+2\left(\frac{1}{72}\right)=1 \qquad \therefore x=\frac{1}{12} \quad \textbf{A1}$$

Answer (a)(i) $x=$ ..... [1]

(ii) from designs A to P, state which is the most common design.

Answer (a)(ii) ..... C **B1** ..... [1]

(b) In **Series II**, consumers will receive either a small black snowflake or a large white snowflake. A carton of **Series II** contains 12 boxes, each with a different design. Two boxes from this carton were purchased. Find the probability of receiving

(i) 2 small black snowflakes,

$$\text{Probability}=\frac{9}{12}\times\frac{8}{11}$$
$$=\frac{6}{11} \quad \textbf{A1}$$

(bi) Wrong assumption, quite a number of students attempted the question assuming "with replacement".

Answer (b)(i) ..... [1]

(ii) at least 1 large white snowflake.

$$\text{Probability}=1-\frac{6}{11}$$
$$=\frac{5}{11} \quad \textbf{A1}$$

(bii) Not many were able to see the shorter method of solving and instead went to calculate the probability of WW + WB + BW.

Answer (b)(ii) ..... [1]

17 Singapore will see her first water price revision in 17 years on 1 July 2017. The price will be increased in 2 steps, on 1 July 2017 and on 1 July 2018.

Water Price Revision 2017

		Current		From 1 July 2017		From 1 July 2018	
		Water Price ($/m³)		Water Price ($/m³)		Water Price ($/m³)	
		0 - 40m³	> 40m³	0 - 40m³	> 40m³	0 - 40m³	> 40m³
Potable Water	Tariff	$1.17	$1.40	$1.19	$1.46	$1.21	$1.52
	Water Conservation Tax (% of water tariff)	$0.35 (30% of $1.17)	$0.63 (45% of $1.40)	$0.42 (35% of $1.19)	$0.73 (50% of $1.46)	$0.61 (50% of $1.21)	$0.99 (65% of $1.52)
# Used Water	Waterborne Fee	$0.28	$0.28	$0.78	$1.02	$0.92	$1.18
	Sanitary Appliance Fee	$2.80 per fitting*		Combined into Waterborne Fee		Combined into Waterborne Fee	
	Total Price	$2.10	$2.61	$2.39	$3.21	$2.74	$3.69

Refers to the water consumed by the household. Waterborne Fee and Sanitary Appliance Fee go towards meeting the cost of treating used water and maintaining the used water network.

The table above is taken from Singapore's National Water Agency, Public Utilities Board's website. It was also stated that key revision to the water price includes "a 30% increase in water price, phased over 2 years, starting from 1 July 2017".

(a) Determine how the 30% increase in water price is calculated. Show your working clearly.

Answer (a)

$$\%\ \text{increase}=\frac{2.74-2.10}{2.10}\times 100\%$$
$$=30\frac{10}{21}\%$$
$$\approx 30\%$$

A1

Many did not even attempt the question!
1. Do not know how to interpret the table
2. Wrong concept: addition of percentage
3. Left answer as 30.5% and not 30% as required by the question
4. Did not show x 100%

[2]

(b) Does this 30% increase in water price apply to all households regardless of water usage? Justify your answer.

For usage $> 40\ m^3$, $\%$ increase $=\frac{3.69-2.61}{2.61}\times 100\%$
$=41\frac{7}{13}\%$ **B1**

Many did not see the hint in the question "regardless of water usage". In justifying, students still missed out on answering "Yes" or "No" to the Qn.

Answer (b) No, the 30% increase does not apply to all regardless of water usage.

For households whose **usage exceed 40 m³**, the **percentage increase in water price** per additional cubic metre **is more than 30%.** **A1** [2]

18 A multiple choice test has a total of 50 questions. All questions must be attempted. There are 2 sections to the paper, Section *A* and Section *B*. The matrix **Q** shows the number of questions attempted by Victoria.

$$\mathbf{Q} = \begin{matrix} \text{Correct} & \text{Incorrect} \\ \begin{pmatrix} 18 & 2 \\ x & 5 \end{pmatrix} & \begin{matrix} \text{Section } A \\ \text{Section } B \end{matrix} \end{matrix}$$

(a) State the value of x.

$x = 50 - 18 - 2 - 5$
$= 25$ **B1**

Answer (a) $x =$ ..... [1]

(b) 2 marks are awarded for each correct answer and 1 mark is deducted for each incorrect answer. Represent the mark allocation in a column matrix **M**.

$\mathbf{M} = \begin{pmatrix} 2 \\ -1 \end{pmatrix}$ **B1**

Surprising badly done. Students do not know what a **column** matrix is?!

Answer (b) **M** = ... [1]

(c) Evaluate the matrix **QM**.

$\mathbf{QM} = \begin{pmatrix} 18 & 2 \\ 25 & 5 \end{pmatrix}\begin{pmatrix} 2 \\ -1 \end{pmatrix}$
$= \begin{pmatrix} 34 \\ 45 \end{pmatrix}$ **A1**

1. Correct **QM** written but wrong multiplication (need first check order of the matrices used)
2. **QM** ≠ **MQ**

Answer (c) **QM** = ... [1]

(d) State what the elements in **QM** represent.

Lack the word "respectively". Follow instructions given by subject teachers!

Answer (d) The elements represent the **marks** obtained by Victoria in the test for **Section *A*** and **Section *B*** **respectively**. **B1** ..... [1]

(e) By matrix multiplication, find the total marks scored by Victoria in the test.

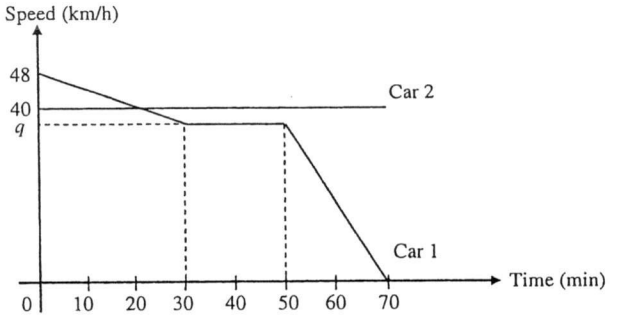
$\begin{pmatrix} 1 & 1 \end{pmatrix}\begin{pmatrix} 34 \\ 45 \end{pmatrix} = (79)$ **A1** (must be 1x1 matrix)

Total = 79 marks

1. Question requested for matrix multiplication!
2. Quite a number simply, $\begin{pmatrix} 1 & 1 \end{pmatrix}\begin{pmatrix} 34 \\ 45 \end{pmatrix} = 79$. The result MUST be a matrix, before giving a concluding statement.

Answer (e) ..... marks [1]

19 The diagram shows the speed-time graphs of 2 cars travelling from Town *A* to Town *B*.



(a) Car 1 travelled at a constant deceleration of 24 km/h^2 for the first 30 minutes of the journey. Calculate the value of q.

$\dfrac{48-q}{\frac{1}{2}} = 24$
$48 - q = 12$
$q = 36$ **A1**

Answer (a) $q =$ ..... [1]

(b) Convert q km/h into m/s.

$36 \text{ km/h} = \dfrac{36000 \text{ m}}{60 \times 60 \text{ s}}$
$= 10 \text{ m/s}$ **B1**

Answer (b) ..... m/s [1]

(c) Find the distance from Town *A* when the 2 cars meet.

Let t h be the time they meet,

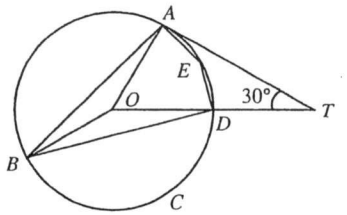
$40t = \frac{1}{2}\left(\frac{1}{2}\right)(48+36) + 36\left(t - \frac{1}{2}\right)$ **B1** (left of equation) **M1** (right of equation)
$40t = 21 + 36t - 18$
$t = \frac{3}{4}$

Badly done!
1. Many assumed the cars meet at $t = 20$, that is the time when they are at same speed.
2. Some even dropped a straight line on the graph to obtain $t = 20$.

Distance $= 40\left(\frac{3}{4}\right)$
$= 30$ km **A1**

Answer (c) ..... km [3]

20



O is the centre of the circle passing through A, B, C, D and E.
TA is a tangent to the circle.
Angle $ATO = 30°$.

Students need to be FAMILIAR with ALL angle properties.

Do NOT create your own shortcuts! If you can't remember, spell the entire word.

(a) Find, giving reasons for each answer,

(i) angle ABD,

$\angle TAO = 90°$ ($\tan \perp$ rad) **B1**

$\angle AOT = 180° - 90° - 30°$ ($\angle$ sum in Δ) **M1**

$= 60°$

$\angle ABD = \frac{1}{2} \times 60°$ ($\angle$ at centre $= 2 \times \angle$at circumference)

$= 30°$ **A1**

Answer (a)(i) Angle ABD = ... [3]

(ii) angle AED.

$\angle AED = 180° - 30°$ (opp $\angle$s in cyclic quad.)

$= 150°$ **A1**

Answer (a)(ii) Angle AED = ... [1]

(b) Given that the radius of the circle is 2 cm and angle BOD = angle AED, show that the area of segment BCD is $\left(\frac{5}{3}\pi - 1\right)$ square centimetres.

Answer (b)

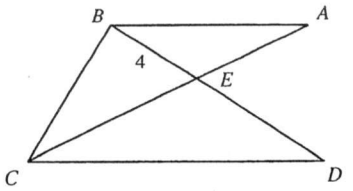
All steps must be **shown** – all relevant substitution into the correct formulae, etc...

Area of segment $= \frac{150°}{360°} \times \pi(2^2) - \frac{1}{2}(2)(2)\sin 150°$ **M1** (sector area) **M1** (triangle area)

$= \left(\frac{5}{3}\pi - 1\right)$ cm^2 If angle $\neq 150°$, 1M

[2]

21 Lines AC and BD intersect at E. It is given that $BE = 4$ cm, $BD = 10$ cm and $3AE = 2EC$.



(a) Prove a pair of similar triangles.

Answer (a) In ΔAEB & ΔCED, $\angle AEB = \angle CED$ (vert. opp. $\angle$s) **B1**

Most lost the first A1 – did not state "given" for AE/CE ratio or did not conclude the equal ratios of the 2 corr. sides.

A LOT of students are not naming the triangles correctly!

$\frac{AE}{CE} = \frac{2}{3}$ (given $3AE = 2EC$)

$\frac{BE}{DE} = \frac{4}{10-4} = \frac{2}{3}$ **A1**

$\therefore \frac{AE}{CE} = \frac{BE}{DE}$

Hence ΔAEB & ΔCED are similar. **A1** [3]

(b) Prove that AB is parallel to CD.

Answer (b) Since ΔAEB & ΔCED are similar, $\angle BAE = \angle DCE$.

Using angle properties of alternate angles, AB is parallel to CD. **A1** [1]

(c) Find the value of

Students needs to learn to phrase their reasoning properly! Do **NOT** use short forms for such "explaining" questions.

(i) $\frac{\text{area of triangle } AEB}{\text{area of triangle } CED}$,

$\frac{\text{area of triangle } AEB}{\text{area of triangle } CED} = \left(\frac{2}{3}\right)^2$

$= \frac{4}{9}$ **A1**

Answer (c)(i) ..... [1]

(ii) $\frac{\text{area of triangle } AEB}{\text{area of triangle } BCD}$.

Badly done, many assumed the triangles are similar!

$\frac{\text{area of triangle } CED}{\text{area of triangle } BCD} = \frac{3}{2+3}$ **M1** (triangle with same height)

$= \frac{3}{5} = \frac{9}{15}$

$\frac{\text{area of triangle } AEB}{\text{area of triangle } BCD} = \frac{4}{15}$ **A1**

$\frac{\text{Area } AEB}{\text{Area } BCD} = \frac{\text{Area } AEB}{\text{Area } CED} \times \frac{\text{Area } CED}{\text{Area } BCD}$

$= \frac{4}{9} \times \frac{3}{5} = \frac{4}{15}$

Answer (c)(ii) ..... [2]

22 The two towers within the One Raffles Place Complex are amongst the most iconic buildings in the Central Business District in Singapore.

Despite being constructed in the 1980s, Tower 1 (**on the left**) remains as one of the tallest buildings on the island. Given that the tower has a triangular cross-sectional area such that $AB = BC = 30$ m and $AC = 40$ m.

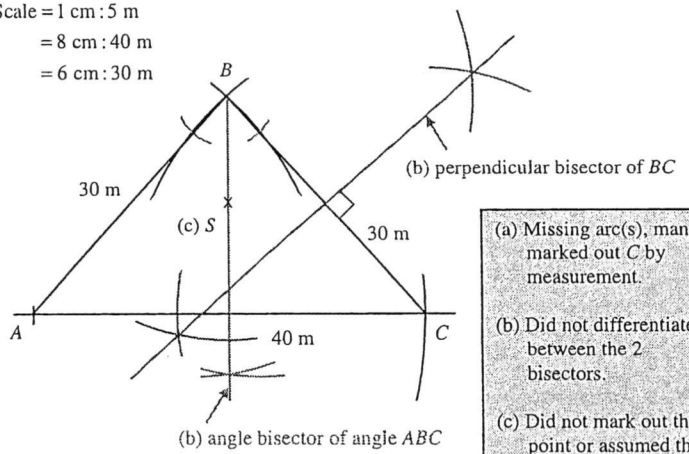
Using ruler and compass,

(a) construct the cross-sectional of the tower using a scale of 1 cm to 5 m, [1]

(b) construct the perpendicular bisector of BC and the angle bisector of angle ABC. [2]

Answer (*a*), (*b*) and (*c*)

Scale = 1 cm : 5 m
= 8 cm : 40 m
= 6 cm : 30 m



(a) Missing arc(s), many marked out C by measurement.

(b) Did not differentiate between the 2 bisectors.

(c) Did not mark out the point or assumed the intersection is the required point.

(a) **B1** (3 arcs + A, B
(b) **B1** ($\perp$ bisector, 4
B1 ($\angle$ bisector, 4
(c) **B1** (on $\angle$ bisecto intersection of both bisectors)

An ideal spot to view the sunset from the rooftop of the tower is equidistant from the edges AB and BC and nearer to point B.

(c) Mark and label a possible spot, S, that is ideal to view the sunset. [1]

End of Paper

Name ______________ Class ____ Register Number ____

4048/02 **17/S4PR1/EM/2**

MATHEMATICS **PAPER 2**

Wednesday **3 May 2017** **2 hours 30 minutes**

NIL SINE LABORE

VICTORIA SCHOOL

PRELIMINARY EXAMINATION ONE
SECONDARY FOUR

Additional Materials: Answer Paper
Graph Paper

READ THESE INSTRUCTIONS FIRST

Write your name, class and register number on all the work you hand in.
Write in dark blue or black pen.
You may use a pencil for any diagrams or graphs.
Do not use paper clips, highlighters, glue or correction fluid.

Answer **all** questions.
If working is needed for any question it must be shown with the answer.
Omission of essential working will result in loss of marks.
You are expected to use a scientific calculator to evaluate explicit numerical expressions.
If the degree of accuracy is not specified in the question, and if the answer is not exact, give the answer to three significant figures. Give answers in degrees to one decimal place.
For π, use either your calculator value or 3.142, unless the question requires the answer in terms of π.

At the end of the examination, fasten all your work securely together.
The number of marks is given in brackets [ ] at the end of each question or part question.
The total number of marks for this paper is 100.

This paper consists of **10** printed pages, including the cover page.

[Turn over

Mathematical Formulae

Compound interest

$$\text{Total amount} = P\left(1+\frac{r}{100}\right)^n$$

Mensuration

$$\text{Curved surface area of a cone} = \pi rl$$

$$\text{Surface area of a sphere} = 4\pi r^2$$

$$\text{Volume of a cone} = \frac{1}{3}\pi r^2 h$$

$$\text{Volume of a sphere} = \frac{4}{3}\pi r^3$$

$$\text{Area of triangle } ABC = \frac{1}{2}ab\sin C$$

$$\text{Arc length} = r\theta, \text{ where } \theta \text{ is in radians}$$

$$\text{Sector area} = \frac{1}{2}r^2\theta, \text{ where } \theta \text{ is in radians}$$

Trigonometry

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$a^2 = b^2 + c^2 - 2bc\cos A$$

Statistics

$$\text{Mean} = \frac{\Sigma fx}{\Sigma f}$$

$$\text{Standard deviation} = \sqrt{\frac{\Sigma fx^2}{\Sigma f} - \left(\frac{\Sigma fx}{\Sigma f}\right)^2}$$

Answer **all** the questions.

1 (a) Expand $\left(w+\frac{1}{w}\right)^2$. [1]

(i) Given that $w+\frac{1}{w}=6$, show that $w^2+\frac{1}{w^2}=34$. [1]

(ii) Hence, find the value of $w-\frac{1}{w}$, if $0<w<1$. [3]

(b) Given that $p=\sqrt{\frac{2m^2+3p}{m^2-p}}$, express m in terms of p. [3]

(c) Given $-4 \le p \le 10$ and $11 \le q \le 20$, where p and q are integers, calculate the

(i) greatest possible value of $q-p$, [1]

(ii) least possible value of $\frac{p}{q}$, if $q \neq 0$, [1]

(iii) least value of $2p^2+q^2$. [1]

---

2 (a) In January 2017, the price of a car was \$96 000. Over the next two months, the price increased by 25% and then decreased by 15%. Mr Ali, a potential car owner, said that the overall increase in the price of the car was 10%.
Do you agree with Mr Ali? Justify your answer. [2]

(b) Alex bought a home entertainment system during the Great Singapore Sale on hire purchase. He paid a deposit of 20% of the selling price. The balance of the payment was \$8000.

(i) Calculate the selling price of the system. [1]

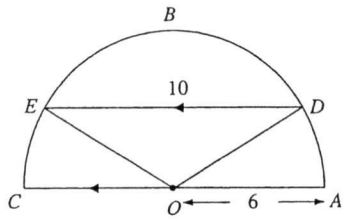
(ii) Alex paid the balance of the payment in monthly instalments of \$380, charged at x % per annum simple interest for 2 years. Find the value of x. [3]

(iii) Alex could have taken a loan to pay the outstanding balance. If the finance company charged him a compound interest of 6.5% per annum with a repayment period of 2 years, would you recommend that he takes up the loan instead of paying in instalments? State your reason clearly. [3]

3 (a) A polygon has n sides. Three of its exterior angles are 60°, 25° and 55°. The remaining angles in the polygon have interior angles of 160°. Find value of n. [2]

(b) John claimed that he can draw a regular polygon with its exterior angle as 70°. Is he right? Explain. [2]

(c)



In the figure above, the chord DE, of length 10 cm, is parallel to the diameter AOC of the semicircle with centre O and a radius 6 cm.

(i) Find $\angle DOE$ in radians. [2]

(ii) Calculate the area of the segment DBE. [3]

---

4 John and Tim begin to walk with a constant speed of x m/s and y m/s respectively towards each other along a pathway of length 20 metres.

(a) Given that they walk past each other 4 seconds later, show that $y=5-x$. [2]

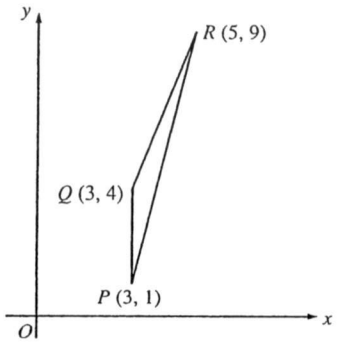
(b) Given also that John takes 3 seconds more than Tim to walk through the pathway, form an equation in x and show that it reduces to

$$3x^2-55x+100=0.$$ [3]

(c) Solve the equation $3x^2-55x+100=0$, giving your answers correct to 2 decimal places. [2]

(d) Calculate the time, to the nearest second, taken by Tim to walk through the pathway. [2]

5 The points P (3, 1), Q (3, 4) and R (5, 9) are shown in the diagram.



Find the

(a) coordinates of S such that $PQRS$ is a parallelogram, [1]

(b) exact length of PR, [1]

(c) area of ΔPRQ and hence the perpendicular distance from Q *to* PR, [4]

(d) equation of the line that passes through P and is parallel to the line $3y = 4x + 5$. [2]

---

6 **(a)** The results of two class tests are shown in the tables below. The total marks for both tests are 80.

Test 1

Marks (x)	$40 \le x < 50$	$50 \le x < 60$	$60 \le x < 70$	$70 \le x < 80$
No. of students	10	12	13	5

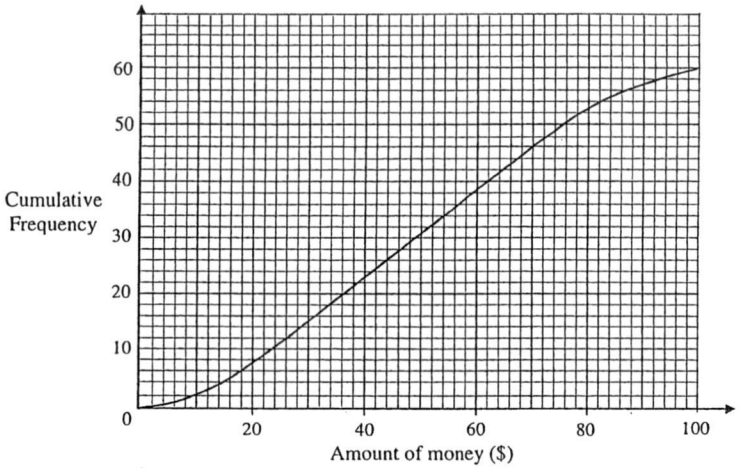
Test 2

Mean = 45 Standard deviation = 15.5

(i) Find the mean mark and the standard deviation of **Test 1**. [3]

(ii) Compare and comment on the marks of the two tests in two different ways. [2]

(b) The cumulative frequency curve below shows the amount of money collected by 60 students in the school's Rugby Team during a fund raising event.



(i) Use the graph to find the

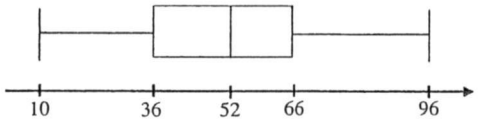
(a) median amount of money raised, [1]

(b) interquartile range of the distribution, [2]

(c) tenth percentile of the distribution. [2]

(ii) A student will be given a T-Shirt if he managed to collect at least $\$y$. Find the value of y if 20% of the students were given the T-Shirt. [2]

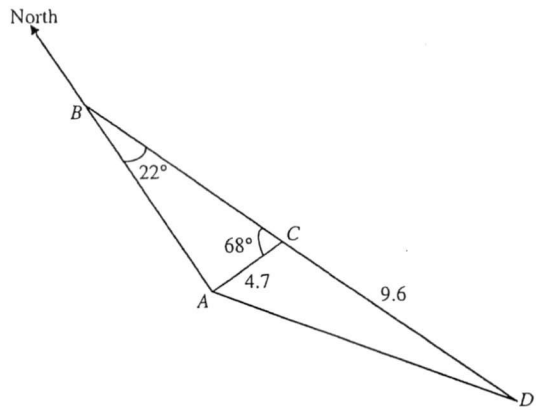
(iii) The box-and-whisker diagram below illustrates the amount raised by 60 students from the Outdoor Club.



Amount of money ($)

Brian commented that the students in the Rugby Team raised more money as compared to the students from the Outdoor Club. Do you agree? Give a reason for your answer. [1]

7 A, B, C and D are points on level ground with A due South of B.
It is given that $\angle ABC = 22°$, $\angle BCA = 68°$, $AC = 4.7$ km and $CD = 9.6$ km.
BCD is a straight line.



Calculate

(a) AD, [2]

(b) $\angle CAD$, [2]

(c) the bearing of C from D, [1]

(d) the shortest distance from C to AD. [2]

A helicopter, H is hovering at a height of 450 m, at a point which is vertically above C.

(e) Find the smallest angle of elevation of the helicopter when observed along AD. [2]

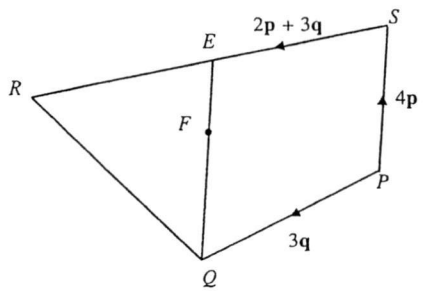
8 (a) R is the point (7, 1) and S is the point (2, 5).

(i) Find the column vector $\overrightarrow{RS}$. [1]

(ii) Calculate the exact value of $\left|\overrightarrow{RS}\right|$. [2]

(iii) If $\overrightarrow{AB} = \frac{1}{4}\overrightarrow{RS}$ and B is the point (3, 6), find the coordinates of A. [2]

(b)



In the diagram, E is a point on SR such that $SR = 2SE$ and F is a point on QE such that $3QF = 2QE$. $\overrightarrow{PS} = 4\mathbf{p}$, $\overrightarrow{PQ} = 3\mathbf{q}$ and $\overrightarrow{SE} = 2\mathbf{p} + 3\mathbf{q}$.

(i) Express each of the following, as simply as possible, in terms of **p** and/or **q**.

(a) $\overrightarrow{PE}$ [1]

(b) $\overrightarrow{QE}$ [1]

(c) $\overrightarrow{PF}$ [1]

(d) $\overrightarrow{PR}$ [1]

(ii) What can you deduce about P, F and R? [1]

(iii) Given that the area of $\Delta QRE = 24\ \text{cm}^2$, find the area of ΔRFE. [2]

9 **Answer the whole of this question on a sheet of graph paper.**

In experiment A, water flows from a tap at a constant rate into a conical flask.

The table below shows the height of water, h cm, at various timings of the water flow.

Time (t s)	0.0	1.0	2.0	3.0	4.0	5.0	6.0
Height (h cm)	0.00	0.50	1.25	2.38	3.90	6.59	10.39

(a) Using a scale of 2 cm to represent 1 second, draw a horizontal t-axis for $0 \le t \le 6$.
Using a scale of 2 cm to represent 1 cm depth, draw a vertical h-axis for $0 \le h \le 11$.
On your axes, plot a smooth graph represented by the data in the given table. [3]

(b) Use your graph to find an estimate for the height of water in the conical flask when $t = 3.4$. [1]

(c) By drawing a tangent at the points $t = 2$ and $t = 5$,

(i) find the gradient of the curve at $t = 2$ and $t = 5$, [2]

(ii) state briefly what these gradients represent, [1]

(iii) explain the significant difference between these gradients. [1]

(d) In experiment B, another container in the shape of a cylinder is being used.
Water flows from the same tap. The height of water can be described by the equation $h = 1.25t + c$, where c is a constant.

(i) If the cylinder is empty at the start of the experiment, determine the value of c. [1]

(ii) By drawing a suitable straight line on the same axes, determine the time taken for the height of water in both containers to be the same. [2]

10 Scuba divers use diving cylinders to help them stay underwater for a long period of time. In this question, the internal compartment of a diving cylinder, as shown in **Figure 1,** can be modelled as a cylinder with a hemisphere on top, as shown in **Figure 2.**

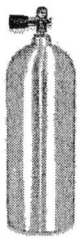


Figure 1

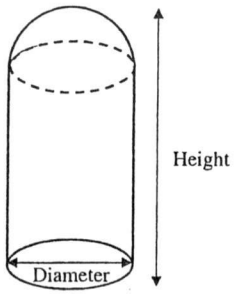


Figure 2

A diver intends to stay underwater at a depth of 20 m for 1.5 hours. He wants to determine whether a particular diving cylinder, with an internal compartment of diameter 20 cm and a height of 75 cm, is sufficient to support the dive.

(a) The volume of the diving cylinder is measured by the volume of its internal compartment. Work out the volume, in litres, of the diving cylinder. [3]

(b) The following equation is used to calculate the volume of oxygen in the cylinder.

$$\text{Volume of oxygen} = \frac{(\text{Volume of cylinder}) \times (\text{Pressure in cylinder})}{(\text{Atmospheric pressure})}$$

The pressure (measured in bars) in the cylinder is 210 bars and the atmospheric pressure (measured in bars) is 1.01 bars.

Find the volume, in litres, of oxygen that the diving cylinder can hold. [2]

(c) To calculate the volume of oxygen a diver requires, the following equation is used.

$$\text{Volume of oxygen required} = (\text{Breathing rate}) \times (\text{Duration}) \times (\text{Ambient pressure})$$

Assuming that the diver's breathing rate is 18 litres per minute and that for every 5 m underwater, the ambient pressure increases by 0.5 bars from the atmospheric pressure, determine whether the diving cylinder is sufficient to support the dive.

Justify your decision with calculations. [4]

End of Paper

Answer Key

1	(ai)	$w^2+2+\frac{1}{w^2}$	6	(bib)	**Interquartile range** = \$39
	(aiii)	-5.66		(bic)	10^{th} percentile of the distribution = \$18
	(b)	$m=\pm\sqrt{\frac{p(3+p^3)}{p^3-2}}$		(bii)	$y=72$
				(biii)	No. The median amount raised by the Rugby Team is lower.
	(ci)	24 (cii) $\frac{-4}{11}$ (ciii) 121	7	(a)	12.2 **km**
2	(a)	Disagree with Mr. Ali.		(b)	47.0°
	(bi)	\$10 000		(c)	338°
	(bii)	$x=7$		(d)	3.44 km
	(biii)	Yes. As the interest/total loan is lower.		(e)	2.7°
3	(a)	$n=14$	8	(ai)	$\begin{pmatrix}-5\\4\end{pmatrix}$
	(b)	No. John is wrong.		(aii)	$\sqrt{41}$ units
	(ci)	1.97 rad		(aiii)	$\left(4\frac{1}{4},5\right)$
	(cii)	18.9 cm^2			
4	(a)	Show $y=5-x$.		(bia)	$3(2p+q)$
	(b)	Show $3x^2-55x+100=0$		(bib)	$6p$
	(c)	$x=16.29$ or 2.05		(bic)	$4p+3q$
	(d)	7 seconds		(bid)	$2(4p+3q)$
				(bii)	P, F and R lie on a straight line.
				(biii)	8 cm^2
5	(a)	$S(5,\ 6)$	9	(b)	$h=2.9$ (ci) 1.02, 3.06
	(b)	$\sqrt{68}$ units		(cii)	The gradients represent the rate of increase of water level in the conical flask at each instant.
	(c)	0.728 units		(ciii)	Water level is rising faster at t = 5 compared to t = 2
				(di)	$c=0$ (dii) 4.8 or 4.9
	(d)	$y=\frac{4}{3}x-3$	10	(a)	22.5 litres
6	(ai)	mean = 58.25 marks SD = 9.85 marks		(b)	4680 litres
	(aii)	The mean mark of Test 1 is higher than of Test 2. This means that students scored better in Test 1 as compared to in Test 2. The standard deviation of Test 2 marks is higher than in Test 1. This means that the spread of marks in Test 2 is bigger/wider than that of Test 1.		(c)	No. It is insufficient.
	(bia)	Median amount = \$49			

1 (a) $\left(w+\frac{1}{w}\right)^2=w^2+2+\frac{1}{w^2}$ ------ [B1]

Students generally did well for this question.

(i)
$$\left(w+\frac{1}{w}\right)^2=w^2+2+\frac{1}{w^2}$$
$$\Rightarrow (6)^2=w^2+2+\frac{1}{w^2}$$
$$\therefore w^2+\frac{1}{w^2}=36-2$$
$$=34\ (\text{shown})$$ ------ [A1]

Students generally did well for this question.

(ii)
$$\therefore\left(w-\frac{1}{w}\right)^2=w^2-2+\frac{1}{w^2}$$ ------ [B1]
$$=w^2+\frac{1}{w^2}-2$$ ------ [M1-for sub
$$=34-2$$
$$\left(w-\frac{1}{w}\right)^2=32$$
$$\therefore w-\frac{1}{w}=\pm\sqrt{32}\ .\ \text{Since } 0<w<1,\quad \Rightarrow w-\frac{1}{w}=-5.66\ (3\text{ SF})$$ ------ [A1]

Question was badly done. Some of the errors include;
1) Did not use part (ai) but solve to find w instead– "Hence question"
2) Did not reject 5.66 (or reject -5.66 instead)
3) Careless. Note that $w^2+\frac{1}{w^2}\neq\left(w-\frac{1}{w}\right)\left(w+\frac{1}{w}\right)$

(b)
$$p=\sqrt[3]{\frac{2m^2+3p}{m^2-p}}$$
$$p^3=\frac{2m^2+3p}{m^2-p}$$ ----- [B1- taking cube on both sides]
$$p^3m^2-p^4=2m^2+3p$$
$$m^2\left(p^3-2\right)=p(3+p^3)$$ ----- [M1- Factorisi
$$m^2=\frac{p(3+p^3)}{\left(p^3-2\right)}$$
$$\therefore m=\pm\sqrt{\frac{p(3+p^3)}{p^3-2}}$$ ----- [A1]

Most students were able to do this question (but lost the last 1 mark). Some of the errors include;
1) Did not factorise $p(3+p^3)$
2) Did not give $\pm\sqrt{}$ as the final form

(c)
(i) greatest possible value of $q-p=20-(-4)=24$ ------ [B1]

(ii) least possible value of $\frac{p}{q}=\frac{-4}{11}$ ---- [B1]

(iii) least value of $2p^2+q^2=2(0)^2+11^2=121$ ----

Questions were well attempted. Some students were penalized because of poor presentation of answers e.g. did not show substitutions and no statements.

2 (a) In January 2017, the price of a car was \$96 000. Over the next two months, the price increased by 25% and then decreased by 15%. Mr Ali, a potential car owner, said that the overall increase in the price of the car was 10%. Do you agree with Mr Ali? Justify your answer.

Disagree with Mr Ali. ---------- [B1]

The actual overall increase = $(1.25\times0.85-1)\times100\%$

$= 6.25\%$ ---------- [B1]

Question was well attempted with different degrees of quality. There were few outstanding reasoning. Some students were penalized because of the lack of Mathematical justifications in their answers. Need to show proper Math workings.

(b) Alex bought a home entertainment system during the Great Singapore Sale on hire purchase. He paid a deposit of 20% of the selling price. The balance of the payment was \$8000.

(i) Calculate the selling price of the system.

Selling Price $= \frac{8000}{80}\times100 = \$10\ 000$ ----------

Question was well attempted. Some students were careless and misinterpret the question i.e. taking 20% as \$8000 resulting in and erroneous answer of \$40 000.

(ii) Alex paid the balance of the payment in monthly instalments of \$380, charged at x % per annum simple interest for 2 years. Find the value of x. [3]

Total repayment amount $= 380\times24 = \$9120$

Total interest amount $= 9120-8000 = 1120$ ---------- [B1]

$\therefore 1120 = \frac{8000\times x\times2}{100}$ ----

$x = 7$ ---------- [A1]

Question was badly done. Some students did not know the difference between Simple (ii) and Compound Interest (iii). Students did not realise the formula

$SI = \frac{PRT}{100}$ (where T is time in YEARS)

(iii) Alex could have taken a loan to pay the outstanding balance. If the finance company charged him a compound interest of 6.5% per annum with a repayment period of 2 years, would you recommend that he takes up the loan instead of paying in instalments? State your reason clearly. [3]

Total loan amount $= 8000\left(1+\frac{6.5}{100}\right)^2 = \$9073.80$---------- [A1]

Yes, he should take up the loan from the finance company. ---------- [A1]

The interest amount is lower. / The total loan amount is lower. ---------- [A1]

Question was quite well attempted. However, there were students who were penalized because;

1) Poor or wrong explanation/reasons – e.g. "The loan is cheaper."
2) Did not understand the formula to be "Total loan amount $= 8000\left(1+\frac{6.5}{100}\right)^2$";
   Instead they took $r = \frac{6.5}{2\times100}$ or $n = 24$.
3) Took Principal amount (P) as \$10 000 instead of \$8000.

3 (a) A polygon has n sides. Three of its exterior angles are 60°, 25° and 55°. The remaining angles in the polygon have interior angles of 160°. Find value of n. [2]

Sum of exterior angles = 360°

$60°+25°+55°+(n-3)\times20° = 360°$ -----[M1]

$140+20n-60 = 360$

$20n = 280$

$n = 14$ -----[A1]

Question was badly done (Poor or no attempt). Some students did not know the formula for either interior or exterior angles of polygons.

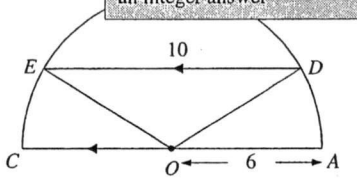
(b) John claimed that he can draw a regular polygon with its exterior angle as 70°. Is he right? Explain. [2]

No. John is wrong. ---------- [B1]

360° is not completely divisible by 70°. ---------- [B1]

Most students knew how answer this question (but with different of accuracy). As a result, marks were deducted. For e.g." 360° cannot be divided by 70°. This is untrue. To make it complete, students should include the phrase "that will result in an integer answer"

(c)



In the figure above, the chord DE, of length 10 cm, is parallel to the diameter AOC of the semicircle with centre O and radius 6 cm.

(i) Find $\angle DOE$ in radians. [2]

$\cos\angle DOE = \frac{6^2+6^2-10^2}{2(6)(6)}$ ------- [M1]

$\angle DOE \approx 1.9702$

$= 1.97$ rad (3 s.f.) ------- [A1]

Question was well attempted. Some common errors include;

1) Carelessness
2) Answer in degree and not in radians

(ii) Calculate the area of the segment DBE. [3]

Area of segment $DBE = \frac{1}{2}(6^2)(1.9702) - \frac{1}{2}(6^2)\sin 1.9702$ ------- [M2]

≈ 18.880

$= 18.9\ \text{cm}^2$ (3 SF) ------- [A1]

Good attempt. Need to be mindful (strongly advised to use 5 SF) and not to use 3SF values in working i.e. use 1.9702 instead of just 1.97.

4 John and Tim begin to walk with a constant speed of x m/s and y m/s respectively towards each other along a pathway of length 20 metres.

(a) Given that they walk past each other 4 seconds later, show that $y = 5 - x$. [2]

$4x + 4y = 20$ ------- [B1]
$x + y = 5$
$y = 5 - x$ -------- [A1]

Question was well attempted. Most students had managed to show proper and sufficient working.

(b) Given also that John takes 3 seconds more than Tim to walk through the pathway, form an equation in x and show that it reduces to

$\frac{20}{x} - \frac{20}{5-x} = 3$ ------[M1 o.e]
$100 - 20x - 20x = 3x(5-x)$ ------[M1- single fraction]
$100 - 40x = 15x - 3x^2$
$\therefore 3x^2 - 55x + 100 = 0$ ------[A1]

Question was well attempted. Most students had managed to show proper and sufficient working.

(c) Solve the equation $3x^2 - 55x + 100 = 0$, giving your answers correct to 2 decimal places. [2]

$3x^2 - 55x + 100 = 0$
$\therefore x = \frac{-(-55) \pm \sqrt{(-55)^2 - 4(3)(100)}}{2(3)}$ ---------- [B1]
$= \frac{55 \pm \sqrt{1825}}{6}$
$\therefore x = 16.29$ or 2.046
$\Rightarrow x = 16.29$ or 2.05 (2 DP) ---------- [A1]

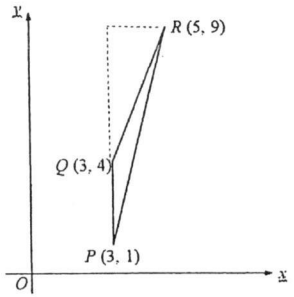
Question was well attempted. Students will be penalized if they did not show the substitution of $-(-55)$ into the QE formula. Some other errors include
1) Rejecting of 16.29 as a valid answer
2) Leaving answers in 3 SF and not in 2 DP as requested

(d) Calculate the time, to the nearest second, taken by Tim to walk through the pathway. [2]

$\therefore x = 16.29$ (rejected)
So $x = 2.046 \Rightarrow y = 2.954$
$\therefore$ Time taken for Tim $= \frac{20}{2.954}$ ------ [M1]
$= 6.7705$
$= 7$ seconds (nearest seconds) ---------- [A1]

Question was well attempted. Only a handful of students did not know what to do or had use the wrong formula i.e. using $\frac{20}{x}$ instead of $\frac{20}{5-x}$. Some did not show working using more than 3 SF values and some did not round answers to nearest seconds (an integer answer is required).

5 The points P (3, 1), Q (3, 4) and R (5, 9) are shown in the diagram.



Find the

(a) coordinates of S such that $PQRS$ is a parallelogram, [1]

$S(5, 6)$. ------ [B1]

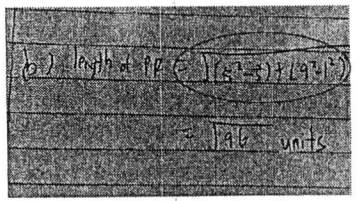
(b) exact length of PR, [1]

$PR = \sqrt{(5-3)^2 + (9-1)^2}$
$= \sqrt{68}$ units (3 SF) ------ [B1]

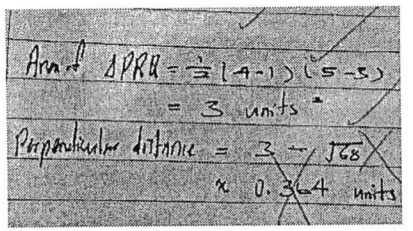
(c) area of ΔPRQ and hence the perpendicular distance from Q to PR, [4]

Area $\Delta PRQ = \frac{1}{2}(2)(3)$ ------ [M1]
$= 3$ units^2 ------ [A1]
Let the perpendicular distance from B to AC be h.
Area $\Delta PRQ = \frac{1}{2}(PR)(h)$
$3 = \frac{1}{2}(\sqrt{68})(h)$ ------ [M1]
$h = \frac{3 \times 2}{\sqrt{68}}$
$= 0.728$ units (3 SF) ------ [A1]

Question 5a, 5b were well attempted. Students will be penalized if they give answer for Q5b to 3sf instead of exact value. There is no need to reduce $\sqrt{68}$ to $2\sqrt{17}$. Some other mistakes for Q5b include
1) Not giving units for length.
2) Using the wrong formula for



Question 5c was generally well attempted. Students should avoid using "shoe lace method" to find area of triangle. Some other mistakes for Q5c include
1) Not writing the ½ for the shoe lace method of finding area of triangle.
2) Not dividing by ½ when finding perpendicular height.
3) Using 3 sf for length PR instead of $\sqrt{68}$



(d) equation of the line that passes through P and is parallel to the line $3y = 4x + 5$. [2]

$3y - 4x = 5$

$\Rightarrow y = \frac{4}{3}x + \frac{5}{3}$

$\therefore$ equation of the line:

$y - 1 = \frac{4}{3}(x - 3)$ ------- [M1]

$y - 1 = \frac{4}{3}x - 4$

$y = \frac{4}{3}x - 3$ ------- [A1]

Question 5c was generally well attempted. Some other mistake for Q5d include
1) Using gradient for the line as 4 instead of $\frac{4}{3}$
2) The equation should not be written as $y = 1\frac{1}{3}x - 3$. Improper fraction should be used when written with x

= 0.727 units

3y = 4x + 5

Gradient of the line = 4

y - 1 = 4(x - 3)

y - 1 = 4x - 12

y = 4x - 11

Equation of line that passes through P:

6 **(a)** The results of two class tests are shown in the tables below. The total marks for both tests are 80.

Test 1

	$40 \le x < 50$	$50 \le x < 60$	$60 \le x < 70$	$70 \le x < 80$
		12	13	5

To find mid value for the following range of data.

$a \le x < b$ or $a < x < b$

Mid value $= \frac{a+b}{2}$

(i) Find the mean mark and the standard deviation of **Test 1**. [3]

$$\text{Mean} = \frac{\sum fx}{\sum f} = \frac{45\times10 + 55\times12 + 65\times13 + 75\times5}{10+12+13+5} = \frac{2330}{40}$$

$= 58.25$ marks -------- [A1]

Question 6a(i) was generally well attempted. Some other mistakes include
1) Using the wrong mid values for the marks.
2) Not writing units for mean and standard deviation.
3) Not showing substitution of of values for $\sum fx^2, \sum fx$ for standard deviation formula.
4) Rounding off the mean to 3sf instead of giving exact value.

$$\text{Standard Deviation} = \sqrt{\frac{\sum fx^2}{\sum f} - \left(\frac{\sum fx}{\sum f}\right)^2} = \sqrt{\frac{139600}{40} - \left(\frac{2330}{40}\right)^2}$$ ------ [M1]

$= \sqrt{96.9375}$

$= 9.85$ marks (3 SF) -------- [A1]

Must show substitution of the values!

(ii) Compare and comment on the marks of the two tests in two different ways. [2]

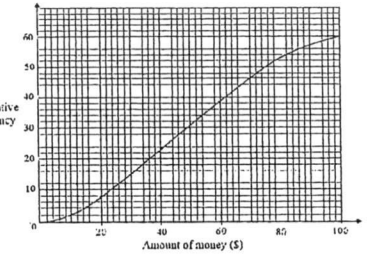
The mean mark of Test 1 is higher than of Test 2. This means that students scored better in Test 1 as compared to in Test 2. [A1]

The standard deviation of Test 2 marks is higher than in Test 1. This means that the spread of marks in Test 2 is bigger/wider than that of Test 1. [A1]

Question 6a(ii) was generally well attempted. Some other mistakes include
1) Simply commenting the marks of test 1 is higher than test 2
2) Using range of marks, more varied marks.

VICTORIA SCHOOL 17/S4PR1/EM/2

(b) The cumulative frequency curve below shows the amount of money collected by 60 students in the school's Rugby Team during a fund raising event.



(i) Use the graph to find the

(a) median amount of money raised, [1]

Median amount = \$49 ------- [B1]

(b) interquartile range of the distribution, [2]

Interquartile range = 69 − 30 -------- [B1]

= \$39 -------- [A1]

Q5b are well answered. Mistakes include
1. reading the graph wrongly,
2. not giving units for median and inter quarter range.

(c) tenth percentile of the distribution. [2]

No. of students in 10th percentile $= \frac{10}{100} \times 60 = 6$ -------- [B1]

10th percentile of the distribution = \$18 -------- [A1]

Must show no. of students at 10th percentile.

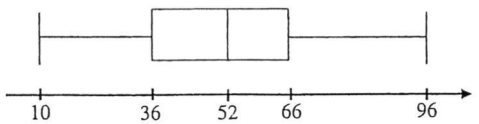
(ii) A student will be given a T-Shirt if he managed to collect at least \$y. Find the value of y if 20% of the students were given the T-Shirt. [2]

No. of students $= \frac{20}{100} \times 60 = 12$ -------- [B1]

Value of $y = 72$ -------- [A1]

Must show no. of students at 20%.

(iii) The box-and-whisker diagram below illustrates the amount raised by 60 students from the Outdoor Club.



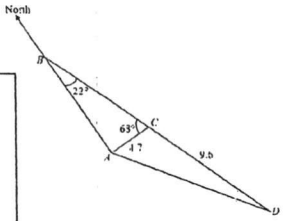
Amount of money (\$)

Brian commented that the students in the Rugby Team raised more money as compared to the students from the Outdoor Club. Do you agree? Give a reason for your answer. [1]

No. The median amount raised by the Rugby Team is lower as compared to the median amount raised by the Outdoor Club students. -------- [B1]

Some students carelessly use mean instead of median.

7 A, B, C and D are points on level ground, with A due South of B.
It is given that $\angle ABC = 22°$, $\angle BCA = 68°$, $AC = 4.7$ km and $CD = 9.6$ km.
BCD is a straight line. Calculate



(a) AD, [2]

$AD^2 = DC^2 + AC^2 - 2(AC)(DC)\cos\angle DCA$

$= (9.6)^2 + (4.7)^2 - 2(9.6)(4.7)\cos(180° - 68°)$ ----- [M1]

$AD \approx 12.168$

$= 12.2$ km (3 SF) ----- [A1]

(b) $\angle CAD$, [2]

$\frac{9.6}{\sin\angle CAD} = \frac{AD}{\sin 112°}$ ------- [M1]

$\sin\angle CAD = 0.73151$

$\angle CAD \approx 47.012°$

$= 47.0°$ (1 DP) ------- [A1]

Generally, well attempted for Q7. Common mistakes include
1. Wrong use of cosine rule.
2. Not using the correct sf in intermediate steps resulting in losing accuracy for final answers.
3. Not providing reasons for 7c.

(c) the bearing of C from D, [1]

Bearing of C from $D = 360° - 22°$ ($\angle$s at a point)

$= 338°$ ------ [A1]

(d) the shortest distance from C to AD. [2]

Need to have 4 to 5 sf

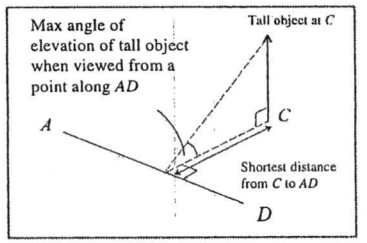
Let the shortest distance from C to AD be h km.

$\frac{1}{2} \times AD \times h = \frac{1}{2} \times DC \times AC \times \sin\angle DCA$

$\frac{1}{2} \times 12.168 \times h = \frac{1}{2} \times 4.7 \times 9.6 \times \sin 112°$ ------ [M1]

$h \approx 3.4381$

$= 3.44$ km (3 SF) ------ [A1]



A helicopter, H is hovering at a height of 450 m, at a point which is vertically above C.

(e) Find the **smallest** angle of elevation of the helicopter when observed along AD. [2]

The smallest angle of elevation occurs when it is observed from point D.
Let the angle of elevation be θ.

$\tan\theta = \frac{450}{9600}$ ------ [M1]

$\theta \approx 2.6838°$

$= 2.7°$ (1 DP) ------ [A1]

The most common mistake students make was to assume the smallest angle of elevation occurs at shortest distance from C to AD.
Final answer to be given to 1 d.p instead of 2 d.p.

8 **(a)** R is the point (7, 1) and S is the point (2, 5).

(i) Find the column vector $\overrightarrow{RS}$. [1]

$\overrightarrow{RS} = \overrightarrow{OS} - \overrightarrow{OR}$

$= \begin{pmatrix}2\\5\end{pmatrix} - \begin{pmatrix}7\\1\end{pmatrix}$

$= \begin{pmatrix}-5\\4\end{pmatrix}$ --------- [A1]

(ii) Calculate the exact value of $\left|\overrightarrow{RS}\right|$. [2]

$\left|\overrightarrow{RS}\right| = \sqrt{(-5)^2 + 4^2}$ -------- [M1]

$= \sqrt{41}$ units -------- [A1]

(iii) If $\overrightarrow{AB} = \frac{1}{4}\overrightarrow{RS}$ and B is the point (3, 6), find the coordinates of A. [2]

$\overrightarrow{AB} = \frac{1}{4}\overrightarrow{RS}$

$\overrightarrow{OB} - \overrightarrow{OA} = \frac{1}{4}\begin{pmatrix}-5\\4\end{pmatrix}$ -------- [M1]

$\overrightarrow{OA} = \overrightarrow{OB} - \overrightarrow{AB}$

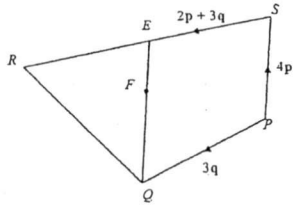
$= \begin{pmatrix}3\\6\end{pmatrix} - \begin{pmatrix}-\frac{5}{4}\\1\end{pmatrix}$

$= \begin{pmatrix}4\frac{1}{4}\\5\end{pmatrix}$

$\therefore$ The coordinates of A are $\left(4\frac{1}{4}, 5\right)$. -------- [A1]

Generally, well attempted for Q8a. Common mistakes include
1. Not giving exact value for $\left|\overrightarrow{RS}\right|$.
2. Unclear method when finding $\overrightarrow{OA}$. Some simply add or subtract directly the elements of the vector.
3. Leaving answer in column vector instead of providing the coordinates of A.

(b) In the diagram, E is a point on SR such that $SR = 2SE$ and F is a point on QE such that $3QF = 2QE$. $\overrightarrow{PS} = 4\mathbf{p}$, $\overrightarrow{PQ} = 3\mathbf{q}$ and $\overrightarrow{SE} = 2\mathbf{p} + 3\mathbf{q}$.



(i) Express each of the following, as simply as possible, in terms of **p** and/or **q**.

(a) $\overrightarrow{PE}$, [1]

$\overrightarrow{PE} = 4\mathbf{p} + 2\mathbf{p} + 3\mathbf{q}$
$= 6\mathbf{p} + 3\mathbf{q}$
$= 3(2\mathbf{p} + \mathbf{q})$ -------- [A1]

(b) $\overrightarrow{QE}$, [1]

$\overrightarrow{QE} = -3\mathbf{q} + 6\mathbf{p} + 3\mathbf{q}$
$= 6\mathbf{p}$ -------- [A1]

Generally, well attempted for Q8b. Common mistakes include
1. Not factorizing the common scalar multiple for the final answer.
2. Not writing the vector with the tilde symbol.
3. Not providing supporting evidence for making deduction about P, F and R.

(c) $\overrightarrow{PF}$, [1]

$\overrightarrow{PF} = 3\mathbf{q} + \frac{2}{3}(6\mathbf{p})$
$= 4\mathbf{p} + 3\mathbf{q}$ -------- [A1]

4p+3q ← tilde , ~

(d) $\overrightarrow{PR}$. [1]

$\overrightarrow{PR} = 4\mathbf{p} + 2(2\mathbf{p} + 3\mathbf{q})$
$= 8\mathbf{p} + 6\mathbf{q}$
$= 2(4\mathbf{p} + 3\mathbf{q})$ -------- [A1]

Wrong use

(ii) What can you deduce about P, F and R? [1]

$\overrightarrow{PR} = 2(4\mathbf{p} + 3\mathbf{q})$
$= 2\overrightarrow{PF}$
Since $\overrightarrow{PR}$ is some scalar multiple of $\overrightarrow{PF}$, $\overrightarrow{PR}$ is parallel to $\overrightarrow{PF}$. P is a common point.
$\therefore P$, F and R lie on a straight line. ------- [A1]

(iii) Given that the area of $\Delta QRE = 24\ \text{cm}^2$, find the area of ΔRFE. [2]

Generally, well attempted for Q8biii. Common mistake include
1. Assuming the 2 triangles are similar.

ΔRFE and ΔQRE share a common height,

$\frac{\text{Area of } \Delta RFE}{\text{Area of } \Delta QRE} = \frac{FE}{QE}$

$\frac{\text{Area of } \Delta RFE}{24} = \frac{1}{3}$ ------- [M1]

Area of $\Delta RFE = \frac{1}{3} \times 24 = 8\ \text{cm}^2$ ------- [A1]

9 Answer the whole of this question on a sheet of graph paper.

In experiment A, water flows from a tap at a constant rate into a conical flask.

The table below shows the height of water, h cm, at various timings of the water flow.

Time (t s)	0.0	1.0	2.0	3.0	4.0	5.0	6.0
Height (h cm)	0.00	0.50	1.25	2.38	3.90	6.59	10.39

(a) Using a scale of 2 cm to represent 1 second, draw a horizontal t-axis for $0 \le t \le 6$.
Using a scale of 2 cm to represent 1 cm depth, draw a vertical h-axis for $0 \le h \le 11$.
On your axes, plot a smooth graph represented by the data in the given table. [3]

(b) Use your graph to find an estimate for the height of water in the conical flask when $t = 3.4$. [1]

$h = 2.9$ cm -------- [B1]

Common mistake - not having units.

(c) By drawing a tangent at the points $t = 2$ and $t = 5$,

Common mistake
- not showing the working for gradients.
- not drawing the tangent line touching the point on the curve.

(i) find the respective gradient of the curve at $t = 2$ and $t = 5$, [2]

At $t = 2$: Gradient $= \frac{3.35 - 0}{4 - 0.7} = 1.02$ -------- [M1 - to find the gradient]

At $t = 5$: Gradient $= \frac{9.5 - 4}{6 - 4.2} = 3.06$ -------- [M1 - to find the gradient]

(ii) state briefly what these gradients represent, [1]

Wrong to say rate of water flow

The gradients represent the rate of increase of water level in the conical flask at each instant. -------- [B1]

(iii) explain the significant difference between these gradients. [1]

Water level is rising faster at $t = 5$ compared to $t = 2$. -------- [B1]

(d) In experiment B, another container in the shape of a cylinder is being used.
Water flows from the same tap. The height of water can be described by the equation $h = 1.25t + c$, where c is a constant.

Only 2 points are needed. To draw the line! (0, 0) and (6, 7.5)

Anglo-Chinese School (Barker Road)

PRELIMINARY EXAMINATION 2017

SECONDARY FOUR EXPRESS / FIVE NORMAL ACADEMIC

MATHEMATICS 4048
PAPER TWO

2 HOURS 30 MINS

Additional Materials: Answer Paper (7 sheets)
Graph Paper (1 sheet)

READ THESE INSTRUCTIONS FIRST

Do not open this booklet until you are told to do so.

Write your class and candidate number on the cover sheet.
Write in dark blue or black pen on both sides of the paper.
You may use a soft pencil for any diagrams or graphs.
Do not use staples, paper clips, highlighters, glue or correction fluid.

Answer all questions.
If working is needed for any question it must be shown with the answer.
[redacted]sential working will result in loss of marks.
[redacted]uld be used where appropriate.
[redacted] accuracy is not specified in the question, and if the answer is not exact, give the answer to [redacted] figures. Give answers in degrees to one decimal place.
For π, use either the calculator value or 3.142, unless the question requires the answer in terms of π.

At the end of the examination, fasten all your work securely together.
The number of marks is given in brackets [ ] at the end of each question or part question.
The total of the marks for this paper is 100.

This paper consists of 11 printed pages inclusive of this page. [Turn over

1 The first three terms in a sequence of numbers, $T_1, T_2, T_3, \ldots$ are given below.

$$T_1 = 1 \times 2 + 10 = 12$$
$$T_2 = 2 \times 3 + 6 = 12$$
$$T_3 = 3 \times 4 + 2 = 14$$

(a) Find T_4. [1]

(b) Show that $T_n = n^2 - 3n + 14$. [2]

(c) Evaluate T_{50}. [1]

(d) Explain why every term in the sequence is even. [2]

---

2 **(a)** It is given that $v^2 = u^2 - 2gh$.

(i) Evaluate v when $u = 30$, $g = 9.8$ and $h = 24$. [2]

(ii) Express u in terms of g, h and v. [2]

(b) Factorise $(x+1)^2 - (y-1)^2$. [2]

(c) Simplify $\dfrac{x^2 - 1}{8 - 3x - 5x^2}$. [3]

(d) Solve the simultaneous equations. [3]

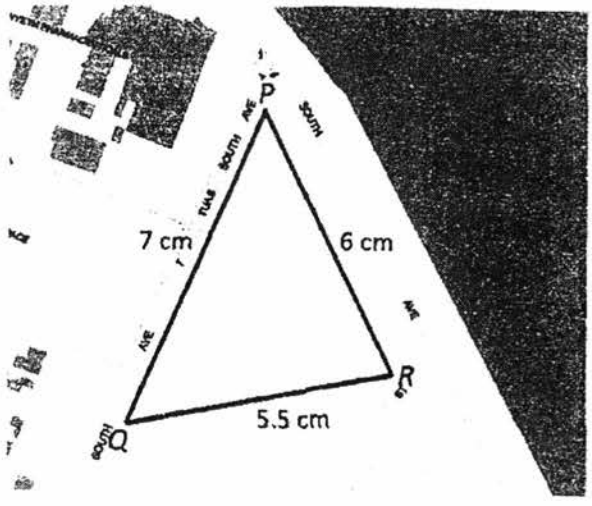
$$1\frac{1}{2}x - 3y = 12$$
$$4y = 3x - 19$$

---

3 (a) The scale of a map is 1 : 7500.

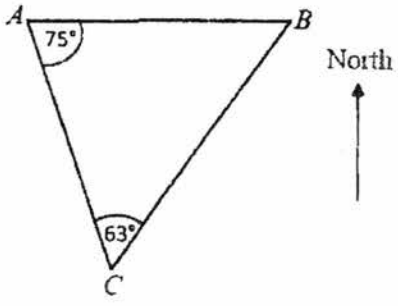
(i) The length of a road on the map is 20.5 cm.
Find the actual length, in kilometres, of the road. [1]

(ii)



On the map, an area formed by a triangle PQR with sides 5.5 cm, 6 cm and 7 cm, is slated for commercial development.
Calculate, in square metres, the actual area. [5]

(b)



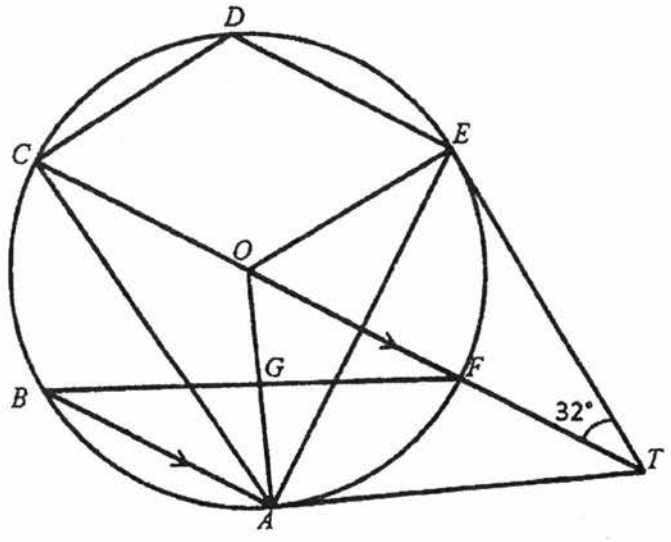
In the diagram, AB is the shoreline. B is due east of A. A boat is at C.
$C = 75°$, angle $ACB = 63°$ and $AB = 35$ m.

(i) Find the bearing of B from C. [2]

(ii) The area of triangle ABC is 444 m^2. Calculate the shortest distance from the boat to the shore. [1]

(iii) A turtle is crawling along the shoreline. An eagle is at a vertical height of 40 m above C. It notices the turtle.
Calculate the greatest angle of depression of the turtle as seen from the eagle. [2]

4 In the diagram, O is the centre of the circle.
TA and TE are tangents to the circle. OA and OE are radii of the circle. COT is a straight line.
OA intersects BF at G. CT is parallel to BA.
Angle $OTE = 32°$.



(a) Find

(i) angle AOF, [2]

(ii) angle CDE, [2]

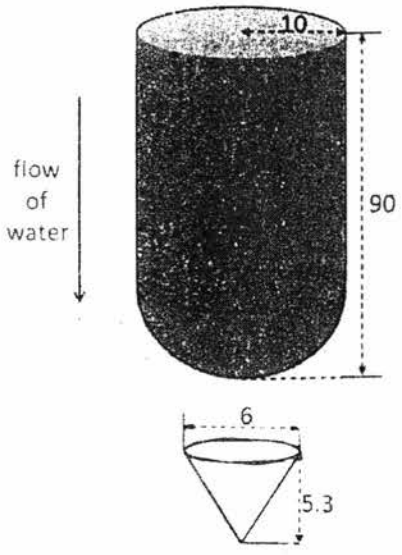
(iii) angle OFG, [2]

(iv) angle AGB. [1]

(b) Explain why points $OETA$ can also be points on the circumference of another circle. [1]

5 The diagram shows a water dispenser that is made up of a cylinder and a hemisphere both of radius 10 cm. The height of the dispenser is 90 cm.

Conical cups of diameter 6 cm and height 5.3 cm are provided to drink the water from the container.



(a) Water is filled to the brim of the dispenser.
Find the amount of water in the dispenser. [2]

(b) Find the capacity of one conical cup.
Give your answer to the nearest cm^3. [2]

(c) Find the external curved surface area of the cup. [2]

(d) Find the height of the water remaining in the dispenser after 250 cups of water has been dispensed. [4]

6 A container can hold 2400 litres of water.

(a) A large tap alone can fill the container in x hours.
Write down an expression, in terms of x, for the amount of water that the large tap can dispense per minute. [1]

(b) A small tap alone will take 1 hour longer than the large tap to fill the container. Write down an expression, in terms of x, for the amount of water that the small tap can dispense per minute. [1]

(c) When both taps are turned on at the same time, they can fill the container in 3 hours.
Form an equation in x and shows that it reduces to $x^2 - 5x - 3 = 0$. [3]

(d) Solve the equation $x^2 - 5x - 3 = 0$, giving your solutions correct to 2 decimal places. [4]

(e) Find the rate of water flow, in litres per minute, of the small tap. [2]

---

7 **Answer the whole of this question on a single sheet of graph paper.**

A stone is thrown from the top of a cliff next to the sea. The height, h metres, of the stone above sea level t seconds after it is released can be modelled by the equation

$$h = 40 + 8t - \frac{5}{2}t^2$$

Some corresponding values of t and h, correct to 1 decimal place, are given in the table below.

t	0	1	2	3	4	5	6
h	40	45.5	46	41.5	32	17.5	p

(a) Calculate the value of p. [1]

(b) Using a scale of 2 cm to represent 1 second, draw a horizontal t-axis for $0 \le t \le 6$.
Using a scale of 1 cm to represent 5 metres, draw a vertical h-axis for $-10 \le h \le 50$.
On your axes, plot the points given in the table and join them with a smooth curve. [3]

raph to estimate

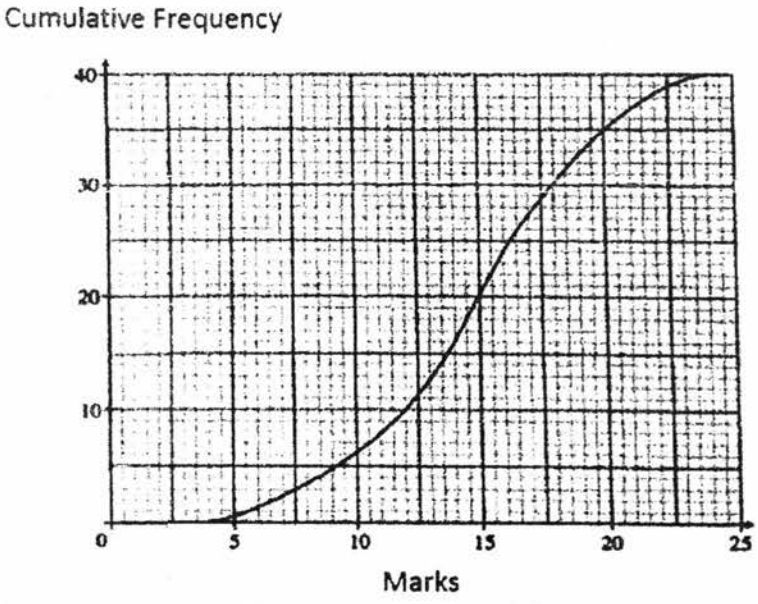
(i) the maximum height of the stone above sea level, [1]

(ii) the length of time that the stone was greater than or equal to 5 m above the top of the cliff, [2]

(iii) the time taken for the stone to hit the water. [1]

(d) By drawing a tangent, find the gradient of the curve at $t = 4$. [2]

8 (a) The marks attained by 40 students in a Mathematics test were recorded.
The cumulative frequency curve shows the distribution of the marks.



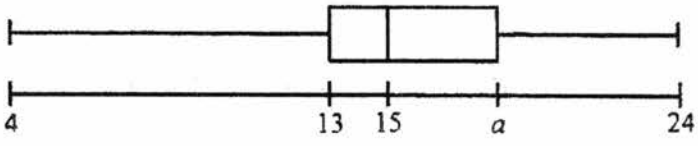
(i) Use the curve to estimate the

(a) the median mark, [1]

(b) the interquartile range. [2]

(ii) 12.5% of students achieved more than x marks in this test. Estimate the value of x. [1]

(iii) The same group of students sat for a Chemistry test. The maximum mark for the test was also 25. The box-and-whisker plot of the distribution of the marks is shown below.



The top 25% of the students for the Chemistry test scored lower than the top 25% in the Mathematics test. Write down the possible range of marks that a can take. [1]

(iv) Describe how the cumulative frequency curve for the marks attained in the Chemistry test may differ from the curve for the Mathematics test. [1]

(b) The weight of 8 students, in kilograms, are listed below:

25, 27, 32, 28, 28, 31, 26, 45

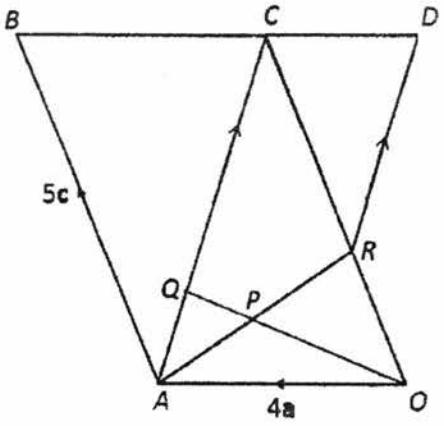
(i) Find the mean weight. [1]

(ii) Explain why the mean may not be an appropriate average to use to summarise the weights of the students. [1]

(iii) Find the standard deviation of the weights. [1]

(iv) Subsequently, it was discovered that the weight of every student was 2 kg less than the actual, due to a faulty weighing scale.
Write down the correct mean and standard deviation of the weights. [2]

9



In the diagram, OA is parallel to DB, AC is parallel to RD and $OABC$ is a parallelogram. $\overrightarrow{OA} = \mathbf{4a}$ and $\overrightarrow{AB} = \mathbf{5c}$ respectively. It is given that $OR : RC = 2 : 3$ and $\overrightarrow{AQ} = \frac{1}{3}\overrightarrow{QC}$.

(a) Find, in terms of **a** and **c**, the vectors

(i) $\overrightarrow{OR}$, [1]

(ii) $\overrightarrow{AR}$, [1]

(iii) $\overrightarrow{OQ}$. [2]

(b) P is a point on OQ such that $OP : PQ = 8 : 3$.

(i) Express $\overrightarrow{AP}$ in terms of **a** and **c**. [2]

(ii) Hence write down two facts about A, P and R. [2]

(c) Name a pair of congruent triangles. [1]

(d) Prove that ΔRCD is similar to ΔCOA. [2]

(e) Find

(i) $\dfrac{\text{Area of } \Delta RCD}{\text{Area of } \Delta COA}$, [1]

(ii) $\dfrac{\text{Area of } \Delta OQA}{\text{Area of } \Delta OCA}$. [1]

10 James has gotten a job that pays him a salary of $60 000 annually. He plans to purchase a car but calculates that he can only afford to set aside 30% of his monthly salary for the expenses incurred in owning the car.

(a) Calculate the sum of money that James can afford to set aside monthly for the expenses incurred in owning the car. [1]

He has set his eyes on two cars. He decides to take a loan from a bank for the purchase. He will repay the loan on a monthly basis. The details are given below:

	Brand A (used car)	**Brand B** (new car)
Engine capacity	1600 cc	1400 cc
Cost	$80 000	$90 000
Intended loan amount	50% of cost price	60% of cost price
Intended loan period	5 years	5 years
Type of interest	compound interest at 2.5% per year, compounded yearly	simple interest at 3% per year

The other major expenses in maintaining a car are as follows:

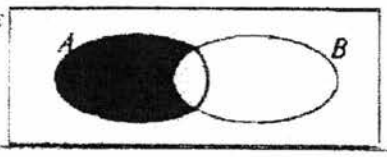
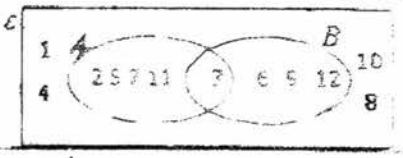
	Brand A (used car)	**Brand B** (new car)
Monthly parking fees	$90	$90
Monthly petrol expenditure	$300	$250
Annual road tax	$744	$626
Annual insurance	$800	$700
Car servicing (twice a year)	$600 each round	$500 each round

(b) Recommend the brand of car that James can purchase, based on the sum of money he can afford to set aside monthly. Justify the decision you make and show your calculations clearly. [7]

End of Paper

Anglo-Chinese School
(Barker Road)

Mathematics Paper 1 Marking Scheme
Secondary 4 Express / 5 Normal Academic
Preliminary Exams 2017

Qn	Steps/Answer			
1	$\sqrt{0.81}$ 0.902 0.86^3 $\frac{399}{441}$			
2	$(256\times10^9)\div(2.5\times10^6)$ (exact answer) $=1.024\times10^5$			
3	$4c(3a+7b)-2d(3a+7b)$ $=(4c-2d)(3a+7b)$ or equivalent $=2(2c-d)(3a+7b)$			
4	$\frac{1}{9}=\$20$ Total sum = \$180			
5	$=\frac{7x}{(x-5)^2}-\frac{1}{x-5}$ $=\frac{7x-(x-5)}{(x-5)^2}$ $=\frac{6x+5}{(x-5)^2}$			
6	$-8\le 2-3x$ and $2-3x<8$ $-2<x\le 3\frac{1}{3}$			
7	12 cooks – 6 hours – 180 people 8 cooks – 9 hours – 180 people 8 cooks – 7.5 hours – 150 people Ans: 7.5 hours			
8	Different scale used for the vertical axis may mislead one to think that more people prefer Brand Y to Brand X.			
9 (a)				
(b)				
10	$=\left(\frac{y^6}{x^6}\right)^{\frac{1}{2}}\times\left(\frac{x^{-5}}{y^{-3}}\right)$ $=\frac{y^3}{x^3}\times\frac{x^5}{y^3}$ $=\frac{x^2}{y}$			
11 (a)	\$6950			
(b)	$\frac{\$6850}{5000}$ US\$1 = [illegible]			

Anglo-Chinese School
(Barker Road)

Mathematics Paper 1 Marking Scheme
Secondary 4 Express / 5 Normal Academic
Preliminary Exams 2017

Qn		Steps/Answer
12	(a)	Cost price of watch for Jimmy $= \frac{80}{100} \times 210$ $= \$168$ Price that Jimmy should sell at $= \frac{120}{100} \times \$168$ $= \$201.60$ Marked price $= \frac{100}{90} \times \$201.60 = \$224$
13		angle FDB = angle CBD = 56° (alternate angles, BC parallel to FD) angle ABD = 180 – (44+56) = 80° angle FAB + angle ABD = 181° **By the property that interior angles of parallel lines are supplementary**, AF is not parallel to BD
14	(a)	720°
	(b)	If the 3 polygons are regular, then angle IKB + angle IKF + angle BKF = 108° + 108°+ 120° = 336° By the property that angles at a point add up to 360°, at least one of the polygons must be irregular.
15	(a)	2 x 7
	(b)	10
	(c)	LCM of 84, 90 and 2450 is 44100. Next flash at 2015
16	(a)	

+	5	6	7	8	9
5	-	11	12	13	14
6	11	-	13	14	15
7	12	13	-	15	16
8	13	14	15	-	17
9	14	15	16	17	-

	(b)	$\frac{1}{10}$
	(c)	$\frac{3}{5} \times \frac{2}{4} \times \frac{1}{3}$ $= \frac{1}{10}$

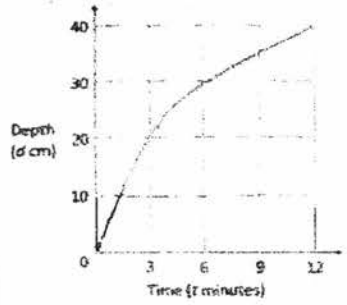
Anglo-Chinese School (Barker Road)

Mathematics Paper 1 Marking Scheme
Secondary 4 Express / 5 Normal Academic
Preliminary Exams 2017

17 (a) $\left(\frac{20}{40}\right)^2 \times 12$

$= 3$ min

(b)



[4]

18 (a) $\mathbf{V} = \begin{pmatrix} 320 & 120 \\ 380 & 100 \\ 410 & 130 \end{pmatrix}$

(b) $\mathbf{P} = \begin{pmatrix} 320 & 120 \\ 380 & 100 \\ 410 & 130 \end{pmatrix} \begin{pmatrix} 2 \\ 0.5 \end{pmatrix} = \begin{pmatrix} 700 \\ 810 \\ 885 \end{pmatrix}$

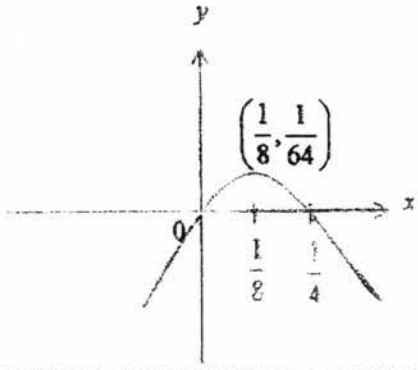
[4]

(c) **P** represents the total charges incurred by all vehicles for each respective day.

(d) $D = (1 \quad 1 \quad 1)$

19 (a) $\left(x - \frac{1}{8}\right)^2 = \frac{1}{64}$

(b)



[4]

(c) $\left(\frac{1}{8}, \frac{1}{64}\right)$

20 (a) $\frac{V}{50} = \frac{204.8}{1}$

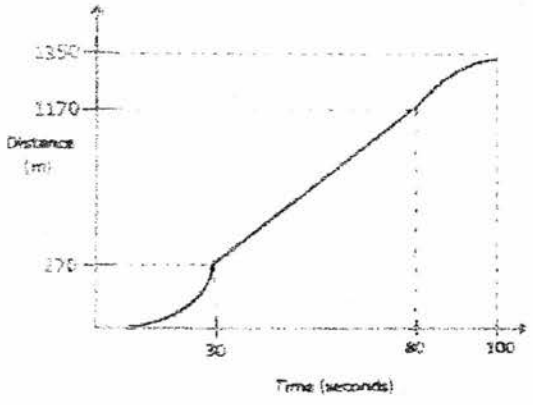
$V_{large} = 204.8$

(b)

Small bottle	Large Bottle
50g - \$5.10	204.8g - \$13.25
1g – 10.2 cents	1g – 6.47

The larger bottle gives more value for money

[4]

21	(a)	angle $ACB = 37 - 20 = 17°$ $\frac{AC}{\sin 143} = \frac{400}{\sin 17}$ $AC = 823\text{m}$
	(b)	$h = \sin 20 \times 823.356$ $= 282$ m
22	(a)	$QP = \frac{8}{\tan 0.7rad} = 9.4979$ Area of triangle $OPQ = \frac{1}{2}(8)(9.4979) = 37.992$ Area of sector $= \frac{1}{2}(8^2)(0.87079rad) = 27.865$ Area of shaded region $= 10.1\ \text{cm}^2$
	(b)	$40.1°$
23	(a)	$\frac{y-6}{x-0} = -\frac{1}{4}$ $4y = -x + 24$
	(b)	$x = 24$ $(24, 0)$
	(c)	24.7 units
	(d)	Identify that D has coordinates $(-12, 0)$ $x = -12$
24	(a)	Speed $= \frac{18}{30} \times 18$ $= 10.8$ m/s $= 38.88$ km/h
	(b)	Distance $= \left(\frac{1}{2} \times 30 \times 18\right) + (50 \times 18) - \left(\frac{1}{2} \times 20 \times 18\right)$ $= 1350$ m
	(c)	

Anglo-Chinese School (Barker Road)

Mathematics Paper 2 Marking Scheme
Secondary 4 Express / 5 Normal Academic
Preliminary Exam 2017

Qn		Steps/Answer			
1	(a)	$T_4 = 18$			
	(b)	$T_n = n(n+1)+10-4(n-1)$ $= n^2+n+10-4n+4$ $= n^2-3n+14$			
	(c)	$T_{50} = 2364$			
	(d)	$n^2-3n+14 = n(n-3)+14$ When n is even n(n-3) is (even x odd) = even When n is odd, n(n-3) is (odd x even) = even. Adding to 14 which is also even, $T_n = n^2-3n+14$ will always be even for all terms.			
2	(ai)	$v^2 = 30^2 - 2(9.8)(24)$ $v = \pm 20.7$			
	(aii)	$v^2 = u^2 - 2gh$ $u^2 = v^2 + 2gh$ $u = \pm\sqrt{v^2+2gh}$			
	(b)	$[(x+1)+(y-1)][(x+1)-(y-1)]$ $= (x+y)(x-y+2)$			
	(c)	$\frac{(x+1)(x-1)}{(1-x)(8+5x)}$			
		$= \frac{(x+1)(x-1)}{-(x-1)(5x+8)}$ or $\frac{-(1-x)(x+1)}{(1-x)(5x+8)}$			
		$= -\frac{(x+1)}{(5x+8)}$ or equivalent			
	(d)	By substitution or elimination method $x = 3$, $y = -2.5$			

3	(ai)	1.5375 km		
	(aii)	Conversion from cm to m or cm^2 to m^2 Using cosine rule, $412.5^2 = 525^2 + 450^2 - 2(525)(450)\cos(angle\,BAC)$ $\cos(angle\,BAC) = \frac{-307968.75}{-472500}$ $angle\,BAC = 49.324°$		
		Area of triangle $= \frac{1}{2}(525)(450)\sin 49.324°$ $= 89\,600\ m^2$		
	(bi)	Bearing of C from $B = 63-(90-75)=048°$		
	(bii)	Shortest distance $= \frac{444 \times 2}{35} = 25.4$ m		
	(biii)	Angle of depression $= \tan^{-1}\left(\frac{40}{25.371}\right)$ $=57.6°$		
4	(ai)	angle OTA = angle $OTE = 32°$ (the line joining an external point to the centre of the circle bisects the angle between the tangents) angle $TAO = 90°$ (tangent perpendicular to radius) angle $AOF = (180 - 90 - 32)° = 58°$ (angles sum of triangle AOT)		
	(aii)	angle $AOE = 58 \times 2 = 116°$ angle $AOC = 180 - 58 = 122°$ angle $CDE = \frac{1}{2}(58 \times 2 + 122°) = 119°$		
	(aiii)	angle $GBA = \frac{1}{2}(58°) = 29°$ (angle at centre is twice angle at circumference) angle OFG = angle $GBA = 29°$ (alternate angles, OF parallel to BA)		
	(aiv)	angle $OGF = (180 - 29 - 58)° = 93°$ angle $AGB = 93°$ (vertically opposite angles)		
	(b)	By the property of 'right-angle in a semi-circle' OT is a diameter and points E and A will lie on the circumference $OETA$ are thus four points on the circumference of this circle Or calculate using 'angles in opposite segments are supplementary'		
5	(a)	volume of water $= \pi(10^2)(80) + (\frac{2}{3})(\pi)(10^3)$ $= [illegible]\ cm^3$		

(b) Capacity of one conical cup = $(\frac{1}{3})(\pi)(3^2)(5.3)$

$= 50\ cm^3$

(c) Slant height of cup = $\sqrt{3^2+5.3^2}$

$= 6.0902$

Curved surface area of cup = $\pi(3)(6.0902)$

$= 57.4\ cm^2$

(d) Volume of water remaining after dispensing 250 cups

$= 8666\frac{2}{3}\pi - (250 \times \frac{1}{3}\pi(3^2)(5.3)$

$= 4619\frac{2}{3}\pi$

Volume of water in cylinder $= 4619\frac{2}{3}\pi - \frac{2}{3}\pi(10^2) = 4025\pi$

Height of water in cylindrical section $= \frac{4025\pi}{\pi(10^2)} = 40.25$

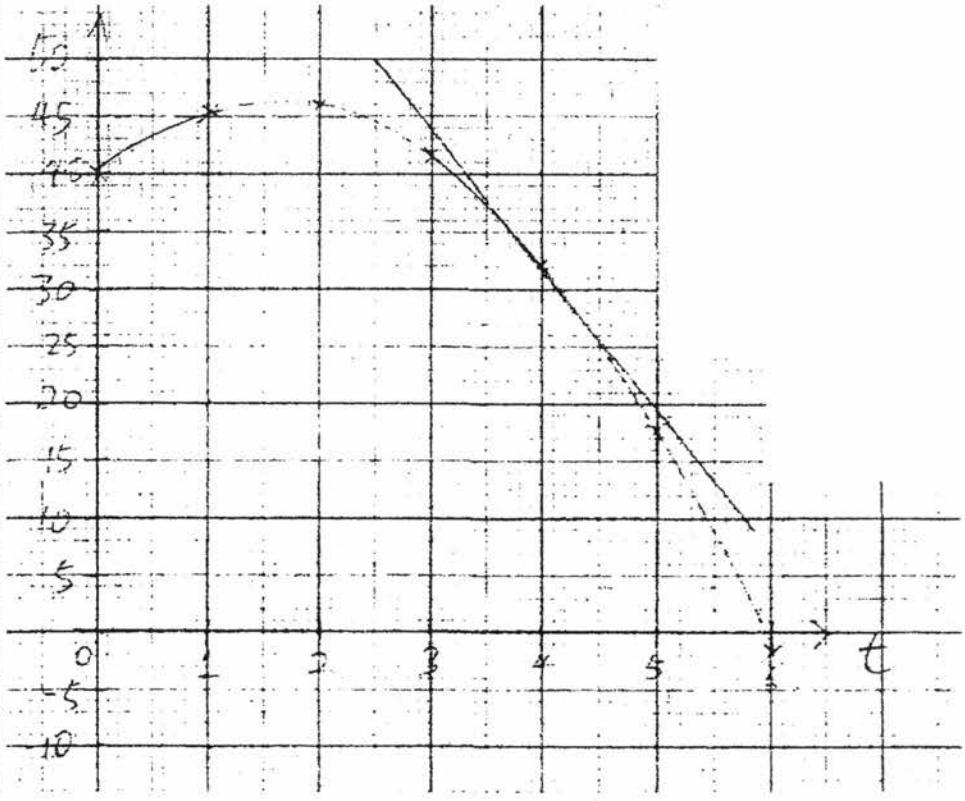
Height of water remaining in dispenser

$= 40.25+10$

$= 50.25$ cm

6	(a)	$\frac{40}{x}$ litres/minute			
	(b)	$\frac{40}{x+1}$ litres/mmute			
	(c)	$180\left(\frac{40}{x+1}+\frac{40}{x}\right)=2400$ $3[40x+40(x+1)]=40x(x+1)$ $40x^2-200x-120=0$ $x^2-5x-3=0$ (shown)			[11]
	(d)	$x=\frac{5\pm\sqrt{(-5)^2-4(1)(-3)}}{2(1)}$ $x=\frac{5\pm\sqrt{37}}{2}$ $x=5.54$ or $x=-0.54$			
	(e)	Rate of water flow for small tap $=\frac{40}{5.54)+1}$ = 6.11 litres per minute			

7	(a)	$p = -2$			
		Please refer to graph			[10]



	(ci)	Maximum height = 46 m			
	(cii)	Length of time = 2.4 – 0-9 = 1.5 s			
	(ciii)	Time taken to hit water = 5 95s			
	(d)	Tangent drawn correctly. Gradient = $\frac{50-0}{2.5-5,8}$ = −12.1			

8	(aia)	15 marks			[11]
	(aib)	18 – 12 = 6 marks			
	(aii)	20 marks			
	(aiii)	$15 \leq a < 18$			
	(aiv)	The curve will be steeper before the median mark of 15 and less steep after the median.			
	(bi)	30.25 kg			
	(bii)	There is an outlier 45 kg which would cause the mean to be skewed.			
	(biii)	Standard deviation = 5.99			
	(biv)	Correct mean = 32.25 kg Standard deviation remains the same			

9	(ai)	$\overrightarrow{OR} = \frac{2}{5}\overrightarrow{OC}$ $= 2\mathbf{c}$			[13]
	(aii)	$\overrightarrow{AR} = \overrightarrow{OR} - \overrightarrow{OA}$ $= 2\mathbf{c} - 4\mathbf{a}$			
	(aiii)	$\overrightarrow{OQ} = \overrightarrow{OA} + \overrightarrow{AQ}$ $= 4\mathbf{a} + \frac{1}{4}\overrightarrow{AC}$ $= 4\mathbf{a} + \frac{1}{4}(5\mathbf{c} - 4\mathbf{a})$ $= 3\mathbf{a} + \frac{5}{4}\mathbf{c}$			
	(bi)	$\overrightarrow{AP} = \overrightarrow{AO} + \overrightarrow{OP}$ $= AO + \frac{8}{11}\overrightarrow{OQ}$ $= -4a + \frac{8}{11}(3\mathbf{a} + \frac{5}{4}\mathbf{c})$ $= \frac{20}{11}\mathbf{a} + \frac{10}{11}\mathbf{c}$			
	(bii)	$AP = \frac{5}{11}AR$ As point A is common, A, P and R are collinear (i.e. lie on the same straight line).			
	(c)	triangle ABC is congruent to triangle COA			
	(d)	$\angle DCR = \angle AOC$ (alt. $\angle$s, $DC \parallel OA$) $\angle DRC = \angle ACO$ (alt. $\angle$s, $DR \parallel CA$) ΔRCD is similar to ΔCOA (AA property)			
	(ei)	$\frac{\text{Area of } \Delta RCD}{\text{Area of } \Delta COA} = \left(\frac{RC}{CO}\right)^2$ $= \left(\frac{3}{5}\right)^2$ $= \frac{9}{25}$			
	(eii)	$\frac{\text{Area of } \Delta OQA}{\text{Area of } \Delta OCA} = \frac{QA}{CA} = \frac{1}{4}$			

(b)

	Brand A		
loan	40000	50% of cost	[8]
Compound amount	$40000(1+\frac{2.5}{100})^5$		
total loan amount	= $45256.33		
monthly instalment	$754.2721419	Divide by 60 months	
Monthly cost of road tax + Insurance + Servicing	$(\frac{744+800+1200}{12})$ =228.67		
Total monthly cost of maintenance	300+90+228.67 =618.67	Adding on monthly petrol and parking costs	
monthly installment + cost of maintenance	1372.94		

	Brand B	
loan	54000	60% of cost
Simple interest	8100	
total loan amount	62100	
monthly instalment	1035	Divide by 60 months
road tax (r)	626	
Insurance (i)	700	
Servicing (s)	1000	
Monthly cost of road tax + Insurance + Servicing	$\frac{626+700+1000}{12}$ =193.83	
	$\frac{250+90+193.83}{12}$	Adding on monthly petrol and parking costs
Total monthly cost of maintenance	$533.83	
monthly installment + cost of maintenance	1568.83	

James can afford Brand A as it is within the sum of money that he can set aside monthly

Register No.	Class

Name : _____

BENDEMEER SECONDARY SCHOOL
2017 PRELIMINARY TWO EXAMINATION
SECONDARY 4 EXPRESS / 5 NORMAL (ACADEMIC)
Elementary Mathematics Paper 1
4048/01

DATE : **22 August 2017**
DURATION : **2 hours**
TOTAL : **80 Marks**

READ THESE INSTRUCTIONS FIRST

Write your name, class and register number on the work you hand in.
Write in dark blue or black pen on both sides of the paper.
You may use a 2B pencil for any diagrams or graphs.
Do not use staples, paper clips, highlighters, glue or correction fluid/tape.

Answer **all** questions.
Write your answers in the spaces provided on the question paper.
All the diagrams in this paper are **not** drawn to scale.
If working is needed for any question, it must be shown with the answer.
Omission of essential working will result in loss of marks.
The use of an approved scientific calculator is expected, where appropriate.
If the degree of accuracy is not specified in the question, and if the answer is not exact, give the answer to three significant figures. Give answers in degrees to one decimal place.
For π, use either your calculator value or 3.142, unless the question requires the answer in terms of π.

At the end of the examination, fasten all your work securely together.
The number of marks is given in brackets [ ] at the end of each question or part question.

FOR EXAMINER'S USE
80

This document consists of **19** printed pages including this cover page.

[Turn over

MATHEMATICAL FORMULAE

Compound Interest

$$\text{Total amount} = P\left(1+\frac{r}{100}\right)^n$$

Mensuration

$$\text{Curved surface area of cone} = \pi rl$$

$$\text{Surface area of a sphere} = 4\pi r^2$$

$$\text{Volume of a cone} = \frac{1}{3}\pi r^2 h$$

$$\text{Volume of sphere} = \frac{4}{3}\pi r^3$$

$$\text{Area of triangle ABC} = \frac{1}{2}ab\sin C$$

$$\text{Arc length} = r\theta, \text{ where } \theta \text{ is in radians}$$

$$\text{Sector area} = \frac{1}{2}r^2\theta, \text{ where } \theta \text{ is in radians}$$

Trigonometry

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$a^2 = b^2 + c^2 - 2bc\cos A$$

Statistics

$$\text{Mean} = \frac{\sum fx}{\sum f}$$

$$\text{Standard Deviation} = \sqrt{\frac{\sum fx^2}{\sum f} - \left(\frac{\sum fx}{\sum f}\right)^2}$$

For Examiner's use

1 (a) By rounding each number to its nearest ten, calculate $\frac{216.1 + 1083.7}{14.99}$.

(b) Write your answer to part (a) correct to 1 significant figure.

Answer (a) .....[1]

(b) .....[1]

For Examiner's use

2 If the length of a rectangle is 340mm and width is 200mm, both are corrected to the nearest 10mm, calculate the

(a) maximum possible area of this rectangle in cm^2,

(b) lowest possible value of the ratio $\frac{width}{length}$.

Answer (a) ..... [2]

(b) .....[1]

3 James was 82kg and 15% above his ideal weight. He exercised and lost 6% of his initial weight. How many percent of his current weight must James lose in order to reach his ideal weight?

Answer..... [3]

4 (a) Solve $4a(a-3) = 2-(20-6a)$.

(b) Factorise $x^2y^2 + 36 - 4x^2 - 9y^2$ completely.

Answer (a) ..... [2]

(b) .....[3]

5 A flight leaving Singapore to London takes about 13 hours and 15 minutes. If the departure time on a Tuesday from Singapore is 1310 hours and Singapore is 7 hours ahead of London, what day and time, in 24 hour format, does the flight reach London?

Answer .....*hours on* ..... [2]

6 In $\Delta DEF, DF = 10cm, EF = 12cm$ and $\angle EDF = 39°$.

(a) Find $\angle DEF$.

(b) Which is the acceptable answer to part (a)? Explain why the other answer is not applicable.

Answer (a) $\angle DEF =$.....° , ...° [2]

(b) .....

..... [2]

7 Given that $\frac{a^2}{c^2} - \frac{b^2}{d^2} = 1$, make b the subject.

Answer ..... [2]

8 (a) Evaluate $(2^{-1} - 5^{-2})$ without using a calculator. Show your working clearly.

(b) Simplify $\frac{\sqrt[3]{b^2} \times b^6}{b^{\frac{2}{3}} \times b}$, giving your answer in the form of b^n.

Answer (a) ..... [2]

(b) ..... [2]

9 Siew Teng is x years old and her brother Victor is 2 years older. Their mother is 6 times older than Victor.

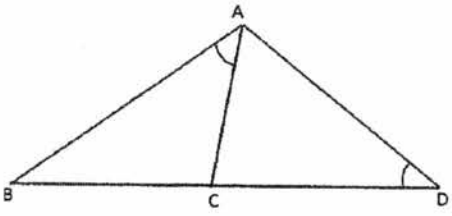
(a) Write down the ratio of Siew Teng's age: Victor's age: Mother's age in terms of x.

(b) Ten years from now, their total ages will be 76. How old was Siew Teng's mother five years ago?

Answer (a) ..... [1]

(b) ..... [2]

10 In the diagram, given that $\angle BAC = \angle BDA$ and C lies on a straight line BD. It is given that AB = 6 cm and BC = 4 cm.



(a) Show that ΔABC and ΔDBA are similar.

.....

.....

.....

.....

..... [2]

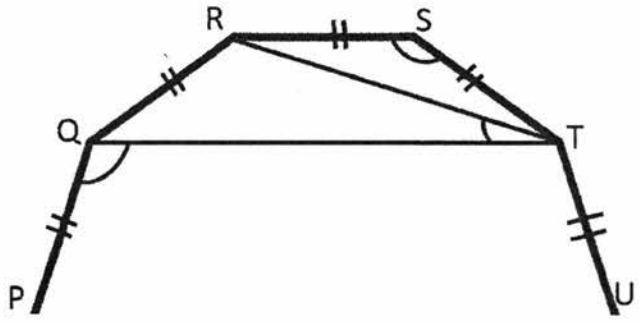
(b) Find BD.

(c) Given the area of ΔABD is 42 cm^2, find the shortest distance from D to AB.

Answer (b) .....*cm* [1]

(c).....cm^2 [2]

11 The below diagram is part of a regular decagon.



Find

(a) $\angle RST$

(b) $\angle RTQ$

(c) $\angle PQT$

Answer (a) .....°[1]

(b) .....° [1]

(c) .....° [2]

12 Two fair six-sided dice are thrown.
Find the probability that

(a) both dice show different numbers,

(b) the sum of the two numbers shown is 12,

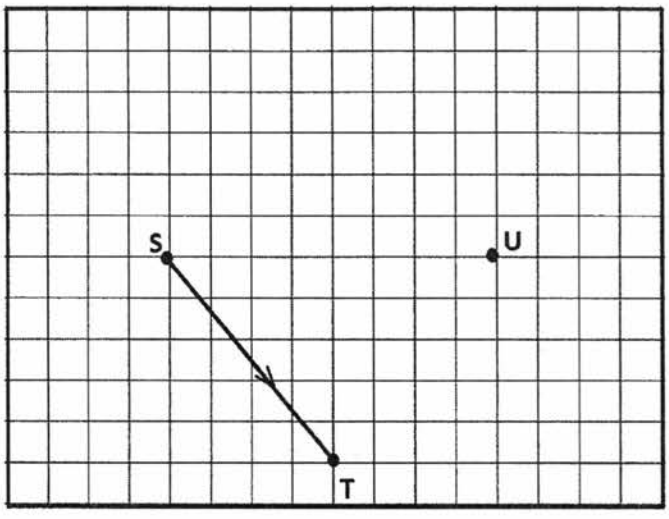
(c) the sum of the two numbers shown is a prime number.

Answer (a)..... [1]

(b)..... [1]

(c)..... [2]

13 The figure below shows the positions of the points S, T and U.



(a) Express $\overrightarrow{ST}$ as a column vector.

(b) V is a point such that STUV is a parallelogram. Draw the parallelogram on the diagram above.

(c) Find the magnitude of $|\overrightarrow{ST}|$ and $|\overrightarrow{TU}|$.

(d) Hence, from your answer in part (c), is $|\overrightarrow{ST}| = |\overrightarrow{TU}|$? What is the specific name of the parallelogram?

Answer (a) ..... [1]

(b) .....See above..... [1]

(c) ..... , ... [2]

(d) .....

.....[2]

14 (a) Hasan invested part of \$8000 at 2.4% per annum simple interest and the remaining at 1.8% per annum simple interest. He received a total interest of \$348 after two years. How much did he invest at 2.4% per annum simple interest?

(b) Amin bought a car at \$70000 and the car depreciated by 25 % at end of first year, 20% at end of second year and 15% at end of third year. What was Amin's car value after 3 years?

Answer (a) \$..... [2]

(b) \$..... [2]

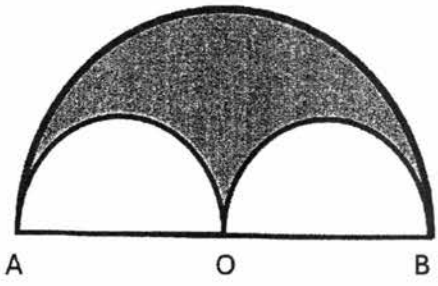
15 The diagram shows 2 small semicircles inside a big semicircle. Given that AB is the diameter of the big semicircle with center O and area of each small semicircle is $\frac{9}{2}\pi\ cm^2$.

Find

(a) the radius of the small semicircle,

(b) the perimeter of the shaded area in terms of π,

(c) the area of the shaded region in terms of π.

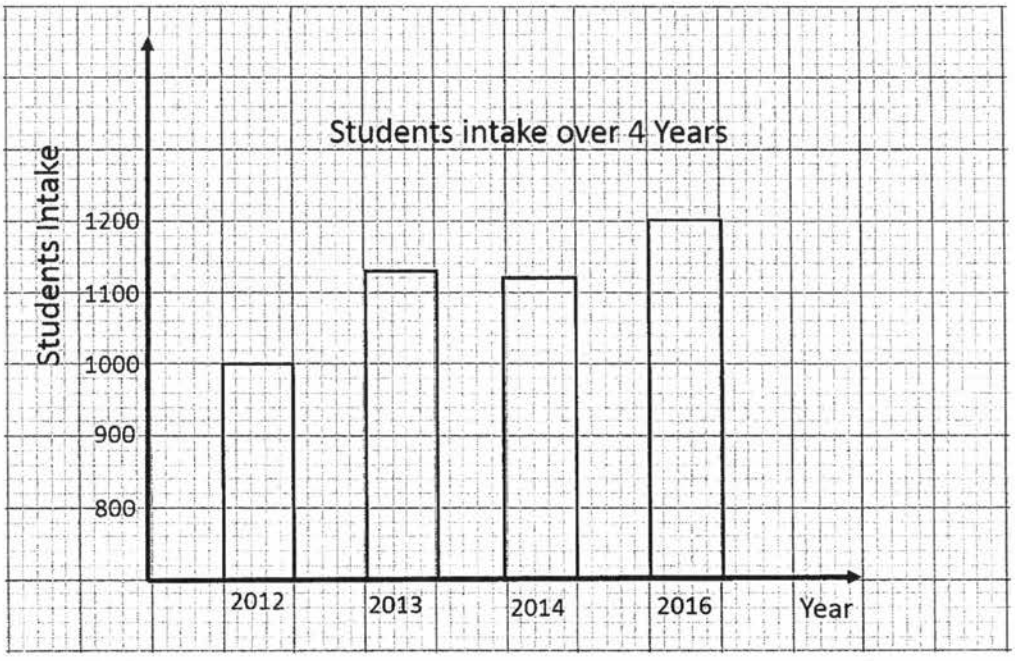


Answer (a) .....*cm* [1]

(b) .....*cm* [1]

(c) .....cm^2 [1]

16 The graph shows the students intake of ABC Secondary school over 4 years.



(a) Express the ratio of the height of the bar representing the students intake in 2012 to that in 2016.

(b) Express the ratio of the student intake in 2012 to the student intake in 2016.

(c) Should both answers you obtain in (a) and (b) be the same?

(d) Explain the similarity or difference in your answers of (a) and (b).

Answer (a) ..... [1]

(b) .....[1]

(c) .....[1]

(d) .....

.....[1]

17 Given the equation of line L_1 is $\frac{1}{2}x - 3y = 9$, find

(a) the coordinates when it cuts the x-axis,

(b) the gradient of the line,

(c) the value of k if the point $(-6, k)$ lies on the line,

(d) the equation of line L_2 that cuts y-axis at 5 and is parallel to L_1.

Answer (a)..... [1]

(b) .....[1]

(c).....[1]

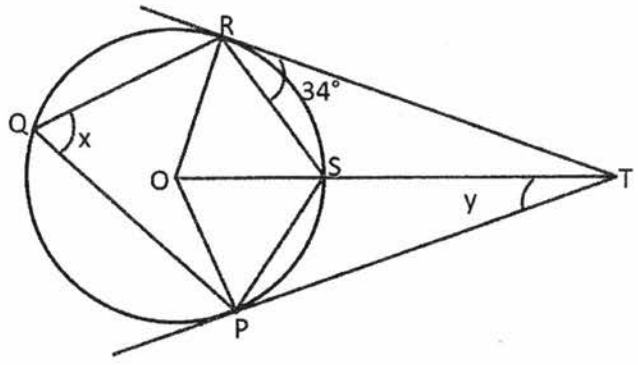
(d).....[1]

18 In the diagram, O is the center of the circle and RT and PT are tangents to the circle at R and P respectively. Find the angles,

(a) x and

(b) y.

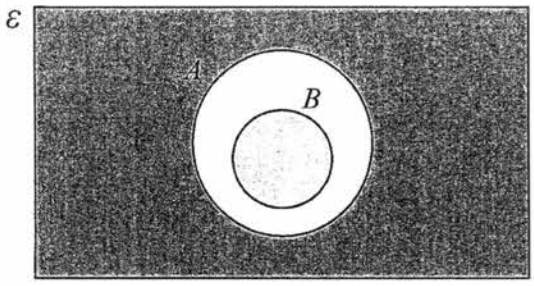
State your reasons clearly.



Answer (a) x = ... [3]

(b) y = ... [1]

19 (a) Use set notation to describe the shaded area in the following Venn diagram.



(b) ε= {numbers from 1 to 10}
A = {even numbers}
B = {prime numbers}
C = {multiples of 2 greater than 6}

(i) List the elements in $A \cap B'$.
(ii) State the relationship between set A and C.

Answer (a) ..... [1]

(b)(i) ..... [1]

(b)(ii) ..... [1]

20 The scale drawing in the answer space below shows the position of towns A and B. Town B is 36 km due South of A. The map scale is given as 1:600 000.

Construct the map of ABCD using the information given below:
(a) Town C which is 54 km from B with a bearing of 085° from B.
(b) Town D is located 18 km from C and on the perpendicular bisector of A and B.
(c) Measure the bearing of Town D from Town A.

Answer (a) .....See above..... [2]

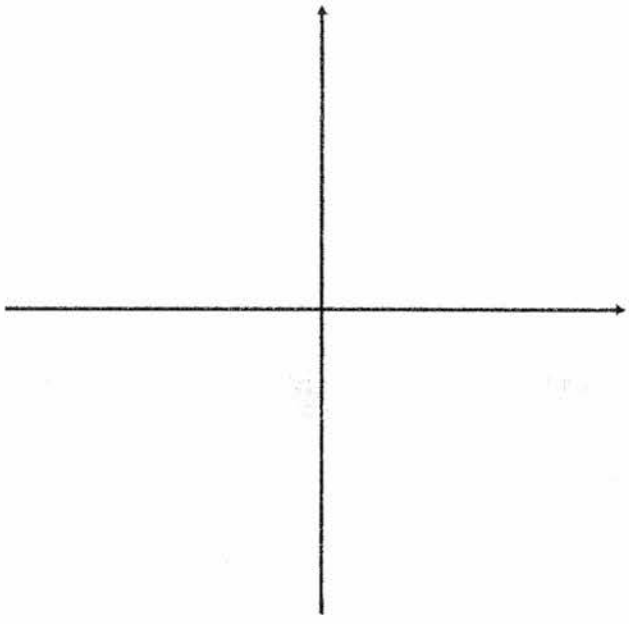
(b) .....See above ..... [2]

(c)..... [1]

21 (a) Express the function $y = -x^2 + 8x - 5$ in the form $y = -(x - h)^2 + k$.

(b) Sketch the graph of the function $y = -x^2 + 8x - 5$. Label the y-intercept and turning point.

(c) Hence, or otherwise, solve the equation $-x^2 + 8x - 5 = -10$



Answer (a) ..... [2]

(b) .....See above ..... [2]

(c) x = ..., ... [2]

--- End of Paper ---

Register No.	Class

Name : _____

BENDEMEER SECONDARY SCHOOL
2017 PRELIMINARY TWO EXAMINATION
SECONDARY 4 EXPRESS / 5 NORMAL (ACADEMIC)
Elementary Mathematics Paper 2
4048/02

DATE : **23 August 2017**
DURATION : **2 hours 30 minutes**
TOTAL : **100 marks**

Additional Materials: Cover page
Answer Paper
Graph Paper (1 sheet)

READ THESE INSTRUCTIONS FIRST
Write your name, class and register number on all the work you hand in.
Write in dark blue or black pen on both sides of the paper.
You may use a 2B pencil for any diagrams or graphs.
Do not use staples, paper clips, highlighters, glue or correction fluid/tape.

Answer **all** questions.
All the diagrams in this paper are **not** drawn to scale.
If working is needed for any question, it must be shown with the answer.
Omission of essential working will result in loss of marks.
The use of an approved scientific calculator is expected, where appropriate.
If the degree of accuracy is not specified in the question, and if the answer is not exact, give the answer to three significant figures. Give answers in degrees to one decimal place.
For π, use either your calculator value or 3.142, unless the question requires the answer in terms of π.

At the end of the examination, fasten all your work securely together.
The number of marks is given in brackets [ ] at the end of each question or part question.

FOR EXAMINER'S USE
100

This document consists of **11** printed pages including this cover page.

[Turn over

MATHEMATICAL FORMULAE

Compound Interest

$$\text{Total amount} = P\left(1+\frac{r}{100}\right)^n$$

Mensuration

Curved surface area of cone = πrl

Surface area of a sphere = $4\pi r^2$

$$\text{Volume of a cone} = \frac{1}{3}\pi r^2 h$$

$$\text{Volume of sphere} = \frac{4}{3}\pi r^3$$

$$\text{Area of triangle ABC} = \frac{1}{2}ab\sin C$$

Arc length = $r\theta$, where θ is in radians

$$\text{Sector area} = \frac{1}{2}r^2\theta, \text{ where } \theta \text{ is in radians}$$

Trigonometry

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$a^2 = b^2 + c^2 - 2bc\cos A$$

Statistics

$$\text{Mean} = \frac{\sum fx}{\sum f}$$

$$\text{Standard Deviation} = \sqrt{\frac{\sum fx^2}{\sum f} - \left(\frac{\sum fx}{\sum f}\right)^2}$$

1 (a) Solve the inequality $\frac{p-2}{4} \leq \frac{1}{2} - \frac{15-2p}{5}$. [3]

(b) (i) Factorise $2q - 18q^3$ completely. [2]

(ii) Hence simplify $\frac{2q-18q^3}{(4q^2-2q)(3q+1)}$. [2]

(c) (i) In January, Joseph's best time to swim 200 metres was 2 minutes 30 seconds.

Calculate his speed in kilometres per hour. [2]

(ii) In December, Joseph's best time is 10% less than his best time in January.

Calculate, in minutes and seconds, his best time in December. [2]

---

2 The first four terms in a sequence of numbers are given below.

$$T_1 = 3 + 2^0 \quad = 4$$
$$T_2 = 5 + 2^1 \quad = 7$$
$$T_3 = 7 + 2^2 \quad = 11$$
$$T_4 = 9 + 2^3 \quad = 17$$

(a) Find T_5. [1]

(b) Find the nth term of the sequence, T_n. [1]

(c) Hence or otherwise, find T_{20}. [1]

(d) Explain why the value of T_n is always odd for all values of n. [1]

(e) T_m and T_{m+1} are consecutive terms in the sequence.

Show that $T_{m+1} - T_m = 2 + 2^{m-1}$. [3]

---

3 A factory produces bottles in both the small and the large size.

(a) It is found that x large bottles can be produced in a minute.
Write down an expression in terms of x, the time taken to produce 1 large bottle, in **seconds**. [1]

(b) 4 more small bottles can be produced in a minute, compared to the large bottles.
Write down an expression in terms of x, the time taken to produce 1 small bottle, in **seconds**. [1]

(c) Given that it takes 2.5 seconds longer to produce a large bottle than a small bottle, form an equation in x and show that it reduces to $x^2 + 4x - 96 = 0$. [3]

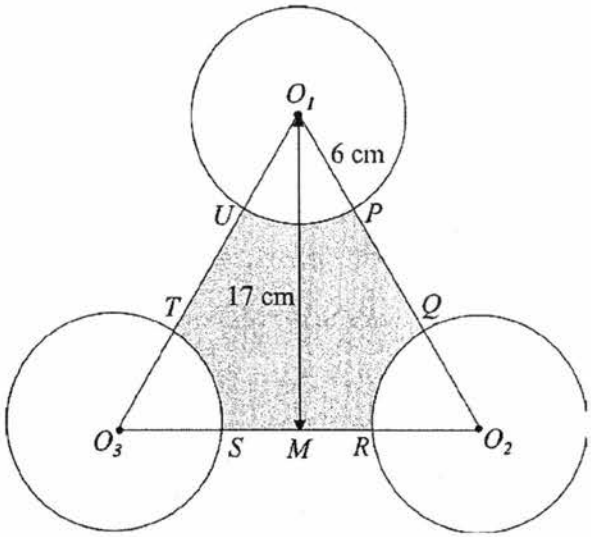
(d) Solve the equation $x^2 + 4x - 96 = 0$. [2]

(e) Hence find the time taken to produce 4000 small bottles, in hours and minutes. [2]

(f) It is known that the factory sells each small bottle at \$0.30 and each large bottle at \$0.50.
Is it more profitable for the factory to produce small or large bottles?
Explain your answer. [3]

---

4 The figure below shows the outline of a spinner toy, which is made up of an equilateral triangle and 3 identical circles with centre O_1, O_2 and O_3 respectively. It is given that the radii of the circles are 6 cm and $O_1M = 17$ cm, where M is the midpoint of SR.



Find (a) PQ, [2]

(b) the perimeter of the shaded region $PQRSTU$ and [3]

(c) the area of the shaded region $PQRSTU$. [3]

---

5 (a) The stem and leaf diagram below shows the marks attained by 15 students in a Mathematics test.

1	3	7				
2	3	6	6			
3	0	4	4	5	7	9
4	1	2	5			
5	0					

Key : 1 | 0 means 10 marks

(i) Using the data given, find the
(a) median mark, [1]
(b) interquartile range and [2]
(c) standard deviation of the marks. [2]

(ii) It was later found that there was a mistake in the marking for the test. As such, every student should get an additional 2 marks.

Describe how the change in marks will affect the median mark and interquartile range. [2]

(b) It is given that a box contains 15 apples and 9 oranges.

Two fruits are then selected from the box at random. If an apple is selected, it is replaced. If an orange is selected, it is not replaced.

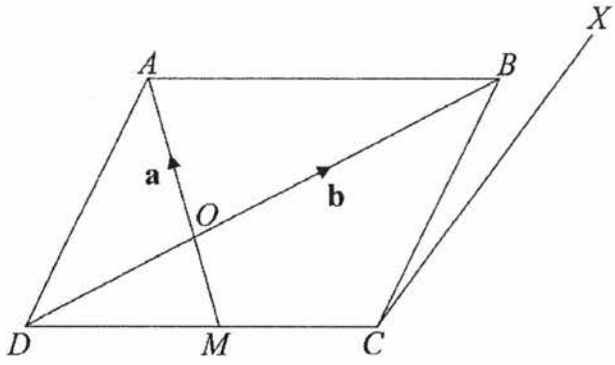
(i) Draw a tree diagram to show the probabilities of the possible outcomes. [2]

(ii) Find, as a fraction in its simplest form, the probability that

(a) both fruits selected are the same, [2]

(b) at least one of the fruit is an apple. [2]

6 In the following diagram, $ABCD$ is a parallelogram where M is the midpoint of CD and $OD = \frac{1}{3}BD$.



Given that $\overrightarrow{OA} = \mathbf{a}$ and $\overrightarrow{OB} = \mathbf{b}$,

(a) express as simply as possible, in terms of **a** and/or **b**,

(i) $\overrightarrow{BD}$, [1]

(ii) $\overrightarrow{AB}$, [1]

(iii) $\overrightarrow{BC}$, [2]

(iv) $\overrightarrow{OM}$. [2]

(b) Given that $\overrightarrow{CX} = \mathbf{a} + \frac{3}{4}\mathbf{b}$, prove that B, D and X are collinear points. [2]

(c) Find the exact value of (i) $\frac{\text{area of } \Delta ODM}{\text{area of } \Delta OAB}$, [2]

(ii) $\frac{\text{area of } \Delta ODM}{\text{area of } ABCD}$. [1]

7 Petrol stations A and B sell two grades of petrol, R92 and P98.

The matrix **L** shows the average amount of petrol sold at the two stations on a day in Week 1.

$$\mathbf{L} = \begin{matrix} \\ \text{Station A} \\ \text{Station B} \end{matrix} \begin{matrix} \begin{matrix} \text{R92} & \text{P98} \end{matrix} \\ \begin{pmatrix} 250 & 180 \\ 280 & 180 \end{pmatrix} \end{matrix}$$

(a) Evaluate the matrix $\mathbf{Q} = 7\mathbf{L}$. [1]

(b) It is given that the petrol price (per litre) of grade R92 and P98 are \$2.00 and \$2.40 respectively.

Represent the petrol prices as a column matrix **P**. [1]

(c) Evaluate the matrix $\mathbf{S} = \mathbf{QP}$. [1]

(d) State what the elements of **S** represent. [1]

(e) In Week 2, the average amount of all petrol sold at both petrol stations dropped by 5%. At the same time, the prices of all grades of petrol increased by 5%.

Calculate the earnings made by Station A and Station B respectively in Week 2. [3]

(f) Write down a matrix **X** such that the total earnings of both petrol stations in Week 2 can be calculated using matrix multiplication.

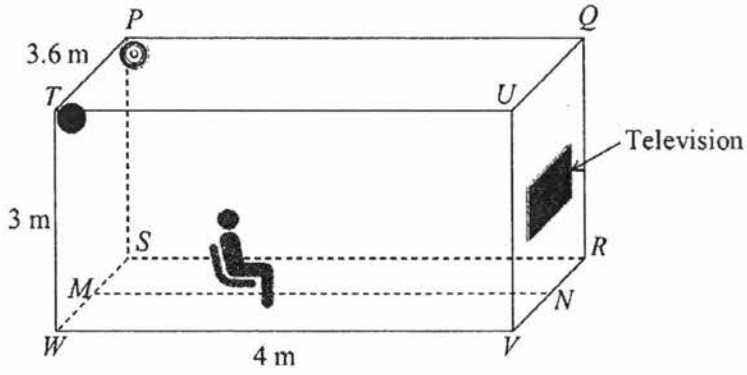
Hence find the total earnings of both petrol stations in Week 2. [2]

---

8 Figure 1 shows the three-dimensional layout of Roy's living room. The room is shaped like a cuboid with dimensions 4 m by 3.6 m by 3 m, where path MN lies across the centre of the room.

- A television is fixed on the wall $QRVU$ such that Y, the centre of the television, is 1.6 m above N.
- Two speakers are fixed at corners P and T respectively.

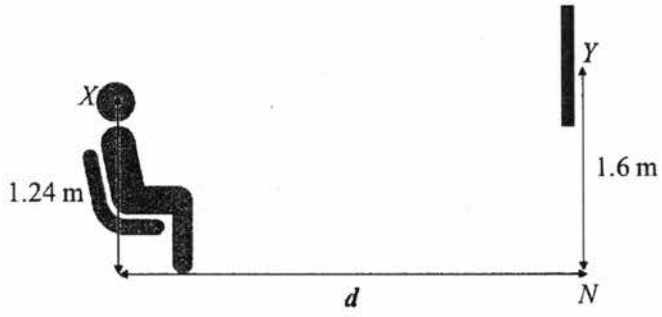
Figure 1



Roy is deciding on the best position to place his armchair along MN. The best position will allow him to have an optimal view of the television when seated in the armchair.

Figure 2 shows Roy's eye level at X, which is 1.24 m when seated at distance $\boldsymbol{d}$ from the television. It is given that $1.8\,\text{m} \leq \boldsymbol{d} \leq 3.8\,\text{m}$ for Roy to have an optimal view of the television.

Figure 2



For this question, the dimensions of the television and speakers are negligible.

(a) If Roy chose to place the armchair at the furthest possible optimal distance, find

(i) TX, [3]

(ii) $\angle PXT$, [2]

(iii) the angle of elevation of Y from X. [2]

(b) When the angle of elevation of Y from X is 12°, will Roy still have an optimal view of the television? Justify your answer. [2]

9 **Answer the whole of this question on a sheet of graph paper.**

The variables x and y are connected by the equation

$$y = 5 + \frac{2}{x} - \frac{1}{4}x^2$$

Some corresponding values of x and y are given in the table below.

x	-6	-5	-4	-3	-2	-1.5	-1	-0.5	-0.3
y	-4.33	-1.65	p	2.08	3	3.10	2.75	0.94	-1.69

(a) Find the value of p. [1]

(b) Using a scale of 2 cm to represent 1 unit, draw a horizontal x-axis for $-6 \le x \le 0$.
Using a scale of 2 cm to represent 1 unit, draw a vertical y-axis for $-5 \le y \le 4$.

On your axes, plot the points given above and join them with a smooth curve. [3]

(c) By drawing a tangent, find the gradient of the curve at $(-1, 2.75)$. [2]

(d) (i) On the same axes, draw the line L with gradient 0.5 and passes through the point $(-4, -3)$. [1]

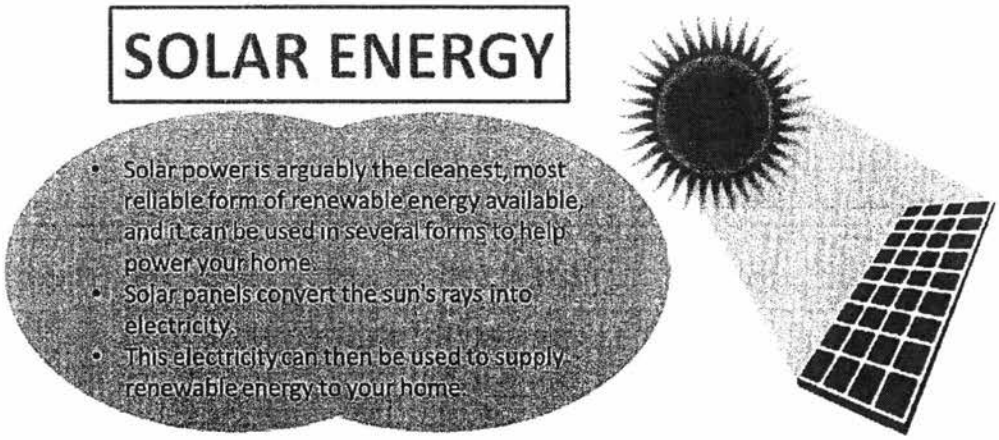
(ii) Write down the equation of the line L. [1]

(iii) The x-coordinate of the point(s) where the line L intersects the curve are the solution(s) to the equation $x^3 + Ax^2 - Bx - 8 = 0$.

Find the values of A and B. [2]

(e) Using the graph, show that $\frac{2}{x} - \frac{1}{4}x^2 + 1 = 0$ has no solution for $x < 0$. [2]

10 Mrs Lim is currently staying at a bungalow with her family. After learning about solar energy from the brochure below, she is thinking of installing solar panels at the bungalow to help reduce the family's electricity bills.



Brochure on Solar Energy

Information that Mrs Lim needs to consider in order to make a decision on the installation can be found under Annex A on the next page.

(a) For the first half of 2017,

(i) calculate the average amount of electricity (in kWh) used by Mrs Lim's family in a month, and [2]

(ii) calculate the average amount (in dollars) paid for electricity usage in a month. [2]

(b) Considering all the information given, should Mrs Lim go ahead with the installation of solar panels for the bungalow?

Justify your answer. [4]

ANNEX A

Table 1: Records of electricity usage by Mrs Lim's family

Electricity Usage for 2017 (in kWh)					
January	February	March	April	May	June
1107.8	**1066.3**	**1123.6**	**1259**	**1249.5**	**1281.6**

Table 2: Charges for electricity usage

Electricity tariff: **21.39 cents per kWh** (Charges subjected to 7% Goods & Services Tax)

Table 3: Details on installing solar panels for Mrs Lim's bungalow

Dimensions of roof area for solar panel installation	**9 m by 4 m**
Dimensions of 1 solar panel	**1.65 m by 1 m**
Cost of installing every 10 solar panels	**$6,250**

Table 4: More about the solar panels

Average amount of electricity produced by 1 solar panel: **19 kWh per month**	Lifespan of solar panels: **20 years**

~END OF PAPER~

Answers:

1a) $p \geq 13\frac{1}{3}$

1b)(i) $2q(1-3q)(1+3q)$ 1b)(ii) $\frac{1-3q}{2q-1}$

1c)(i) 4.8 km/h 1c)(ii) 2 min 15 sec

2a) 27

2b) $2n+1+2^{n-1}$

2c) 524329

3a) $\frac{60}{x}$ s

3b) $\frac{60}{x+4}$ s

3d) $x=-12,\ 8$

3e) 5 h $33\frac{1}{3}$ min or 5 h 34 min

3f) It is more profitable for the factory to produce **large** bottles.

4a) PQ = 7.63 cm

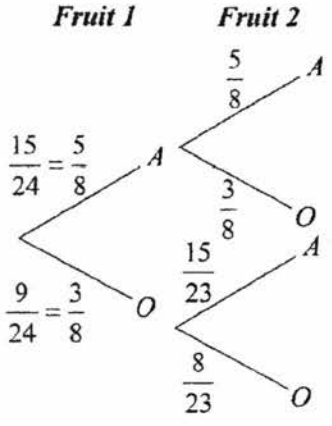
4b) Perimeter = 41.7 cm

4c) Area = 110 cm^2

5a)(i)(a) Median = 34 marks 5a)(i)(b) IQR = 15 marks 5a)(i)(c) SD = 9.99 marks

5a)(ii) The median will increase by 2 and the interquartile range will remain the same.

5b)(i)



5b)(ii)(a) P(both are the same) = $\frac{767}{1472}$ 5b)(ii)(b) P(at least 1 apple) = $\frac{20}{23}$

6a)(i) $\overrightarrow{BD} = -\frac{3}{2}\mathbf{b}$ 6a)(ii) $\overrightarrow{AB} = -\mathbf{a}+\mathbf{b}$

6a)(iii) $\overrightarrow{BC} = -\mathbf{a}-\frac{1}{2}\mathbf{b}$ 6a)(iv) $\overrightarrow{OM} = -\frac{1}{2}\mathbf{a}$

6c)(i) $\dfrac{\text{area of } \Delta ODM}{\text{area of } \Delta OAB} = \left(\dfrac{1}{2}\right)^2 = \dfrac{1}{4}$ 6c)(ii) $\dfrac{\text{area of } \Delta ODM}{\text{area of } ABCD} = \dfrac{1}{4} \times \dfrac{2}{3} \times \dfrac{1}{2} = \dfrac{1}{12}$

7a) $\mathbf{Q} = \begin{pmatrix} 1750 & 1260 \\ 1960 & 1260 \end{pmatrix}$ 7b) $\mathbf{P} = \begin{pmatrix} 2.00 \\ 2.40 \end{pmatrix}$ 7c) $\mathbf{S} = \begin{pmatrix} 6524 \\ 6944 \end{pmatrix}$

7d) The earnings of Station A ($6,524) and Station B ($6,944) respectively for Week 1.

7e) The earnings of Station A ($6,507.69) and Station B ($6,926.64) respectively for Week 2.

7f) $\mathbf{X} = \begin{pmatrix} 1 & 1 \end{pmatrix}$

Total earnings $= \begin{pmatrix} 1 & 1 \end{pmatrix} \begin{pmatrix} 6507.69 \\ 6926.64 \end{pmatrix}$

$= \begin{pmatrix} 13434.33 \end{pmatrix}$

Total earnings of both stations (Week 2) = $13,434.33

8a)(i) TX = 2.53 m

8a)(ii) $\angle PXT \approx 90.9°$

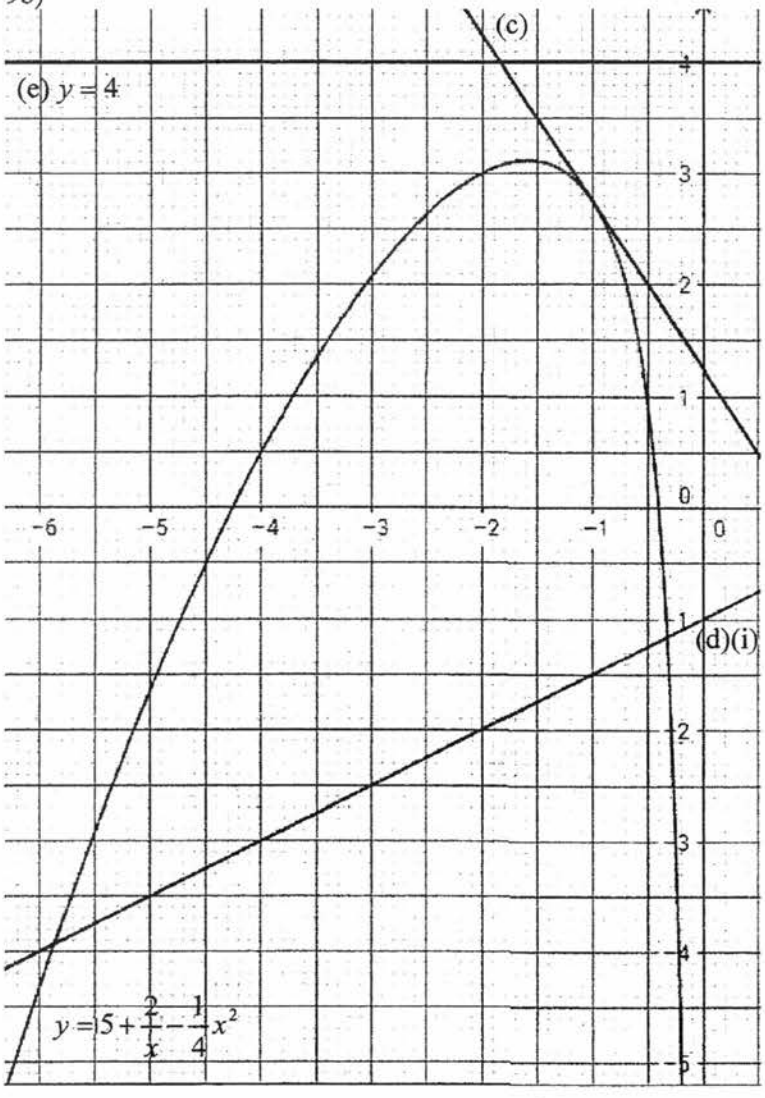
8a)(iii) Angle of elevation = 5.4°

8b) $\tan 12° = \dfrac{0.36}{d}$ $\rightarrow d \approx 1.69$ m

Since 1.69 m is less than the minimum optimal distance 1.8 m, Roy will not have an optimal view of the TV in this case.

9a) $p = 0.5$

9b)



9c) Gradient $= -1.5 \quad (\pm 0.2)$

9d)(ii) $y = \frac{1}{2}x - 1$

9d)(iii) $A = 2$ and $B = 24$

10a)(i) 1181.3 kWh

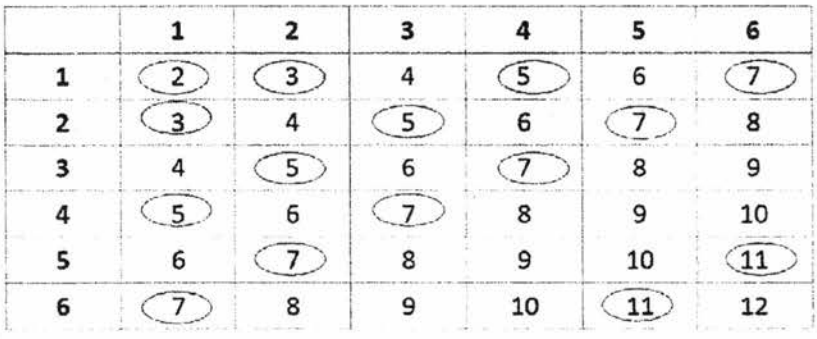
10a)(ii) \$270.37

10b) Since the average amount paid by Mrs Lim per month will be lesser than what she is currently paying for electricity usage, she should go ahead with the installation.

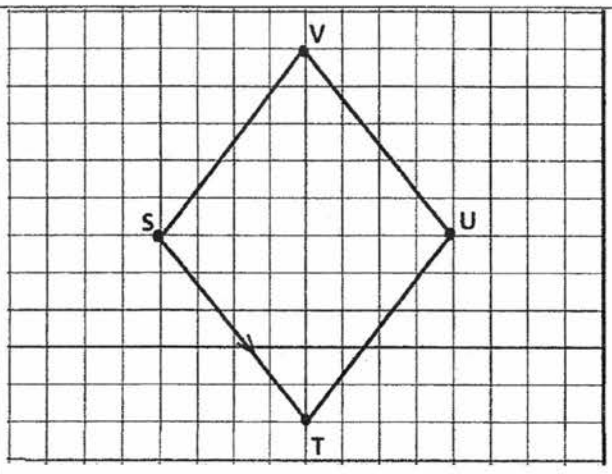
2017 Sec 4E/5NA Preliminary One Mathematics Marking Scheme

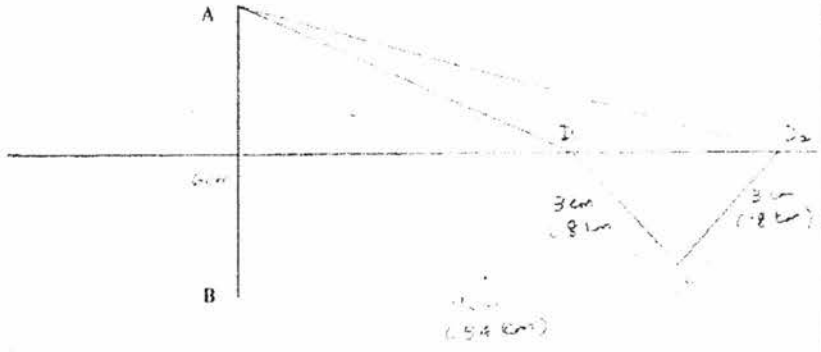
Qn	Answer	Marks
1(a)	130	B1
1(b)	100	B1
2(a)	344 x 204 = 70176 mm^2 1 mm^2 = 0.1^2 cm^2 71196 mm^2 = 0.1 x 0.1 x 70176 cm^2 = 701.76 cm^2 = 702 cm	M1 A1 (accept exact value)
2(b)	$\frac{195}{344}$	B1
3	Ideal weight = $\frac{82}{115} \times 100 = 71.30kg$ Current weight = $\frac{94}{100} \times 82 = 77.08kg$ Per cent = $\frac{77.08-71.30}{77.08} \times 100 = 7.4987 = 7.50\%$	M1 M1, A1
4(a)	$4a^2 - 12a = 2 - 20 + 6a$ $4a^2 - 18a + 18 = 0$ $2a^2 - 9a + 9 = 0$ $(2a - 3)(a - 3) = 0$ $a = \frac{3}{2}, 3$	 M1 A1
4(b)	$x^2y^2 + 36 - 4x^2 - 9y^2$ $= x^2y^2 - 4x^2 - (9y^2 - 36)$ $= x^2(y^2 - 4) - 9(y^2 - 4)$ $= (x^2 - 9)(y^2 - 4)$ $= (x + 3)(x - 3)(y + 2)(y - 2)$	M1 M1 A1
5	Singapore Tuesday 1310 => London Tuesday 0610 Flight 13 hours and 15 minutes => Arrival Tuesday 1925 Or Flight 13 hours and 15 minutes => Arrival 0225 Wednesday Singapore time Singapore 0225 Wednesday => London Tuesday 1925	M1 B1 Or M1 B1 (If no working,

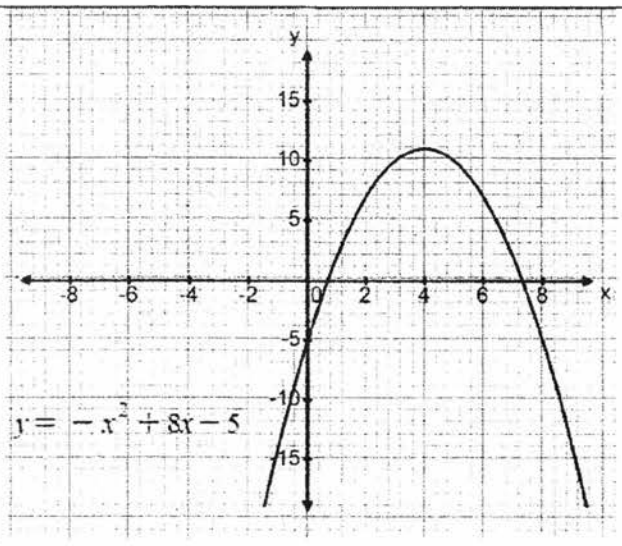
6(a)	$\frac{\sin 39}{12} = \frac{\sin \angle DEF}{10}$ $\angle DEF = \sin^{-1}\frac{10sin39}{12}$ $\angle DEF = 31.63, 180 - 31.63$ $= 31.6, 148.4$	M1 A1
6(b)	Acceptable answer => 31.6°. Reject 148.4° because (148.4 + 39) >180 which is more than angle sum of a triangle.	B1 B1
7	$a^2d^2 - b^2c^2 = c^2d^2$ $b^2c^2 = d^2(a^2 - c^2)$ $b = \pm\frac{d}{c}\sqrt{(a^2 - c^2)}$	M1 A1 No mark if no $\pm$.
8(a)	$\frac{1}{2} - \frac{1}{5^2} = \frac{25-2}{50} = \frac{23}{50}$	M1, A1
8(b)	$b^{\frac{2}{3}+6-\frac{2}{3}-1} = b^5$	M1, A1
9(a)	$x : x + 2 : 6(x + 2)$	B1
9(b)	$(x + 10) + (x + 12) + (6x + 22) = 76$ $x = 4$ $Mother's\ age = 6(4 + 2) - 5 = 31\ years\ old$	M1 A1
10(a)	$In\ \Delta\ ABC\ and\ \Delta DBA$ $\angle BAC = \angle BDA\ (given)$ $\angle ABC = \angle DBA\ (Common\ \angle)$ $\Delta ABC\ is\ similar\ to\ \Delta DBA\ (AA\ Simiarlity)$	}B1(order of vertices must be in corresponding order B1 (statement and reason) No reason no mark
10(b)	$\frac{BC}{BA} = \frac{BA}{BD}$ $\frac{4}{6} = \frac{6}{BD}$	

	$BD = 9$	B1
10(c)	Let shortest distance be s. $\frac{1}{2} \times 6 \times s = 42$ $s = 14cm$	 M1 A1
11(a)	$\angle RST = \frac{(10-2)180}{10} = 144$	B1
11(b)	$\angle SRT = \frac{180-144}{2} = 18° \ (base\ of\ issos.\Delta)$ $\angle RTQ = 18° \ (alt\ \angle)$	B1
11(c)	$\angle QRT = \angle QRS - \angle SRT = 144 - 18 = 126°$ $\angle RQT = 180 - 126 - 18 = 36° \ (\angle\ sum\ of\ \Delta)$ $\angle PQT = 144 - 36 = 108°$	M1 A1
12(a)	$\frac{5}{6}$	B1
12(b)	$\frac{1}{36}$	B1
12(c)	 $\frac{5}{12}$	M1 A1
13(a)	$\begin{pmatrix} 4 \\ -5 \end{pmatrix}$	B1
13(b)		B1

1	2 3 4 5 6 7
2	3 4 5 6 7 8
3	4 5 6 7 8 9
4	5 6 7 8 9 10
5	6 7 8 9 10 11
6	7 8 9 10 11 12

		
13(c)	$\left\|\overrightarrow{ST}\right\| = \sqrt{4^2 + (-5)^2} = 6.40\ unit$ $\left\|\overrightarrow{TU}\right\| = \sqrt{4^2 + 5^2} = 6.40\ unit$	B2
13(d)	Yes. ST=TU. Parallelogram is a rhombus.	B1 B1
14(a)	Let p be the amount invested at 2.4% p.a. $\frac{2.4 \times 2}{100} \times p + \frac{1.8 \times 2}{100} \times (8000 - p) = 348$ $4.8p + 3.6p = 34800 - 28800$ $p = \$5000$	M1 A1
14(b)	$70000 \times 0.75 \times 0.8 \times 0.85 = \$35700$	M1, A1
15(a)	3cm	B1
15(b)	2(Arc length of small semicircle) = $2(\pi \times 3) = 6\pi\ cm$ Radius of big semicircle = 6cm Arc length of big semicircle = $\pi \times 6 = 6\pi\ cm$ Perimeter = $6\pi + 6\pi = 12\pi cm$	 B1
15(c)	Area = $\frac{1}{2}\pi(6^2) - 2(\frac{1}{2}\pi 3^2) = 9\pi\ cm^2$	B1
16(a)	$\frac{3}{5}$	B1
16(b)	$\frac{5}{6}$	B1
16(c)	Yes. Both answers are supposed to be the same.	B1
16(d)	There is a difference in answer because the scale of the vertical	B1

	axis does not start from zero	
17(a)	(18,0)	B1
17(b)	$\frac{1}{6}$	B1
17(c)	k = -4	B1
17(d)	$y = \frac{1}{6}x + 5$	B1
18(a)	$\angle ORS = 90 - 34 = 56\ (radius\ perpendicular\ to\ tangent)$ $\angle ROS = 180 - 2(56) = 68\ (angle\ sum\ of\ issos.triangle)$ $\angle ROP = 2(68) = 136$ $\angle ROP = 2x\ \ (angle\ at\ center = 2\ angles\ at\ circumference)$ $x = 68°$	M1 M1 A1 (If more than 2 reasons not given, deduct 1m overall)
18(b)	$y = 180 - 90 - 68 = 22°\ (angle\ sum\ of\ triangle)$	B1
19(a)	$A' \cup B$	B1
19(b)(i)	{4, 6, 8, 10}	B1
19(b)(ii)	$C \subset A$	B1
20(a) / 20(b)		C1 – Correct angle measurement C1- Correct scale conversion C1 – Perpendicular bisector C1 – Label of Town D (accept either D_1 or D_2)
20(b)	$104° \pm 1°, 114° \pm 1°$	B1
21(a)	$y = -(x^2 - 8x + 5)$ $y = -[(x-4)^2 + 5 - 4^2)]$ $y = -(x-4)^2 + 11$	M1 A1
21(b)		P1- correct shape P1 – correct intercepts and

		turning point.
21(c)	8.58, -0.583	B2

Name : _____

Register No.	Class

BENDEMEER SECONDARY SCHOOL
2017 PRELIMINARY TWO EXAMINATION
SECONDARY 4 EXPRESS / 5 NORMAL (ACADEMIC)
Elementary Mathematics
4048 / 02

DATE : **23 August 2017**
DURATION : **2 hours 30 minutes**
TOTAL : **100 marks**

MARK SCHEME

[Turn over

MARK SCHEME

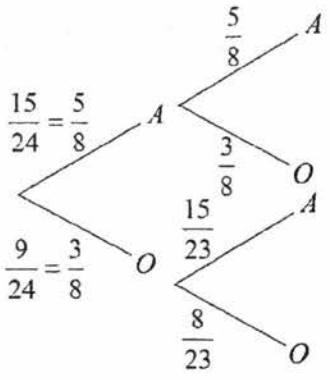
Qn.	Solutions	Remarks
1(a)	$\frac{p-2}{4} \le \frac{1}{2} - \frac{15-2p}{5}$ $\frac{p-2}{4} \le \frac{-25+4p}{10}$ [B1] $10(p-2) \le 4(-25+4p)$ $-6p \le -80$ [B1] $\therefore p \ge 13\frac{1}{3}$ [B1]	
1(b)	(i) $2q-18q^3 = 2q(1-9q^2)$ [B1] $= 2q(1-3q)(1+3q)$ [B1] (ii) $\frac{2q-18q^3}{(4q^2-2q)(3q+1)} = \frac{2q(1-3q)(1+3q)}{(4q^2-2q)(3q+1)}$ $= \frac{2q(1-3q)}{2q(2q-1)}$ [B1] $= \frac{1-3q}{2q-1}$ [B1]	
1(c)	(i) 200 m → 0.2 km, 2 min 30 s → $\frac{1}{24}$ h Speed $= 0.2 / \frac{1}{24}$ [M1] $= 4.8$ km/h [A1] (ii) Best time (Dec) $= 0.9 \times \frac{1}{24}$ [B1] $= \frac{3}{80}$ h $= 2$ min 15 seconds [B1]	
		Total Marks: 11

2(a)	$T_5 = 11 + 2^4 = 27$	[B1]	
2(b)	n^{th} term $= 2n + 1 + 2^{n-1}$	[B1]	
2(c)	$T_{20} = 2(20) + 1 + 2^{20-1} = 524\,329$	[B1]	
2(d)	Since $2n$ and 2^{n-1} are even, then $T_n = 2n + 1 + 2^{n-1}$ = even + 1 + even = odd	[B1]	
2(e)	$T_{m+1} - T_m = 2(m+1) + 1 + 2^{m+1-1} - (2m + 1 + 2^{m-1})$ $= 2m + 2 + 1 + 2^m - 2m - 1 - 2^{m-1}$ $= 2 + 2^m - 2^{m-1}$ $= 2 + 2^m - \frac{1}{2}(2^m)$ $= 2 + \frac{1}{2}(2^m)$ $= 2 + 2^{m-1}$ (shown)	[B1] [B1] [B1]	
			Total Marks: 7
3(a)	Time taken to produce 1 large bottle $= \frac{60}{x}$ s	[B1]	
3(b)	Time taken to produce 1 small bottle $= \frac{60}{x+4}$ s	[B1]	
3(c)	$\frac{60}{x} - \frac{60}{x+4} = 2.5$ $60(x+4) - 60x = 2.5x(x+4)$ $240 = 2.5x^2 + 10x$ $x^2 + 4x - 96 = 0$ (shown)	[B1] [B1] [B1]	
3(d)	$x^2 + 4x - 96 = 0$ $(x-8)(x+12) = 0$ $\therefore x = -12$ (*N.A*), 8	[M1] [A1]	
3(e)	Time taken to produce 4000 small bottles $= 4000 \times \frac{60}{8+4}$ $= 5 \text{ h } 33\frac{1}{3} \text{ min}$	[B1] [B1]	Accept: ≈ 5 h 33 min
3(f)	In the same duration of time y seconds, Amount earned for selling large bottles $= \$0.50 \times (y/7.5)$ $\approx \$0.067y$ Amount earned for selling small bottles $= \$0.30 \times (y/5)$ $= \$0.06y$ $\therefore$ It is more profitable for the factory to produce **large** bottles.	[B1] [B1] [B1]	

	or Amount earned in 1 min (Large) = 8($0.50) = $4.00 [B1] Amount earned in 1 min (Small) = 12($0.30) = $3.60 [B1] ∴ It is more profitable for the factory to produce **large** bottles. [B1]	
		Total Marks: 12
4(a)	Since M is the midpoint, then O_1M is perpendicular to $O_2\ O_3$. So, $\sin 60^\circ = \dfrac{17}{6+PQ+6}$ [M1] $\therefore PQ = \dfrac{34}{\sqrt{3}} - 12 \approx 7.629909152$ ≈ 7.63 cm [A1] ***or*** Let $O_1\ O_2$ be $2x$. $(2x)^2 = x^2 + 17^2 \rightarrow 3x^2 = 17^2$ $\rightarrow x = \sqrt{96\frac{1}{3}}$ [B1] $\therefore PQ = 2\times\sqrt{96\frac{1}{3}} - 2(6) \approx 7.63$ cm [B1]	
4(b)	Arc length PU $= 6\left(\dfrac{\pi}{3}\right)$ or $\pi\times 2(6)\times\left(\dfrac{60^\circ}{360^\circ}\right)$ ≈ 6.283185307 cm [B1] Perimeter of shaded region $PQRSTU$ $= (6.283185307\times 3) + (7.629909152\times 3)$ [B1] ≈ 41.7 cm [B1] ***or*** Perimeter of shaded region $PQRSTU$ $= 7.629909152 + (3\times\dfrac{\pi}{3}\times 6)$ [B2] ≈ 41.7 cm [B1]	

4(c)	Area of $\Delta O_1O_2O_3$ $= \frac{1}{2}\times 17\times(6+6+\frac{34}{\sqrt{3}}-12)$ ≈ 166.8542278 cm^2 [B1] Area of sector O_1PU $= \frac{1}{2}\times 6^2\times\frac{\pi}{3}$ or or $\pi\times 6^2\times\left(\frac{60^\circ}{360^\circ}\right)$ ≈ 18.84955592 cm^2 [B1] Area of shaded region $PQRSTU$ $= 166.8542278-3(18.84955592)$ ≈ 110 cm^2 [B1] ***or*** Area of shaded region $PQRSTU$ $= \frac{1}{2}\times 17\times 2(\sqrt{96\frac{1}{3}})-\frac{1}{2}\pi(6^2)$ [B2] ≈ 110 cm^2 [B1]	
		Total Marks: 8
5(a)	(i)(a) Median $= 34$ marks [B1] (i)(b) IQR $= 41-26$ [M1] $= 15$ marks [A1] (i)(c) Mean $= \frac{492}{15}$ $= 32.8$ marks S.D. $= \sqrt{\frac{17636}{15}-32.8^2}$ [M1] ≈ 9.99 marks [A1] (ii) The median will increase by 2 and become 36 marks. [B1] The interquartile range will remain the same at 15 marks. [B1]	

5(b) (i)



[B1] Correct branches
[B1] Correct probabilities

(ii)(a) P(both are the same) $= \left(\frac{5}{8}\times\frac{5}{8}\right)+\left(\frac{3}{8}\times\frac{8}{23}\right)$ [B1]

$= \frac{767}{1472}$ [B1]

(ii)(b) P(at least 1 apple) $= 1-\left(\frac{3}{8}\times\frac{8}{23}\right)$ [B1]

$= \frac{20}{23}$ [B1]

Total Marks: 13

6(a) (i) $\overrightarrow{BD} = -\frac{3}{2}\mathbf{b}$ [B1]

(ii) $\overrightarrow{AB} = \overrightarrow{AO}+\overrightarrow{OB} = -\mathbf{a}+\mathbf{b}$ [B1]

(ii) $\overrightarrow{BC} = \overrightarrow{BD}+\overrightarrow{DC} = -\frac{3}{2}\mathbf{b}+(-\mathbf{a}+\mathbf{b})$ [B1]

$= -\mathbf{a}-\frac{1}{2}\mathbf{b}$ [B1]

(iv) $\overrightarrow{OM} = \overrightarrow{OD}+\overrightarrow{DM} = -\frac{1}{2}\mathbf{b}+\frac{1}{2}(-\mathbf{a}+\mathbf{b})$ [B1]

$= -\frac{1}{2}\mathbf{a}$ [B1]

6(b) $\overrightarrow{XB} = \overrightarrow{XC}+\overrightarrow{CB} = -\mathbf{a}-\frac{3}{4}\mathbf{b}+\mathbf{a}+\frac{1}{2}\mathbf{b} = -\frac{1}{4}\mathbf{b}$ [B1]

Since $\overrightarrow{BD} = 6\overrightarrow{XB}$ → BD // XB and B is a common point, then B, D and X must be collinear points. [B1]

6(c) (i) $\frac{\text{area of } \Delta ODM}{\text{area of } \Delta OAB} = \left(\frac{1}{2}\right)^2 = \frac{1}{4}$ [B1, B1]

(ii) $\frac{\text{area of } \Delta ODM}{\text{area of } A\text{BCD}} = \frac{1}{4}\times\frac{2}{3}\times\frac{1}{2} = \frac{1}{12}$ [B1]

Total Marks: 11

7(a)	$\mathbf{Q} = \begin{pmatrix} 1750 & 1260 \\ 1960 & 1260 \end{pmatrix}$ [B1]	
7(b)	$\mathbf{P} = \begin{pmatrix} 2.00 \\ 2.40 \end{pmatrix}$ [B1]	
7(c)	$\mathbf{S} = \begin{pmatrix} 1750 & 1260 \\ 1960 & 1260 \end{pmatrix}\begin{pmatrix} 2.00 \\ 2.40 \end{pmatrix} = \begin{pmatrix} 6524 \\ 6944 \end{pmatrix}$ [B1]	
7(d)	The earnings of Station A ($6,524) and Station B ($6,944) respectively for Week 1. [B1]	
7(e)	Amount of petrol sold (Week 2) $= 0.95\begin{pmatrix} 1750 & 1260 \\ 1960 & 1260 \end{pmatrix}$ $= \begin{pmatrix} 1662.5 & 1197 \\ 1862 & 1197 \end{pmatrix}$ [B1] Prices of petrol (Week 2) $= 1.05\begin{pmatrix} 2.00 \\ 2.40 \end{pmatrix}$ $= \begin{pmatrix} 2.10 \\ 2.52 \end{pmatrix}$ [B1] Earnings (Week 2) $= \begin{pmatrix} 6507.69 \\ 6926.64 \end{pmatrix}$ The earnings of Station A ($6,507.69) and Station B ($6,926.64) respectively for Week 2. [B1]	
7(f)	$\mathbf{X} = \begin{pmatrix} 1 & 1 \end{pmatrix}$ [B1] Total earnings $= \begin{pmatrix} 1 & 1 \end{pmatrix}\begin{pmatrix} 6507.69 \\ 6926.64 \end{pmatrix}$ $= \begin{pmatrix} 13434.33 \end{pmatrix}$ Total earnings of both stations (Week 2) = $13,434.33 [B1]	
		Total Marks: 9

8(a) (i) At furthest possible optimal distance, $\boldsymbol{d} = 3.8$ m,
$\rightarrow M$ to foot of $X = 4 - 3.8 = 0.2$ m [B1]

By Pythagoras' Theorem,
W to foot of $X = \sqrt{0.2^2 + (3.6 \div 2)^2} = \sqrt{3.28}$ [B1]

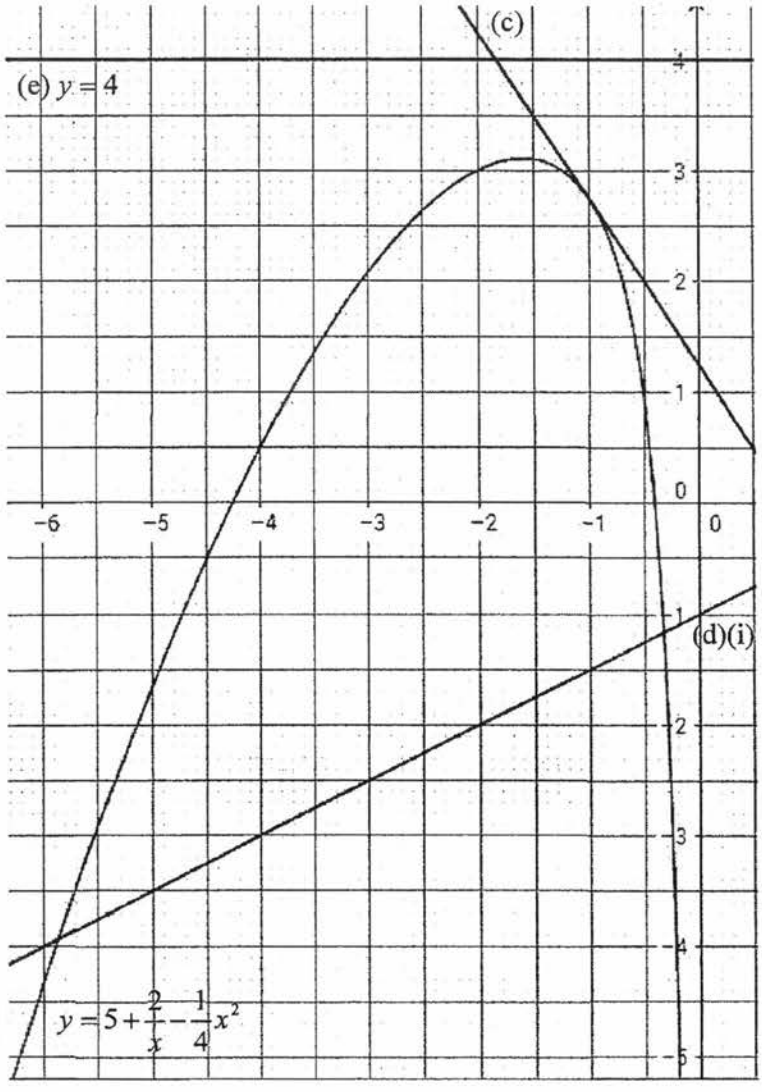
So, $TX = \sqrt{3.28 + (3 - 1.24)^2} = \sqrt{6.3776}$
≈ 2.53 m [B1]

(ii) By Cosine Rule,
$3.6^2 = 6.3776 + 6.3776 - 2(6.3776)\cos \angle PXT$ [M1]
$\angle PXT \approx 90.9°$ [A1]

(iii) Let the angle of elevation here be θ.
$\tan\theta = \dfrac{1.6 - 1.24}{3.8}$ [M1]
$\theta \approx 5.4°$ [A1]

8(b) $\tan 12° = \dfrac{0.36}{d}$ $\rightarrow d \approx 1.69$ m [B1]
Since 1.69 m is less than the minimum optimal distance 1.8 m, Roy will not have an optimal view of the TV in this case. [B1]

Total Marks: 9

9(a)	$p = 0.5$	[B1]	
9(b)	 Correct scale + labeling Correct plotting of points Smooth curve	[B1] [B1] [B1]	
9(c)	Drawing of suitable tangent at $x = -1$ Gradient $= \frac{3.5 - 2.75}{-1.5 - (-1)}$ $= -1.5 \quad (\pm 0.2)$	[B1] [B1]	M1 is given if tangent not accurate but correct formula used to find gradient
9(d)	(i) Drawing of correct straight line (ii) $y = \frac{1}{2}x - 1$	[B1] [B1]	

	(iii) For $5+\frac{2}{x}-\frac{1}{4}x^2=\frac{1}{2}x-1$ [B1] $x^3+2x^2-24x-8=0$ So, $A=2$ and $B=24$ [B1]	
9(e)	For $\frac{2}{x}-\frac{1}{4}x^2+1=0 \rightarrow \frac{2}{x}-\frac{1}{4}x^2+5=4$ [B1] For $x<0$, No point of intersection with $y=4$. → No solution (shown) [B1]	
		Total Marks: 12
10(a)	(i) Ave. amount of electricity used per month = (1107.8 + 1066.3 + 1123.6 + 1259 + 1249.5 + 1281.6)/6 [M1] = 1181.3 kWh [A1] (ii) Ave. amount paid per month $= 1181.3 \times \$0.2139 \times 1.07$ [B1] $\approx \$270.37$ [B1]	
10(b)	Max. no. of solar panels that can be installed = 20 [B1] (Based on calculations $(9 \div 1.65) \approx 5$ [length] and $(4 \div) = 4$ [width]) ***After installation,*** Ave. amount of electricity saved per month $= 19 \times 20$ $= 380$ kWh Ave. amount paid per month $= (1181.3 - 380) \times \$0.2139 \times 1.07$ $\approx \$183.40$ [B1] Ave. cost of solar panels per month $= (2 \times \$6250) \div (20 \times 12)$ $\approx \$52.08$ [B1] Total ave. amount paid per month = \$183.40 + \$52.08 = \$235.48 (< \$270.37) Since the average amount paid by Mrs Lim per month will be lesser than what she is currently paying for electricity usage, she should go ahead with the installation. [B1]	
		Total Marks: 8

Name _____( ) Class:_____

CHIJ KATONG CONVENT
PRELIMINARY EXAMINATION 2017
SECONDARY 4 EXPRESS / 5 NORMAL (ACADEMIC)

MATHEMATICS
PAPER 1

4048/01
Duration: 2 hours

Classes: 401, 402, 403, 404, 405, 406, 501, 502

READ THESE INSTRUCTIONS FIRST

Write your name, class and registration number on all the work you hand in.
Write in dark blue or black pen.
You may use an HB pencil for any diagrams or graphs.
Do not use staples, paper clips, glue or correction fluid/tape.

Answer **all** questions.
If working is needed for any question it must be shown with the answer.
Omission of essential working will result in loss of marks.
The use of an approved scientific calculator is expected, where appropriate.
If the degree of accuracy is not specified in the question, and if the answer is not exact, give the answer to three significant figures. Give answers in degrees to one decimal place.
For π, use either your calculator value or 3.142, unless the question requires the answer in terms of π.

At the end of the examination, hand in separately:
1. **Section A**
2. **Section B**
3. **Section C**

The number of marks is given in brackets [ ] at the end of each question or part question.
The total number of marks for this paper is 80.

FOR EXAMINER'S USE	
Total marks	**/80**

This question paper consists of **17** printed pages. **[Turn over**

Mathematical Formulae

Compound interest

$$\text{Total amount} = P\left(1+\frac{r}{100}\right)^n$$

Mensuration

Curved surface area of a cone $= \pi rl$

Surface area of a sphere $= 4\pi r^2$

$$\text{Volume of a cone} = \frac{1}{3}\pi r^2 h$$

$$\text{Volume of a sphere} = \frac{4}{3}\pi r^3$$

$$\text{Area of triangle } ABC = \frac{1}{2}ab\sin C$$

Arc length $= r\theta$, where θ is in radians

$$\text{Sector area} = \frac{1}{2}r^2\theta, \text{ where } \theta \text{ is in radians}$$

Trigonometry

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$a^2 = b^2 + c^2 - 2bc\cos A$$

Statistics

$$\text{Mean} = \frac{\Sigma fx}{\Sigma f}$$

$$\text{Standard deviation} = \sqrt{\frac{\Sigma fx^2}{\Sigma f} - \left(\frac{\Sigma fx}{\Sigma f}\right)^2}$$

Name: _____ ( ) Class: _____

Answer **all** the questions.

Section A [22 marks]

1 (a) Simplify $\frac{x+1}{x^2-9}-\frac{2}{3-x}$.

Answer ..... [4]

(b) Simplify $\frac{(abc^{-2})^3}{(a^{-4}b^{-1})^{-1}}\times\frac{a^{-6}b^{-7}}{(bc^2)^{-4}}$, leave your answer in positive indices.

Answer ..... [3]

---

2 Given that $\frac{k}{3}=\sqrt{\frac{A-3b^2}{cA}}$, express A in terms of b, c and k.

Answer $k=$ ..... [3]

---

3 Factorise the following completely.

(a) $18x^2y+27xy-9xy^3$

Answer ..... [1]

(b) $27a^2-12b^2$

Answer ..... [1]

(c) $3rs-3s-r+1$

Answer ..... [1]

---

4 Given that $-5 \le x \le 2$ and $-6 \le y \le -1$, find

(a) the largest possible value of $x-y$,

Answer ..... [1]

(b) the smallest possible value of y^2-x^2,

Answer ..... [1]

(c) the smallest possible value of $(x-y)^2$.

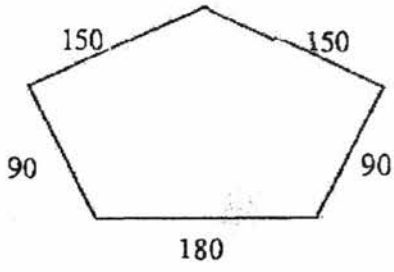
Answer ..... [1]

---

5 A small bus interchange has 2 feeder buses. Bus number 801 leaves the interchange at 15-minute intervals while number 802 at 25-minutes intervals. If both buses leave together on a particular day, how many times will they leave together in the next 5 hours?

Answer ..... times [3]

6 A pond with the shape of a pentagon is shown below (measurements are given in metres and not drawn to scale).



Lamp posts are to be constructed around the pond with the following requirements:

(I) The lamp posts are to be equally spaced from each other.
(II) One lamp post must be constructed at each vertex of the pentagon.
(III) Minimum number of lamp posts are to be constructed to save cost.

Find

(a) the distance between any two lamp posts.

Answer ..... [1]

(b) the number of lamp posts to be constructed.

Answer ..... [2]

Section B [18 marks]

7 When written as the product of their prime factors,

$A = 2^{m+2} \times 3^n$

$B = 2^m \times 3^{n+1} \times 5$, where m and n are positive constants.

Find the lowest common multiple of A and B, giving your answer as a product of its prime factors.

Answer ..... [2]

8 Solve the simultaneous equations.

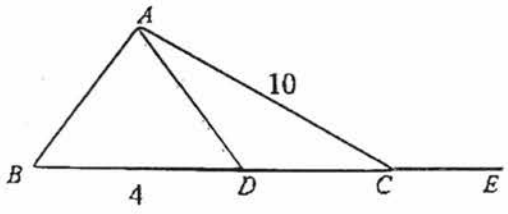
$$\frac{1}{2}x + y = 1,$$

$$\frac{1}{4}x - 3y = 11$$

Answer $x =$ .....

$y =$ ..... [3]

9 In the diagram, $BDCE$ is a straight line, $BD = 4$ cm, $AC = 10$ cm and $AB = AD$.
Given that the area of triangle ABD is 16 cm^2, calculate



(a) the vertical height of triangle ABD. [2]

(b) the value of sin $\angle ACD$. [1]

Answer vertical height = ... cm [2]
sin $\angle ACD$ = ... [1]

(c) the value of cos $\angle ACE$.

Answer cos $\angle ACE$ = ... [2]

10 During their quest to reach the South Pole on the first day of the new millennium, the Singapore Antarctica 2000 Expedition team experienced temperatures ranging from -35°C to -5°C while their family members in Singapore experienced temperatures ranging from a°C to b°C, where $a < b$.
Find, in terms of a and/or b,

(a) the greatest difference in temperatures between the South Pole and Singapore.

Answer ..... $^{\circ}$C [1]

(b) the smallest difference in temperatures between the South Pole and Singapore.

Answer ..... $^{\circ}$C [1]

11 Two maps of a new town are drawn. On the first map, a school is represented by an area of 3 cm^2.

The school is represented by an area of 12 cm^2 on the second map.

Given that the scale of the first map is 1 : 80000, find the scale of the second map in the form of 1: n.

Answer 1 : ..... [4]

---

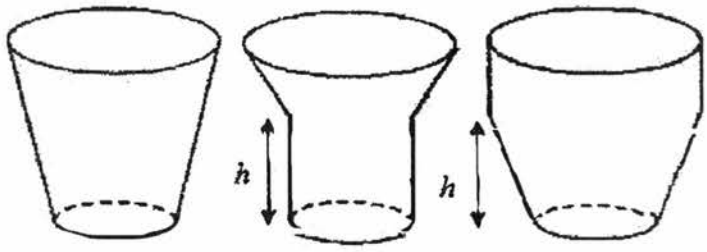
12 Mrs Ang invested \$36 000 in a bank that pays compound interest of 3.2 % per annum, payable every 3 months.

Calculate the amount that Mrs Ang has in the bank after 6 years.

Answer \$..... [2]

Section C [40 marks]

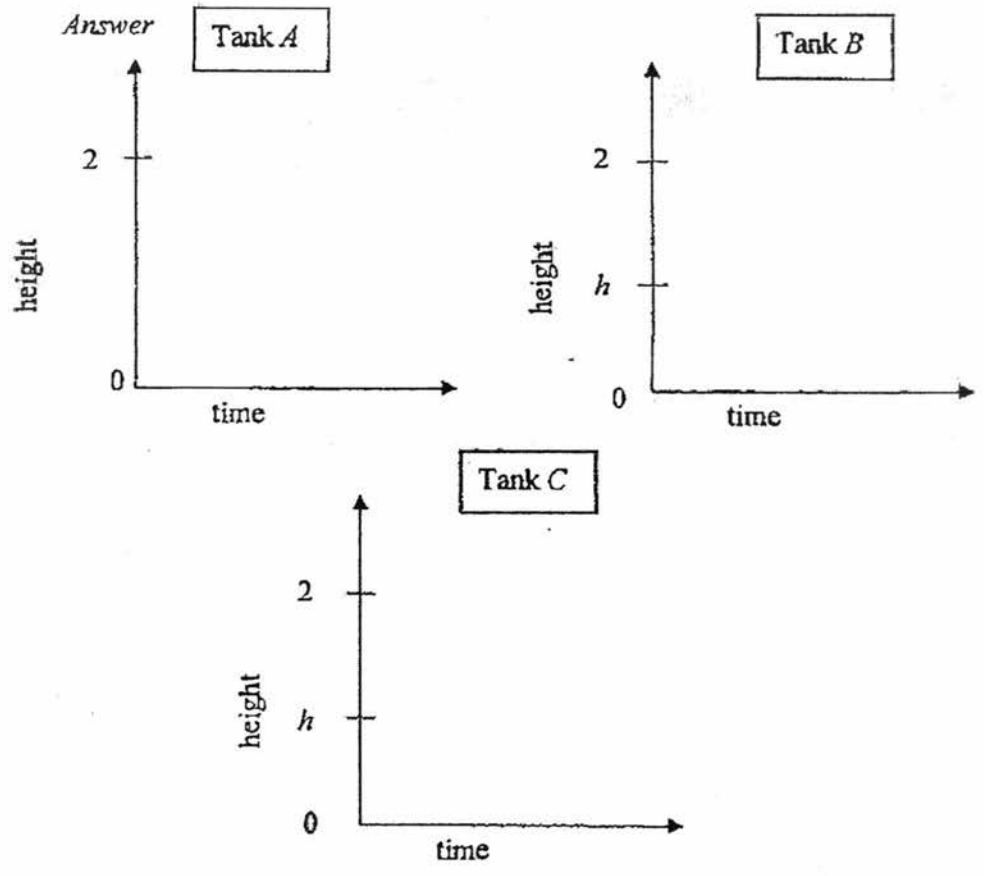
13 Liquid X is poured into three different tanks at a constant rate.
The height of each tank is 2 metres.



Tank A Tank B Tank C

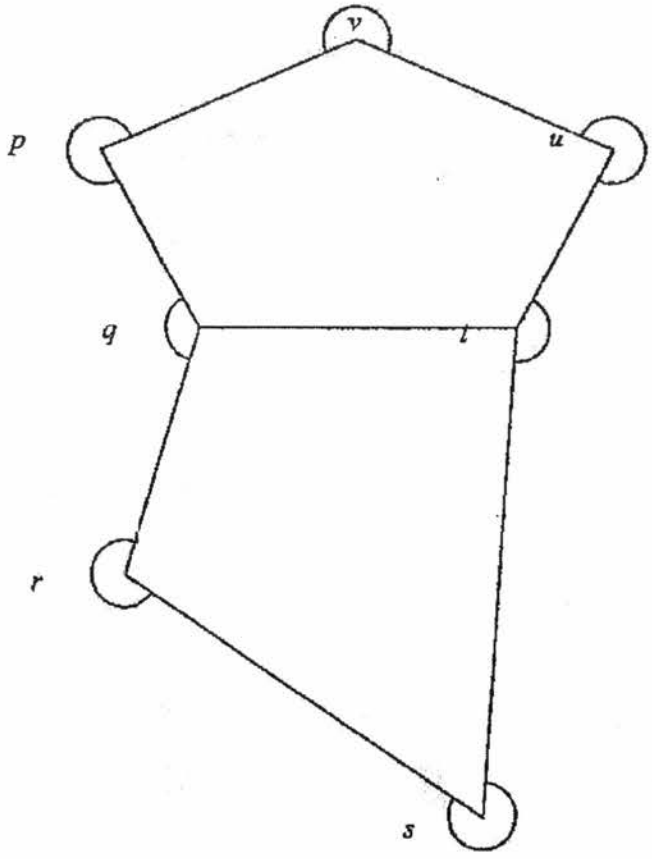
On each of the grids below, sketch the graphs to show how the height of the water changes with time for each tank.

Answer



[3]

14 (a) Calculate the sum of the angles p, q, r, s, t, u and v shown in the diagram.



Answer .....° [2]

(b) A regular polygon has n sides.

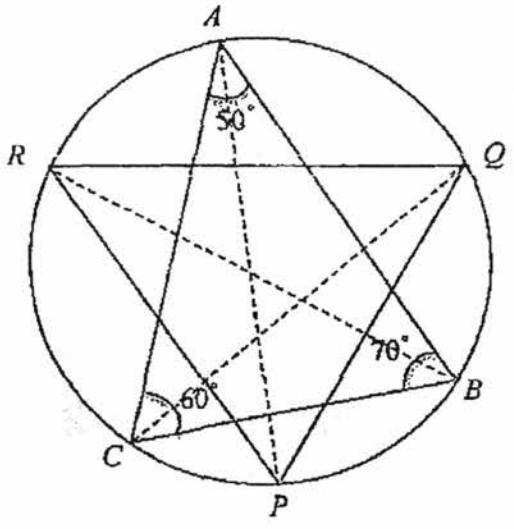
Each exterior angle is $\frac{n}{40}$ degrees.

Find the size of each interior angle in this polygon.

Answer .....° [2]

15 In the figure, the vertices of triangle ABC and triangle PQR touch the circumference of the circle.

Given that angle $CAB = 50^\circ$, angle $ABC = 70^\circ$ and angle $BCA = 60^\circ$ and AP, BR and CQ are angle bisectors of angle CAB, angle ABC and angle BCA respectively, find the values of angles RPQ, PQR and PRQ.



Answer angle RPQ = ...° [2]

angle PQR = ...° [1]

angle PRQ = ...° [1]

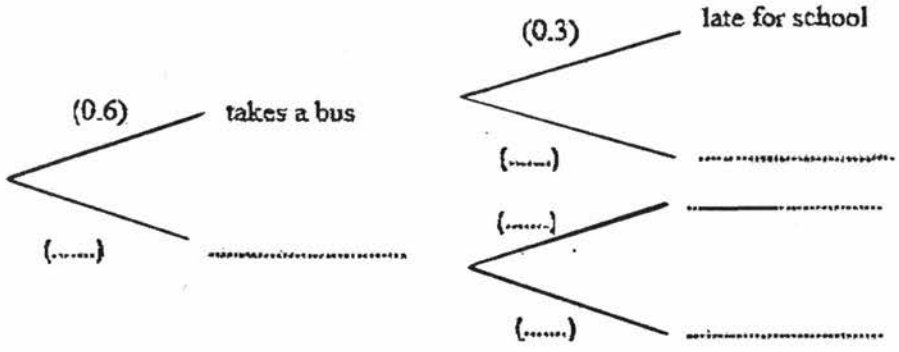
16 The probability that Katie takes a bus is 0.6.

If she takes a bus, the probability that she is late for school is 0.3.

If she does not take a bus, the probability that she is late for school is 0.2.

(a) Complete the probability tree given below

Answer



[3]

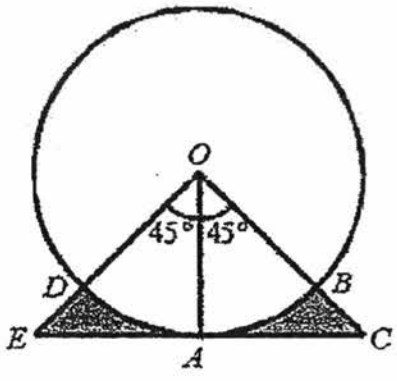
(b) Calculate the probability that Katie is not late to school.

Answer ..... [2]

17 In the diagram, the circle, centre O, passes through D, A and B.

The tangent at A meets OB produced at C and OD produced at E.

The radius of the circle is 4 cm and angle AOB = angle AOE = 45°.



(a) The area of the shaded region can be expressed as $(a - b\pi)$ cm^2, where a and b are constants.

Find the values of a and b.

Answer a = ... [2]

b = ... [2]

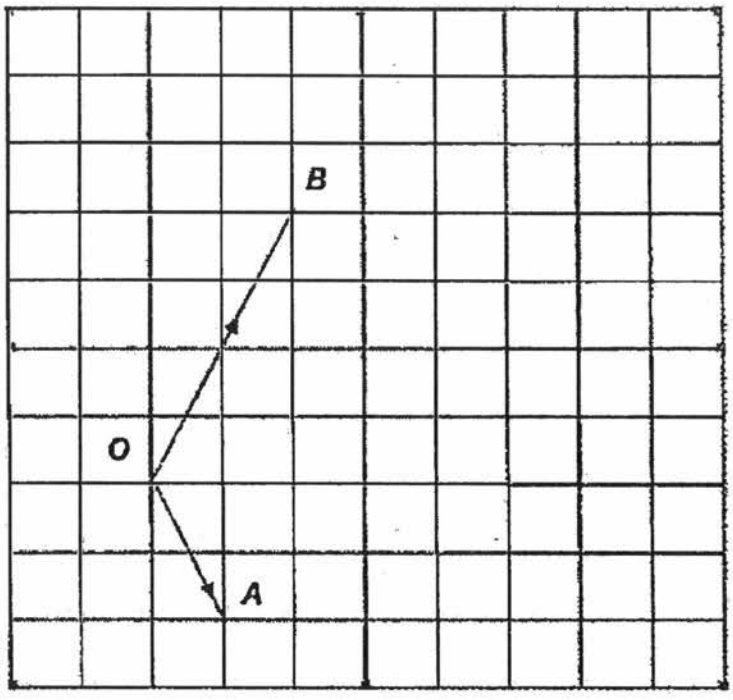
(b) The perimeter of the shaded region can be expressed as $\left(p\pi + 2\sqrt{q}\right)$ cm.

Find the values of p and q.

Answer p = ... [2]

q = ... [2]

18 Vectors $\overrightarrow{OB}$ and $\overrightarrow{OA}$ are drawn below.



Given that $\overrightarrow{OP} = \begin{pmatrix} -1 \\ -2 \end{pmatrix}$.

(a) (i) locate point P on the grid, mark it with a cross X and label it, [1]

(ii) express $\overrightarrow{OP}$ in terms of $\overrightarrow{OB}$ and/or $\overrightarrow{OA}$.

Answer $\overrightarrow{OP}$ = ... [1]

(b) $OBQA$ is a parallelogram.

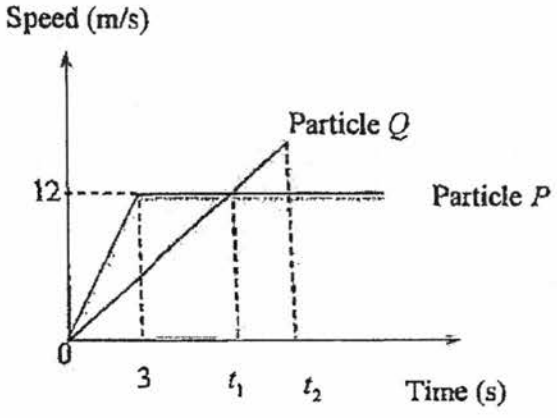
(i) locate point Q on the grid, mark it with a cross X and label it, [1]

(ii) find the column vector representing $\overrightarrow{OQ}$.

Answer $\overrightarrow{OQ}$ = ... [1]

19 The diagram shows the speed-time graphs of two particles P and Q. Both particles

P and Q start from rest. P accelerates uniformly for 3 seconds until it reaches a speed of 12 m/s. It then continues to travel at this constant speed. Q starts from the same point as P but accelerates from rest at a constant rate of 3 m/s^2.



(a) Write down the value of t_1, where the speeds P and Q are the same.

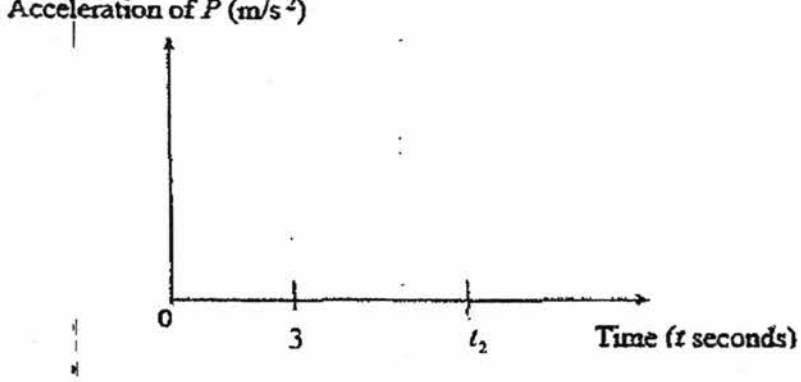
Answer $t_1 =$ ..... [1]

(b) Given that Q overtakes P t_2 seconds after the start of the motion, find the value of t_2

Answer $t_2 =$ ..... [3]

(c) In the answer space below, sketch the acceleration-time graph of P for $0 \le t \le t_2$.

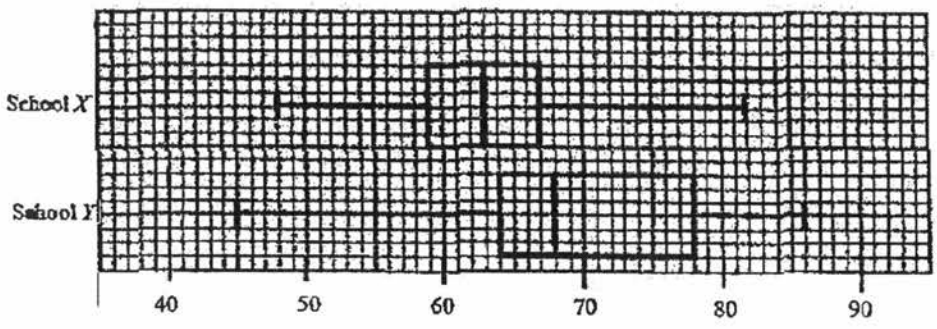
Answer



[1]

20 All the students from 2 schools X and Y took the same examination paper.

The box-and -whisker diagram below shows the results for the two schools.



(a) State, with a reason, which school achieved a better result.

Answer .....

.....

..... [1]

(b) State, with a reason, which school has a more uniformly-distributed mark.

Answer .....

.....

.....[1]

21 The numbers in the Number Triangle are consecutive even numbers.

Row	Number Triangle	Sum of row (R)	No. of even numbers (E)	Average of Row (A)
1	2	2	1	2
2	4 6	10	2	5
3	8 10 12	30	3	10
4	14 16 18 20	68	4	p
5	22 24 26 28 30	130	5	26
6	32 34 36 38 40 42	q	6	37

(a) Find the values of p and q.

Answer $p =$ ....., $q =$ ..... [2]

(b) Write down a formula connecting A and E.

Answer ..... [1]

(c) Write down a formula connecting R and E.

Answer ..... [1]

(d) Justify, with reason why the number 6400 could not appear in the column A.

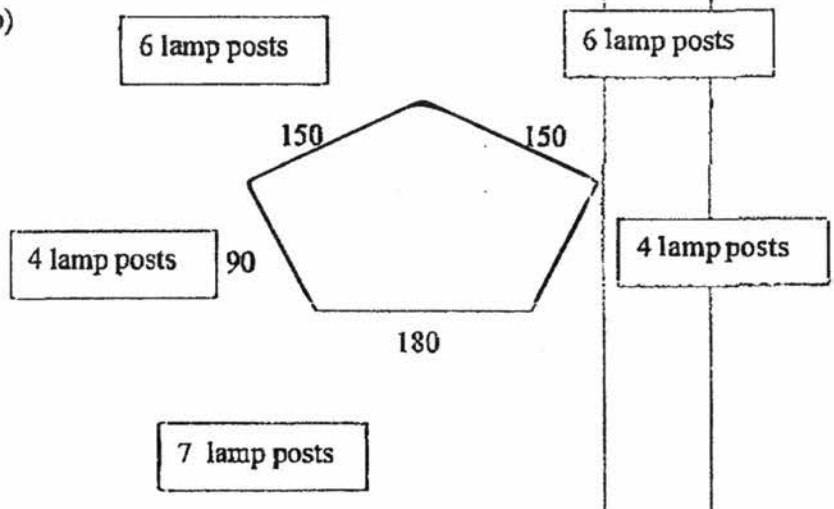
Answer .....

.....

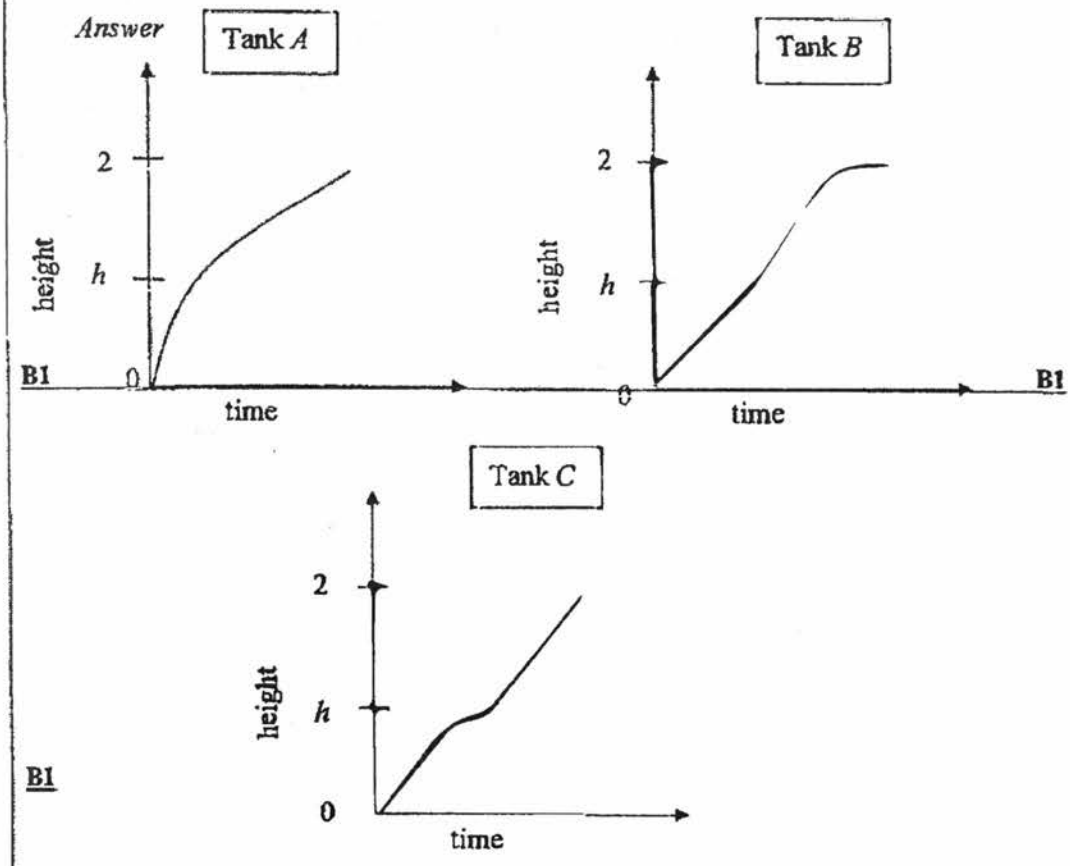
..... [1]

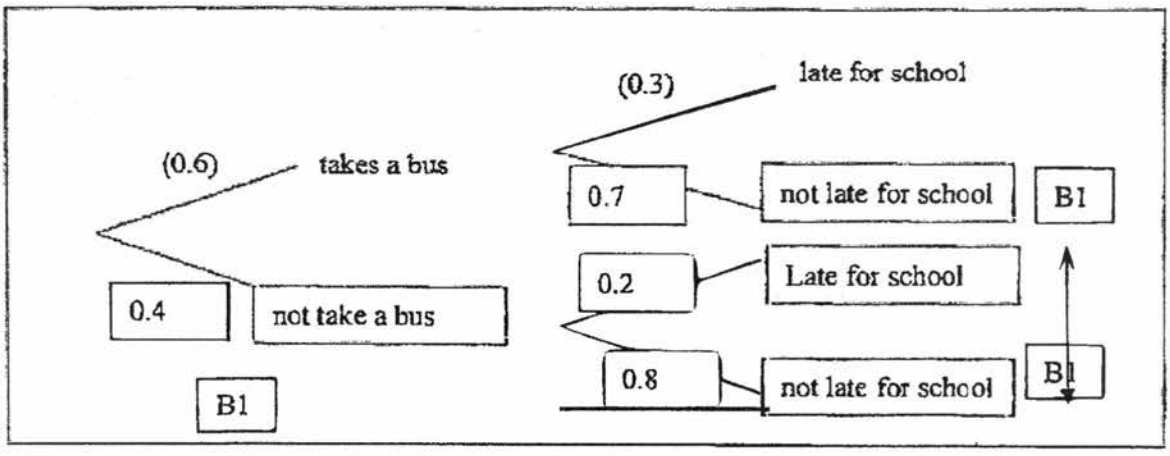
End of Paper

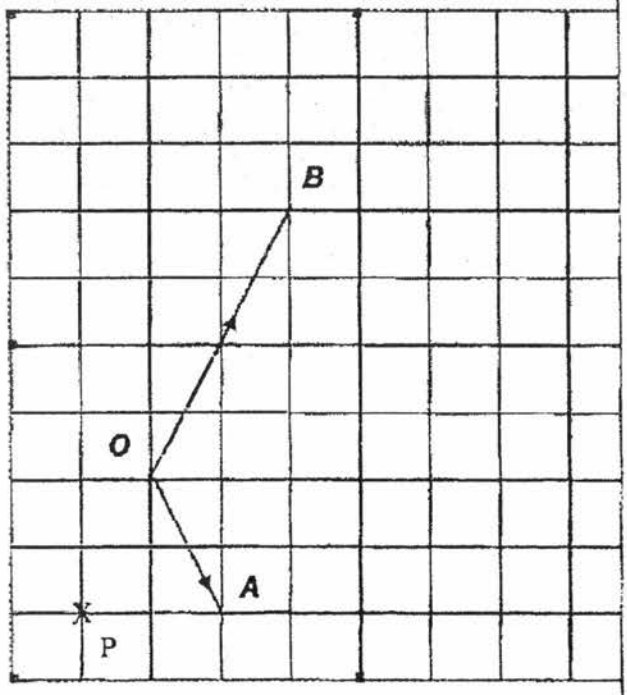
Qn	Solution		
1a	Section A $\frac{x+1}{x^2-9}-\frac{2}{3-x}=\frac{x+1}{(x-3)(x+3)}+\frac{2}{x-3}$ $=\frac{x+1+2(x+3)}{(x-3)(x+3)}$ $=\frac{3x+7}{(x+3)(x-3)}$		
1b	$\frac{(abc^{-2})^3}{(a^{-4}b^{-1})^{-1}}\times\frac{a^{-6}b^{-7}}{(bc^2)^{-4}}=\frac{a^3b^3c^{-6}}{a^4b^1}\times\frac{a^{-6}b^{-7}}{b^{-4}c^{-8}}$ $=\frac{a^{-3}b^{-4}c^{-6}}{a^4b^{-3}c^{-8}}$ $=a^{-7}b^{-1}c^2$ $=\frac{c^2}{a^7b}$		
2	$\frac{k}{3}=\sqrt{\frac{A-3b^2}{cA}}$ $\frac{k^2}{9}=\frac{A-3b^2}{cA}$ $k^2cA=9A-27b^2$ $A(k^2c-9)=-27b^2$ $A=\frac{27b^2}{9-k^2c}$ OR $A=\frac{-27b^2}{(ck^2-9)}$		
3	(a) $9xy(2x+3-y^2)$ (b) $3(3a-2b)(3a+2b)$ (c) $(r-1)(3s-1)$		

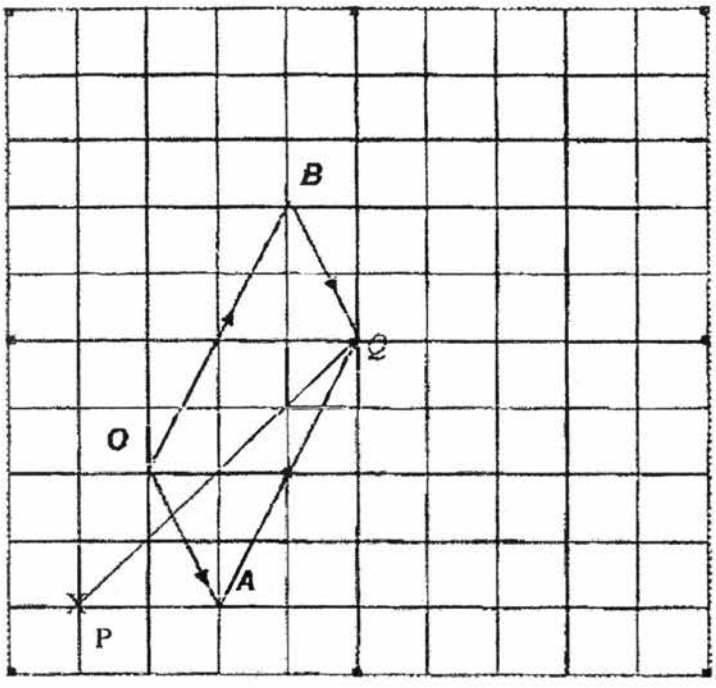
Qn	Solution
4	(a) 8 (b) -24 (c) 0
5	Bus 801 Bus 802 3 | 15 25 5 | 5 25 5 | 1 5 1 1 LCM is 75 5 hours = 300 mins $\frac{300}{75}$ = 4 *times*
6	(a) HCF of 150, 90, 180 is 30m (b)  6 + 6 + 4 + 4 + 7 = 27 lamp posts Double counting answer 27 – 5 = 22 lamp posts
	SECTION B [18m]

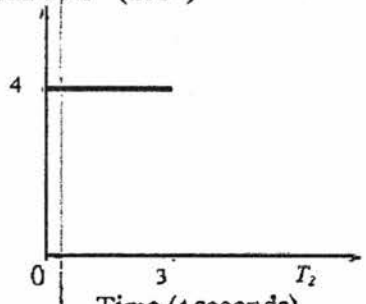
Qn	Solution		
7	$A = 2^m \times 2^2 \times 3^n$ $B = 2^m \times 3^n \times 3 \times 5$ $LCM = 2^{m+2} \times 3^{n+1} \times 5$		
8	$x = 2 - 2y$ $\frac{1}{4}(2-2y) = 11 + 3y$ $y = -3$ $x = 8$		
9	(a) $\frac{1}{2} \times 4 \times h = 16$ $h = 8$ (b) $\sin \angle ACD = \frac{8}{10} = \frac{4}{5}$		
	(c) $XC = \sqrt{10^2 - 8^2} = 6$ $\cos \angle ACE = -\frac{6}{10} = -\frac{3}{5}$		
10	(a) $35 + b$ (b) $5 + a$		
11	1 cm^2: 64×10^8 cm^2 Map 1 3 cm^2 : 192 X 10^8 cm^2 Map 2 12 cm^2 : 192 X 10^8 cm^2 1 cm^2 : 16 X 10^8 cm^2 1: 40000		
12	Amount = $\$36000\left(1+\frac{\frac{3.2}{4}}{100}\right)^{24} = \$43586.83$		

Qn	Solution
13	**Section C [40m]** *Answer*  B1 B1 B1
14	(a) Total angles in the 2 polygons $= 540^0 + 360^0$ $= 900^0$ sum of all required angles $= 7 \times 360^0 - 900^0$ $= 1620^0$
	$\frac{360^0}{n} = \frac{n}{40}$ (b) $n^2 = 14400$ $n = 120$
15	angle RPQ = angle RPA + angle APQ = angle RPA + angle ACQ (angles in the same segment) $= 35^0 + 30^0 = 65^0$ angle PQR = angle PQC + angle CQR = angle PAC + angle CBR = 25 + 35 = 60° angle PRQ = 25 + 30 = 55°

Qn	Solution		
16 a			
	16b $0.6 \times 0.7 + 0.4 \times 0.8 = 0.74$ **M1** **A1** **(multiplication of probability from the tree)**		
17	(a) Angle $OAE = 90^0$ $OA = AE = AC$ Area of shaded region $= \frac{1}{2} \times 8 \times 4 - \frac{1}{2} \times 4^2 \times \pi/2$ $= 16 - 4\pi$ $x = 16$ $y = 4$		
17 (b)	$OE = \sqrt{4^2 + 4^2} = \sqrt{32}$ $4\left(\frac{\pi}{2}\right) + 8 + 2\left(\sqrt{32} - 4\right)$ $= 2\pi + 8 + 2\sqrt{32} - 8$ $= 2\pi + 2\sqrt{32}$ $p = 2$ $q = 32$		

Qn	Solution		
18 ai			A1

18 aii	$\overrightarrow{OP} = -\frac{1}{2}\overrightarrow{OB}$		A1
18 b			A1

	18bii $\overrightarrow{PQ} = \begin{pmatrix} 4 \\ 4 \end{pmatrix}$ A1		
19	(a) 4s (b) $\frac{1}{2} \times T_2 \times 3T_2 = \frac{1}{2} \times 3 \times 12 + 12(T_2 - 3)$ $T_2^2 - 8T_2 + 12 = 0$ $(T_2 - 6)(T_2 - 2) = 0$ $T_2 = 6$ (c) Acceleration of P (m/s^2)  		
20	(a) Sch Y achieved better results because it has a higher median of 68 as compared to 63 for X. (b) Sch X is more uniform because of a smaller interquartile range of 8 as compared to 14 for Y.		
21	a. p = 17 q = 222 b. $A = E^2 + 1$ c. $R = E^3 + E$ d. $6400 = 80^2$, a perfect square number, but the number in column A are not perfect square numbers.		

Answer all the questions.

Section A [30 marks]

1 (a) Expand and simplify $(4x-1)^2-(8x+1)(2x-1)$. [2]

(b) Express $\frac{4x^2-9}{x^2+x-20} \div \frac{4x^2-6x}{16-x^2}$ as a fraction in its lowest term. [3]

(c) Solve the equation $\frac{x}{3}-\frac{2x-1}{x-3}=-2$, leaving your answer correct to 3 decimal places. [3]

(d) y is directly proportional to x^2.
It is known that $y = 144$ for a particular value of x.
Find the percentage change in y when the value of x decreases by 25%. [3]

---

2 During a school's sports day, the number of first, second and third positions won by the different houses are given in the table below.
The number of points won for individual and group events are also given in the table.

Houses	Individual events			Group events		
	First	Second	Third	First	Second	Third
Blue	7	5	4	3	2	0
Green	5	4	6	1	2	1
Red	4	5	5	1	2	2
Yellow	4	6	5	1	0	3
Points	5	3	1	10	6	2

(a) It is given that $\mathbf{A} = \begin{pmatrix} 7 & 5 & 4 \\ 5 & 4 & 6 \\ 4 & 5 & 5 \\ 4 & 6 & 5 \end{pmatrix}$ and $\mathbf{B} = \begin{pmatrix} 5 \\ 3 \\ 1 \end{pmatrix}$, evaluate the matrix $\mathbf{P} = \mathbf{AB}$. [2]

(b) Given matrix $\mathbf{C} = \begin{pmatrix} 3 & 2 & 0 \\ 1 & 2 & 1 \\ 1 & 2 & 2 \\ 1 & 0 & 3 \end{pmatrix}$.

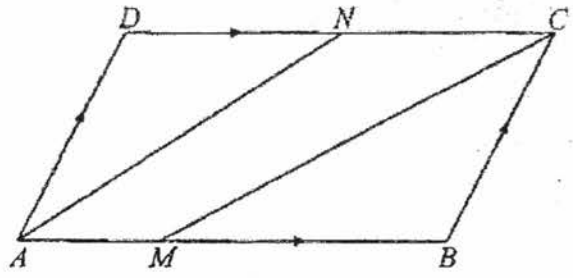
(i) Represent the group event scoring system in a 3×1 matrix $\mathbf{D}$. [1]

(ii) Evaluate the matrix $\mathbf{Q} = \mathbf{CD}$ and explain what do the elements of $\mathbf{Q}$ represent. [2]

(c) The scores of individual events and group events are added for each house.
Using matrix manipulation, determine which house won the overall championship. [2]

---

3 $ABCD$ is a parallelogram.
N is the midpoint of DC and M is the point on AB such that $2AM = MB$.



Given that $\overrightarrow{AB} = 6\mathbf{a}$ and $\overrightarrow{AD} = 4\mathbf{b}$,

(a) Express as simply as possible, in terms of **a** and/or **b**.

(i) $\overrightarrow{AM}$ [1]

(ii) $\overrightarrow{MC}$ [1]

(iii) $\overrightarrow{AN}$ [1]

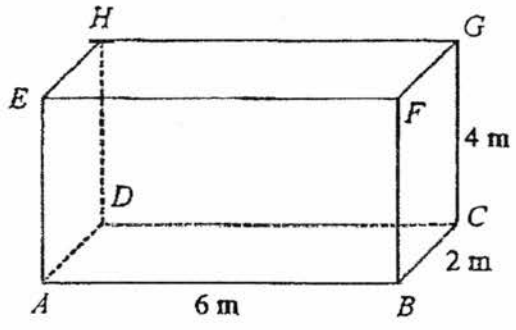
(b) Find the numerical value of

(i) $\dfrac{\text{area of triangle } ADN}{\text{area of parallelogram } ABCD}$, [1]

(ii) $\dfrac{\text{area of triangle } ADN}{\text{area of triangle } AMN}$. [2]

---

4 The diagram shows a rectangular cuboid $ABCDEFGH$.
$AB = 6$ m, $BC = 2$ m and $CG = 4$ m.



(a) Show that angle $HBD = 32.3°$, correct to 1 decimal place. [2]

(b) Calculate angle AFC. [3]

(c) Calculate the greatest angle of elevation of the point H when viewed from the line AB. [1]

Section B [70 marks]
Please begin Question 5 on a NEW sheet of paper

5 (a) Chloe has a total of 126 marks in x tests.
In the next two tests, she scored 9 marks and 8 marks respectively.

Find, in terms of x, her mean mark for the

(i) first x tests, [1]

(ii) $(x + 2)$ tests. [1]

Her mean mark for the first x tests was one greater than her mean mark for the $(x + 2)$ tests.

(iii) write an equation in x to represent this information and show that it reduces to $x^2 + 19x - 252 = 0$. [3]

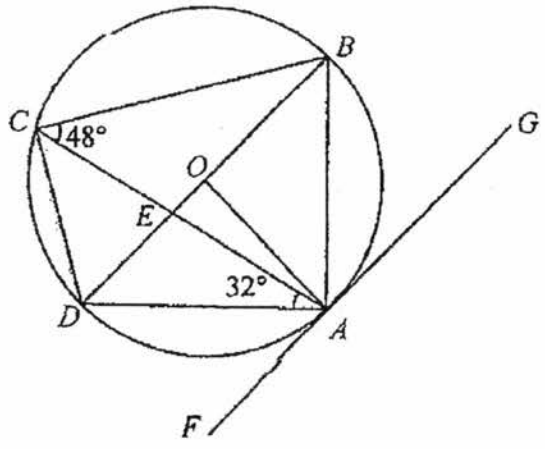
(iv) Solve the equation to find the number of tests Chloe took initially. [3]

(b) Amanda has a mean of 13.5 marks for the first $(x + 1)$ tests, but her mark on the last test gave her a mean of 14 marks for the $(x + 2)$ tests.

Calculate the number of marks Amanda scored in the last test. [2]

---

6 In the diagram, O is the centre of the circle through A, B, C and D.
FG is the tangent to the circle at A.
AC intersects BD at E.
Angle $ACB = 48°$ and angle $CAD = 32°$.



(a) Calculate the following angles, stating your reasons clearly.

(i) Angle ABO [2]

(ii) Angle CDA [2]

(iii) Angle GAB [2]

(b) Explain why BD is not parallel to GF. [2]

---

7 (a) The frequency table shows the weekly expenditure on food of n students from School X.

Weekly expenditure ($x)	Frequency
$30 < x \leq 40$	8
$40 < x \leq 50$	17
$50 < x \leq 60$	34
$60 < x \leq 70$	p
$70 < x \leq 80$	3

(i) If $\frac{5}{16}$ of the n students have a weekly expenditure of at most \$50, show that the value of p is 18. [2]

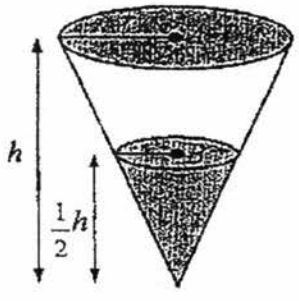
(ii) Calculate an estimate of

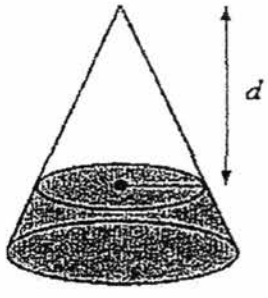
(a) the mean weekly expenditure on food, [1]

(b) the standard deviation. [1]

(iii) The standard deviation of the weekly expenditure on food of students from School Y was \$5.62.
Using this information, comment on one difference between the two distributions. [1]

(b) The diagram shows an inverted cone of height h and radius r.
It contains water to a depth of $\frac{1}{2}h$.



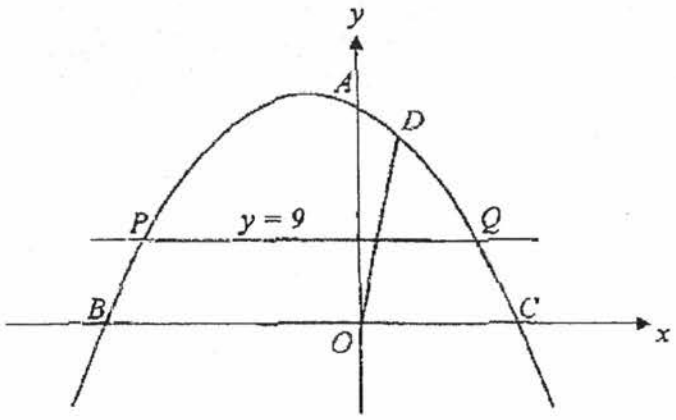


(i) Find the ratio of area of surface B to area of surface A. [1]

(ii) Find the volume of the water if the cone can hold 480 cm^3 of water when full. [2]

(iii) The cone is now inverted such that the liquid rests on the flat circular base of the cone, as shown in the diagram on the right.
Find, in terms of h, an expression for d, the vertical distance of the liquid surface from the tip of the cone. [3]

8 The diagram shows the curve $y = (4 - x)(x + k)$, where k is a constant.
The curve cuts the y-axis at the point $A(0, 24)$, and the x-axis at B and C.



(a) Show that the value of k is 6. [1]

(b) Write down the coordinates of B and C. [2]

(c) Find the coordinates of the maximum point on the curve. [2]

(d) $D(1, m)$ is a point on the given curve.
Find the value of m and the equation of the line OD. [3]

(e) The line $y = 9$ intersects the curve at P and Q. Find the coordinates of P and Q. [3]

9 A student needed to make a circular face mask for a school performing arts event. She took a circular sheet of radius 10 cm and removed two circles, each of radius 2.5 cm for two eyes and an isosceles triangle of base 2 cm and equal sides of 3 cm each for a nose, as shown in **Diagram I**.

The mouth is shown in the **Diagram II**.
It is formed by an arc, AXB, of a circle, centre O and radius 3 cm.
AYB is the arc of another circle with diameter, AB, 3 cm.

She painted the remaining area.

Diagram I

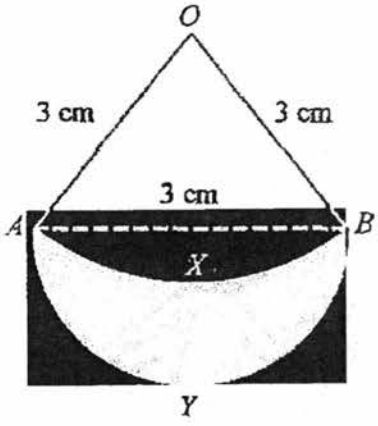


Diagram II

(a) Calculate the area removed. [7]

(b) Calculate the area of mask that was painted. [2]

10 Answer the whole of this question on a sheet of graph paper.

The variables x and y are connected by the equation $y=5-\frac{x^2}{10}-\frac{4}{x}$. Some corresponding values are given in the following table.

x	0.5	0.7	1	2	3	4	5	6	7	8
y	−3.0	−0.8	0.9	2.6	2.8	k	1.7	0.7	−0.5	−1.9

(a) Calculate the value of k. [1]

(b) Taking 2 cm to represent 1 unit on each axis, draw a horizontal x-axis for $0 \le x \le 8$ and a vertical y-axis for $-3 \le y \le 3$, draw the graph of $y=5-\frac{x^2}{10}-\frac{4}{x}$ for the values of x in the range $0.5 \le x \le 8$. [3]

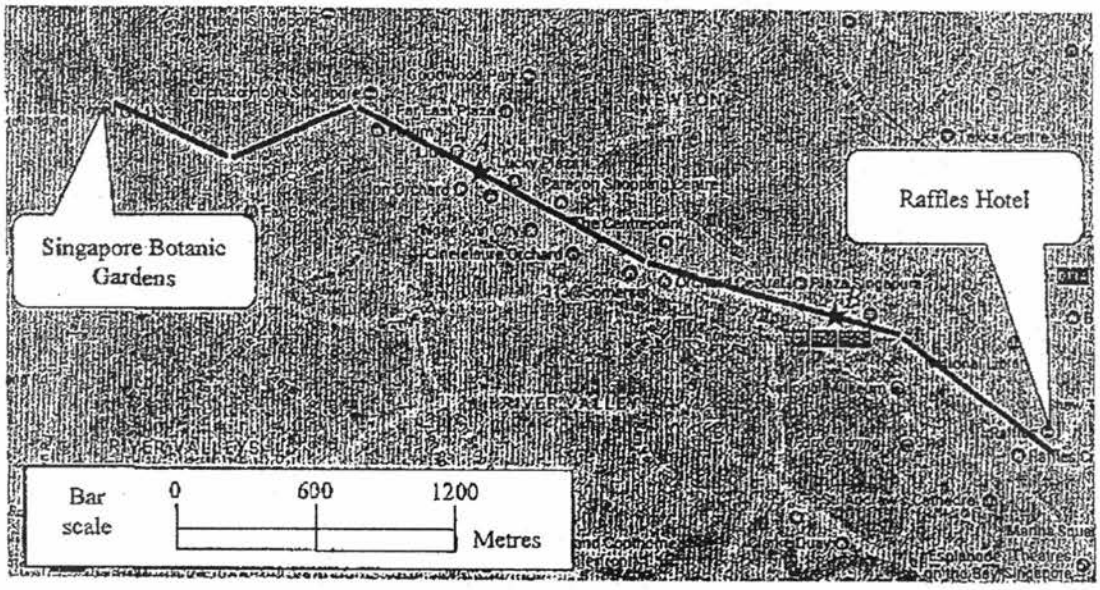
(c) Use your graph to find the greatest value of $5-\frac{x^2}{10}-\frac{4}{x}$ in the interval $0.5 \le x \le 8$. [1]

(d) By drawing a tangent, find the gradient of the graph at the point where $x=2$. [2]

(e) Use your graph to solve $5-\frac{x^2}{10}-\frac{4}{x}=2$ in the range $0.5 \le x \le 8$. [3]

(f) By drawing a suitable straight line, find the range of values of x in the interval $0.5 \le x \le 8$ for which $5-\frac{x^2}{10}-\frac{4}{x} \ge x$. [2]

11 Cheryl works at the Singapore Botanic Gardens.
She needs to rush down to meet a client at Raffles Hotel.
The quickest route from Cheryl's location to Raffles Hotel is indicated on the map with black solid lines.
The bar scale on the lower left corner of the map provides the corresponding actual ground distance.



(a) Calculate the actual travelling distance, in kilometres, between Cheryl's location and Raffles Hotel, giving your answer correct to 2 significant figures. [2]

(b) At 6.14 pm, Cheryl decided to call for a ride from Singapore Botanic Gardens to Raffles Hotel.

Information about FastDel Cab and Aber services and other travelling details are on the opposite page.
Along the way, there are two ERP gantries, indicated by *A* and *B* with a star each on the map.

Determine which service Cheryl should choose.
Justify your answer with relevant working. [7]

Travelling time

From	To	Duration
Singapore Botanic Gardens	Orchard ERP (*A*)	6 minutes
Orchard ERP	Handy Road ERP (*B*)	5 minutes
Handy Road ERP	Raffles Hotel	4 minutes

ERP Charges

Orchard (*A*)		**Handy Road Gantry (*B*)**	
12.00 pm – 5.29 pm	$0.50	12.00 pm – 12.04 pm	$0.50
5.30 pm – 5.34 pm	$1.00	12.05 pm – 1.59 pm	$1.00
5.35 pm – 5.59 pm	$1.50	2.00 pm – 2.04 pm	$1.50
6.00 pm – 6.54 pm	$2.00	2.05 pm – 2.54 pm	$2.00
6.55 pm – 6.59 pm	$1.50	2.55 pm – 2.59 pm	$1.50
7.00 pm – 7.59 pm	$1.00	3.00 pm – 5.29 pm	$1.00
		5.30 pm – 5.59 pm	$0.50
		6.00 pm – 7.54 pm	$1.00
		7.55 pm – 7.59 pm	$0.50

FastDel Cab Service

The first 1 km or less		$3.20	
Every 400 m thereafter or less up to 10 km		$0.22	
Every 350 m thereafter or less after 10 km		$0.22	
Current Booking Peak Period ($3.30) Monday to Friday (Except Public Holidays):	6.00 am – 9.29 am	Monday to Sunday & Public Holidays:	6.00 pm – 11.59 pm
Peak Period Surcharge (25% of metered fare) Monday to Friday (Except Public Holidays):	6.00 am – 9.29 am	Monday to Sunday & Public Holidays:	6.00 pm – 11.59 pm
ERP Charge Passengers are required to bear the ERP charge shown on the upper display of the In-vehicle Unit. The ERP charge is deducted each time the taxi passes under the ERP gantry, payable on top of metered fare			

Aber Service

Base Fare	$3.00
Travelling time per minute	$0.20
Travelling distance per km	$0.45
6 pm to 8 pm peak period surge	2.5× of normal fare

End of Paper

4E5N Mathematics Preliminary Exam 2017 (Paper 2)

Section A

1(a)	$(4x-1)^2-(8x+1)(2x-1)$ $=16x^2-8x+1-(16x^2-6x-1)$ $=16x^2-8x+1-16x^2+6x+1$ $=-2x+2$	
1(b)	$\frac{(4x^2-9)}{(x^2+x-20)}\div\frac{(4x^2-6x)}{(16-x^2)}$ $=\frac{(2x-3)(2x+3)}{(x+5)(x-4)}\div\frac{2x(2x-3)}{-(x-4)(x+4)}$ $=\frac{(2x-3)(2x+3)}{(x+5)(x-4)}\times\frac{-(x-4)(x+4)}{2x(2x-3)}$ $=\frac{-(2x+3)(x+4)}{2x(x+5)}$	
1(c)	$\frac{x}{3}-\frac{2x-1}{x-3}=-2$ $\frac{x(x-3)-3(2x-1)}{3(x-3)}=-2$ $x^2-3x-6x+3=-6(x-3)$ $x^2-9x+3=-6x+18$ $x^2-3x-15=0$ $x=\frac{-(3)\pm\sqrt{(-3)^2-4(1)(-15)}}{2(1)}$ $=5.653 \quad \text{or} \quad -2.653$	
1(d)	$y=kx^2$ $144=kx^2$ Original value: x New value: $0.75x$ $Y=kX^2$ $Y=k(0.75x)^2$ $=0.5625kx^2$ $=0.5625(144)$ $=81$ Percentage change $=\frac{81-144}{144}\times 100$ $=-43.75\%$	

2(a)

$$P = \begin{pmatrix} 7 & 5 & 4 \\ 5 & 4 & 6 \\ 4 & 5 & 5 \\ 4 & 6 & 5 \end{pmatrix} \begin{pmatrix} 5 \\ 3 \\ 1 \end{pmatrix}$$

$$= \begin{pmatrix} 54 \\ 43 \\ 40 \\ 43 \end{pmatrix}$$

2(bi)

$$D = \begin{pmatrix} 10 \\ 6 \\ 2 \end{pmatrix}$$

2(bii)

$$Q = \begin{pmatrix} 3 & 2 & 0 \\ 1 & 2 & 1 \\ 1 & 2 & 2 \\ 1 & 0 & 3 \end{pmatrix} \begin{pmatrix} 10 \\ 6 \\ 2 \end{pmatrix}$$

$$= \begin{pmatrix} 42 \\ 24 \\ 26 \\ 16 \end{pmatrix}$$

The elements of **Q** represent the **total score from group events for each house respectively.**

2(c)

$$\text{Total score} = \begin{pmatrix} 54 \\ 43 \\ 40 \\ 43 \end{pmatrix} + \begin{pmatrix} 42 \\ 24 \\ 26 \\ 16 \end{pmatrix}$$

$$= \begin{pmatrix} 96 \\ 67 \\ 66 \\ 59 \end{pmatrix}$$

Blue house won overall championship.

3(ai)	$2\overrightarrow{AM} = \overrightarrow{MB}$ $\frac{AM}{MB} = \frac{1}{2}$ $\overrightarrow{AM} = \frac{1}{3}\overrightarrow{AB}$ $= \frac{1}{3}(6\mathbf{a})$ $= 2\mathbf{a}$	
3(aii)	$\overrightarrow{MC} = \overrightarrow{MB} + \overrightarrow{BC}$ $= \frac{2}{3}(6\mathbf{a}) + 4\mathbf{b}$ $= 4\mathbf{a} + 4\mathbf{b}$	
3(aiii)	$\overrightarrow{DN} = \frac{1}{2}\overrightarrow{DC}$ $= 3\mathbf{a}$ $\overrightarrow{AN} = \overrightarrow{AD} + \overrightarrow{DN}$ $= 3\mathbf{a} + 4\mathbf{b}$	
3(bi)	$\frac{\text{area of triangle } ADN}{\text{area o parallelogram } ABCD} = \frac{\frac{1}{2}(h)(DN)}{(h)(DC)}$ $= \frac{\frac{1}{2}(DN)}{(DC)}$ $= \frac{1}{2} \times \frac{1}{2}$ $= \frac{1}{4}$	
3(bii)	$\frac{\text{area of triangle } ADN}{\text{area of triangle } AMN} = \frac{DN}{AM}$ $= \frac{\frac{1}{2}(DC)}{\frac{1}{3}(DC)}$ $= \frac{3}{2}$	

4(a)

$DB^2 = 6^2 + 2^2$

$= 40$

$DB = \sqrt{40}$

$= 6.3245$

$\tan \angle HBD = \dfrac{4}{\sqrt{40}}$

$\angle HBD = \tan^{-1}\left(\dfrac{4}{\sqrt{40}}\right)$

$= 32.311°$

$= 32.3°$ (1 d.p.)

4(b)

$AF^2 = 6^2 + 4^2$ $\qquad$ $FC^2 = 2^2 + 4^2$

$= 52$ $\qquad$ $= 20$

$AF = \sqrt{52}$ $\qquad$ $FC = \sqrt{20}$

$= 7.2111$ $\qquad$ $= 4.4721$

$AC = DB$

$= \sqrt{40}$

$= 6.3245$

$AC^2 = AF^2 + FC^2 - 2(AF)(FC)\cos \angle AFC$

$\cos \angle AFC = \dfrac{AF^2 + FC^2 - AC^2}{2(AF)(FC)}$

$= \dfrac{52 + 20 - 40}{2\left(\sqrt{52}\right)\left(\sqrt{20}\right)}$

$\angle AFC = \cos^{-1}\left(\dfrac{32}{2\left(\sqrt{52}\right)\left(\sqrt{20}\right)}\right)$

$= 60.255°$

$= 60.3°$ (1 d.p.)

4(c)

$\tan \angle HAD = \dfrac{4}{2}$

$\angle HAD = \tan^{-1}(2)$

$= 63.434°$

$= 63.4°$ (1 d.p.)

$\therefore$ greatest angle of elevation is 63.4°

Section B

5(a)	Mean mark for first x tests $= \frac{126}{x}$	
5(b)	Mean mark for first $(x+2)$ tests $= \frac{126+9+8}{x+2}$ $= \frac{143}{x+2}$	
5(c)	$\frac{126}{x} - \frac{143}{x+2} = 1$ $\frac{126(x+2)-143x}{x(x+2)} = 1$ $126x+252-143x = x^2+2x$ $252-17x = x^2+2x$ $x^2+19x-252 = 0$ (shown)	
5(d)	$x^2+19x-252 = 0$ $(x-9)(x+28) = 0$ $x = 9$ or -28 (reject) $\therefore$ Chloe took 9 tests initially.	
5(e)	Number of marks Amanda scored in the last test $= 14(x+2)-13.5(x+1)$ $= 14(11)-13.5(10)$ $= 19$	
6(ai)	$\angle BDA = 48°$ (angles in the same segment) $\angle ABO = 90° - 48°$ (right angle triangle in semicircle) $= 42°$ OR $\angle DCE = 90° - 48°$ (right angle triangle in semicircle) $= 42°$ $\angle ABO = 42°$ (angles in the same segment) OR $\angle AOB = 48° \times 2$ $= 96°$ (angle at centre is twice angle at circumference) 42° (isosceles triangle AOB)	

6(aii)	$\angle DCE = 42°$ (angles in the same segment) $\angle CDA = 180° - 42° - 32°$ (sum of angles in triangle) $= 106°$ OR $\angle CBD = 32°$ (angles in the same segment) (angles in opposite segement are supplementary) $\angle CDA = 180° - 32° - 42°$ $= 106°$	
6(aiii)	$\angle OAB = 42°$ (base angles of isosceles triangle) $\angle OAG = 90°$ (tangent perpendicular to radius) $\angle GAB = 90° - 42°$ $= 48°$ OR $\angle GAB = 48°$ (alternate segment theorem)	
6(b)	Since $\angle OBA \neq \angle GAB$, it does not satisfy the property of alternate angles with a set of parallel line. Hence, BD is not parallel to GF OR If BD is parallel to GF, $\angle OBA = \angle GAB$, based on alternate angles. Since $\angle OBA \neq \angle GAB$, BD is not parallel to GF.	
7(ai)	$\frac{5}{16}$ ------ $8 + 17 = 25$ students $\therefore 8+17+34+p+3 = \frac{25}{5} \times 16$ $62 + p = 80$ $p = 18$ (shown)	
7(aiia)	$\text{Mean} = \frac{\sum fx}{\sum f}$ $= \$53.875$ $= \$53.88$ (2 d.p.)	
7(aiib)	$\text{Standard deviation} = \sqrt{\frac{\sum fx^2}{\sum f} - \left(\frac{\sum fx}{\sum f}\right)^2}$ $= 9.8734$ $= 9.87$ (3 s.f.)	

7(aiii)	The weekly expenditure on food for School X has a **wider spread (less consistent)** than that for School Y as the standard deviation for School X is greater than that of School Y.	
7(bi)	$\frac{\text{area of surface B}}{\text{area of surface A}} = \left(\frac{\frac{1}{2}h}{h}\right)^2$ $= \frac{1}{4}$	
7(bii)	$\frac{\text{Volume of water}}{\text{Volume of full cone}} = \left(\frac{1}{2}\right)^3$ $\frac{\text{Volume of water}}{480} = \frac{1}{8}$ $\text{Volume of water} = \frac{1}{8} \times 480$ $= 60 \text{ cm}^3$	
7(biii)	Remainder volume $= 480 - 60 = 420 \text{ cm}^3$ $\frac{\text{Volume of empty part}}{\text{Volume of full cone}} = \left(\frac{d}{h}\right)^3$ $\frac{420}{480} = \left(\frac{d}{h}\right)^3$ $\frac{d}{h} = \sqrt[3]{\frac{7}{8}}$ $d = 0.95647h$ $= 0.956h$ (3 s.f.)	
8(a)	At $A(0, 24)$, $24 = (4-0)(0+k)$ $24 = 4k$ $k = 6$	
8(b)	$B(-6, 0)$ $C(4, 0)$	
8(c)	Line of symmetry: $x = \frac{-6+4}{2} = -1$ At $x = -1$, $-1+6)$ $\therefore$ Coordinate of maximum point $= (-1, 25)$	

8(d)

At $x = 1$,

$m = (4-1)(1+6)$

$= 21$

$$\text{gradient} = \frac{21}{1}$$

$= 21$

$\therefore$ Equation of line: $y = 21x$

8(e)

Sub. y = 9 into equation of graph,

$9 = (4-x)(x+6)$

$9 = -x^2 - 2x + 24$

$x^2 + 2x - 15 = 0$

$(x-3)(x+5) = 0$

$x = 3$ or -5

P(−5, 9)

Q(3, 9)

9(a)

Area of eyes $= 2 \times \pi r^2$

$= 2 \times (2.5)^2 \pi$

$= 12.5\pi$ cm^2

For isosceles triangle,

$$\cos\alpha = \frac{3^2 + 3^2 - 2^2}{2(3)(3)}$$

$$= \frac{14}{18}$$

$$\alpha = \cos^{-1}\left(\frac{14}{18}\right)$$

$= 38.942°$

Area of nose $= \frac{1}{2}(3)(3)\sin 38.942°$

$= 2.8284$ cm^2

OR

$h = \sqrt{3^2 - 1^2} = \sqrt{8}$

$\text{igle} = \frac{1}{2} \times 2 \times \sqrt{8}$

$= 2.8284$ cm^2

For mouth, $\beta = 60°$

Area of semicircle $= \frac{1}{2}\pi(1.5)^2$

$= \frac{9}{8}\pi \quad cm^2$

Area of sector $= \frac{60}{360}\pi(3)^2$

$= \frac{3}{2}\pi \quad cm^2$

Area of triangle $= \frac{1}{2}(3)(3)\sin 60°$

$= 3.89711 \quad cm^2$

OR

$h = \sqrt{3^2 - 1.5^2} = \sqrt{\frac{27}{4}}$

Area of triangle $= \frac{1}{2} \times 3 \times \sqrt{\frac{27}{4}}$

$= 3.89711 \ cm^2$

Area of mouth $= \frac{9}{8}\pi - \left(\frac{3}{2}\pi - 3.89711\right)$

$= 2.71901 \quad cm^2$

Total area removed $= 12.5\pi + 2.8284 + 2.71901$

$= 44.8173$

$= 44.8 \ cm^2 \quad (3 \text{ s.f.})$

9(b)

Area of whole mask $= \pi r^2$

$= 100\pi \quad cm^2$

Area of mask painted $= 100\pi - 44.8173$

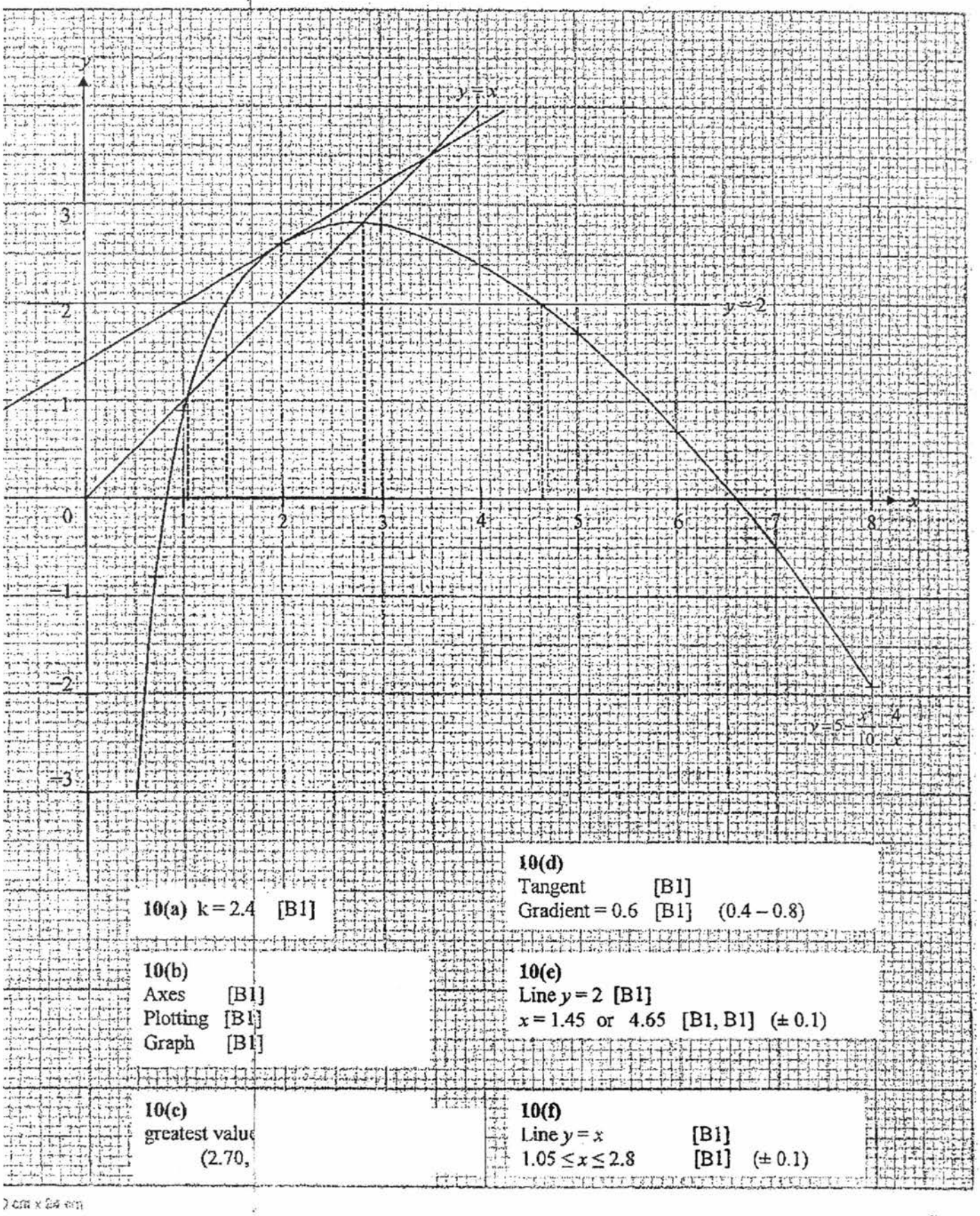
$= 269.341$

$= 269 \quad cm^2 \quad (3 \text{ s.f.})$

11(a)	Total distance on map $= 1.8 + 1.9 + 4.7 + 3.8 + 2.8$ $= 15$ cm Actual distance $= \frac{15}{2} \times 600$ $= 4500$ m $= 4.5$ km	
11(b)	**FastDel service** Base fare = \$3.20 400m thereafter or less: $\frac{3500\text{ m}}{400\text{ m}} = 8.75 \approx 9$ Normal fare = $\$3.20 + 9 \times \$0.22$ = \$5.18 **A** Normal fare + peak surcharge = $\$5.18 \times 1.25$ **B** = \$6.475 Total metered fare = \$6.475 + booking + ERP = \$6.475 + \$3.30 + \$3.00 = \$12.775 = \$12.78 (2 d.p.) **Aber service** Base fare = \$3.00 Travelling time fare = $\$0.20 \times 15 = \$3.00$ Distance fare = $\$0.45 \times 4.5 = \$2.025$ Normal fare = \$3 + \$3 + \$2.025 = \$8.025 Total fare = $\$8.025 \times 2.5$ = \$20.0625 = \$20.06 (2 d.p.) Cheryl should choose **FastDel** service .	

Name _____ Index No. _____

Subject _____ Class _____ Date _____



10(a) k = 2.4 [B1]

10(b)
Axes [B1]
Plotting [B1]
Graph [B1]

10(c)
greatest valu
(2.70,

10(d)
Tangent [B1]
Gradient = 0.6 [B1] (0.4 – 0.8)

10(e)
Line $y = 2$ [B1]
$x = 1.45$ or 4.65 [B1, B1] (± 0.1)

10(f)
Line $y = x$ [B1]
$1.05 \leq x \leq 2.8$ [B1] (± 0.1)

) cm x 24 cm

Sols

Answer **all** the questions.

1 **(a)** Evaluate $\dfrac{\sqrt{239}-17^2}{34.79^3 \times 13}$, giving your answer correct to 5 significant figures.

Answer _____ [1]

(b) Simplify $5x-2(x+2)$.

Answer _____ [1]

---

2 An estimated number of 36 000 people were present at a concert.

(a) If the estimated number was actually rounded off to 3 significant figures, state the maximum possible number of people at the concert.

Answer _____ [1]

(b) If the estimated number was actually rounded off to 2 significant figures, state the minimum possible number of people at the concert.

Answer _____ [1]

---

3 Factorise completely $6ax-2bx+9ay-3by$.

Answer _____ [2]

---

4 The equation of a curve is $y = x^2 + bx + c$ where b and c are constants.

(a) Given that (2 , 0) is a point on the curve, show that $b = -\frac{4+c}{2}$.

Answer

[2]

(b) If the y-intercept of the curve is 14, find the values of b and c.

Answer $b =$ _____ $c =$ _____ [2]

---

5 Triangle ABC is a right angled triangle. Given that $AB = 13$ cm and $BC = 12$ cm, find two possible lengths for the side AC.

Answer _____ or _____ cm [3]

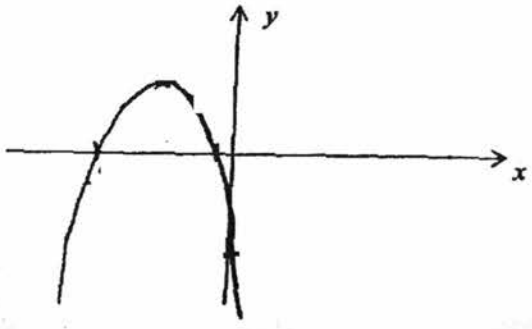
---

6 (a) Express $-x^2 - 5x - 6$ in the form $-(x + a)(x + b)$, where a and b are constants.

Answer _____ [1]

(b) Hence sketch the curve of $y = -x^2 - 5x - 6$, indicating clearly the intercepts and turning point.

Answer



[3]

---

7 Write as a single fraction in its simplest form $\frac{3x}{(x-2)^2} - \frac{2}{2-x}$.

Answer _____ [2]

---

8 The number of apples, oranges and pears at a fruit stall is given by the ratio 2 : 3 : 7.

(a) If there are 126 pears at the fruit stall, find the number of apples at the fruit stall.

Answer _____ [1]

(b) If half the number of oranges at the fruit stall is replaced by papayas, find the fraction of papayas at the fruit stall.

Answer _____ [1]

9 Some values of x and y are given in the table below.

x	3	4	6	12
y	8	6	4	2

State whether x and y could be in direct or inverse proportion, and explain why this is so.

Answer _____

_____

_____ [2]

10 Solve the following equations.

(a) $5(x - 4) = 4 - 2(3x + 1)$

Answer $x =$ _____ [2]

(b) $\frac{3x+1}{5} = \frac{1}{x-2} - 1$

Answer $x =$ _____ [3]

11 Factorise the following.

(a) $25x - 30x^2$

Answer _____ [1]

(b) $5x^2 + 13x - 6$

Answer _____ [2]

(c) $12x^2 - 3$

Answer _____ [2]

---

12 A bag costs $3500 in Singapore.
On a trip to the US, Amy manages to find an identical bag that costs US$3000.

1 US dollar = 1.36 Singapore dollars.

Is the bag cheaper in the US or Singapore? You must show your calculations.

Answer _____ [2]

13 The length of a road from one end to the other is 34.1 km.

(a) On a map, the same road measures 5.5 cm. Write down the scale of the map in the form $1 : n$.

Answer 1 : _____ [2]

(b) A plot of land of area 88.412 km^2 has been marked out for construction of commercial buildings. What is the area on the map that is marked out for construction of commercial buildings?

Answer _____ cm^2 [3]

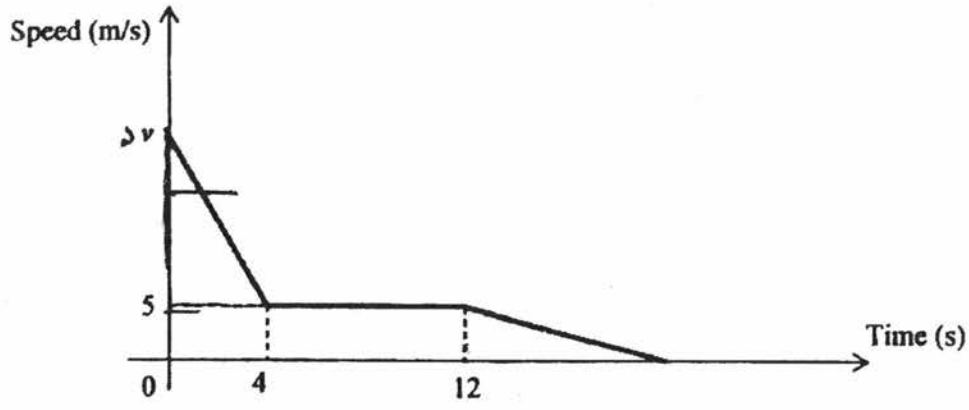
---

14 A computer costs $2300.
During a sale, David buys the computer for $1782.50.
Calculate the percentage discount of the computer during the sale.

Answer _____ % [2]

---

15 A car travelling at an initial speed of v m/s decelerates uniformly for 4 seconds, then travels at a uniform speed of 5 m/s for 8 seconds before decelerating uniformly until it comes to a complete rest. The speed-time graph for the car is shown below.



(a) A van, starting at the same time as the car from the same initial point travels along the same route at a uniform speed of 11 m/s throughout the journey. On the graph above, draw the line representing the speed-time graph of the van, given that $v > 11$. [1]

(b) It is given that deceleration is represented by the gradient of the speed-time graph. The deceleration of the car during the first 4 seconds is 3.75 m/s^2. Show that $v = 20$.

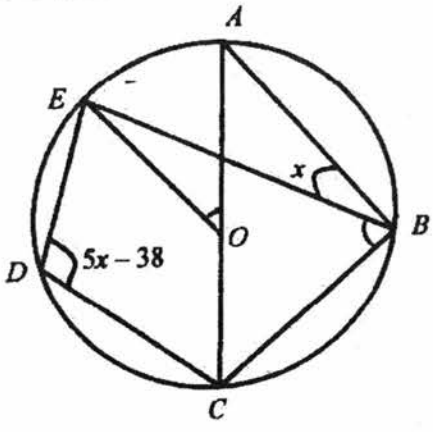
Answer

[2]

(c) It is given that the area under the speed-time graph represents the distance travelled. At how many seconds, after the van and car started from the initial point, will the van overtake the car?

Answer _____ s [4]

16



O is the centre of the circle passing through A, B, C, D and E.
Angle $ABE = x°$, and angle $EDC = (5x - 38)°$.

(a) Find, in terms of x, angle AOE.

Answer _____ ° [1]

(b) Find, in terms of x, angle EBC.

Answer _____ ° [1]

(c) Find x.

Answer $x =$ _____ [2]

17 David's wages, W, varies directly as the square of the number of sales he makes in a month. In January, he makes x number of sales. In February, the number of sales he makes increases by 150% as compared to January. *Calculate the percentage change in David's wages in February as compared to January.*

Answer _____ % [3]

---

18 A class of 40 students had their individual weights taken and the mean and standard deviation of the weights were calculated. It was later found out that the weighing machine used was faulty and every student should be heavier by 2 kg. Describe the effect, if any, it would have on the mean and standard deviation that was calculated.

Answer _____

_____

_____ [2]

---

19 (a) Express 600 as a product of its prime factors, giving your answer in index notation.

Answer _____ [2]

(b) p and q are not prime numbers.

Given that $600 \times pq$ is a perfect square, and that p and q are positive integers smaller than 10, find the smallest possible value of $p - q$.

Answer _____ [2]

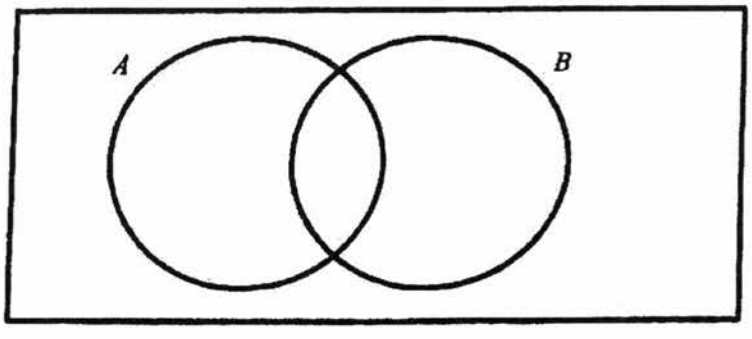
---

20 It is given that

$\xi = \{x : x \text{ is a positive integer smaller than } 10\}$,
$A = \{x : x \text{ is a prime number}\}$,
$B = \{x : x \text{ is an even number}\}$.

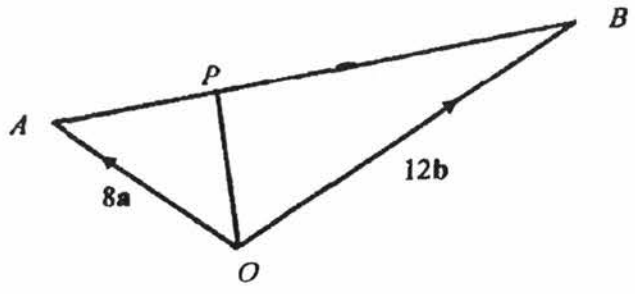
Write down all the numbers in the universal set in the Venn Diagram below.

Answer



[3]

21



OAB is a triangle.
$\overrightarrow{OA} = 8\mathbf{a}$ and $\overrightarrow{OB} = 12\mathbf{b}$.
P is a point on AB such that $AP : PB = 1 : 3$.

(a) Write each of the following in terms of **a** and **b**.
Give your answers in their simplest form.

(i) $\overrightarrow{AB}$.

Answer _____ [1]

(ii) $\overrightarrow{AP}$.

swer _____ [1]

(b) A line is drawn from O to Q where Q lies on the line AB extended. Given that B is the mid-point of PQ, express $\overrightarrow{OQ}$ in terms of **a** and **b**, giving your answer in its simplest form.

Answer _____ [2]

(c) Find the value of $\frac{\text{Area of triangle } OBQ}{\text{Area of triangle } OAQ}$.

Answer _____ [2]

22 The coordinates of A is $(-3, 5)$ and the coordinates of B is $(7, 10)$.
$\overrightarrow{AC} = \begin{pmatrix} 4 \\ -7 \end{pmatrix}$.

(a) Find $\overrightarrow{AB}$, expressing your answer as a column matrix.

Answer _____ [1]

(b) Find $\left|\overrightarrow{AC}\right|$.

Answer _____ [1]

(c) Find the coordinates of C.

Answer _____ [2]

23 An architect designing a walkway draws a scale drawing of the walkway below. The drawing is drawn accurately to a scale of 1 : 10 000.
Point B is directly east of Point A.

Answer

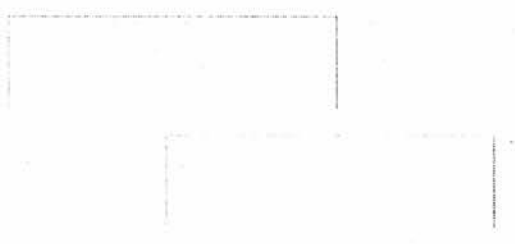
(a) The architect plans to extend the walkway by 0.8 km at a bearing of 145° from point B. Use the scale drawing above to draw the extension of the walkway and label the end of the walkway as Point C. [2]

(b) The walkway is then further extended from Point C back to Point A. By measurement, find the length of the walkway from A to C in kilometres.

Answer _____ km [1]

(c) The architect intends to put a notice board along BC, equidistant from points A and C. By constructing a perpendicular bisector on the scale drawing, indicate and label the position of the notice board with the letter N. [2]

F PAPER



Answer **all** the questions.

1 **(a)** Express as a single fraction in its simplest form

$$\frac{1}{p-2}-\frac{2}{4p+3}.$$ [3]

(b) The formula used in an experiment is

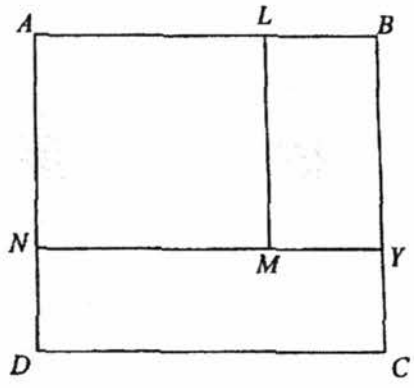
$$T=\frac{k(x-a)}{a}.$$

(i) Express x in terms of T, k and a. [2]

(ii) Find, in terms of k, the value of T when $x = 3a$. [1]

---

2 In the given diagram, $ABCD$ and $ALMN$ are squares.
$AB = (3x - 1)$ cm and $AN = (x + 2)$ cm.



(a) Write down the length of LB in terms of x. [1]

(b) The area of the rectangle $LBYM$ is 10 cm^2.
Write down an equation in x and show that it reduces to $2x^2 + x - 16 = 0$. [2]

(c) Solve the equation $2x^2 + x - 16 = 0$, giving your solutions correct to two decimal places. [4]

(d) Which value of x do you have to reject and why? [2]

(e) Hence, calculate the perimeter of $LBYM$, giving your answer to the nearest millimetre. [2]

---

3 Singapore and Kuala Lumpur are 350.7 km apart.

(a) Ms Wong travelled by car from Singapore to Kuala Lumpur (KL) at an average speed of 90 km/h. How long did the journey take? [1]

(b) Ms Wong left Singapore at 0600. If she had a meeting to attend in KL at 1000, was she early or late for this meeting? [1]

(c) After the 3-hour meeting, Ms Wong took a one-hour lunch-break before making her return journey. She wanted to reach Singapore before the evening peak-hour commenced at 4pm. If the speed limit is 100 km/h, would she be able to reach Singapore by 4pm? [3]

(d) The upcoming Singapore-KL high-speed-rail (HSR) train line boasts a travelling time of 99 minutes in a single direction between the two cities. What is the average speed of the train? [1]

(e) The maximum speed of the train is expected to be 300 km/h. What is the percentage decrease in speed as mentioned in (d), compared to the expected speed? [2]

---

4 A bag contains 6 tennis-balls comprising of 4 green balls and 2 red balls.

Amy selects a ball at random from the bag and then replaced. She randomly selects another ball from the same bag.

(a) Draw a probability-tree diagram to represent the outcomes. [1]

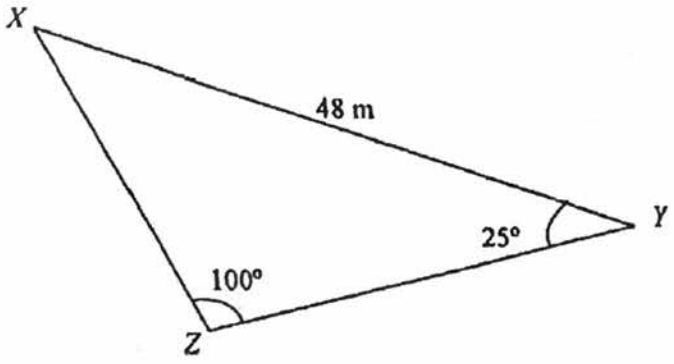
(b) Find, in its simplest form, the probability that the selected balls

(i) are green, [1]

(ii) are of different colours, [2]

(iii) include at least one red ball. [2]

5



X, Y and Z are on level horizontal ground. The bearing of Y from X is 100°.
$XY = 48$ m, angle $XZY = 100°$ and angle $XYZ = 25°$.

(a) Calculate

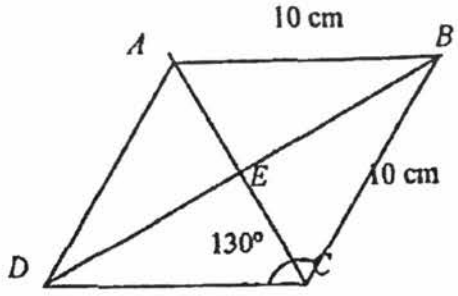
(i) the bearing of X from Y, [1]

(ii) the bearing of Z from X, [2]

(iii) the shortest distance from Z to XY. [3]

(b) If there is a tower of height 10 m at X, calculate the angle of depression of Y from the top of the tower. [2]

6



The diagram shows a cross-section of a rhombus cookie-box, $ABCD$, and E is the intersection-point of AC and BD.
AB // DC and AD // BC, $AB = CD = 10$ cm and angle $BCD = 130°$.

(a) **(i)** Explain why angle AEB is a right-angle. [1]

(ii) Calculate BD. [2]

(iii) Calculate the length of EC. [1]

(iv) Hence, calculate the area of triangle BCD. [1]

(b) A geometrically similar smaller version of the cookie-box is necessary for smaller quantities of cookies. In the smaller cookie-box, $AB = 8$ cm.

Find the cross-sectional area of the smaller cookie-box. [2]

7 (a) The following table shows the scores of 30 students from Secondary 4 Ace in their Mathematics Examination.

80	88	96	60	59	70	88	97	69	60
39	37	69	74	47	92	72	49	58	66
88	82	100	95	56	77	99	62	79	63

(i) Calculate the mean score for the students in Secondary 4 Ace. [1]

(ii) Calculate the standard deviation for the scores above. [1]

(b) The mean and standard deviation of Secondary 4 Bravo for the same examination are as follow:-

Mean Score	71.75
Standard Deviation	15.6

(i) Which class performed better? Support your claim with evidence. [2]

(ii) Which class had more consistent results? Support your claim with evidence. [2]

8 A funnel is in the form of an inverted right circular cone. Figure 1 shows a vertical cross-section of the funnel. It contains oil and water (which do not mix). The depths of water and oil are all 10 cm, with water at the bottom. It is given that the height of the funnel is 30 cm and the base radius is 9 cm.

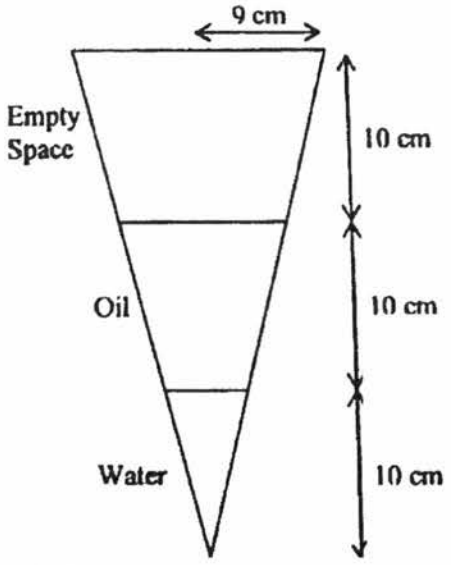


Figure 1

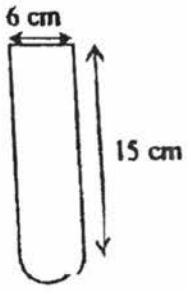


Figure 2

(a) Find the volume of the funnel in terms of π. [1]

(b) Find the fraction of

(i) $\dfrac{\textit{volume of oil}}{\textit{volume of water}}$, [2]

(ii) $\dfrac{\textit{surface area of the funnel in contact with water}}{\textit{total surface area of the interior of the funnel}}$. [2]

(c) All the water in the funnel is then drained through the tap at the vertex of the funnel, into another container formed by a cylinder of diameter 6 cm and surmounted by a hemisphere at the lower part of the cylinder, as shown in Figure 2. The height of the cylindrical part of the container is 15 cm.
Find the depth of water in this container.
(Note: Only the water is drained; the oil remains in the funnel.) [3]

9 Two outlets of a new fast-food chain sell three types of soft drinks, namely Coke, Sprite and Lemon Tea. The tables below show the sales of the soft drinks in the afternoon and evening respectively.

	Afternoon		
	Coke	Sprite	Lemon Tea
Outlet *A*	280	200	150
Outlet *B*	200	300	350

	Evening		
	Coke	Sprite	Lemon Tea
Outlet *A*	420	300	260
Outlet *B*	350	420	540

The sales of the soft drinks in the afternoon are represented by the matrix **A**, where

$$\mathbf{A} = \begin{pmatrix} 280 & 200 & 150 \\ 200 & 300 & 350 \end{pmatrix}.$$

(a) Write down the 2×3 matrix **E** representing the sales in the evening for the two outlets respectively. [1]

The cost price of supplying the soft drinks to the fast-food chain is \$1.20, \$1.00 and \$1.50 for Coke, Sprite and Lemon Tea respectively. The selling price for each soft drink is \$2.00, \$2.00 and \$3.50.
The cost price of supplying the soft drinks is represented by matrix **C**, where

$$\mathbf{C} = \begin{pmatrix} 1.20 \\ 1.00 \\ 1.50 \end{pmatrix}.$$

(b) Write down the column matrix **S** representing the selling price of the soft drinks for the three types of soft drinks respectively. [1]

(c) Calculate $\mathbf{T} = \mathbf{A} + \mathbf{E}$, and describe what matrix **T** represents. [2]

(d) Evaluate **AC** and describe what is represented by the elements of **AC**. [2]

(e) Evaluate $\mathbf{T}(\mathbf{S} - \mathbf{C})$, and explain what the elements of $\mathbf{T}(\mathbf{S} - \mathbf{C})$ represent. [2]

(f) (i) If the fast-food chain's general manager would like to evaluate the combined total amount in sales for both outlets for the day, write down the matrix operation he needs to calculate. [1]

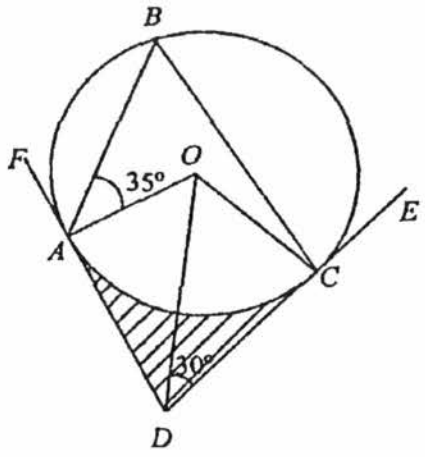
(ii) Evaluate the matrix that you have specified in part (i) above. [1]

10 (a) (i) Find the value of each interior angle of a regular 15-sided polygon. [2]

(ii) An n-sided polygon has 3 interior angles measuring 140° each.
The remaining interior angles all measure $y°$ each.

Find an expression for y in terms of n. [2]

(b)



The diagram shows a circle ABC, with centre O.
FAD and DCE are tangents to the circle, and $OA = OC = 8$ cm.
Angle $OAB = 35°$ and angle $CDO = 30°$.

(i) Name the pair of congruent triangles. [1]

(ii) Find

(a) angle DOA, [1]

(b) angle CBA, [1]

(c) angle ECB. [1]

(d) the area of the shaded region. [2]

11 **Answer the whole of this question on a sheet of graph paper.**
From the top of a mountain, Barry fires a pellet from an air gun upwards into the air. The height, h metres, of the pellet from Barry t seconds after it is released can be modelled by the equation $h = 1 + 10t - 3t^2$.

Some corresponding values of t and h are given in the table below.

t	0	1	2	3	4	5	6
h	1	8	9	4	m	-24	-47

(a) Calculate the value of m. [1]

(b) Using a scale of 2 cm to represent 1 second, draw a horizontal t-axis for $0 \le t \le 6$.
Using a scale of 1 cm to represent 5 metres, draw a vertical h-axis for $-50 \le h \le 10$.

On your axes, plot the points given in the table and join them with a smooth curve. [3]

(c) Use your graph to estimate

(i) the maximum height of the pellet above ground level, [1]

(ii) the length of time that the pellet was more than 2 metres above ground level, [2]

(iii) the time elapsed before the pellet reaches the same level as it was fired from. [1]

(d) By drawing a tangent, find the gradient of the curve at $(5, -24)$. [3]
State the units of your answer.

12 From July 2017 onwards, the price of water to households will be increased in two steps, on 1 July 2017 and on 1 July 2018. At the same time, the Government will be increasing the annual GST Voucher – U-Save rebate for eligible HDB households by between $40 and $120, depending on the flat type. The average change in water bill after the increased U-Save rebates is given in Table A on the next page.

(a) Show that for a 4-room HDB flat, the U-Save Rebate given in July 2017 is $7. [1]

Table B shows how the water tariffs will be increased between 2017 and 2018.
Charlie owns a new 4-room build-to-order (BTO) HDB flat in Woodleigh.
Read and understand the contents of the utility bill dated June 2017 in Table C.

(b) Assuming that the amount of water Charlie used in July 2017 is the same as that for June 2017, calculate the individual charges in July 2017 for

(i) water usage (reading), [1]
(ii) waterborne fee, [1]
(iii) water conservation tax, [1]
(iv) total cost of water services (after deduction of U-Save Rebate). [1]

(c) Assuming that the amount of water Charlie uses for July 2018 is the same as that for June 2017, calculate the total cost of water services in July 2018 (before the U-Save Rebate). [3]

(d) Why do you think that average changes in 2017 and 2018 bills are increasing from 1-room HDB flats to the executive/multi-generation flats? [1]

Table A: Average Change in Water Bill after Increased U-Save Rebates (by HDB Flat Type)

Source: https://www.pub.gov.sg/Documents/WaterPriceRevisionsBrochure.pdf

Water Bill	1-room HDB flat	2-room HDB flat	3-room HDB flat	4-room HDB flat	5-room HDB flat	Executive / Multi-Generation HDB flat
Before price increase	$23	$29	$33	$42	$44	$49
After price increase (2017)	$26	$34	$37	$47	$50	$55
After increased U-Save rebates (2017)	$16	$24	$29	$40	$45	$51
Average change in 2017 Bill	– $7	– $5	– $4	– $2	+ $1	+ $2
Average change in 2018 Bill	– $3	$0	+ $2	+ $5	+ $8	+ $11

Table B: Water Price Revisions

Source: https://www.pub.gov.sg/Documents/WaterPriceRevisionsBrochure.pdf

		Before 1 July 2017		From 1 July 2017		From 1 July 2018	
		Water Price ($/m³)		Water Price ($/m³)		Water Price ($/m³)	
		0 – 40m³	> 40m³	0 – 40m³	> 40m³	0 – 40m³	> 40m³
Potable Water	Tariff	$1.17	$1.40	$1.19	$1.46	$1.21	$1.52
	Water Conservation Tax (% of water tariff)	$0.35 (30% of $1.17)	$0.63 (45% of $1.40)	$0.42 (35% of $1.19)	$0.73 (50% of $1.46)	$0.61 (50% of $1.21)	$0.99 (65% of $1.52)
Used Water	Waterborne Fee	$0.28	$0.28	$0.78	$1.02	$0.92	$1.18
	Sanitary Appliance Fee	$2.80 per fitting*		Combined into Waterborne Fee		Combined into Waterborne Fee	
Total Price		$2.10	$2.61	$2.39	$3.21	$2.74	$3.69

Note: Water is charged per cubic metre (m^3), which is equivalent to 1000 litres.
All figures are before GST.
*For the calculation of total price, the Sanitary Appliance Fee is converted to its volumetric equivalent.

Table C: Utility Bill for June 2017

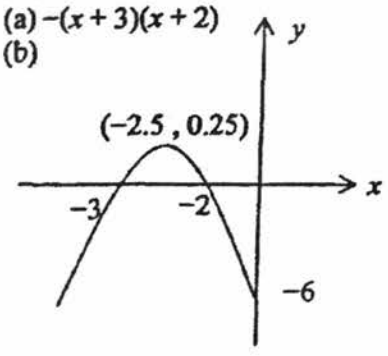
June 2017 Bill
Account No. ########

Breakdown of Current Charges	Usage	Rate ($)	Amount ($)	Total ($)
Electricity Services				
Reading taken on 28 Jun 2017: 53902	736 kWh	0.2139	157.43	157.43
Water Services by Public Utilities Board				
Reading taken on 28 Jun 2017 : 5094.9	35.8 Cu M	1.1700	41.89	
Waterborne Fee	35.8 Cu M	0.2803	10.03	
Water Conservation Tax	$41.89	30%	12.57	
Sanitary Appliance Fee	2 Fittings	2.8032	5.61	70.10
Refuse Removal by Veolia ES Singapore P L	1 Qty	7.71	7.71	7.71
Subtotal			235.24	235.24
GST	$235.24	7%	16.46	16.46
Current Charges: (Inclusive of GST)				$251.70

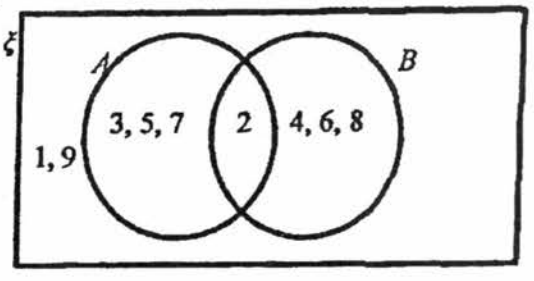
d of Paper

Answer Key

1. (a) −0.00 049 971
   (b) $3x-4$
2. (a) 36 049
   (b) 35 500
3. $(2x+3y)(3a-b)$
4. (a) $b=-\dfrac{4+c}{2}$
   (b) $b=-9$; $c=14$
5. $AC=5$ cm or 17.7 cm
6. (a) $-(x+3)(x+2)$
   (b)



7. $\dfrac{5x-4}{(x-2)^2}$
8. (a) 36
   (b) $\dfrac{1}{8}$
9. –
10. (a) $x=2$
    (b) $x=\dfrac{1}{3}$ or $x=3$
11. (a) $5x(5-6x)$
    (b) $(5x-2)(x+3)$
    (c) $3(2x+1)(2x-1)$
12. cheaper in Singapore
13. (a) 620000
    (b) 2.3 cm^2
14. 22.5%
15. (a) -
    (b) –
    (c) $t=5$
16. (a) $2x$
    (b) $90-x$ or $218-5x$
    (c) $x=32$
17. 525%
18. –
19. (a) $600=2^3\times3\times5^2$
    (b) −5
20.



21. (ai) 12b – 8a
    (aii) 3b – 2a
    (b) 21b – 6a
    (c) $\dfrac{3}{7}$
22. (a) $\begin{pmatrix}10\\5\end{pmatrix}$
    (b) 8.06
    (c) (1 , −2)
23. (b) 1.16 km

Answer Key

1. (a) $\frac{2p+7}{(p-2)(4p+3)}$

   (bi) $x=\frac{aT}{k}+a$

   (bii) $T=2k$

2. (a) $(2x-3)$ cm

   (b) –

   (c) $x=2.59$ or -3.09

   (d) –

   (e) 13.5 cm

3. (a) 3.90 h

   (b) She was early for the meeting.

   (c) She would not be able to reach Singapore by 4 pm.

   (d) $212.\dot{5}\dot{4}$ *km / h or* 213 *km / h* (*to* 3 *s.f.*)

   (e) $29.\dot{1}5\%$ *or* 29.2% (*to* 3 *s.f.*)

4. (a) –

   (bi) $\frac{4}{9}$

   (bii) $\frac{4}{9}$

   (biii) $\frac{5}{9}$

5. (ai) 280°

   (aii) 165°

   (aiii) 16.9 m

   (b) 11.8°

6. (ai) –

   (aii) 18.1 cm

   (aiii) 4.23 cm

   (aiv) 38.3 cm^2

   (b) 49.0 cm^2

7. (ai) $72.3\dot{6}$ *or* 72.4 (*to* 3 *s.f.*)

   (aii) 17.6

   (bi) –

   (bii) –

8. (a) $810\pi\ cm^3$

   (bi) $\frac{7}{l}$ *or* 7

   (bii) $\frac{l}{9}$

   (c) 4.33 cm

9. (a) $E=\begin{pmatrix} 420 & 300 & 260 \\ 350 & 420 & 540 \end{pmatrix}$

[Turn over

(b) $S=\begin{pmatrix} 2.00 \\ 2.00 \\ 3.50 \end{pmatrix}$

(c) $T=\begin{pmatrix} 700 & 500 & 410 \\ 550 & 720 & 890 \end{pmatrix}$

Matrix T represents the sales of Coke, Sprite and Lemon Tea in the afternoon and evening at outlets A and B respectively.

(d) $AC=\begin{pmatrix} 761 \\ 1065 \end{pmatrix}$

Matrix AC represents the total cost price of supplying soft drinks to the fast-food chain in the afternoon at outlets A and B respectively.

(e) $T(S-C)=\begin{pmatrix} 1880 \\ 2940 \end{pmatrix}$

Matrix $T(S-C)$ represents the total profits in the afternoon and evening at outlets A and B respectively.

(fi) $\begin{pmatrix} 1 & 1 \end{pmatrix}\left[T\begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix}\right]$

(fii) (3770)

10. (ai) 156°

(aii) $y=\dfrac{180n-780}{n-3}$ or $180-\dfrac{240}{n-3}$

(bi) –

(biia) 60°

(biib) 60°

(biic) 65°

(biid) 43.8 cm^2

11. (a) m = –7

(b) –

(ci) 9.4 m

(cii) 3.15s

(ciii) 3.35s

(d) -22.64 m/s

12. (a) $7

(bi) $42.60

(bii) $27.92

(biii) $14.91

(biv) $78.44

(c) $97.91

(d) -

Name	Register Number	Class	Calculator Model

智行茲願

MANJUSRI SECONDARY SCHOOL
文 殊 中 學

PRELIMINARY EXAMINATION 2017

Subject:	Mathematics
Paper:	4048/01
Level:	Secondary 4 Express / 5 Normal (Academic)
Date:	7 August 2017
Duration:	2 hours
Setter:	Mr Lee Beng Huat

Candidates answer on the Question Paper
Additional materials: Geometrical Instruments

READ THESE INSTRUCTIONS FIRST

Write your Name, Register Number and Class on all the work you hand in.
Write in dark blue or black pen in the spaces provided on the Question Paper.
You may use a pencil for any diagrams or graphs.
Do not use staples, paper clips, highlighters, glue or correction fluid.

Answer **all** questions.
If working is needed for any question it must be shown with the answer.
Omission of essential working will result in loss of marks.
Calculators should be used where appropriate.
If the degree of accuracy is not specified in the question, and if the answer is not exact, give the answer to three significant figures.
For π, use either your calculator value or 3.142, unless the question requires the answer in terms of π.

At the end of the examination, fasten all your work securely together.
The number of marks is given in brackets [ ] at the end of each question or part question.
The total number of marks for this paper is **80**.

Marks Obtained
80

This paper consists of **15** printed pages including this cover page.

Mathematical Formulae

Compound Interest

$$\text{Total amount} = P\left(1+\frac{r}{100}\right)^n$$

Mensuration

Curved surface area of a cone $= \pi rl$

Surface area of a sphere $= 4\pi r^2$

$$\text{Volume of a cone} = \frac{1}{3}\pi r^2 h$$

$$\text{Volume of a sphere} = \frac{4}{3}\pi r^3$$

$$\text{Area of triangle } ABC = \frac{1}{2}ab\sin C$$

Arc length $= r\theta$, where θ is in radians

$$\text{Sector area} = \frac{1}{2}r^2\theta, \text{ where } \theta \text{ is in radians}$$

Trigonometry

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$a^2 = b^2 + c^2 - 2bc\cos A$$

Statistics

$$\text{Mean} = \frac{\sum fx}{\sum f}$$

$$\text{Standard deviation} = \sqrt{\frac{\sum fx^2}{\sum f} - \left(\frac{\sum fx}{\sum f}\right)^2}$$

Answer all the questions.

1 (a) Estimate, correct to the nearest whole number, the value of $\frac{4.97^2 - \sqrt{15}}{\sqrt[3]{30}}$ without the use of a calculator.

Answer ..... [1]

(b) Write down the following in order of size, smallest first.

$\sqrt{0.35}$ 35% 3.5 $\frac{35}{53}$

Answer ....., ..., ..., ... [2]

2 (a) Solve $\frac{x}{3} + 15 = 9$.

Answer $x =$ ..... [1]

(b) Simplify $15(x-13) + 14(13-x)$.

Answer ..... [2]

3 During a sale, there is a discount of 15% on all items selling in a shop. If the discounted price of a watch is \$182.75, find the original price of the watch before the discount.

Answer \$ ..... [2]

4 (a) Simplify $18a^3b \div 6ab^{-3}$.

Answer ..... [1]

(b) Given that $\sqrt{2} \times 4^n = 1$, find the value of n.

Answer $n =$ ..... [2]

---

5 $\xi = \{\text{integers } x : 11 \leq x < 19\}$
$A = \{\text{multiples of 3}\}$
$B = \{\text{prime numbers}\}$

List the elements in

(a) A',

Answer ..... [1]

(b) $A' \cap B$,

Answer ..... [1]

(c) $(A \cup B)'$.

Answer ..... [1]

---

6 Factorise completely $3ap + 8bq - 12aq - 2bp$.

Answer ..... [2]

7 The plan of a museum is drawn to a scale of 1 : 500.

(a) Find the length, in metres, of a corridor which is represented by a line 10.5 cm long on the plan.

Answer ..... m [1]

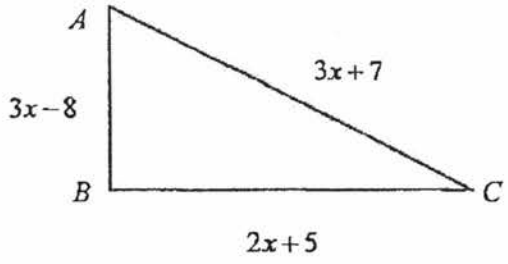
(b) The area of the floor of a bookshop is 500 m^2. Find, in square centimeters, its area on the plan.

Answer ..... cm^2 [2]

8 After Pluto is no longer considered a planet, Mercury is now the smallest planet while Jupiter is still the biggest planet in our solar system.
Planet Mercury has a mass of 3.3×10^{23} kg and Jupitar has a mass of 1.898×10^{27} kg.
How many times is the mass of Jupiter compare to the mass of Mercury?
Give your answer in standard form, correct to 3 significant figures.

Answer ..... [2]

9 The diagram shows a triangle ABC.



(a) One property of a triangle is that the length of the longest side must be less than the sum of the lengths of the two shorter sides.
Form an inequality in x and solve it.

Answer ..... [2]

(b) Given also that the perimeter of the triangle is no more than 85 cm.
Find the largest possible length of the longest side, given x is a prime number.

Answer ..... cm [3]

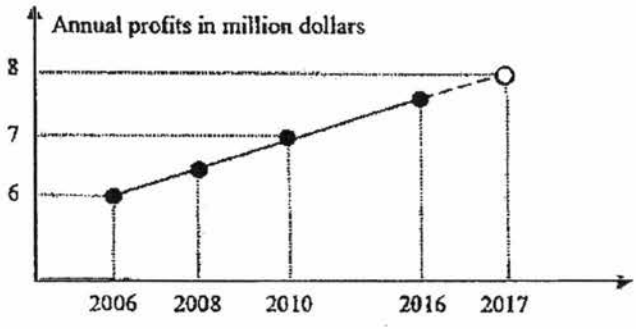
10 Write as a single fraction in its simplest form $\frac{x}{x^2-4}-\frac{2}{2-x}$.

Answer ..... [2]

11 Given that n is a positive integer and $n-\frac{1}{n}=5$. Find the value of $n^2+\frac{1}{n^2}$.

Answer ..... [2]

12 The CEO used the following line graph to show the annual profits made by the company over a number years.



State one aspect of the graph that may be misleading and explain how the annual profits in 2017 can be projected wrongly.

Answer .....

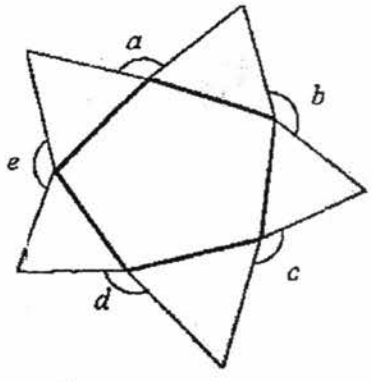
.....

..... [2]

13 Given that $x \quad y = 0.2 : 0.5$ and $y : z = \frac{1}{3} : \frac{1}{2}$, find $x : y : z$.

Answer ..... [3]

14 The diagram shows a pentagon and five equilateral triangles.
Calculate the sum of the angles a, b, c, d and e.

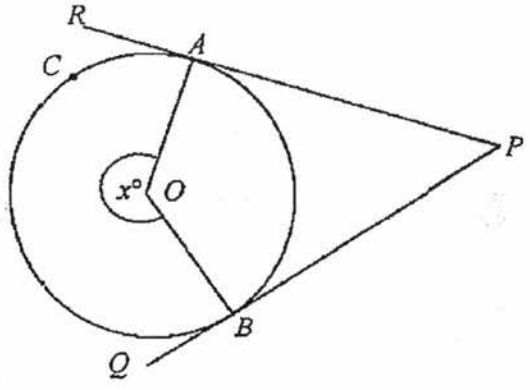


Answer ..... [3]

15 Jane can make 8 dresses in 7 hours. Judy can make 7 dresses in 6 hours.
If Jane and Judy continue to make dresses at the same rate, how long will it take them to make 20 dresses? Give your answer in hours and minutes, to the nearest minutes.

Answer ..... hours ..... minutes [3]

16 A, B and C are points on the circle centre O. PBQ and PAR are tangents to the circle.
Reflex $\angle AOB = x°$.



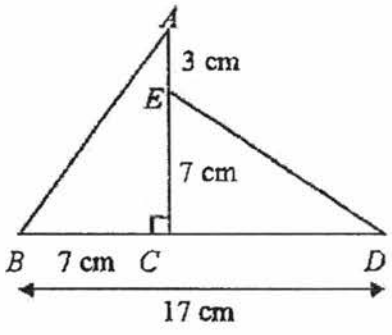
(a) Given C is a point along the major arc AB, express $\angle ACB$ in terms of x.

Answer $\angle ACB =$ ..... [2]

(b) Express $\angle APB$ in terms of x.

Answer $\angle APB =$ ..... [2]

17 In the diagram, $AE = 3$ cm, $EC = 7$ cm, $BC = 7$ cm and $BD = 17$ cm.
Name a pair of congruent triangles, stating your case of congruency.



Answer .....

.....

.....

..... [3]

18 (a) Express 168 as a product of its prime factors.

Answer $168 =$ ..... [2]

(b) Find the smallest positive integer m such that $\frac{168}{\sqrt{m}}$ is a perfect cube.

Answer $m =$ ..... [2]

(c) Alice uses all 168 cubes of side 1 unit to make a cuboid. Each of the sides of the cuboid is made up of more than 3 cubes. Find the number of cubes on each side of the cuboid.

Answer ..... by ..... by ..... [2]

19

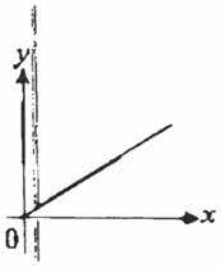


Figure 1

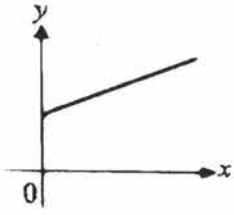


Figure 2

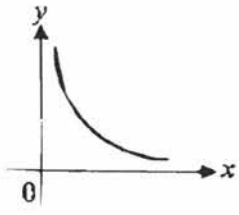


Figure 3

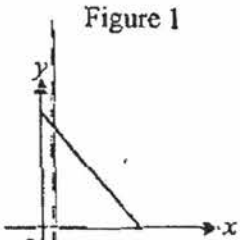


Figure 4

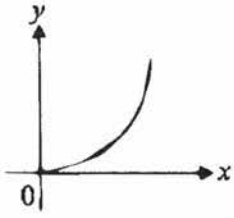


Figure 5

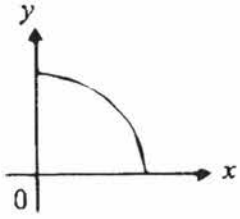


Figure 6

From the graphs above, select one which illustrates each of the following statements.

(a) The amount of pressure y, exerted is inversely proportional to the surface area of a cube, of sides x cm.

Answer Figure ..... [1]

(b) The surface area y, of a sphere is proportional to the square of the radius, x cm.

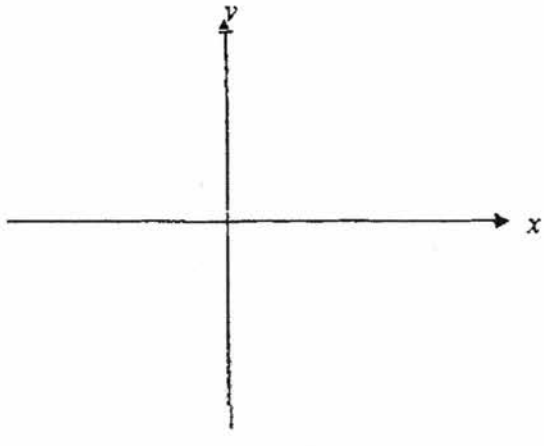
Answer Figure ..... [1]

(c) The total taxi fare \$$y$, of a fixed flag down fees plus x metres of distance travelled, given 1 cent is charged for every metre travelled.

Answer Figure ..... [1]

---

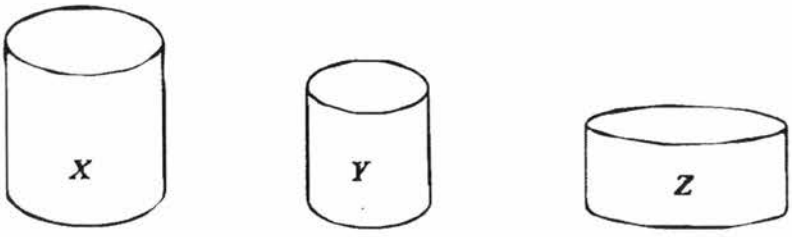
20 Sketch the graph of $y = (x+3)(5-x)$ on the axes below, indicating its turning point and all the intercepts on the axes clearly.



[3]

---

21 There are three mugs X, Y and Z. Mugs X and Y are geometrically similar. The volume of X and Y are 512 cm^3 and 216 cm^3 respectively.



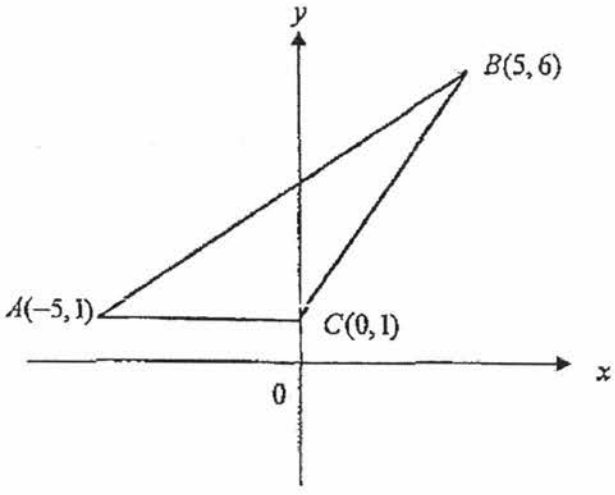
(a) Find the ratio of the surface area of X to Y.

Answer ..... : ..... [2]

(b) The volume of Y is given by the formula $V = \pi r^2 h$ where h is the height of the mug and r the radius of the circular base. Find the volume of Z which has $\frac{2}{3}$ the height of Y and twice the radius of the circular base of Y.

Answer ..... cm^3 [2]

22 In the diagram, the vertices of a triangle A, B and C are $(-5,1)$, $(5, 6)$ and $(0,1)$ respectively.



Find

(a) the equation of line BC,

Answer ..... [2]

(b) the equation of the line which passes through A and is parallel to $3x+6y=5$,

Answer ..... [2]

(c) the area of the triangle ABC.

Answer ..... units^2 [2]

23 A frustum and a cone were obtained by slicing a conical container, height $2h$, as shown in Diagram I at the midway of the height. These figures were then attached to a cylinder, height h, to form a new container as shown in Diagram II. Water was poured into the empty container in Diagram II at a constant rate from the top and it took 33 seconds to fill to the brim.

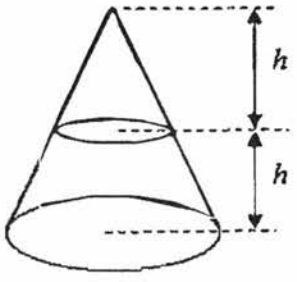


Diagram I

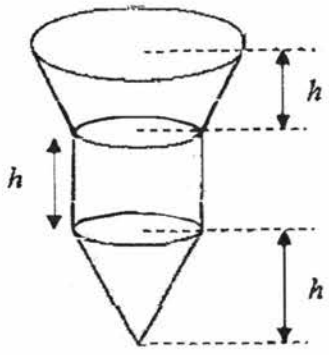


Diagram II

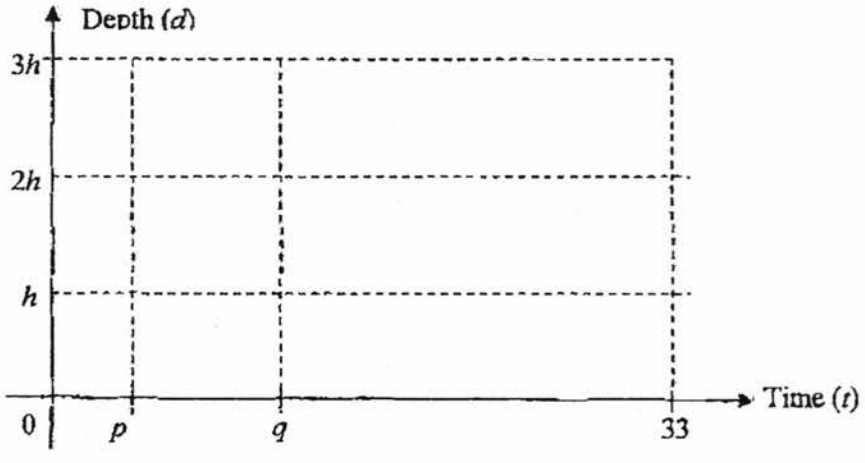
Given that it took p seconds for the water to reach the container to a height of h and q seconds to reach the height $2h$.

(a) Find the value of p and of q.

Answer $p =$ ..... , $q =$ ..... [3]

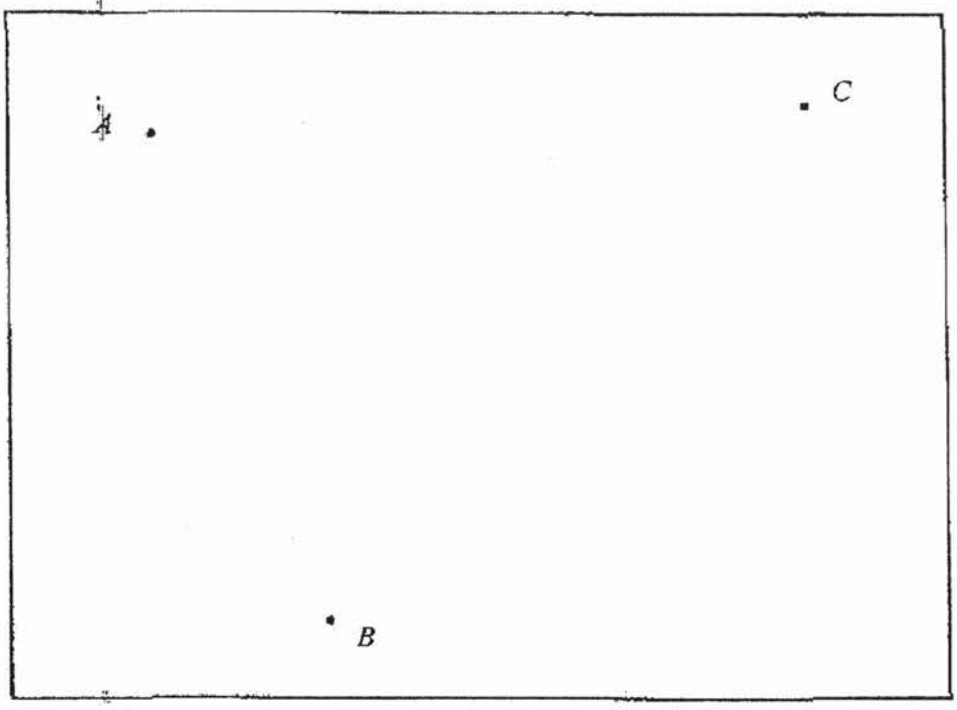
(b) On the grid in the answer space, sketch the graph of the depth of water (d) against the time (t).

Answer



[2]

24 The diagram below is part of the scale drawing of a rectangular field showing the position of 3 soccer players, A, B and C. In the drawing, 1 cm represents 5 m.



(a) The ball is placed in the field equidistant from A and B and 30 m from C.
By constructing suitable lines and arcs in the answer space above, mark and label clearly the position of the ball X.

[2]

(b) Measure and state the distance between player A and the ball X.

Answer ..... m [1]

(c) Both players A and C are to run for the ball.
Player A can run at a speed of 6 m/s while player C's top speed is 7 m/s.
Who will get the ball first? Show your working clearly.

Answer Player ..... [2]

------ *End of Paper* ----

Answer all the questions.

1 (a) It is given that $H = \frac{k}{\sqrt{m-n}}$.

(i) Find H when $k = 12$, $m = 6$ and $n = -3$. [1]

(ii) Express n in terms of H, k and m. [2]

(b) Simplify $\frac{9a^2b}{(2a)^2} \div \frac{12ab^3}{8b^5}$, leaving your answer in positive indices. [2]

(c) Solve the equation $\frac{5}{x+7} + \frac{4}{11-x} = 1$. [3]

(d) Solve the following simultaneous equations:

$$5x - 3y = 22$$
$$y - 4x + 12 = 0$$ [3]

---

2 (a) Alex needs a loan of \$45 000 to buy a new car.
Bank *ABC* charges an interest rate of 2.45% per annum compounded monthly.
Bank *XYZ* charges a simple interest rate of 2.65% per annum.

If Alex plans to take a five year loan, which bank should he loan from?
Justify your answer. [4]

(b) Alex buys the new car on hire purchase. He uses the \$45 000 loan to pay the 30% down payment and then makes monthly payments of \$1950 for 5 years.

(i) Calculate the cash price of the new car. [1]

(ii) Calculate the interest Alex has to pay in this hire purchase scheme. [2]

(iii) Calculate the rate of simple interest charged for hire purchase.
Leave your answer in 3 decimal places. [1]

(c) Alex took his new car for a road trip from Singapore to Bangkok.
Before the trip. Alex paid S\$109 for 50 litres of petrol to fill up the tank.
In Bangkok, Alex paid a total of 9 408 Thai bahts for 320 litres of petrol he pumped into his car.

Given S\$1 = 24.5 Thai bahts.

Alex said that the petrol price in Bangkok is less than half the petrol price in Singapore.
Do you agree? Justify your answer. [3]

---

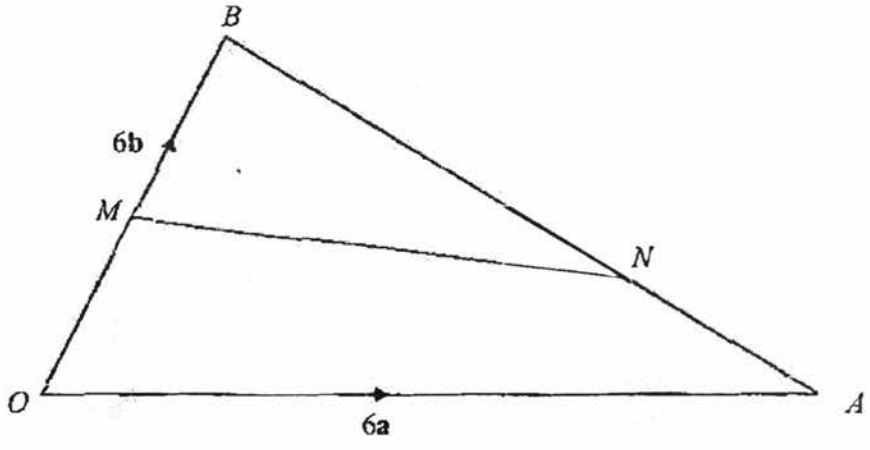
3 (a) Given $\overrightarrow{PQ} = \begin{pmatrix} -7 \\ 24 \end{pmatrix}$ and $\overrightarrow{PS} = \begin{pmatrix} k \\ 12 \end{pmatrix}$.

(i) Find $\left|\overrightarrow{PQ}\right|$. [1]

(ii) Find the value of k such that P, Q and S are collinear. [2]

(iii) Find the coordinates of Q if P is the point $(10, -15)$ [1]

(b) In the diagram, $\overrightarrow{OA} = 6\mathbf{a}$, $\overrightarrow{OB} = 6\mathbf{b}$ and $3\overrightarrow{AN} = \overrightarrow{AB}$. M is the mid-point of OB.



Express, as simply as possible, in terms of **a** and/or **b**,

(i) $\overrightarrow{AN}$, [1]

(ii) $\overrightarrow{ON}$, [1]

(iii) $\overrightarrow{NM}$. [1]

P is a point not shown in the diagram such that $\overrightarrow{MP} = 3\overrightarrow{MN}$.

(iv) Find the position vector of P. [1]

(v) Make two statements about the points O, A and P. [2]

Calculate the value of

(vi) $\dfrac{\text{area of } \Delta AMN}{\text{area of } \Delta BMN}$, [1]

(vii) $\dfrac{\text{area of } \Delta BMN}{\text{area of } \Delta BOA}$. [1]

4 A photocopier prints pages in either 'black and white' or in 'colour'.

(a) In one minute, this photocopier prints x pages in black and white. Write down an expression in terms of x, for the number of seconds it takes to print one page in black and white. [1]

(b) In one minute, this photocopier prints 2 more copies in black and white than it does in colour. Write down an expression, in terms of x, for the number of seconds it takes to print one page in colour. [1]

(c) It takes 1.2 seconds longer to print one page in colour than it takes to print one page in black and white. Form an equation in terms of x and show that it reduces to

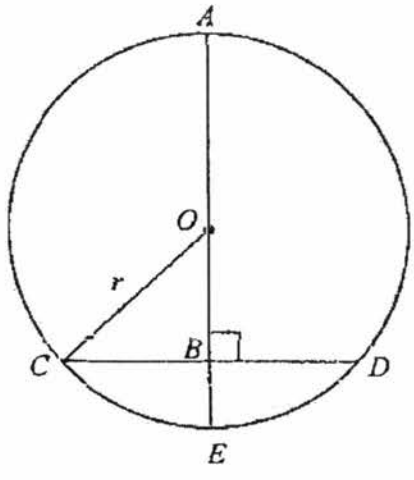
$$x^2 - 2x - 100 = 0.$$ [3]

(d) Solve the equation $x^2 - 2x - 100 = 0$, leaving your answers in 2 decimal places. [2]

(e) Hence, find the time taken in minutes and seconds to print 85 pages in colour. Give your answer corrected to the nearest second. [2]

---

5 The diagram shows a circle, centre O and radius r cm. AB is perpendicular to the CD. Given that $AB = 9$ cm and $CD = 6$ cm.



(a) Express OB in terms of r. [1]

(b) Show that the radius of the circle = 5 cm. [3]

(c) Calculate the area of the minor segment CDE. [4]

6 (a) The first four terms in a sequence of numbers, $u_1, u_2, u_3, u_4, \ldots$, are given below

$$u_1 = 1^2 + 1 = 2$$
$$u_2 = 2^2 + 3 = 7$$
$$u_3 = 3^2 + 5 = 14$$
$$u_4 = 4^2 + 7 = 23$$

(i) Write down an expression for u_5 and show that $u_5 = 34$. [1]

(ii) Find an expression, in terms of n, for u_n. [2]

(iii) Evaluate u_{30}. [1]

(b) A toy manufacturing company makes toy boats and toy cars.
The following table is used in calculating the cost of manufacturing each toy boat and toy car.

	Labour (hours)	Wood (blocks)	Paint (tins)
Boat	6	4	5
Car	4	2	3

This information can be represented by the matrix $T = \begin{pmatrix} 6 & 4 & 5 \\ 4 & 2 & 3 \end{pmatrix}$.

(i) Labour cost \$8 per hour, wood cost \$5 per block and paint costs \$3 per tin.
Represent the cost by a 3×1 column matrix C. [1]

(ii) Evaluate the matrix $V = TC$. [2]

(iii) State what the elements of V represent. [1]

(iv) Given that $W = (80 \quad 50)$,
evaluate WV and explain what the answer represents. [2]

7 The stem and leaf diagram below shows the mass of 21 students.

Stem	Leaf
4	7 7 8
5	0 3 3 4 6 6 6 8 9
6	1 2 4 7 7 8 8
7	0 1

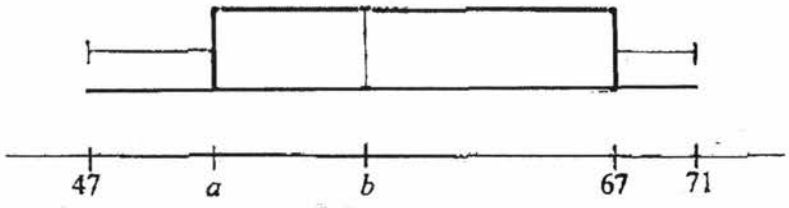
Key : 5|2 means 52 kg

(a) Find

(i) the modal mass, [1]

(ii) the percentage of students more than 62 kg. [1]

(b) The box-and-whisker plot for the above distribution is shown below.



(i) Write down the value of a and of b. [2]

(ii) Find the interquartile range. [1]

(c) Two students are selected from the group.
Calculate the probability that only one student is at least 50 kg. [2]

8 The diagram shows three markers A, B and C placed on a horizontal ground.
The marker A is 250 m from C and the marker B is 400 m due West from A.
Angle $BAC = 65°$

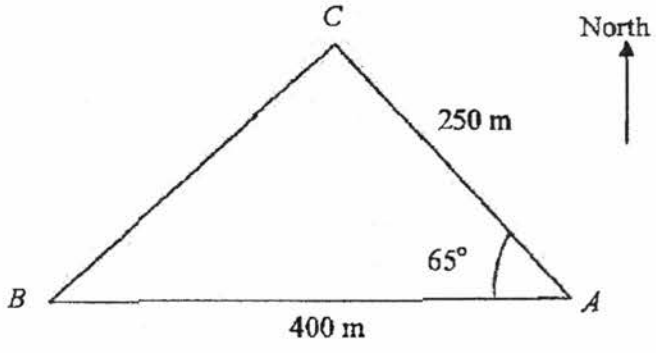


Diagram is not drawn to scale

(a) Calculate

(i) the length BC, [3]

(ii) the area of the triangle ABC, [2]

(iii) the angle ABC and [2]

(iv) the bearing of C from B. [1]

(b) An eagle is hovering vertically above A.
The angle of elevation of the eagle from B is 18°.

Find the angle of depression of C from the eagle. [3]

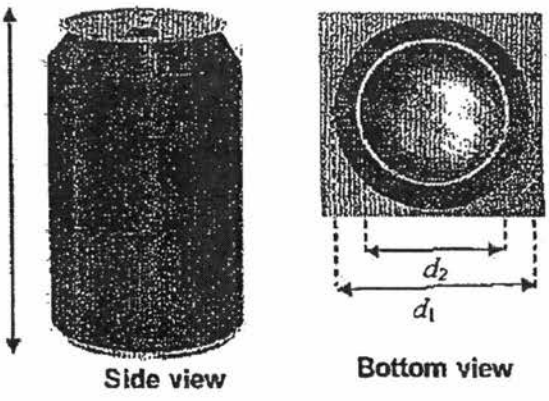
9 Some information about a soda can is shown below.

Soda Can

Height (h): 11.4 cm

Outer Diameter (d_1): 6.4 cm

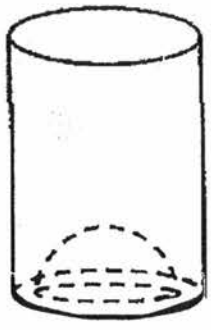
Inner Diameter (d_2): 5.0 cm



Density of soda: 1.2 g/cm^3

Safety Information: the soda can is to be filled to a maximum of 95% of its total volume.

In this question, the soda can (above) can be modelled as a cylinder with an inner hemisphere that is hollowed inwards (concave) at the base of the can.



(a) Calculate

(i) the base area, in square centimetres, of the soda can and [2]

(ii) the total volume, in cubic centimetres, of the soda can. [2]

(b) The material used to make the wall of the soda can must be carefully chosen such that the total mass of each filled soda can is below 620 g.
The manager of the soda manager proposed to use an alloy which has a mass of 0.8 g for every 1 cm^2 to make the can.
If the thickness of the soda can is negligible, will you accept his proposal?
Justify your answer with suitable calculation.

[6]

10 **Answer the whole of this question on a sheet of graph paper.**

The table below gives the values of x and y connected by the equation $y = \frac{x^2}{6} + \frac{12}{x} - 6$.

The table below shows some corresponding values of x and y.

x	1	1.5	2	3	4	5	6	7
y	6.2	2.4	0.7	–0.5	–0.3	0.6	k	3.9

(a) Calculate the value of k. [1]

(b) Using a scale of 2 cm to 1 unit, draw a horizontal x-axis for $0 \le x \le 8$.
Using a scale of 2 cm to 1 unit, draw a vertical y-axis for $-1 \le y \le 7$.
On your axes, plot the points given in the table and join them with a smooth curve. [3]

(c) By drawing a tangent, find the gradient of the curve at $x = 1.5$. [2]

(d) (i) On the same axes, draw the line $y = \frac{x}{6}$. [1]

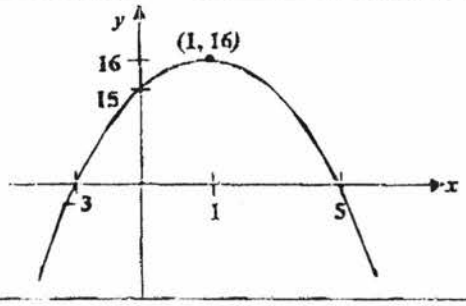
(ii) Write down the x-coordinate of the points where the line intersects the curve. [2]

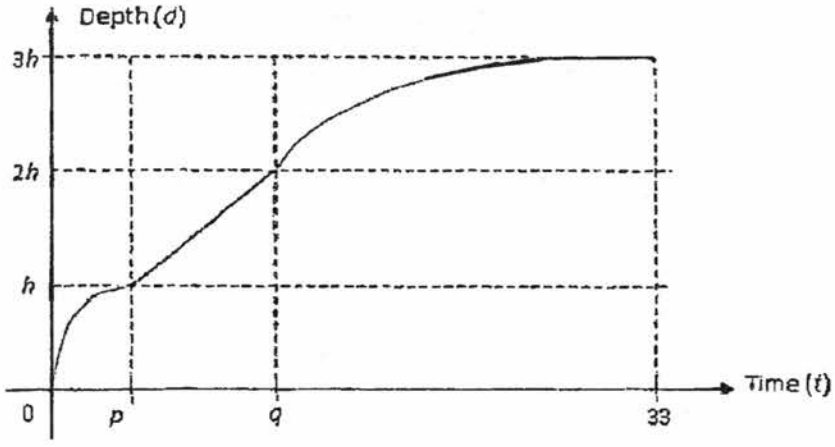
(iii) These values of x is a solution of the equation $x^3 - x^2 + Ax + B = 0$.
Find the value of A and value of B. [2]

... End of Paper 2 ...

Manjusri Secondary School
Preliminary Examination 2017
Elementary Mathematics 4048 Paper 1
Answer key

1(a)	7
1(b)	$35\%, \sqrt{0.35}, \frac{35}{53}, 3.5$
2(a)	-18
2(b)	$x-13$
3	\$215
4(a)	$3a^2b^4$
4(b)	$-\frac{1}{4}$
5(a)	11, 13, 14, 16, 17
5(b)	11, 13, 17
5(c)	14, 16
6	$(3a-2b)(p-4q)$
7(a)	52.5 cm
7(b)	3.6cm
	$20\ cm^2$
8	5.75×10^3
9(a)	$x>5$
9(b)	28 cm
10	$\frac{3x+4}{(x+2)(x-2)}$ or $\frac{3x+4}{x^2-4}$
11	27
12	Data from Year 2007, 2009, 2011 to 2015 are missing. The scale in horizontal axis is not consistent. The line graph may not be sloping upward as it seem to be.
13	4 : 10 : 15
14	660°

15	8 hours 40 minutes
16(a)	$\frac{1}{2}(360^\circ - x)$ or $180^\circ - \frac{1}{2}x$
16(b)	$x - 180^\circ$
17	$BC = EC = 7$ cm $CD = CA = 10$ cm $\angle ACB = \angle DCE = 90^\circ$ $\therefore \Delta ABC \equiv \Delta DEC$ (SAS)
18(a)	$2^3 \times 3 \times 7$
18(b)	441
18(c)	$4 \times 6 \times 7$
19(a)	Figure 3
19(b)	Figure 5
19(c)	Figure 2
20	
21(a)	16 : 9
21(b)	576 cm^3
22(a)	$y = x + 1$
22(b)	$y = -\frac{1}{2}x - \frac{3}{2}$ or $2y = -x - 3$
22(c)	12.5 units^2
23(a)	$p = 3$, $q = 12$

23(b)	
24(b)	25 ± 0.5 m
24(c)	Player A

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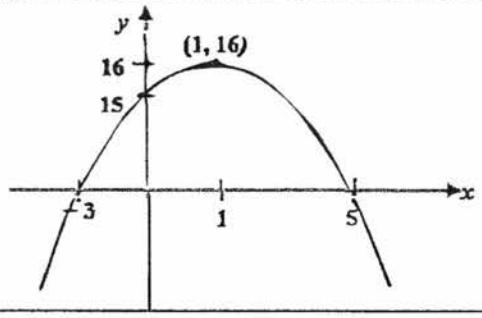
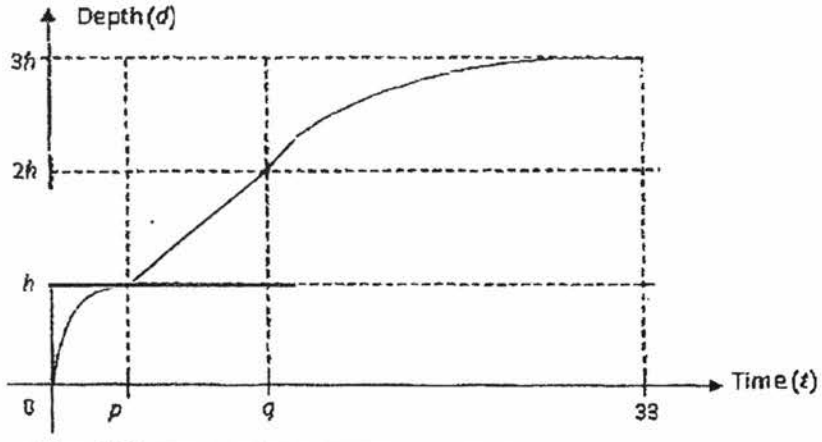
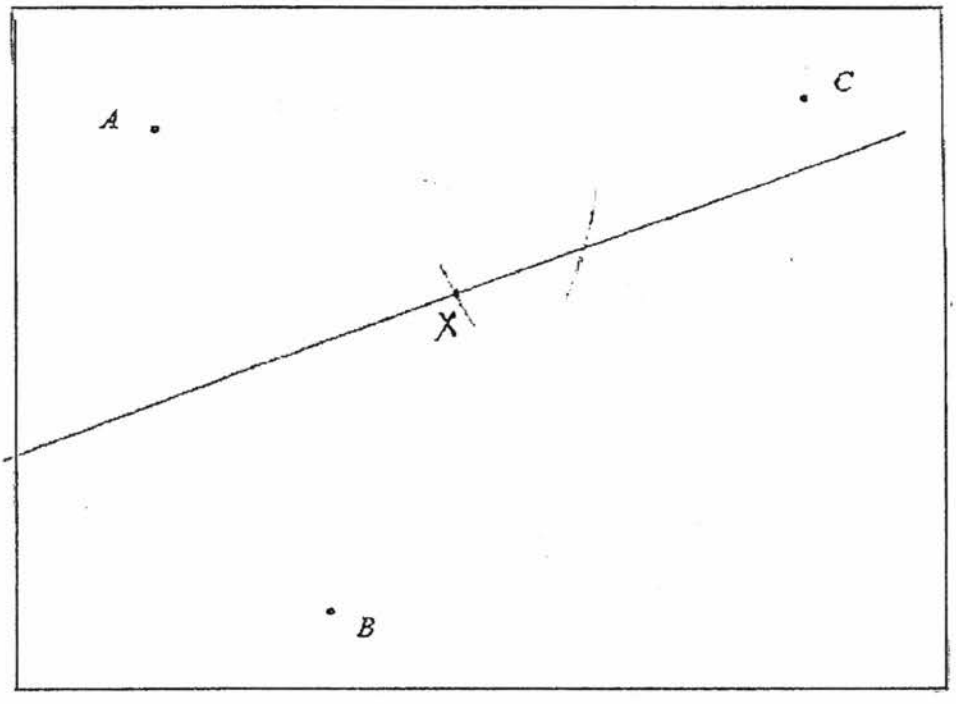
Marking Scheme

1 (a)	$\frac{5^2-\sqrt{16}}{\sqrt[3]{27}}=7$	B1
1 (b)	$\sqrt{0.35}\approx 0.59$ $35\%=0.35$ $\frac{35}{53}\approx 0.66$ $35\%,\ \sqrt{0.35},\ \frac{35}{53},\ 3.5$	M1 A1
2 (a)	$x+45=27$ $x=-18$	B1
2 (b)	$15(x-13)-14(x-13)$ $=x-13$	M1 A1
3	$\frac{100}{85}\times 182.75$ $=\$215$	M1 A1
4 (a)	$3a^2b^4$	B1
4 (b)	$2^{\frac{1}{2}}\times 2^{2n}=2^0$ $\frac{1}{2}+2n=0$ $n=-\frac{1}{4}$	M1 A1
5 (a)	11, 13, 14, 16, 17	B1
5 (b)	11, 13, 17	B1
5 (c)	14, 16	B1
6	$3ap-12aq+8bq-2bp$ $=3a(p-4q)+2b(4q-p)$ $=(3a-2b)(p-4q)$	M1 A1

7 (a)	1 cm : 500 cm 1 cm : 5 m 10.5 cm : $10.5 \times 5 = 52.5$ m	B1
7 (b)	$1 \text{ cm}^2 : 25 \text{ m}^2$ $500 \text{ m}^2 : \frac{500}{25} = 20 \text{ cm}^2$	M1 A1
8	$\frac{1.898 \times 10^{27}}{3.3 \times 10^{23}} \approx 5751$ $= 5.75 \times 10^3$	M1 A1
9 (a)	$3x - 8 + 2x + 5 > 3x + 7$ $x > 5$	M1 A1
9 (b)	$(3x-8) + (2x+5) + (3x+7) \leq 85$ $x \leq 10\frac{1}{8}$ Largest possible length = 3×7+7 = 28 cm	M1 B1 A1
10	$\frac{x}{x^2-4} + \frac{2}{x-2}$ $= \frac{x+2(x+2)}{(x+2)(x-2)}$ $= \frac{3x+4}{(x+2)(x-2)}$ or $\frac{3x+4}{x^2-4}$	M1 A1
11	$(n - \frac{1}{n})^2 = n^2 - 2 + \frac{1}{n^2}$ $n^2 + \frac{1}{n^2} = 5^2 + 2$ $= 27$	M1 A1
12	Data from Year 2007, 2009, 2011 to 2015 are missing. The scale in horizontal axis is not consistent. The line graph may not be sloping upward as it seem to be. (Do not accept: the vertical axis does not start from 0)	B1 B1
13	$x : y = 2 : 5$ $y : z = 2 : 3$ $x : y : z = 4 : 10 : 15$	B1 B1 B1

14	Sum of interior angles in pentagon $= (5-2)\times 180^\circ$ $= 540^\circ$	M1
	Sum of angles a, b, c, d and $e = 5(360^\circ) - 540^\circ - 10(60^\circ)$ $= 660^\circ$ (deduct one mark if student assumed regular pentagon)	M1 A1
15	In 1 hour, Jane made $\frac{8}{7}$ dresses. Judy made $\frac{7}{6}$ dresses. Both made $(\frac{8}{7}+\frac{7}{6})=\frac{97}{42}$ dresses. Time to make 20 dresses $= 20 \div \frac{97}{42}$ $= 8.659$ hour $= 8$ hours 40 minutes	 M1 M1 A1
16 (a)	$\angle AOB = 360^\circ - x$ $\angle ACB = \frac{1}{2}(360^\circ - x)$ or $180^\circ - \frac{1}{2}x$	M1 A1
16(b)	$\angle OAP = \angle OBP = 90^\circ$ $\angle APB = 180^\circ - (360^\circ - x) = x - 180^\circ$	M1, A1
17	$BC = EC = 7$ cm $CD = CA = 10$ cm $\angle ACB = \angle DCE = 90^\circ$ $\therefore \Delta ABC \equiv \Delta DEC$ (SAS)	M1 M1 A1
18 (a)	2 | 168 2 | 84 2 | 42 3 | 21 7 | 7 1 $168 = 2^3 \times 3 \times 7$	M1 A1
18 (b)	$\frac{168}{3\times 7} = 2^3$ $\sqrt{m} = 21$ $m = 441$	M1 A1
18 (c)	$168 = 2^3 \times (2\times 3)\times 7$ $= 4\times 6\times 7$	M1 A1

19 (a)	Figure 3	B1
19 (b)	Figure 5	B1
19 (c)	Figure 2	B1
20		B1 – correct shape B1– indicating turning point B1 – x and y-intercepts
21(a)	$\sqrt[3]{\frac{512}{216}} = \frac{4}{3}$ $\frac{\text{Surface area of } x}{\text{Surface area of } y} = \left(\frac{4}{3}\right)^2 = \frac{16}{9}$ Ratio = 16 : 9	 M1 A1
21(b)	Volume of Z $= \pi(2r)^2 \frac{2}{3}h$ $= \frac{8}{3} \times \pi r^2 h$ $= \frac{8}{3} \times 216$ $= 576\ \text{cm}^3$	 M1 A1
22(a)	Gradient $BC = \frac{6-1}{5-0} = 1$ Equation: $y = x + 1$	B1 B1
22(b)	$m = -\frac{1}{2}$ $y = mx + c$ $1 = -\frac{1}{2}(-5) + c \quad \Rightarrow \quad c = -\frac{3}{2}$ $y = -\frac{1}{2}x - \frac{3}{2}$ or $2y = -x - 3$	 M1 A1
22(c)	Area $= \frac{1}{2}(6-1)(0+5)$ $= 12.5\ \text{units}^2$	M1 A1

23(a)	$\frac{\text{Vol of small cone}}{\text{Vol of big cone}} = \left(\frac{1}{2}\right)^3 = \frac{1}{8}$ $\frac{\text{Vol of small cone}}{\text{Vol of frustum}} = \frac{1}{7}$ $\frac{\text{Vol of small cone}}{\text{Vol of cylinder}} = \frac{1}{3}$ $p = 33 \div 11 = 3$ $q = 3 \times 4 = 12$	M1 A1 A1
23(b)		B1- correct shape for $0-p$ or $q-33$s B1 - correct shapes for $p-q$ s
		

24(a)	Construct the perpendicular bisector of AB	B1
	Mark the point X 6 cm from C.	B1
24(b)	$5 \times 5 = 25 \pm 0.5$ m	B1
24(c)	Time taken to reach the ball A: $\frac{25}{6} = 4.17$ sec C: $\frac{30}{7} = 4.28$ Player A will get the ball first.	M1 A1

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Answer key

1	(a)(i)	4		
	(a)(ii)	$n = m - \left(\frac{k}{H}\right)^2$		
1	(b)	$\frac{3b^3}{2a}$		
1	(c)	2 or 3		
1	(d)	$x = 2$ and $y = -4$		
2	(a)	Bank *ABC*.		
2	(b)(i)	\$150 000		
2	(b)(ii)	\$12 000		
2	(b)(iii)	2.286%		
2	(c)	No		
3	(a)(i)	25 units		
3	(a)(ii)	−3.5		
3	(a)(iii)	(3, 9)		
3	(b)(i)	$2\mathbf{b} - 2\mathbf{a}$		
3	(b)(ii)	$4\mathbf{a} + 2\mathbf{b}$		
3	(b)(iii)	$\mathbf{b} - 4\mathbf{a}$		
3	(b)(iv)	$12\mathbf{a}$		
3	(b)(v)	Points *O*, *A* and *P* are collinear points/ form a straight line . *A* is a mid-point of OP / $OA = \frac{1}{2}OP$.		
3	(b)(vi)	$\frac{1}{2}$		

3	(b)(vii)	$\frac{1}{3}$		
4	(a)	$\frac{60}{x}$		
4	(b)	$\frac{60}{x-2}$		
4	(c)	$\frac{60}{x-2}-\frac{60}{x}=1.2$		
4	(d)	$x=-9.05$ or 11.05		
4	(e)	9 min 24 sec		
5	(a)	$9-r$		
	(b)	$(9-r)^2+3^2=r^2$		
	(c)	4.09 cm^2		
6	(a)(i)	$u_5=5^2+9=34$		
6	(a)(ii)	$u_n=n^2+2n-1$		
6	(a)(iii)	959		
6	(b)(i)	$\begin{pmatrix}8\\5\\3\end{pmatrix}$		
6	(b)(ii)	$\begin{pmatrix}83\\51\end{pmatrix}$		
6	(b)(iii)	Elements of V represent the cost of manufacturing each toy boat and toy car respectively.		
6	(b)(iv)	(9190) The answer represents the total cost of manufacturing 80 toy boats and 50 toy cars.		
7	(a)(i)	56 kg		
7	(a)(ii)	$33\frac{1}{3}$% or 33.3%		
7	(b)(i)	$a=53$, $b=58$		
7	(b)(ii)	14 kg		

7	(c)	$\frac{9}{35}$		
8	(a)(i)	371 m		
8	(a)(ii)	45 300 m^2		
8	(a)(iii)	37.6°		
8	(a)(iv)	052.4°		
8	(b)	27.5°		
9	(a)(i)	51.8 cm^2		
9	(a)(ii)	334 cm^3		
9	(b)	Total mass of each filled soda can = 631.308 g Will NOT accept the proposal,		
10	(a)	$k = 2$		
10	(c)	Gradient = -4.8 ± 0.5 (Range accepted from -5.1 to -4.3)		
10	(d)(i)	Draw the line $y = \frac{x}{6}$		
10	(d)(ii)	$x = 2.1 \pm 0.1$ or $x = 5.2 \pm 0.1$		
10	(d)(iii)	$A = -36$, $B = 72$		

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Marking Scheme

1	(a)(i)	$H = \dfrac{12}{\sqrt{6-(-3)}}$ $= 4$	 B1	
	(a)(ii)	$H\sqrt{m-n} = k$ $m-n = \left(\dfrac{k}{H}\right)^2$ $n = m-\left(\dfrac{k}{H}\right)^2$	 M1 A1	
	(b)	$\dfrac{9a^2b}{4a^2} \times \dfrac{8b^5}{12ab^3} = \dfrac{9\times 8}{4\times 12} a^{2-3} b^{1+5-3}$ $= \dfrac{3b^3}{2a}$	 M1 A1	
	(c)	$5(11-x)+4(x+7) = (x+7)(11-x)$ $x^2 - 5x + 6 = 0$ $(x-2)(x-3) = 0$ $x = 2$ or $x = 3$	M1 M1 A1	 Factorise
	(d)	Substitute $y = 4x-12$ into $5x-3y = 22$ $5x-3(4x-12) = 22$ $x = 2$ and $y = -4$	M1 A1 A1	Elimination method can be used
				11 Marks
2	(a)	Bank *ABC*: Amount $= 45\,000\left[1+\dfrac{2.45}{12(100)}\right]^{5\times 12}$ $= \$50\,858$ Bank *XYZ*: Interest $= 45\,000 \times \dfrac{2.65}{100} \times 5 = \$5\,962.50$ Amount $= \$50\,962$ Alex should loan from Bank *ABC*.	M1 M1 M1 A1	

	(b)(i)	Cash Price: $\frac{100}{30}\times 450\,000$ $=\$150\,000$	B1	
	(b)(ii)	Hire Purchase Price $45\,000+(1950\times 5\times 12)=\$162\,000$ Interest = \$12 000	M1 A1	
	(b)(iii)	Rate $=\frac{12\,000\times 100}{105\,000\times 5}$ $=2.286\%$ (3 d.p)	B1	
	(c)	Price of 1 litre of petrol in Singapore: $\frac{109}{50}=S\$2.18$ Bangkok: $\frac{9408}{320}=29.4$ Thai bahts $=\frac{29.4}{24.5}=S\$1.20$ Half of Singapore price $=\frac{1}{2}\times 2.18=\$1.09$ Since 1.20 > 1.09, I do not agree.	M1 M1 A1	
				11 Marks
3	(a)(i)	$\sqrt{(-7)^2+24^2}=25$ units	B1	
	(a)(ii)	$\overrightarrow{PQ}=n\overrightarrow{PS}$ $\begin{pmatrix}-7\\24\end{pmatrix}=n\begin{pmatrix}k\\12\end{pmatrix}$ $n=2$ $k=-3.5$	M1 A1	Accept $\frac{12}{k}=\frac{24}{-7}$ but not $\frac{k}{12}=\frac{-7}{24}$
	(a)(iii)	$\overrightarrow{OQ}=\overrightarrow{OP}+\overrightarrow{PQ}$ $=\begin{pmatrix}10\\-15\end{pmatrix}+\begin{pmatrix}-7\\24\end{pmatrix}=\begin{pmatrix}3\\9\end{pmatrix}$ Coordinates of $Q=(3, 9)$	B1	
	(b)(i)	$\overrightarrow{AN}=\frac{1}{3}\overrightarrow{AB}$ $=2\mathbf{b}-2\mathbf{a}$	B1	

	(b)(ii)	$\overrightarrow{ON} = \overrightarrow{OA} + \overrightarrow{AN}$ $= 4\mathbf{a} + 2\mathbf{b}$	B1	
	(b)(iii)	$\overrightarrow{NM} = \overrightarrow{OM} - \overrightarrow{ON}$ $= \mathbf{b} - 4\mathbf{a}$	B1	
	(b)(iv)	$\overrightarrow{MP} = 3\overrightarrow{MN}$ $\overrightarrow{OP} = \overrightarrow{OM} - 3\overrightarrow{NM}$ $= 3\mathbf{b} - 3(\mathbf{b} - 4\mathbf{a})$ $= 12\mathbf{a}$	B1	
	(b)(v)	Points O, A and P are collinear points/ form a straight line . A is a mid-point of OP / $OA = \frac{1}{2}OP$.	B1 B1	
	(b)(vi)	$\frac{\text{Area of } \Delta AMN}{\text{Area of } \Delta BMN} = \frac{1}{2}$	B1	
	(b)(vii)	$\frac{\text{Area of } \Delta BMN}{\text{Area of } \Delta BOA} = \frac{1}{3}$	B1	
				12 Marks
4	(a)	$\frac{60}{x}$	B1	
	(b)	$\frac{60}{x-2}$	B1	
	(c)	$\frac{60}{x-2} - \frac{60}{x} = 1.2$ $60x - 60(x-2) = 1.2x(x-2)$ $x^2 - 2x - 100 = 0$ (shown)	M1 M1 A1	Form equation Attempt to simplify
	(d)	$x = \frac{2 \pm \sqrt{(-2)^2 - 4(1)(-100)}}{2(1)}$ $x = -9.05$ or 11.05 (2 d.p)	M1 A1	
	(e)	Time taken $= 85\left(\frac{60}{11.05-2}\right)$ $= 564$ seconds $= 9$ min 24 sec	M1 B1	
				9 Marks

5	(a)	$OB = 9 - r$	B1	
	(b)	$(9-r)^2 + 3^2 = r^2$ $81 - 18r + r^2 + 9 = r^2$ $r = 5$ cm (Shown)	B1 M1 A1	
	(c)	$\sin \angle BOC = \frac{3}{5}$ $\angle BOC = 36.869°$ or 0.6435 rad $\angle COD = 73.739°$ or 1.287 rad Area of sector $= \frac{73.739}{360} \times \pi \times 5^2$ or $\frac{1}{2} \times 5^2 \times 1.287$ $= 16.0875$ cm^2 Area of $\Delta OCD = \frac{1}{2} \times 4 \times 6 = 12$ cm^2 Area of req. segment $= 4.09$ cm^2. (3 s.f.)	B1 M1 M1 A1	
				8 Marks
6	(a)(i)	$u_5 = 5^2 + 9 = 34$	B1	
	(a)(ii)	$u_n = n^2 + 2n - 1$	B1 B1	B1 for n^2 B1 for $2n - 1$
	(a)(iii)	$U_{30} = 30^2 + 2(30) - 1$ $= 959$	B1	
6	(b)(i)	$\begin{pmatrix} 8 \\ 5 \\ 3 \end{pmatrix}$	B1	
	(b)(ii)	$V = \begin{pmatrix} 6 & 4 & 5 \\ 4 & 2 & 3 \end{pmatrix} \begin{pmatrix} 8 \\ 5 \\ 3 \end{pmatrix}$ $= \begin{pmatrix} 83 \\ 51 \end{pmatrix}$	B1 B1	
	(b)(iii)	Elements of V represent the cost of manufacturing each toy boat and toy car respectively.	B1	
	(b)(iv)	$WV = \begin{pmatrix} 80 & 50 \end{pmatrix} \begin{pmatrix} 83 \\ 51 \end{pmatrix} = \begin{pmatrix} 9190 \end{pmatrix}$ The answer represents the total cost of manufacturing 80 toy boats and 50 toy cars.	B1 B1	

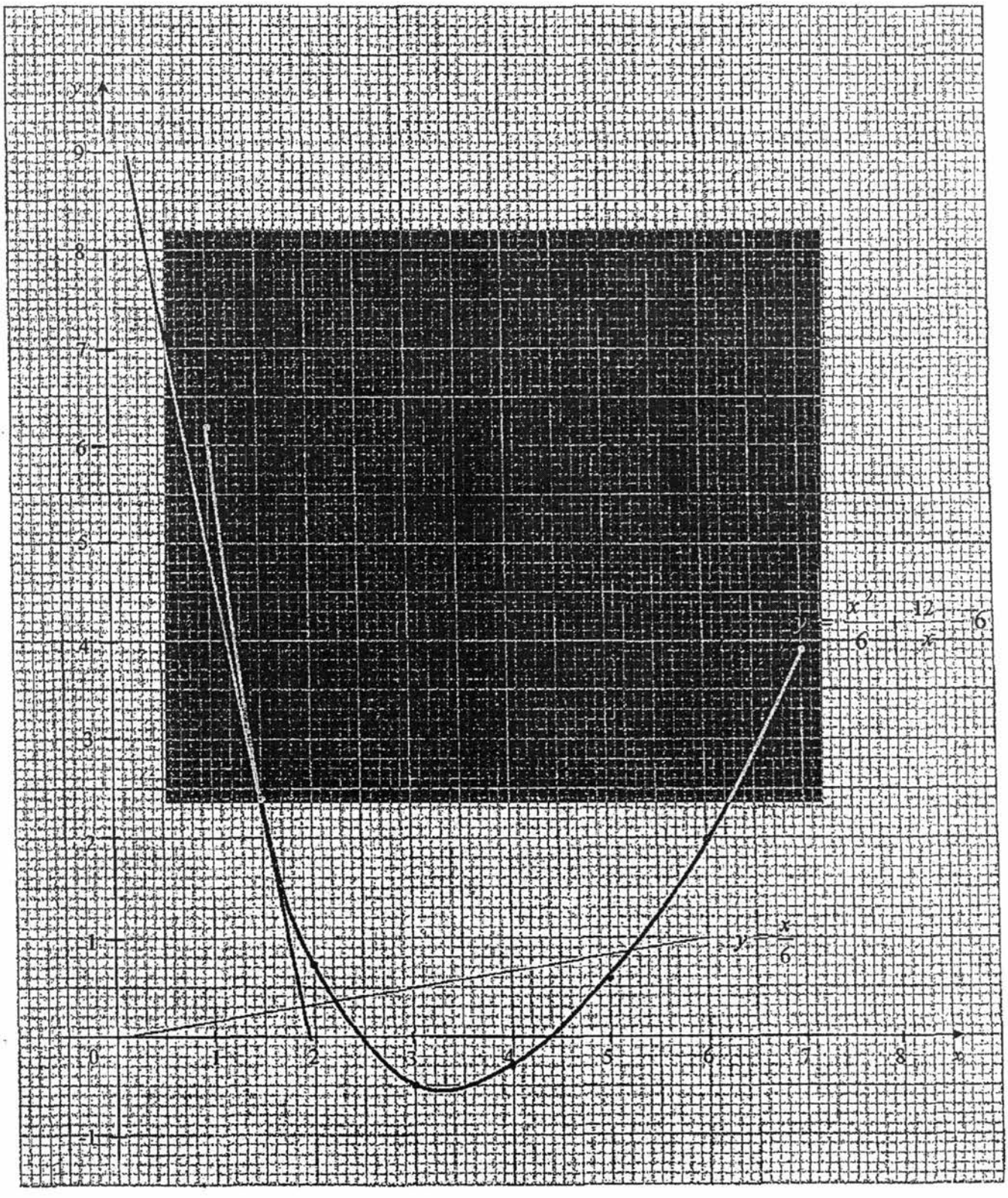
				10 Marks
7	(a)(i)	Modal mass = 56 kg	B1	
	(a)(ii)	$\frac{7}{21}\times 100\% = 33\frac{1}{3}\%$ or 33.3%	B1	
	(b)(i)	$a = 53$ $b = 58$	B1 B1	
	(b)(ii)	Interquartile range $= 67 - 53$ $= 14$ kg	B1	
	(c)	$\left(\frac{18}{21}\times\frac{3}{20}\right)+\left(\frac{3}{21}\times\frac{18}{20}\right)$ $=\frac{9}{35}$	M1 A1	
				7 Marks
8	(a)(i)	$BC^2 = 250^2 + 400^2 - 2(250)(400)\cos 65°$ $BC = 371.45$ $= 371$ m (3 s.f.)	B1 B1 A1	
	(a)(ii)	Area $= \frac{1}{2}(250)(400)\sin 65°$ $= 45\,315.38$ $= 45\,300$ m^2 (3 s.f.)	M1 A1	
	(a)(iii)	$\frac{\sin \angle ABC}{250} = \frac{\sin 65°}{371.45}$ $\angle ABC = 37.588$ $\approx 37.6°$ (1 d.p.)	M1 A1	
	(a)(iv)	Bearing $= 90° - 37.6°$ $= 052.4°$	B1	
	(b)	Let h be the height of eagle above the ground $\frac{h}{400} = \tan 18°$ $h = 129.967$ m $\tan \angle ACE = \frac{129.967}{250}$ $\angle ACE = 27.46°$ Angle of depression = 27.5° (to 1 d.p.)	 B1 M1 A1	
				11 Marks

9	(a)(i)	Area of hemisphere $= 2\pi(2.5)^2$ $= 39.2699\ cm^2$ Area of ring $= \pi(3.2^2 - 2.5^2)$ $= 12.534\ cm^2$ Area of the base $= 51.8048$ $= 51.8\ cm^2$	M1 A1	Any one part of working shown
	(a)(ii)	Volume of hemisphere $= \frac{1}{2} \times \frac{4}{3} \times \pi(2.5)^3$ $= 32.7249\ cm^3$ Volume of cylinder $= \pi \times 3.2^2 \times 11.4)$ $= 366.73696\ cm^3$ Volume of the soda can $= 334.01$ $= 334\ cm^3$ (3 s.f.)	M1 A1	Any one part of working shown
	(b)	Surface area of the can $= 2\pi(3.2) \times 11.4 + \pi(3.2)^2 + 51.8048$ $= 313.185$ $= 313\ cm^2$ Mass of the empty can using the proposed material $= 313.185 \times 0.8$ $= 250.548$ g Mass of soda in each can $= 95\% \times 334 \times 1.2$ $= 380.76$ g Total mass of each filled soda can $= 250.548 + 380.76$ $= 631.308$ g Since $631.308 > 620$ g, $\therefore$ I will NOT accept the proposal.	M1 B1 B1 M1 M1 A1	Allow error from part (a).to carry forward in this whole part of question.
				10 Marks
10	(a)	$k = 2$	B1	
	(b)	Refer to attached graph.	B1 – Axes drawn to scale B1 – Points are plotted correctly B1 – Smooth curve plotted	

10	(c)	Tangent is drawn at the point $x = 1.5$ Refer to attached graph Gradient $= -4.8 \pm 0.5$ (Range accepted from -5.3 to -4.3)	B1 B1	
	(d)(i)	Draw the line $y = \frac{x}{6}$ Refer to attached graph.	B1	
	(d)(ii)	$x = 2.2 \pm 0.1$ or $x = 5.2 \pm 0.1$	B1 B1	
	(d)(iii)	$\frac{x^2}{6} + \frac{12}{x} - 6 = \frac{x}{6}$ $x^3 - x^2 - 36x + 72 = 0$ $A = -36$, $B = 72$	M1 A1	 Both correct
				11 Marks

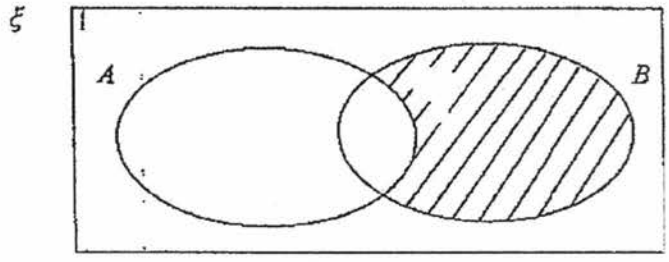
Name : _____ ( )

lass: _____ Date: _____



Answer all the questions.

1 Express in set notation, the set shaded in the following Venn diagram.



Answer ..... [1]

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2 (a) Simplify $(3+2x)(1+x)$.

Answer ..... [1]

(b) Factorise completely $32a^2-18b^2$.

Answer .... [2]

---

3 Factorise completely $12bx-6ay+8by-9ax$.

Answer ..... [2]

---

4 Write as a single fraction in its simplest form $\frac{5}{2+x}+\frac{6x}{x^2-4}$.

Answer ..... [2]

---

5 Show that for all p, where p is a positive integer, $(7p-3)^2-4p(p-3)+6$ is divisible by 15.

Answer

[2]

---

6 (a) Express $5-6x-x^2$ in the form $p-(x+q)^2$.

Answer ..... [2]

(b) Hence, sketch the graph of $y=5-6x-x^2$ indicating the y-intercept and the coordinates of the turning point on the graph.

Answer

[2]

---

7 A bicycle rental shop uses the formula $C = 5.5 + 3.5h$ to calculate charges for rental of bicycles, where C is the cost of rental and h is the number of hours of rental.

(a) State the basic charge to be paid regardless of the number of hours of rental.

Answer $ ..... [1]

(b) Mathew and Ethan both rented a bicycle each for different number of hours.
The difference in the cost of rental between the two of them is $14.
Find the difference in the number of hours of rental between the two boys.

Answer ..... hours [2]

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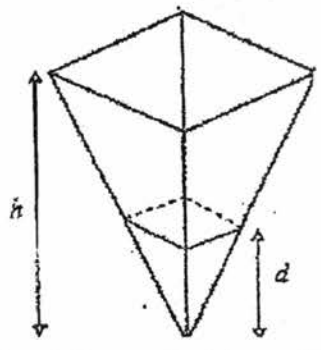
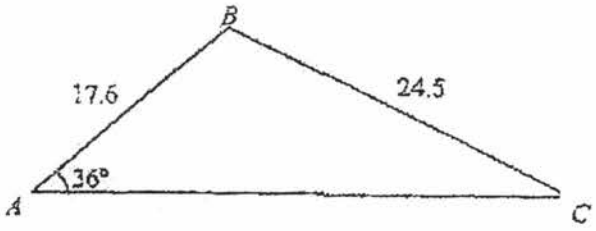
8 The diagram shows an inverted pyramid with a capacity of 800 cm³.



The depth of the liquid in the inverted pyramid, d, is one-third the height, h, of the pyramid.
Calculate the volume of the liquid.

Answer ..... cm³ [2]

9



ABC is a triangle, where $AB = 17.6$ cm, $BC = 24.5$ cm and angle $BAC = 36°$

Find angle ABC.

Answer angle ABC = ... [3]

10 Jane plans to travel back to Singapore from the United States.

In Singapore, the exchange rate is SGD \$1 = USD \$0.71.

In the United States, the exchange rate is USD \$100 = SGD \$153.

Jane wants to change USD \$1426 into Singapore dollars.

Which country should Jane change her money in order to get a better deal?

You must show your calculations.

Answer ..... [3]

11 Hector was arranging 315 one-centimetre cubes into a cuboid.

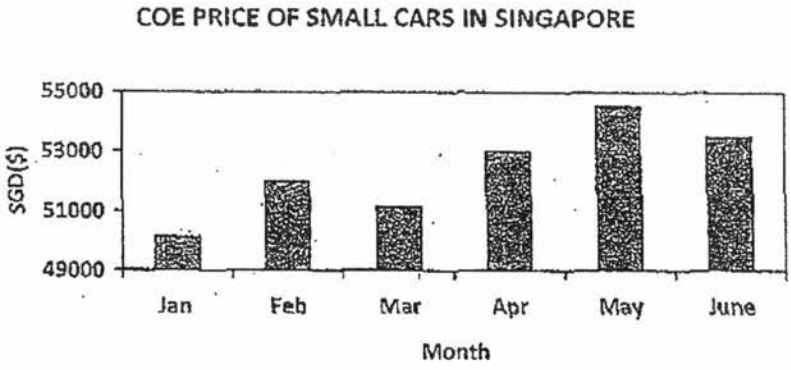
The perimeter of the base of the cuboid is 28 cm.

Each side of the cuboid has a length greater than 3 cm.

Find the height of the cuboid.

Answer .. cm [2]

12 The bar graph shows the COE price of small cars in Singapore over a period of 6 months.



State one aspect of the graph that may be misleading and explain how this may lead to a misinterpretation of the graph.

Answer .....

.....

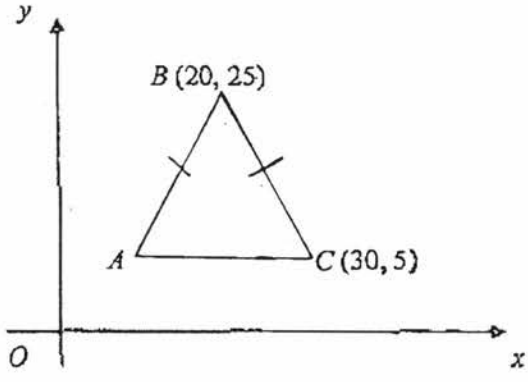
..... [2]

---

13 The diagram shows an isosceles triangle.

AC is parallel to the x-axis.

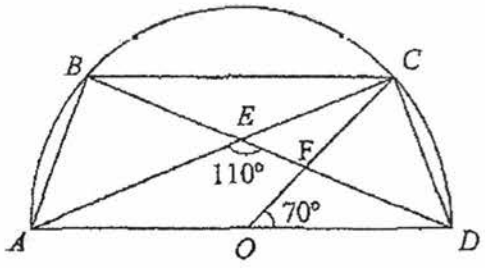
Point B has coordinates (20, 25) and C has coordinates (30, 5)



Find the coordinates of A.

Answer (....., ...) [1]

14



$ABCD$ is a semicircle with centre O.

BED and AEC are straight lines.

Angle $COD = 70°$ and angle $AED = 110°$.

(a) Stating your reasons clearly, calculate

(i) angle ACD,

Answer angle ACD = ...[1]

(ii) angle ADC,

Answer angle ADC = ...[1]

(iii) angle ABC,

Answer angle ABC = ...[1]

(iv) angle BFC.

Answer angle BFC = ...[3]

(b) Explain why BC is parallel to AD.

Answer .....

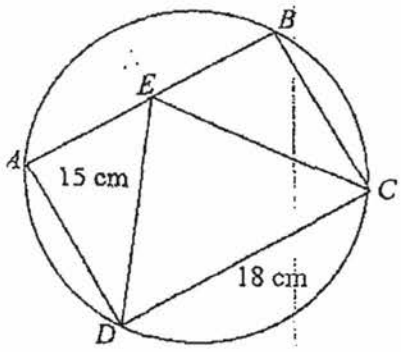
.....[1]

15 The diagram shows a circle $ABCD$.

E is the midpoint of the chord AB.

$ABCD$ is a rectangle.

$DE = 15$ cm and $DC = 18$ cm.



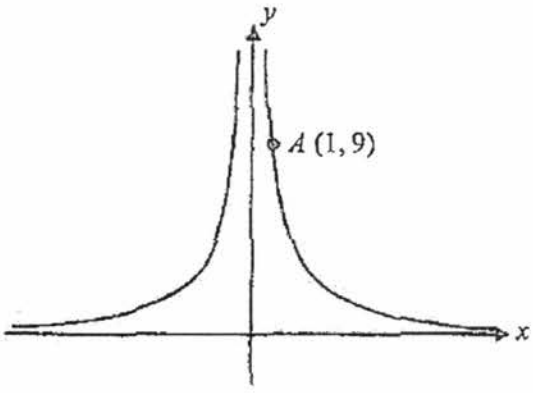
(a) Calculate the area of triangle ADE.

Answer ..... cm² [2]

(b) Calculate the circumference of the circle.

Answer ..... cm [2]

16 The sketch shows the graph of $y = 3^k \times x^{-n}$.
The graph passes through the point A (1, 9).



(a) (i) State a possible value of n.

Answer $n =$ ..... [1]

(ii) Find the value of k.

$k =$ ..... [1]

(b) Given that the coordinates of B is $(-2, 2.25)$, find the length of the line segment AB.

Answer ..... [2]

---

17 (a) Express 3780 as the product of its prime factors.

Answer ..... [1]

(b) Using your answer to **part (a)**, explain why 3780 is **not** multiple of 49.

Answer .....

..... [1]

(c) c is a composite number and p is a prime number.
Find the values of p and c such that $3780 \times \frac{c}{p}$ is a perfect square and c has the least value.

Answer $p =$ .....

$c =$ ..... [2]

---

18 A map of Singapore is such that 9 cm^2 on the map represents the actual area of 36 km^2 on the land.

(a) Express the scale of the map in the form $1 : n$.

Answer 1 : ..... [2]

(b) The length of Bukit Timah Expressway on the map is 5 cm.
Calculate the actual distance, in kilometres, of the Bukit Timah Expressway.

Answer ..... km [1]

19 The table shows the prices of one litre of petrol and the discounts offered by leading petrol companies

Company	Petrol price per litre	Discount
A	\$1.723	18%
B	\$1.689	15%
C	\$1.702	12% discount plus \$3 off for every \$30 sale after discount

(a) Ronn wants to fill up his car with 55 litres of petrol at Company *C*.
Calculate the total amount Ronn paid for the petrol.

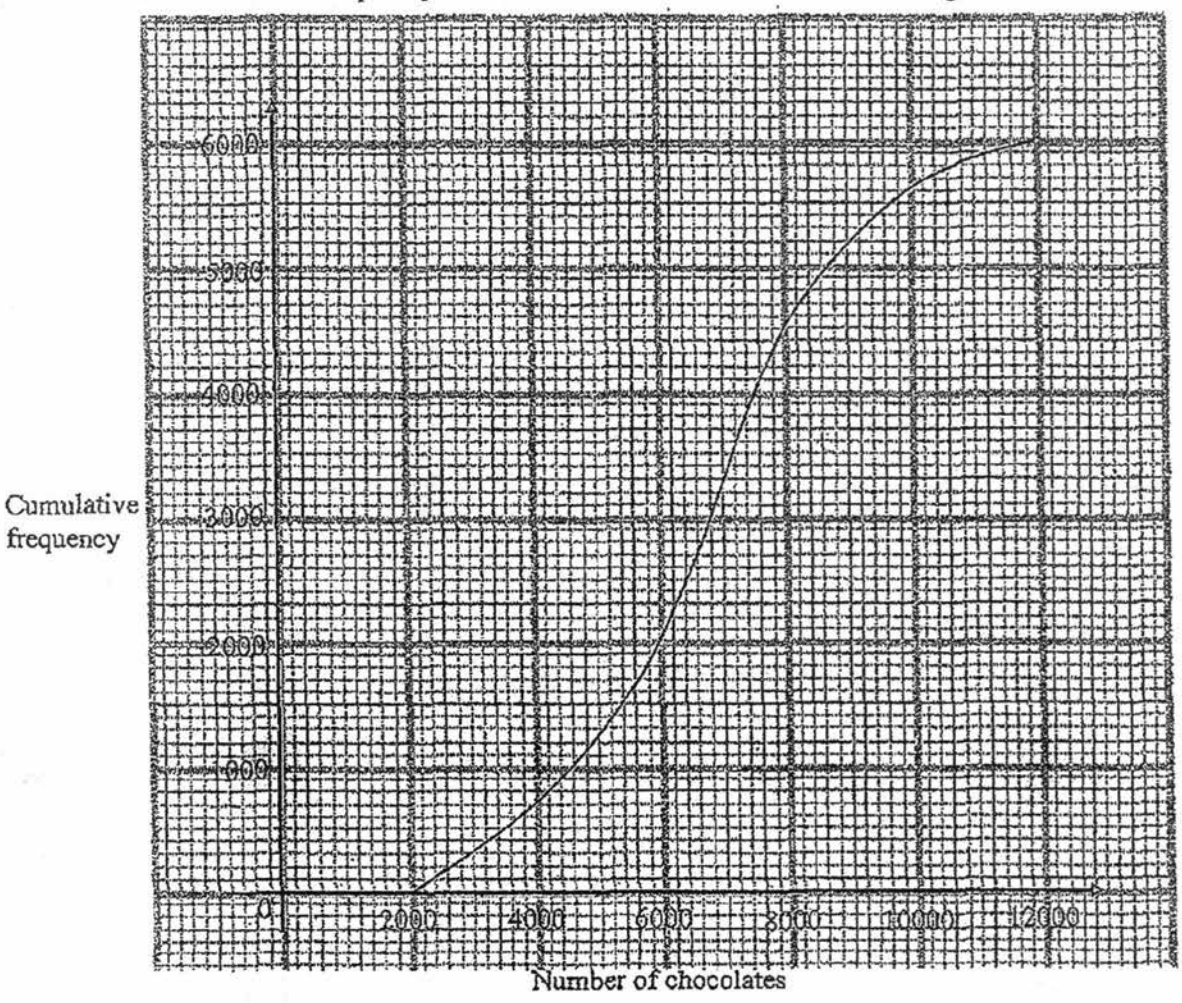
Answer \$ ..... [2]

(b) Comparing Company *A* and *B*, show clearly which company offers a better deal.

Answer ..... [2]

20 6000 customers participated in a contest where they have to guess the number of chocolates in a big glass container.

The cumulative frequency curve below shows the distribution of their guesses.



The actual number of chocolates is 6000.

(a) Find the median.

Answer ..... chocolates [1]

(b) Find the interquartile range.

Answer ..... chocolates [1]

(c) Find the probability that a customer, chosen at random, gave an estimate within 10% of the actual number of chocolates.

Answer ..... [3]

21 Gate B and Gate C are 400 m apart in a park. Gate A is such that angle $ACB = 105°$ and $AB = 550$ m.

(a) Using a scale of 1 cm to 50 m and the line BC is drawn for you, complete the scale drawing of triangle ABC. [1]

(b) A pavilion, inside the park, is located equidistant from the three gates.
By construction, find and label the position of the pavilion P. [2]

(c) Measure and calculate the actual distance between Gate A and the pavilion P.

Answer ..... m [1]

22 The position vectors of A and B are $\begin{pmatrix} 3 \\ 4 \end{pmatrix}$ and $\begin{pmatrix} -3 \\ 4 \end{pmatrix}$ respectively.

(a) Find the length of $\overrightarrow{OB}$.

Answer ..... [1]

(b) C is the point $(0, p)$ where $p > 0$.
$\overrightarrow{OC} = 4\overrightarrow{OA} + 4\overrightarrow{OB}$.
Find the value of p.

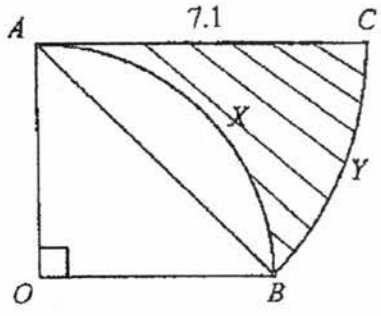
Answer $p =$ ..... [2]

(c) What type of quadrilateral is $OACB$?

Answer ..... [1]

---

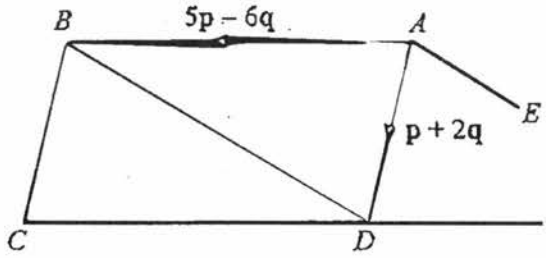
23 In the diagram, angle $AOB = 90°$, AC is parallel to OB and $AC = 7.1$ cm.
AXB is an arc of a circle with centre O and CYB is an arc of a circle with centre A.
Find the area of the shaded region.



Answer ..... cm^2 [5]

---

24 In the diagram, $ABCD$ is a parallelogram, $\overrightarrow{AD} = \mathbf{p} + 2\mathbf{q}$ and $\overrightarrow{AB} = 5\mathbf{p} - 6\mathbf{q}$.



(a) Express, as simply as possible, in terms of p and q,

(i) $\overrightarrow{CB}$,

Answer ..... [1]

(ii) $\overrightarrow{DB}$.

Answer ..... [2]

(b) E is a point such that $\overrightarrow{EA} = \mathbf{p} - 2\mathbf{q}$.

(i) Explain why $\overrightarrow{DB}$ is parallel to $\overrightarrow{EA}$.

Answer .....

..... [1]

(ii) Find the ratio of the area of triangle ADE to the area of triangle DBA.

Answer ..... : ..... [2]

End of Paper

MATHEMATICAL FORMULAE

Compound Interest

$$\text{Total amount} = P\left(1+\frac{r}{100}\right)^n$$

Mensuration

Curved surface area of cone = πrl

Surface area of a sphere = $4\pi r^2$

$$\text{Volume of a cone} = \frac{1}{3}\pi r^2 h$$

$$\text{Volume of sphere} = \frac{4}{3}\pi r^3$$

$$\text{Area of triangle ABC} = \frac{1}{2}ab\sin C$$

Arc length = $r\theta$, where θ is in radians

$$\text{Sector area} = \frac{1}{2}r^2\theta, \text{ where } \theta \text{ is in radians}$$

Trigonometry

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$a^2 = b^2 + c^2 - 2bc\cos A$$

Statistics

$$\text{Mean} = \frac{\sum fx}{\sum f}$$

$$\text{Standard Deviation} = \sqrt{\frac{\sum fx^2}{\sum f} - \left(\frac{\sum fx}{\sum f}\right)^2}$$

Answer all the questions.

1 (a) (i) Factorise $-3x^2-2x+5$. [1]

(ii) Simplify $\dfrac{6x+12}{3x^2-15x-42}$. [2]

(b) It is given that $d=\sqrt{\dfrac{5e-f}{ef}}$.

(i) Find d when $e=4$ and $f=2$. [1]

(ii) Express e in terms of d and f. [2]

(c) Solve the equation $\dfrac{3x+2}{5}-\dfrac{1}{2}=\dfrac{x}{2}$. [2]

(d) Solve these simultaneous equations.

$$7x+4y=-37$$
$$x-5y=17$$ [3]

2 In one small packet of gummies, there are both gummy bears and gummy snakes in two colours; red and green. In a large packet, there are 10 small packets.

The information can be represented by the matrix $A = \begin{matrix} \text{Green} & \text{Red} \\ \begin{pmatrix} 5 & 5 \\ 4 & 6 \end{pmatrix} & \begin{matrix} \text{Bear} \\ \text{Snake} \end{matrix} \end{matrix}$

(a) Evaluate the matrix $B = 10A$. [1]

(b) It costs \$0.10 and \$0.12 to produce 1 green and red gummy respectively.

Represent the cost of each colour of gummy in a 2×1 column matrix C in dollars. [1]

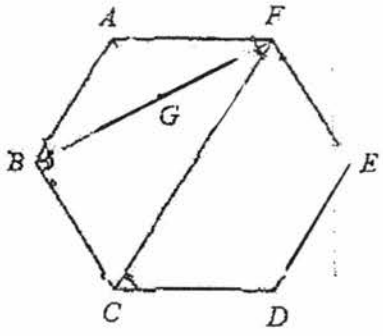
(c) Evaluate the matrix $D = BC$. [1]

(d) State what the elements of D represent. [1]

(e) Another gummy-making company, Company *Y*, packs 6 green gummy bears, 4 red gummy bears, 7 green gummy snakes and 3 red gummy snakes in one small packet. The costs to produce one green gummy and one red gummy remain the same. One large packet is also made up of 10 small packets.

Calculate the total cost for Company *Y* to produce one large packet. [3]

3 (a) The diagram shows a regular hexagon.



(i) Calculate the interior angle of a regular hexagon. [2]

(ii) It is given that $2AG = BC$. Find $\frac{\text{area of triangle } ABF}{\text{area of triangle } BFC}$ [2]

(b) (i) Simplify $\frac{(mn^2)^3}{p^5} \div \frac{n^5}{p^4}$. [2]

(ii) Given that $\frac{2^{q+5}}{4^{2q}} = \frac{1}{16}$, find the value of q. [3]

4 The first five terms in a sequence of numbers are given below.

0, 3, 8, 15, 24...

(a) Find the next two terms. [2]

(b) Find an expression, in terms of n, for the nth term, T_n, of the above sequence. [1]

(c) T_n and T_{n+1} are consecutive terms in the sequence.

Find and simplify an expression, in terms of n, for $T_{n+1} - T_n$. [3]

(d) Explain why two consecutive terms of the sequence cannot have a difference of 8. [2]

5 **Answer the whole of this question on a sheet of graph paper.**

The variables x and y are connected by the equation

$$y = x^3 - 4x^2 + \frac{5}{2}$$

Some corresponding values of x and y are given in the table below.

x	−1.5	−1	−0.5	0	0.5	1	1.5	2
y	−9.875	−2.5	1.375	2.5	p	−0.5	−3.125	−5.5

(a) Find the value of p. [1]

(b) Using a scale of 4 cm to represent 1 unit, draw a horizontal x-axis for $-1.5 \le x \le 2$.

Using a scale of 1 cm to represent 1 unit, draw a vertical y-axis for $-12 \le y \le 4$.

On your axes, plot the points given in the table and join them with a smooth curve. [3]

(c) Use your graph to find the coordinates of the maximum point of $y = x^3 - 4x^2 + \frac{5}{2}$, in the range of $-1.5 \le x \le 2$. [1]

(d) Use your graph to find the solutions to the equation $x^3 - 4x^2 + 6 = 0$, in the range $-1.5 \le x \le 2$. [3]

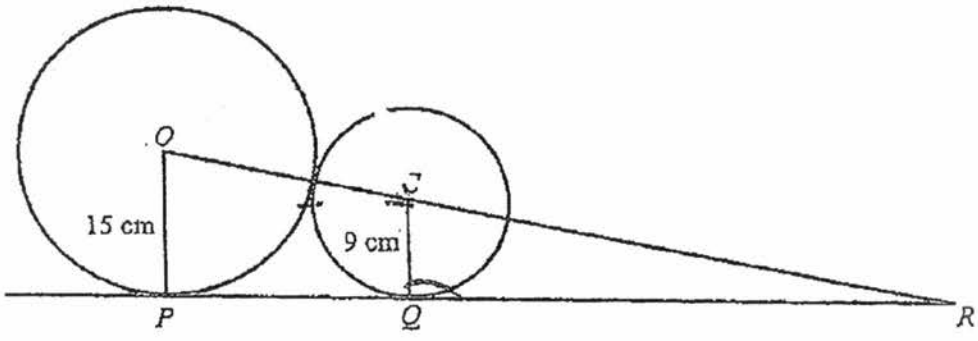
(e) By drawing a tangent, find the gradient of the curve at $(-1, -2.5)$. [2]

(f) (i) On the same axes, draw the line $y = -3x - 4$ for $-1.5 \le x \le 2$. [1]

(ii) Write down the coordinates of the point where this line intersects the curve. [1]

6 The diagram shows a circle, centre O, with radius 15 cm touching another circle, centre C, with radius 9 cm.

OCR and PQR are straight lines and PQR is a tangent to both the circles at points P and Q.



(a) State the value of angle CQR and explain your answer. [2]

(b) Show that triangles OPR and CQR are similar.
Give a reason for each statement you make. [2]

(c) Find the value of $\dfrac{\text{area of triangle } CQR}{\text{area of trapezium } OCQP}$ [2]

(d) Find the difference in the areas of the two circles.
Leave your answer in terms of π. [2]

7 A company manufactures and sells posters for decoration and display.

(a) The posters manufactured by the company are sold in local shops and department stores. In a particular week, the number of posters available for sale in local shops and department stores are in the ratio 3 : 7.
Given that 160 more posters are available for sale in department stores, find the total number of posters available for sale in that week. [2]

(b) A shop owner bought x posters for \$60 from the company.

(i) Write down an expression, in terms of x, for the cost of each poster in dollars. [1]

The shop owner decides to sell the posters at a profit of \$1 each.

(ii) Write down an expression, in terms of x, for the selling price of each poster in dollars. [1]

The shop owner managed to sell 10 posters at the selling price in (ii).
He decided to sell the rest of the posters at \$5 each.

(iii) Write down an expression, in terms of x, for the total amount of money in dollars, that he collected from the sale of all posters. [1]

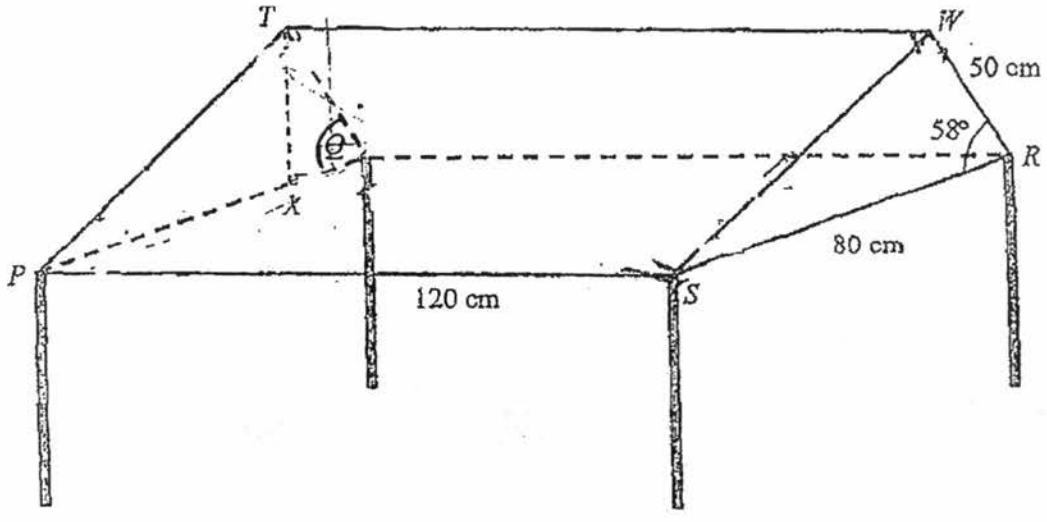
(iv) Given that the shop owner collected a total of \$130 from the sale of all posters, write down an equation in x to represent this information and show that it reduces to

$$x^2 - 34x + 120 = 0$$ [3]

(v) Solve the equation $x^2 - 34x + 120 = 0$. [3]

(vi) Find the cost price of each poster. [1]

8 The diagram shows a table used by an interior designer.
It is made up of a prism and 4 table legs for support.
The rectangle $PQRS$ lies on a horizontal plane.
T is vertically above X.
$PS = 120$ cm, $RS = 80$ cm and $WR = 50$ cm.
Angle $WRS = 58°$.



Calculate

(a) WS, [3]

(b) the volume of the prism, [3]

(c) TX, [2]

(d) XS, [4]

(e) the angle of elevation of T from S. [2]

9 (a) The amount of money, in dollars, spent by a group of 20 students (Group *A*) in the month of May is shown in the stem-and-leaf diagram below.

5	2	5			
6	2	3	7		
7	1	1	5	8	9
8	0	4	5	6	
9	2	3	8	9	
10	5	8			

Key 5|6 means \$56

(i) Find the mean amount of money spent by the 20 students. [1]

(ii) Find the standard deviation of the amount of money spent by the 20 students. [1]

(iii) The mean and standard deviation of the amount of money spent by another group of 20 students (Group *B*) in May were \$70 and \$12 respectively.
Use the information to comment on two differences between the two distributions. [2]

(b) John plays a game at a carnival. In this game, he has to pick 2 coloured balls from two bags, *A* and *B*. He is only allowed to pick one ball from each bag. He has to pick one ball from Bag *A*, followed by another ball from Bag *B*.
Bag *A* contains 2 red balls, 3 blue balls and 6 yellow balls.
Bag *B* contains 4 red balls, 1 blue ball and 4 yellow balls.

(i) Draw a tree diagram to show the probabilities of the possible outcomes. [2]

(ii) John will win a large prize if he picks 2 balls that are blue, a small prize if he picks only one ball that is blue and goes home empty-handed otherwise.
Find, as a fraction in the simplest form, the probability that

(a) John will win a large prize, [1]

(b) John will win a small prize, [1]

(c) John will not win anything. [1]

10 A group of students are tasked to design, print and distribute brochures containing tips to save water to students in school, as part of the school's effort to raise awareness of the importance of saving water in school.

The students have been allocated a budget of $1200 to complete this task.

The students are required to print and distribute a copy of the brochure to each student and teacher in the school.

Each brochure is printed on both sides of 2 sheets of A4 size paper.

Students will be given brochures printed in black and white and teachers will be given brochures printed in colour. They will have to purchase the sheets of A4 size paper and toner cartridges from *ABC* bookstore, which will be delivered to school.

In addition, the students are also tasked to design and print 50 copies of A3 size coloured posters containing tips to save water, to be put up in all classrooms and various areas in the school. They have sourced for an external supplier, *XYZ* supplier, to print the posters. The posters will be delivered to school as well.

The information that the students require is found in Annex A, on the opposite page.

The students estimates that they have to distribute the brochures to 1360 students and 90 teachers.

(a) How many sheets of A4 size paper will the students require to purchase to print the brochures for all students and teachers? [1]

(b) How many toner cartridges will the students require to purchase to print the brochures for all students and teachers? [3]

(c) Given that one of the students in the group is a member of *ABC* bookstore and that the students aim to reduce the cost as far as possible, determine if the amount of budget allocated is sufficient to cover all costs.
Justify your answer with relevant mathematical working. [6]

Annex A

1) Cost of purchasing stationaries from *ABC* Bookshop:

Item	Description	Unit Cost (excluding GST)
A4 Paper	White paper	
	1 pack of 100 sheets	$2.00
	1 pack of 500 sheets	$5.00
	5 packs of 500 sheets each	$22.50
	10 packs of 500 sheets each	$42.00
Toner Cartridges	Black printing (each cartridge is able to print 1200 pages)	$136.00
	Colour printing (each cartridge is able to print 900 pages)	$140.00
The above prices are subjected to 7% Goods and Services Tax (GST). Member discount: 10% off total bill, after 7% GST Delivery cost: $30 per trip (not subjected to 7% GST) (Free delivery for minimum purchase of $200 in total bill, inclusive of 7% GST and after member discount.)		

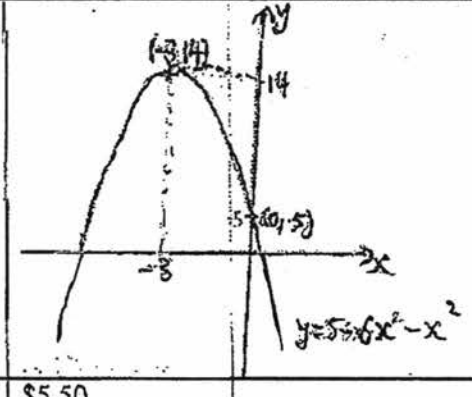
2) Cost of printing A3 size coloured posters

Supplier: *XYZ* Printing

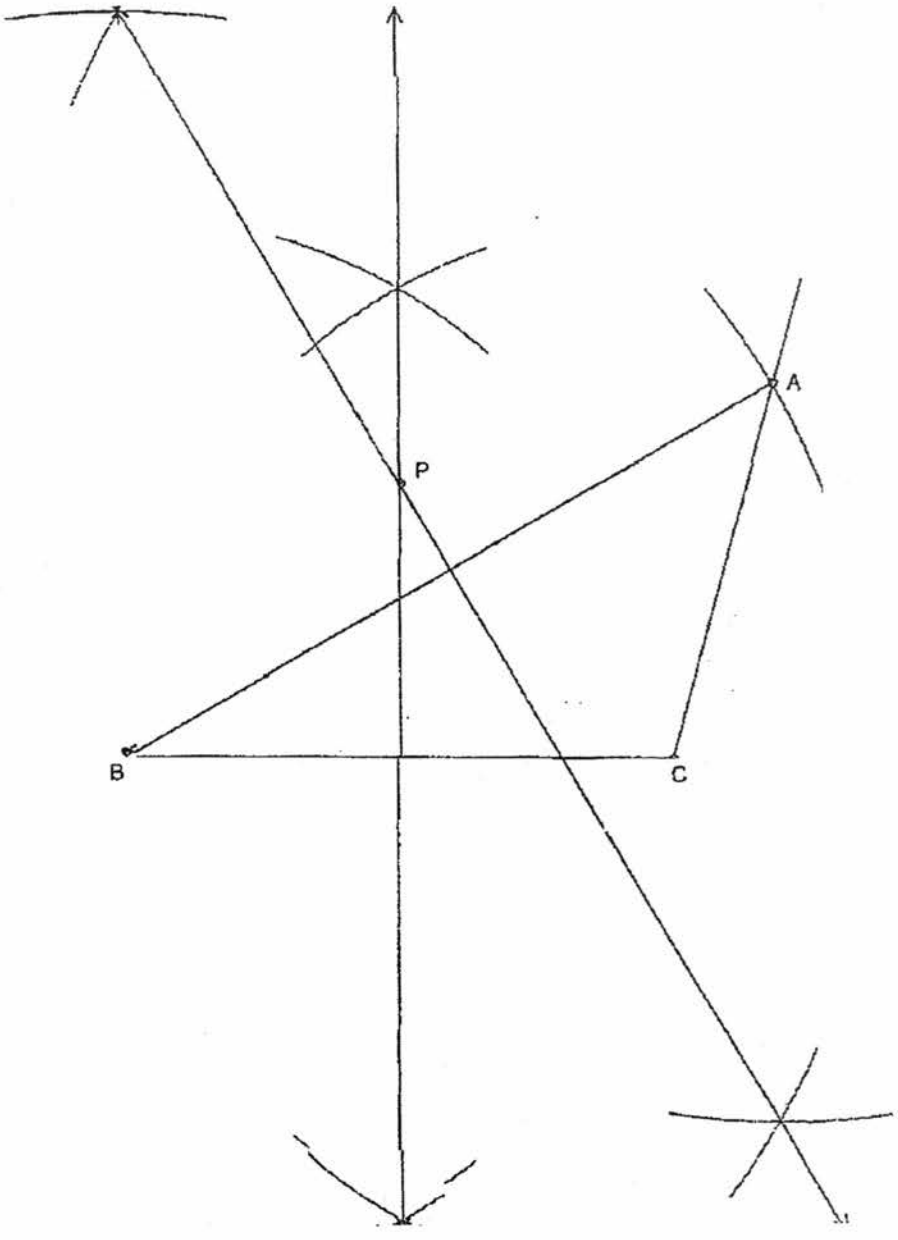
Item	Description	Unit Cost (excluding GST)
Black and White Posters	10 sheets	$25.00
	50 sheets	$120.00
Coloured Posters	10 sheets	$35.00
	50 sheets	$170.00
The above prices are subjected to 7% Goods and Services Tax (GST). Delivery cost: $20 per trip (not subjected to 7% GST) (Free delivery for minimum purchase of $200 in total bill, inclusive of 7% GST.)		

End of Paper

Pei Hwa Secondary School
Mid Year Examination 2017
Sec 4E & 5N Mathematics Paper 1
Answer Key

1(a)	$A' \cap B$
2(a)	$-2x^2+x+3$
2(b)	$2(4a+3b)(4a-3b)$
3	$(4b-3a)(3x+2y)$
4	$\dfrac{11x-10}{(x+2)(x-2)}$
5	$(7p-3)^2-4p(p-3)+6$ $=49p^2-42p+9-4p^2+12p+6$ $=45p^2-30p+15$ $=15(3p^2-2p+1)$ $\therefore$ for all p, $(7p-3)^2-4p(p-3)+6$ is divisible by 15. (Shown)
6(a)	$14-(x+3)^2$
6(b)	
7(a)	\$5.50
7(b)	4 hours
8	$29.6cm^3$ (3s.f.)
9	119.0° (1d.p.)
10	Amount of money Jane will get in Singapore $=\dfrac{1426}{0.71}$ $=SGD\$2008.45$ Amount of money Jane will get in the United States $=\dfrac{153}{100}\times 1426$ $=SGD\$2181.78$

	Jane will change her money in the **United States** as she will get back more Singapore dollars.
11	7 cm
12	In the graph, the data doesn't start at \$0, but somewhere around \$49000. This makes the differences appear much larger proportionally.
13	(10, 5)
14(a)(i)	90°
14(a)(ii)	55°
14(a)(iii)	125°
14(a)(iv)	75°
14(b)	Angle $BCE = 35°$ (Angles in the same segment) Since angle BCE = angle CAO (by property of alternate angles), BC is parallel to AD
15(a)	$54cm^2$
15(b)	$68.0cm$
16(a)(i)	$n = -2$
16(a)(ii)	$9 = 3^k \times \left(\frac{1}{1}\right)^{-2}$ $k = 2$
16(b)	7.39 units
17(a)	$2^2 \times 3^3 \times 5 \times 7$
17(b)	Index of 7 is not at least 2
17(c)	$c = 15$ $p = 7$
18(a)	1 : 200000
18(b)	10 km
19(a)	\$76.38
19(b)	Company B offers a better deal.
20(a)	6800
20(b)	2600
20(c)	$\frac{1}{5}$

21a,b	
21(c)	Distance = 5.7 (±0.1) × 50 = 285 (±5) m
22(a)	5 units
22(b)	$p = 32$

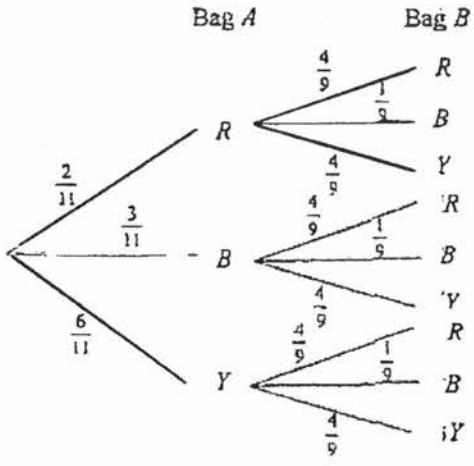
22(c)	Kite
23	12.6 cm^2
24(a)(i)	$-p-2q$
24(a)(ii)	$4p-8q$
24(b)(i)	$\overrightarrow{DB}$ $=4(p-2q)$ $=4\ \overrightarrow{EA}$
24(b)(ii)	$\frac{1}{4}$

PHSS 4E EM MYE Paper 2 2017 Answer Key

No.	Answer
1(a)(i)	$-3x^2-2x+5=(3x+5)(1-x)$
1(a)(ii)	$\frac{2}{x-7}$
1(b)(i)	$d=1.5$ or $d=1\frac{1}{2}$
1(b)(ii)	$e=\frac{f}{5-d^2f}$
1(c)	$x=1$
1(d)	$x=-3,\ y=-4$
2(a)	$\mathbf{B}=\begin{pmatrix}50 & 50\\ 40 & 60\end{pmatrix}$
2(b)	$\mathbf{C}=\begin{pmatrix}0.10\\ 0.12\end{pmatrix}$
2(c)	$\mathbf{D}=\begin{pmatrix}11\\ 11.2\end{pmatrix}$
2(d)	The elements of D represent the cost to produce all the gummy bears and gummy snakes in a large packet respectively.
2(e)	Total cost = \$10.80 + \$10.60 = \$21.40
3(a)(i)	$120°$
3(a)(ii)	$\frac{1}{2}$
3(b)(i)	$\frac{m^3n}{p}$
3(b)(ii)	$q=3$
4(a)	$T_6=35$ $T_7=48$
4(b)	$T_n=n^2-1$ or $(n+1)(n-1)$
4(c)	$T_{n+1}-T_n=n^2+2n-(n^2-1)$ $=2n+1$

No.	Answer
4(d)	$2n+1=8$ $n=3.5$ Assuming that the difference between two terms is 8, the first consecutive term is 3.5, which does not exist. Therefore, two consecutive terms cannot have a difference of 8. OR The difference $(2n+1)$ is an odd number. Therefore, two consecutive terms cannot have a difference of 8, which is an even number.
5(a)	$p=1.625$
5(b)	If all 8 points plotted correctly, otherwise, at least 6 points plotted correctly. Smooth curve
5(c)	Maximum point = (0, 2.5)
5(d)	From the graph, $x=-1.10\pm0.10$ and $x=1.55\pm0.10$
5(e)	Gradient $=8.67\pm3$
5(f)(i)	Correctly drawn line
5(f)(ii)	$(-0.85, -1.4)$
6(a)	$\angle CQR=90°$ tangent perpendicular to radius
6(b)	$\angle OPR=90°$ (tangent perpendicular to radius) $\angle OPR=\angle CQR$ $\angle PRO=\angle QRC$ (common angle) $\angle POR=\angle QCR$ (corresponding angles, $OP//CQ$) Hence, triangle OPR is similar to triangle CQR. (AA Similarity)
6(c)	$\frac{9}{16}$
6(d)	144π cm^2
7(a)	400
7(b)(i)	$\$\left(\frac{60}{x}\right)$
7(b)(ii)	$\$\left(\frac{60}{x}+1\right)$
7(b)(iii)	$\frac{600}{x}+5x-40$

7(b)(iv)	$\frac{600}{x}+10+5x-50=130$ $\frac{600}{x}+5x-170=0$ $600+5x^2-170x=0$ $5x^2-170x+600=0$ $x^2-34x+120=0$ (shown)
7(b)(v)	$x=30$ or $x=4$
7(b)(vi)	\$2
8(a)	68.3 cm
8(b)	204000 cm^3
8(c)	$TX = 42.4$ cm
8(d)	$XS = 131$ cm
8(e)	$\theta = 17.9°$
9(a)(i)	\$80.15
9(a)(ii)	\$15.60
9(a)(iii)	1. The mean amount of money spent by students in Group *A* is higher than that of Group *B*. On average, students in Group *A* spent more money than students in Group *B*. 2. The standard deviation of the amount of money spent by students in Group *B* is lower than that of Group *A*. There is a smaller spread in the amount of money spent by students in Group *B*./ The amount of money spent by students in Group *B* is more consistent.

9(b)(i)	
9(b)(ii)(a)	$\frac{1}{33}$
9(b)(ii)(b)	$\frac{32}{99}$
9(b)(ii)(c)	$\frac{64}{99}$
10(a)	2900
10(b)	6
10(c)	Cost of purchase from *ABC* Bookstore Total cost with delivery cost, after member discount = \$816.1425 Cost of purchase from *XYZ* Printing Total cost with delivery = \$20 + \$181.90 = \$201.90 Grand total cost = \$816.1425 + \$201.90 = \$1018.04 The amount of budget of \$1200 is sufficient to cover all costs.

SERANGOON GARDEN SECONDARY SCHOOL

Vision: Critical Thinkers, Thoughtful Leaders
Mission: Love to Learn, Learn to Lead

PRELIMINARY EXAMINATION 2017

CANDIDATE NAME			
CLASS		REGISTER NUMBER	

MATHEMATICS 4048/02
Paper 2 22 August 2017
Secondary 4 Express/ 5 Normal Academic 2 hours 30 minutes
0800 – 1030

Additional Materials: Writing Paper
Graph Paper (1 sheet)

READ THESE INSTRUCTIONS FIRST

Write your name, class and class register number on all the work you hand in.
Write in dark blue or black pen.
You may use an HB pencil for any diagrams or graphs.
Do not use staples, paper clips, highlighters, glue or correction fluid.

Answer all questions.
If working is needed for any question, it must be shown with the answer.
Omission of essential working will result in loss of marks.
The use of an approved scientific calculator is expected, where appropriate.
If the degree of accuracy is not specified in the question, and if the answer is not exact, give the answer to three significant figures. Give answers in degrees to one decimal place.
For π, use either your calculator value or 3.142, unless the question requires the answer in terms of π.

At the end of the examination, fasten all your work securely together.
The number of marks is given in brackets [ ] at the end of each question or part question.
The total of the marks for this paper is 100.

Areas for Improvement		
Error	Penalty	Qn. No.(s)
Accuracy of non-exact answers	– 1	
Missing/ wrong units (for Paper 2 only)	– 1	
Presentation/ Not using ink	– 1	

Name/Signature of Parent/Guardian _____ Date _____

FOR MARKER'S USE

100

This question paper consists of 13 printed pages and 1 blank page.

Setter: Mr Ng HJ Vetter: Mr Ko TH

MATHEMATICAL FORMULAE

Compound Interest

$$\text{Total amount} = P\left(1+\frac{r}{100}\right)^n$$

Mensuration

Curved surface area of cone = πrl

Surface area of a sphere = $4\pi r^2$

$$\text{Volume of a cone} = \frac{1}{3}\pi r^2 h$$

$$\text{Volume of sphere} = \frac{4}{3}\pi r^3$$

$$\text{Area of triangle ABC} = \frac{1}{2}ab\sin C$$

Arc length = $r\theta$, where θ is in radians

$$\text{Sector area} = \frac{1}{2}r^2\theta, \text{ where } \theta \text{ is in radians}$$

Trigonometry

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$a^2 = b^2 + c^2 - 2bc\cos A$$

Statistics

$$\text{Mean} = \frac{\sum fx}{\sum f}$$

$$\text{Standard Deviation} = \sqrt{\frac{\sum fx^2}{\sum f} - \left(\frac{\sum fx}{\sum f}\right)^2}$$

Answer all the questions.

1 (a) n is a positive integer. Show that $n^2 + n$ is always even. [2]

(b) Solve the equation $p^2 - 7p + 12 = 0$. [2]

Hence solve the equation $q^4 - 7q^2 + 12 = 0$. [2]

(c) A 2.5 km^2 lake has an area of 40 cm^2 on a map.

(i) If the scale of the map is such that 1 cm represents n km, find the value of n. [2]

(ii) The distance between the hospital and the village town on the map is 30 cm. Find the actual distance, in kilometres, between the hospital and the village town. [1]

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2 Mr Kia is going on a business trip to a province in the same country. There are two options for him to go to the province: by domestic flight or by car.

- If he decides to drive, he would cover a distance of 400 km at a speed of x km/h.
- If he decides to take a domestic flight, he would cover a distance of 300 km at a speed of $(x + 250)$ km/h.

(i) Find an expression, in terms of x, for the time taken to travel from home to the province if Mr Kia decides to drive. [1]

(ii) Find an expression, in terms of x, for the time taken to travel from home to the province if Mr Kia decides to take a domestic flight. [1]

(iii) If the flight time is 210 minutes less than the driving time, form an equation in x and show that it reduces to $7x^2 + 1550x - 200000 = 0$. [3]

(iv) Solve the equation $7x^2 + 1550x - 200000 = 0$, giving your answers correct to 1 decimal place. [3]

(v) If Mr Kia needs to meet his client punctually at 1400, find the latest time that he needs to leave home if he decides to drive. **Assume that time has been factored in for the usual traffic conditions.** [2]

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3 (a) A set of 10 cards is made as shown.

S	T	A	T	I	S	T	I	C	S

The cards are shuffled and placed face down on a desk. A card is drawn at random from the set of cards. It is then replaced and shuffled again before another card is being drawn again.

Calculate the probability that

(i) both cards show the letter T, [2]

(ii) exactly one of the cards shows the letter T. [2]

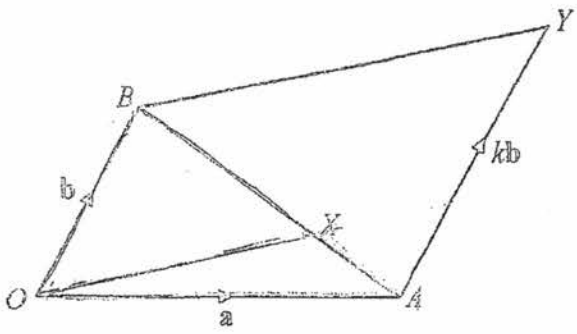
(b) The table shows the ages of 1100 people who entered a 10-km run

Age (x years)		$20 \le x < 30$	$30 \le x < 40$	$40 \le x < 50$
Frequency	Men	375	186	99
	Women	250	122	68

(i) One person is chosen at random. Find, as a fraction in its lowest term, the probability that the person is a man aged less than 40 years old. [1]

(ii) Two persons are chosen at random. Find the probability that both of them are women aged 30 or more. [2]

4 In the diagram, $\overrightarrow{OA} = \mathbf{a}$, $\overrightarrow{OB} = \mathbf{b}$ and $\overrightarrow{AY} = k\mathbf{b}$. X lies on the line AB such that $\overrightarrow{AX} = \frac{1}{3}\overrightarrow{AB}$.



(i) Express $\overrightarrow{AX}$ and $\overrightarrow{OX}$ in terms of $\mathbf{a}$ and $\mathbf{b}$. [2]

(ii) Express $\overrightarrow{BY}$ in terms of k, $\mathbf{a}$ and $\mathbf{b}$. [1]

(iii) Given that OX is parallel to BY, find the value of k. [2]

(iv) The line OX when produced, meets AY at Z. Express $\overrightarrow{AZ}$ in terms of $\mathbf{b}$. [1]

(v) Find the value of

(a) $\dfrac{\text{area of } \Delta OAX}{\text{area of } \Delta OBX}$, [1]

(b) $\dfrac{\text{area of } \Delta AXZ}{\text{area of quadrilateral } XBYZ}$. [2]

5 The following shows the work done by a student in calculating the sum of the first n natural numbers.

n	Series	Sum	Formula
1	1	1	$\frac{1}{2}(1)(1+1)$
2	$1+2$	3	$\frac{1}{2}(2)(2+1)$
3	$1+2+3$	6	$\frac{1}{2}(3)(3+1)$
4	$1+2+3+4$	10	$\frac{1}{2}(4)(4+1)$
⋮	⋮	⋮	⋮
6	$1+2+3+4+5+6$	a	b
⋮	⋮	⋮	⋮
n	$1+2+3+\cdots+n$	c	

(i) Study the pattern and write down the values of a and b. [2]

(ii) Find in terms of n, the value of c. [1]

After doing some additional calculations, the student realised that

$$1^3+2^3+3^3=36=6^2,$$
$$1^3+2^3+3^3+4^3=100=10^2.$$

(iii) Determine the sum of the series

(a) $1^3+2^3+3^3+4^3+5^3+6^3$, [1]

(b) $1^3+2^3+3^3+\cdots+n^3$ in terms of n. [1]

(iv) Hence, using (iii)(b), determine the exact value of the sum of the series

$$3^3+6^3+9^3+12^3+\cdots+300^3.$$ [2]

6 (a) State the order and name of each matrix. [2]

	Matrix	Order	Name of matrix
(i)	$\begin{pmatrix} 2 \\ 5 \\ 12 \end{pmatrix}$		
(ii)	$\begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix}$		

(b) The Tan family owns two cars. Every week (Monday to Friday) on average, Mr Tan spends \$150, \$70 and \$10 on petrol, carpark charges and road pricing (ERP) respectively. Every week (Monday to Friday) on average, Mrs Tan spends \$80, \$45 and \$30 on petrol, carpark charges and road pricing (ERP) respectively.

The information can be represented by the matrix

$$\mathbb{P} = \begin{matrix} \text{Mr Tan} & \text{Mrs Tan} \\ \begin{pmatrix} 150 & 80 \\ 70 & 45 \\ 10 & 30 \end{pmatrix} & \begin{matrix} \text{Petrol} \\ \text{Carpark charges} \\ \text{Road pricing (ERP)} \end{matrix} \end{matrix}.$$

During weekends, the Tan family drives the weekend car and spends on average \$20, \$10 and \$2 on petrol, carpark charges and ERP respectively.

In a year, on average, both Mr Tan and Mrs Tan work for 48 weeks.

(i) Represent the average weekend car expenses of the Tan family by a matrix $\mathbb{R}$. [1]

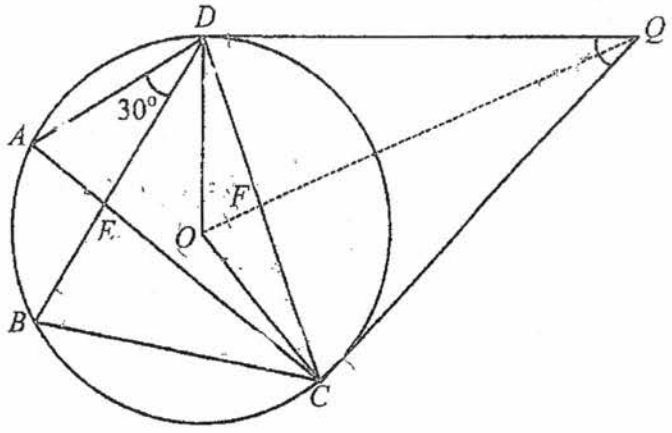
(ii) Evaluate $\mathbb{Q} = \mathbb{P}\begin{pmatrix} 1 \\ 1 \end{pmatrix}$ and $\mathbb{S} = 48\mathbb{Q} + 52\mathbb{R}$. [3]

(iii) State what the elements of $\mathbb{S}$ represent. [1]

(iv) The matrix $\mathbb{T}$ is given by $\mathbb{T} = \begin{pmatrix} 1 & 1 & 1 \end{pmatrix}\mathbb{S}$. Evaluate matrix $\mathbb{T}$ and describe in a sentence what the element(s) of the matrix $\mathbb{T}$ represent. [2]

(v) A recent credit card promotion entitles Mr and Mrs Tan 12.5% savings on petrol every time they pump petrol.

Calculate the new expenses for petrol, carpark charges and ERP for the Tan family in a year. [2]



In the diagram above, AEC and BED are chords of the circle with centre O. $\angle ADE = 30^\circ$ and $\angle CQD = 50^\circ$. CQ and DQ are tangents to the circle and F is the midpoint of chord CD.

(i) Explain why ΔADE is similar to ΔBCE. [2]

(ii) Name a pair of congruent triangles. [1]

(iii) Find, stating your reasons clearly,

(a) $\angle DAC$, [2]

(b) $\angle BEC$. [1]

(iv) Is it possible to draw a circle that passes through C, O, D and Q? Explain your answer clearly. [1]

8 **Answer the whole of this question on a single sheet of graph paper.**

The table below gives the values of x- and y-coordinates of some points on the graph of $y = \frac{ax}{x+b}$.

x	-0.5	0	1	2	3	4	5
y	-2	0	2	3	3.6	4	4.3

(a) By formulating two equations, find the values of a and b. [3]

(b) Using a scale of 2 cm to represent 1 unit on both the x-axis and y-axis, plot the points given in the table and join them with a smooth curve for $-0.5 \le x \le 5$. [3]

(c) By drawing a suitable tangent, find the gradient of the curve at the point $x = 1.5$. [2]

Using the values of a and b found in (a),

(d) find the solution(s) of the equation

$$\frac{ax}{x+b} = -\frac{1}{3}x + 1,$$

by drawing a suitable straight line on the same axes, [2]

(e) find the range of values of x such that $\frac{ax}{x+b} < 2.5$. [2]

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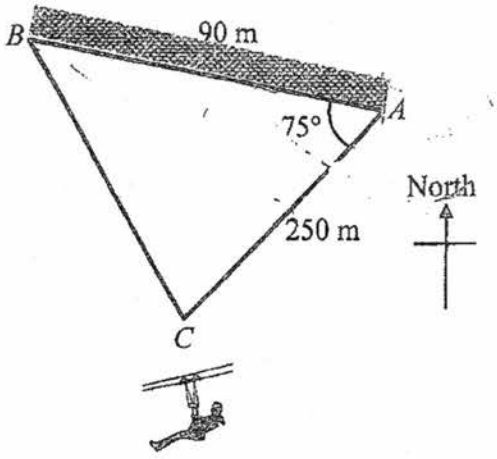
9 Points A and B are points at the bottom a cliff 50 metres tall in height. Point C on a flat ground is 250 metres away from A with AB making an angle of 75° with the line AC. The bearing of C from A is 217° and A and B are 90 m apart.



Calculate the

(a) bearing of B from A, [1]

(b) area of the land formed by the points A, B and C, [2]

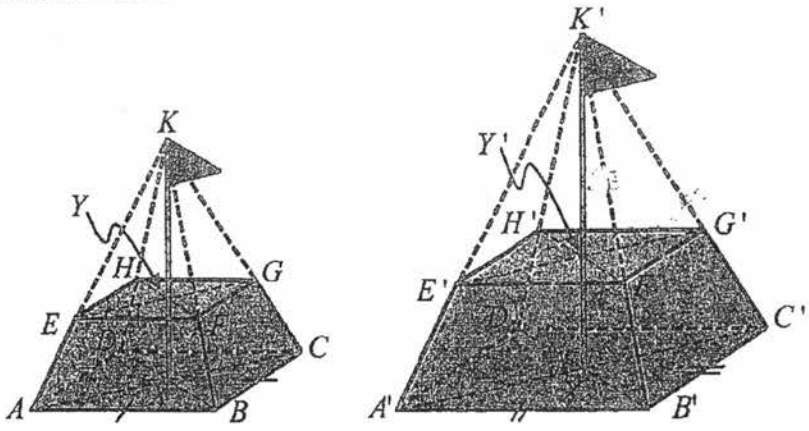
(c) shortest distance from C to the bottom of the cliff. [2]

An outdoor adventure company wants to build a flying fox using a metal cable with the starting point X on the cliff and the landing point at C.

(d) Find the distance away from B vertically below X such that the slope is the greatest. [2]

(e) Find the angle that the metal cable makes with the ground at point C. [2]

10 A company manufactures geometrically similar flagpole bases of two different sizes as shown below.



The bases are made of cement and are in the shape of truncated right pyramids. If each pyramid could be completed, its vertex would be the top of the flagpole at K and K' respectively. The height of the flagpole for the bigger-sized base is 2.5 metres and the ratio of the side length of the bottom surfaces $ABCD$ and $A'B'C'D'$ is $3:5$.

(a) The area of the bottom surface $A'B'C'D'$ is $2500\,\text{cm}^2$. What is the area of the bottom surface $ABCD$? [2]

(b) Given that $E'F' = F'G' = 40\,\text{cm}$, find the length $K'Y'$ and the volume of the base **(as represented by the shaded part)** for the bigger-sized flagpole base. [3]

(c) Hence, find the volume of the base for the smaller-sized flagpole base. [2]

(d) If it costs \$15 to buy a smaller-sized flagpole base and \$25 to buy a bigger-sized flagpole base, which flagpole base is more value for money? Explain with clear working. [2]

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11 The concert band of a school intends to rent a concert venue for their annual performance as their school hall is undergoing a renovation.

Information that the chairperson Peter and his committee need is on the opposite page.

As shown in **Figure 1**, seats in the concert hall are arranged along arcs of concentric circles of equal spacing. There are three rows of seats in front and one row of limited seats behind the stage.

(i) Show that angle $COD = 1.55$ radians and find the area taken up by the stage. [3]

(ii) Each normal concert chair takes up 80 cm of the arc. Show that row 1 can fit a maximum of 47 normal concert chairs. [2]

Peter and his committee decide that they will have a total of 3 rehearsals (including the rehearsal on the actual performance day) and a total of 30 VIP guests. They need to decide whether they should take up Package A or Package B of the concert hall rental offered by the venue management.

(iii) Assuming that Peter and his committee decide to charge \$20, \$15, \$12 and \$25 for Row 1, 2, 3 and 4 respectively, help Peter to decide which package he should take up. Justify the decision with clear calculations and assumption(s) so that Peter can present the proposal to his teacher-in-charge. [5]

Details of the stage

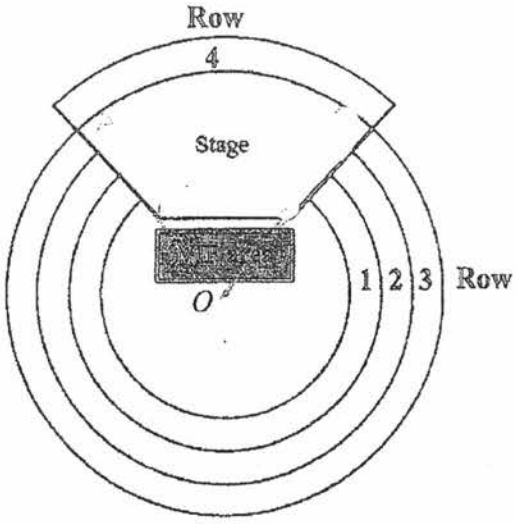


Figure 1

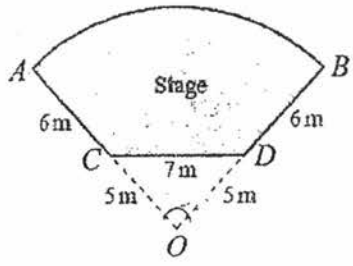


Figure 2

Cost of rental of items

Package	Details (All prices in this column are nett prices)	Cost of renting one normal concert chair (excluding 7% GST)	Cost of renting one VIP concert chair (excluding 7% GST)
A	▫ Basic rental cost: $2800 ▫ Freebies: ◦ Free 150 normal concert chairs ◦ Free 25 VIP chairs ◦ 1st rehearsal (unlimited time usage on day of event): $100 ◦ 2nd rehearsal: 20% off normal rehearsal price ◦ 3rd rehearsal and beyond: 10% off normal rehearsal price	$8	$18
B	▫ Basic rental cost: $1500 ▫ Freebies: ◦ Free 100 normal concert chairs ◦ Free 10 VIP chairs ▫ All rehearsals cost $120 each with unlimited time usage ▫ Terms and Condition: Row 4 cannot be opened for selling of tickets.	$12	$20

END OF PAPER

Sec 4E/5NA Prelims P2 Suggested Mark Scheme

Qn	Solution	
Algebra		
1(a)	$n^2+n=n(n+1)$ If n is odd, then $(n+1)$ is even. If n is even, then $(n+1)$ is odd. Product of an odd number and an even number is even. Thus $n(n+1)$ is even. **Alternative:** If n is odd, then n^2 is odd. Then sum of two odd numbers n and n^2 is even. If n is even, then n^2 is even. Then sum of two even numbers n and n^2 is even.	
(b)	$p^2-7p+12=0$ $(p-3)(p-4)=0$ $p=3$ or $p=4$ $q^4-7q^2+12=0$ Let $p=q^2$. $q^2=3\Rightarrow q=\pm\sqrt{3}$ or $q^2=4\Rightarrow q=\pm 2$	
(c) (i)	$40\,\text{cm}^2$: $2.5\,\text{km}^2$ $1\,\text{cm}^2$: $0.0625\,\text{km}^2$ $1\,\text{cm}$: $0.25\,\text{km}$ $n=0.25$	
(ii)	Actual distance between the hospital and the village town $=30\times 0.25\,\text{km}$ $=7.5\,\text{km}$	
		Total for Q1: 9

Word problem and quadratic equations		
2(i)	Time taken to travel from home to the province if Mr Kia decides to drive = $\frac{400}{x}$ h.	
(ii)	Time taken to travel from home to the province if Mr Kia decides to take a domestic flight = $\frac{300}{x+250}$ h.	

(iii)	$\frac{400}{x}-\frac{300}{x+250}=\frac{7}{2}$ $400(x+250)-300(x)=\frac{7}{2}(x)(x+250)$ $400x+100000-300x=\frac{7}{2}x^2+875x$ $\frac{7}{2}x^2+775x-100000=0$ $7x^2+1550x-200000=0$ (shown)	
(iv)	$7x^2+1550x-200000=0$ $x=\frac{-1550\pm\sqrt{(1550)^2-4(7)(-200000)}}{14}$ $=\frac{-1550\pm\sqrt{8002500}}{14}$ $=94.919$ or -312.776 $=94.9$ or -312.8 (1d.p.)	
(v)	x must be positive, thus $x=94.919$ If Mr Kia drives, time taken $=\frac{400}{94.919}\text{h}=4.2141\text{h}$ 0947 hrs $\xrightarrow{13\text{ minutes}}$ 1000 hrs $\xrightarrow{4\text{ hr}}$ 1400 hrs He must leave home latest by 0947.	
		Total for Q2: 10

Probability

S	T	A	T	I	S	T	I	C	S

3S, 3T, A, 2I, C

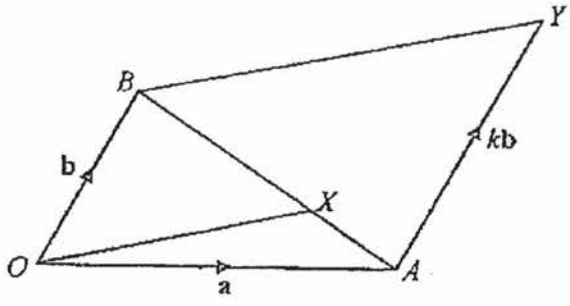
3(a)(i)	P(both cards show the letter T) $=\frac{3}{10}\times\frac{3}{10}$ $=\frac{9}{100}$	
(ii)	P(exactly one of the cards shows the letter T) $=\frac{3}{10}\times\frac{7}{10}+\frac{7}{10}\times\frac{3}{10}$ $=\frac{42}{100}=\frac{21}{50}$	
(b)(i)		

Age (x years)		$20\le x<30$	$30\le x<40$	$40\le x<50$
Frequency	Men	375	186	99
	Women	250	122	68

Probability that the person is a man aged less than 40 years old

	$=\frac{375+186}{1100}$ $=\frac{561}{1100}$ $=\frac{51}{100}$	
(ii)	Probability that both of them are women aged 30 or more $=\frac{122+68}{1100}\times\frac{122+68-1}{1099}$ $=\frac{190}{1100}\times\frac{189}{1099}$ $=\frac{513}{17270}=0.0297$	
		Total for Q3: 7

Vectors



4(i)	$\overrightarrow{AX}=\frac{1}{3}\overrightarrow{AB}=\frac{1}{3}(\mathbf{b}-\mathbf{a})$ $\overrightarrow{OX}-\overrightarrow{OA}=\frac{1}{3}(\mathbf{b}-\mathbf{a})$ $\overrightarrow{OX}=\frac{1}{3}(\mathbf{b}-\mathbf{a})+\mathbf{a}=\frac{1}{3}(2\mathbf{a}+\mathbf{b})$	
(ii)	$\overrightarrow{BY}=-\mathbf{b}+\mathbf{a}+k\mathbf{b}=\mathbf{a}+(k-1)\mathbf{b}$	
(iii)	OX is parallel to $BY \Rightarrow m\overrightarrow{OX}=\overrightarrow{BY}$ $\frac{1}{3}m(2\mathbf{a}+\mathbf{b})=\mathbf{a}+(k-1)\mathbf{b}$ $\begin{cases}\frac{2}{3}m=1\Rightarrow m=\frac{3}{2}\\ \frac{1}{3}m=k-1\Rightarrow k=1+\frac{1}{3}(\frac{3}{2})=\frac{3}{2}\end{cases}$	
(iv)	$\overrightarrow{AZ}=\frac{1}{2}\mathbf{b}$ since $\overrightarrow{OZ}=\overrightarrow{BY}$ and $\overrightarrow{OB}=\overrightarrow{ZY}$.	
(v)(a)	$\frac{\text{area of }\Delta OAX}{\text{area of }\Delta OBX}=\frac{AX}{BX}=\frac{(\frac{1}{3})}{(\frac{2}{3})}=\frac{1}{2}$	
(b)	$\frac{\text{area of }\Delta AXZ}{\text{area of }\Delta ABY}=\left(\frac{1}{3}\right)^2=\frac{1}{9}$ $\frac{\text{area of }\Delta AXZ}{\text{area of quadrilateral }XBYZ}=\frac{1}{8}$	
		Total for Q4: 9

Number patterns

n	Series	Sum	Formula
1	1	1	$\frac{1}{2}(1)(1+1)$
2	$1+2$	3	$\frac{1}{2}(2)(2+1)$
3	$1+2+3$	6	$\frac{1}{2}(3)(3+1)$
4	$1+2+3+4$	10	$\frac{1}{2}(4)(4+1)$
⋮	⋮	⋮	⋮
6	$1+2+3+4+5+6$	a	b
⋮	⋮	⋮	⋮
n	$1+2+3+\cdots+n$	c	

5(i) $b=\frac{1}{2}(6)(6+1)=21$

$a=21$

(ii) $c=\frac{1}{2}(n)(n+1)$

(iii)(a) $1^3+2^3+3^3+4^3+5^3+6^3=21^2=441$

(b) $1^3+2^3+3^3+\cdots+n^3=\left[\frac{1}{2}n(n+1)\right]^2$

(iv)
$$3^3+6^3+9^3+12^3+\cdots+300^3$$
$$=(3\times1)^3+(3\times2)^3+(3\times3)^3+(3\times4)^3+\cdots+(3\times100)^3$$
$$=3^3\left[1^3+2^3+3^3+4^3+\cdots+100^3\right]$$
$$=27\left[\frac{1}{2}(100)(101)\right]^2$$
$$=688567500$$

Total for Q5: 7

Matrices

6(a)

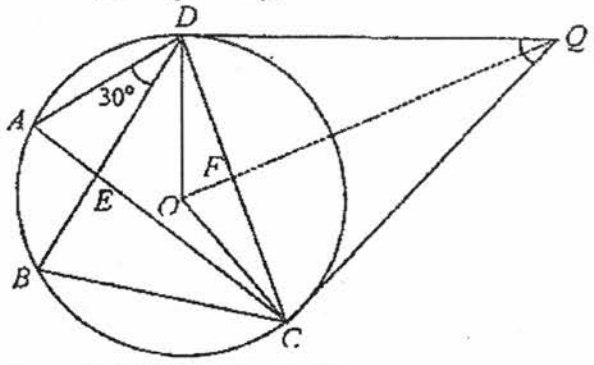
	Matrix	Order	Name of matrix
(i)	$\begin{pmatrix}2\\5\\12\end{pmatrix}$	3×1 **OR** 3 by 1	Column matrix
(ii)	$\begin{pmatrix}0&0\\0&0\end{pmatrix}$	2×2 **OR** 2 by 2	Square matrix **OR** Null matrix **OR** Zero matrix

(b)(i) $\mathbf{R}=\begin{pmatrix}20\\10\\2\end{pmatrix}$

(ii) Given $\mathbf{P}=\begin{pmatrix}150&80\\70&45\\10&30\end{pmatrix}$

	$\mathbf{Q} = \mathbf{P}\begin{pmatrix}1\\1\end{pmatrix} = \begin{pmatrix}150 & 80\\70 & 45\\10 & 30\end{pmatrix}\begin{pmatrix}1\\1\end{pmatrix} = \begin{pmatrix}230\\115\\40\end{pmatrix}$ $\mathbf{S} = 48\mathbf{Q} + 52\mathbf{R} = 48\begin{pmatrix}230\\115\\40\end{pmatrix} + 52\begin{pmatrix}20\\10\\2\end{pmatrix} = \begin{pmatrix}12080\\6040\\2040\end{pmatrix}$	
(iii)	The elements 12080, 6040 and 2040 represent the Tan family's yearly car expenses on petrol, carpark charges and ERP respectively.	
(iv)	$\mathbf{T} = \begin{pmatrix}1 & 1 & 1\end{pmatrix}\mathbf{S}$ $= \begin{pmatrix}1 & 1 & 1\end{pmatrix}\begin{pmatrix}12080\\6040\\2040\end{pmatrix} = (12080+6040+2040)$ $= (20160)_{1\times1}$ It represents the Tan family's total car expenses in a year.	
(v)	<u>Method 1:</u> New yearly expenses for petrol = $0.875\times12080 = \$10570$ carpark charges = \$6040 ERP = \$2040 <u>Method 2:</u> Given $\mathbf{P}_{new} = \begin{pmatrix}131.25 & 70\\70 & 45\\10 & 30\end{pmatrix}$ $\mathbf{Q}_{new} = \mathbf{P}_{new}\begin{pmatrix}1\\1\end{pmatrix} = \begin{pmatrix}131.25 & 70\\70 & 45\\10 & 30\end{pmatrix}\begin{pmatrix}1\\1\end{pmatrix} = \begin{pmatrix}201.25\\115\\40\end{pmatrix}$ $\mathbf{S}_{new} = 48\mathbf{Q}_{new} + 52\mathbf{R}_{new} = 48\begin{pmatrix}201.25\\115\\40\end{pmatrix} + 52\begin{pmatrix}17.5\\10\\2\end{pmatrix} = \begin{pmatrix}10570\\6040\\2040\end{pmatrix}$	
		Total for Q6: 11

Circle Properties (& similarity, congruency)



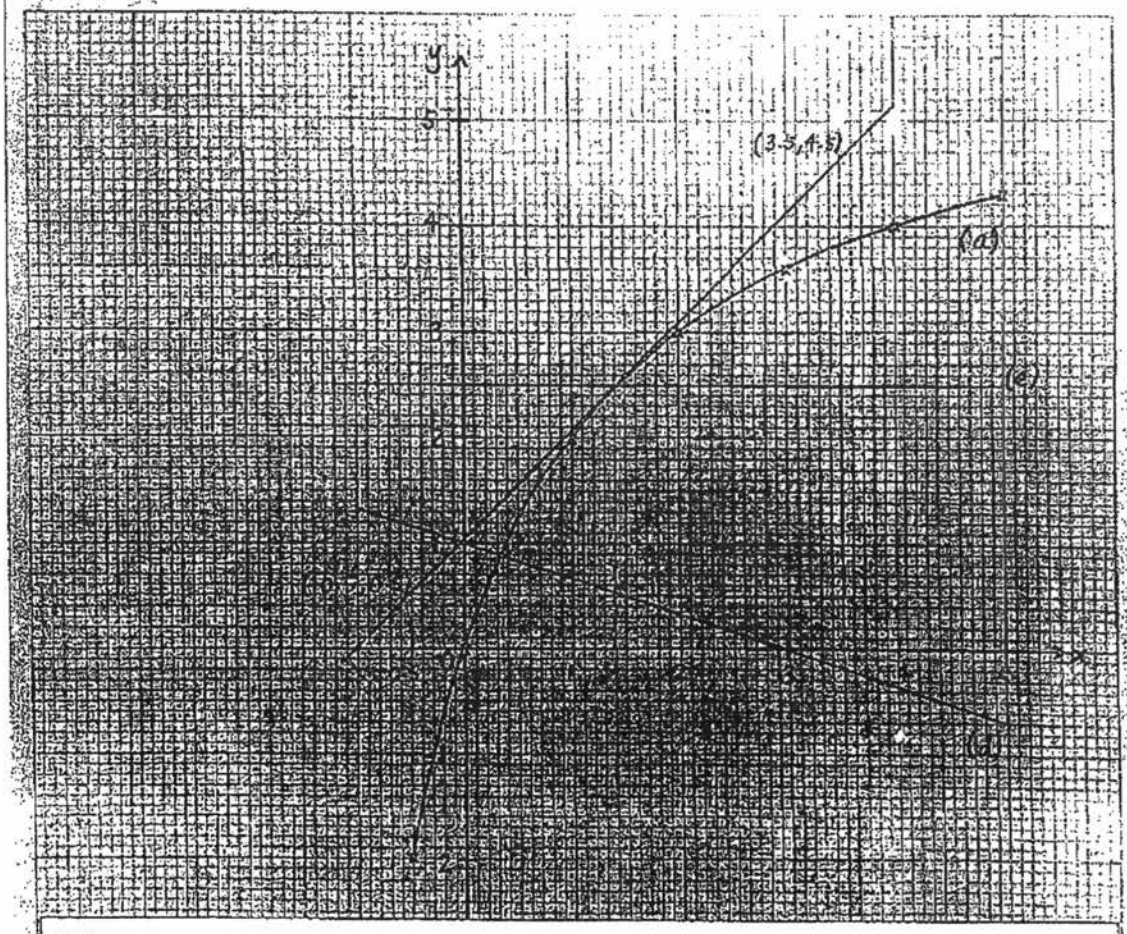
7(i)	$\angle ADE = \angle BCE$ (angle in the same segment) $\angle DAE = \angle CBE$ (angle in the same segment) By the **AA similarity test**, ΔADE is similar to ΔBCE.	
(ii)	Any of the following answers: ▫ ΔDOF is congruent to ΔCOF **OR** ▫ ΔDOQ is congruent to ΔCOQ **OR** ▫ ΔDFQ is congruent to ΔCFQ	
(iii) (a)	$\angle ODQ = \angle OCQ = 90°$ **(tan ⊥ rad)** $\angle DOQ = 360° - 90° - 90° - 50° = 130°$ $\angle DAC = \frac{1}{2}\angle DOC$ $= 65°$ **(angle at centre = twice angle at circumference)**	
(b)	$\angle BEC = \angle AED$ (vertically opposite angles) $= 180° - 30° - 65°$ ($\angle$ sum of triangle) $= 85°$	
(iv)	It is possible to draw a circle that passes through C, O, D and Q since $\angle ODQ = \angle OCQ = 90°$ **(tan ⊥ rad) and angle in a semicircle**. In this case, OQ is the diameter of the circle.	
		Total for Q7: 7

Graph

8(a)	$y = \frac{ax}{x+b}$ (1,2): $2 = \frac{a}{1+b} \Rightarrow a - 2b = 2$ (2,3): $3 = \frac{2a}{2+b} \Rightarrow 2a - 3b = 6$ Solving the two equations simultaneously, $a = 6, b = 2$	
(b)	Refer to graph on page 7.	
(c)		
(d)		
(e)		
		Total for Q8: 12

Name: _____

Subject: _____



(c) Gradient at $x = 1.5$

$$= \frac{4.5 - 0.5}{3.5 - (-0.5)}$$

$$= \frac{4}{4}$$

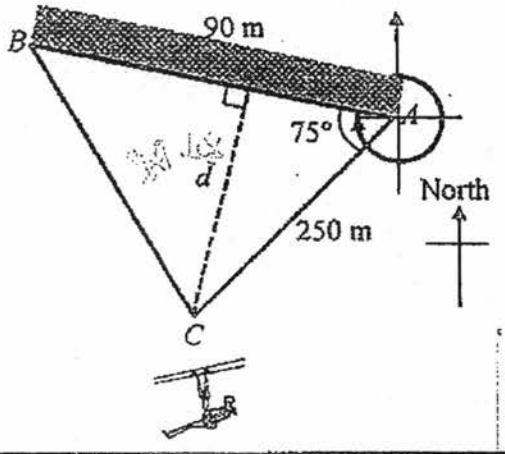
$$= 1$$

(d) $x = 0.35$

(e) $-0.5 \leq x < 1.4$

Trigonometry

9(a) Bearing of B from $A = 217° + 75° = 292°$



(b) Area of the land formed by the points A, B and C

$= \frac{1}{2}(90)(250)\sin 75°$

$= 10866.67$

$= 10900\ \text{m}^2$ (3 s.f.)

(c) Shortest distance from C to the bottom of the cliff

$= d$

$= 250\sin 75°$

$= 241.48$

$= 241\ \text{m}$ (3 s.f.)

(d) Slope is greatest when angle of elevation is the greatest from C.

Distance away from B

$= 90 - 250\cos 75°$

$= 25.295$

$= 25.3\ \text{m}$ (3 s.f.)

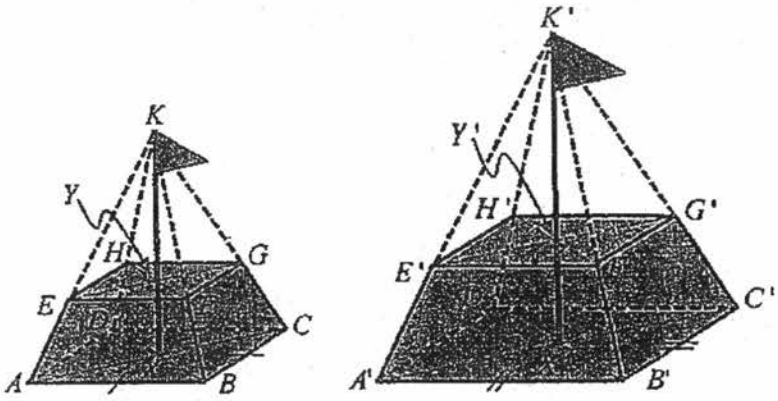
(e) Required angle

$= \tan^{-1}\left(\frac{50}{241.48}\right)$

$= 11.7°$ (1 d.p.)

Total for Q9: 9

Mensuration and similarity involving areas and volumes



10 (a)

$$\frac{\text{area of } ABCD}{\text{area of } A'B'C'D'} = \left(\frac{3}{5}\right)^2$$

$$\frac{\text{area of } ABCD}{2500} = \left(\frac{3}{5}\right)^2$$

$$\Rightarrow \text{area of } ABCD = \frac{9}{25} \times 2500 = 900 \text{ cm}^2$$

(b)

$$\frac{K'Y'}{K'X'} = \frac{40}{50} \Rightarrow K'Y' = \frac{4}{5} \times 2.5 = 2 \text{ m}$$

$$\text{Volume of the base} = \frac{1}{3} \times 2500 \times 250 - \frac{1}{3} \times 1600 \times 200$$

$$= 101666\frac{2}{3} \text{ cm}^3$$

(c)

$$\frac{\text{volume of smaller base}}{\text{volume of bigger base}} = \left(\frac{3}{5}\right)^3$$

$$\frac{\text{volume of smaller base}}{101666\frac{2}{3}} = \left(\frac{3}{5}\right)^3$$

$$\Rightarrow \text{volume of smaller base} = 21960 \text{ cm}^3$$

(d)

1 cm^3 of the smaller base costs $\$\frac{15}{21960} \approx \$0.000683$

1 cm^3 of the bigger base costs $\$\frac{25}{101666\frac{2}{3}} \approx \$0.000246$

Since 1 cm^3 of the bigger base costs cheaper, the bigger-sized flagpole base is more value for money.

Total for Q10: 9

PRWC – Arc length and area of sector, segment

Details of the stage

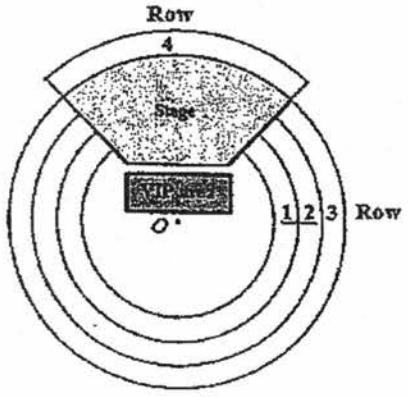


Figure 1

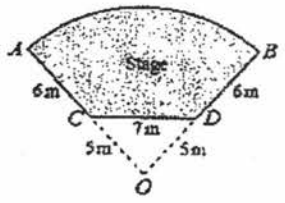


Figure 2

11 (i)

$$\angle COD = \cos^{-1}\left(\frac{5^2+5^2-7^2}{2(5)(5)}\right)$$

$$= \cos^{-1}\left(\frac{1}{50}\right)$$

$$= 1.5508 \text{ rad}$$

$$= 1.55 \text{ rad (3 s.f.)} \quad \text{(shown)}$$

$$\text{Area of stage} = \frac{1}{2}(11)^2(1.5508) - \frac{1}{2}(5)(5)\sin 1.5508$$

$$= 81.326 \text{ m}^2$$

$$= 81.3 \text{ m}^2$$

(ii)

Length of first row

$$= (5+3)(2\pi - 1.5508)$$

$$= 37.859 \text{ m}$$

Number of normal concert seats that can be accommodated in the first row $= \frac{37.859 \text{ m}}{0.80 \text{ m}} = 47.324 \approx 47$ (shown)

(iii)

Row	Length of row	No. of normal concert chairs	Cost of earnings
1	37.859 m	47	47×20=940
2	44.958 m	56	56×15=840
3	52.056 m	65	65×12=780
4	12.5(1.5 =19.38		24×25=600
Total earnings if package A taken up			$3160
Total earnings if package B taken up			$2560

Total number of normal concert chairs needed
$= 47 + 56 + 65 + 24 = 192$

Package	Cost of renting VIP chairs	Cost of renting normal concert chairs	Cost of rehearsals
A	$5 \times 18 \times 1.07$ $= \$96.30$	$(192-150) \times 8$ $\times 1.07$ $= \$359.52$	$100+80+90$ $= \$270$
Total cost for using package A $= \$2800 + 96.30 + 359.52 + 270$ $= \$3525.82$			
B	$20 \times 20 \times 1.07$ $= \$428$	$(192-100) \times 12$ $\times 1.07$ $= \$1181.28$	120×3 $= \$360$
Total cost for using package B $= \$1500 + 428 + 1181.28 + 360$ $= \$3469.28$			

Profit after taking up package A
$= \$3160 - 3525.82$
$= -\$365.82$

Profit after taking up package B
$= \$2560 - 3469.28$
$= -\$909.28$

Although package B seems cheaper than package A, taking into consideration the earnings, package A has a smaller loss than package B. Thus Peter and his committee should take up package A.

Assumptions:
- Other factors are not taken into consideration. The decision is made purely based on the profit made.

Total for Q11: 10

TANJONG KATONG SECONDARY SCHOOL
Preliminary Examination 2017
Secondary 4

CANDIDATE NAME			
CLASS		INDEX NUMBER	

MATHEMATICS

4048/01

Paper 1

Friday 18 August 2017

2 hours

Candidates answer on the Question Paper.

READ THESE INSTRUCTIONS FIRST

Write your name, class and register number on all the work you hand in.
Write in dark blue or black pen.
You may use a pencil for any diagrams or graphs.
Do not use staples, paper clips, highlighters, glue or correction fluid.

Answer **all** questions.
If working is needed for any question it must be shown with the answer.
Omission of essential working will result in loss of marks.
You are expected to use a scientific calculator to evaluate explicit numerical expressions.
If the degree of accuracy is not specified in the question, and if the answer is not exact, give the answer to three significant figures. Give answers in degrees to one decimal place.
For π, use either your calculator value or 3.142, unless the question requires the answer in terms of π.

At the end of the examination, fasten all your work securely together.
The number of marks is given in brackets [ ] at the end of each question or part question.
The total of the marks for this paper is 80.

For Examiner's Use

This document consists of 16 printed pages.

[Turn over

Mathematical Formulae

Compound Interest

$$\text{Total Amount} = P\left(1+\frac{r}{100}\right)^n$$

Mensuration

Curved surface area of a cone $= \pi r\ell$

Curved surface area of a sphere $= 4\pi r^2$

$$\text{Volume of a cone} = \frac{1}{3}\pi r^2 h$$

$$\text{Volume of a sphere} = \frac{4}{3}\pi r^3$$

$$\text{Area of triangle } ABC = \frac{1}{2}ab\sin C$$

Arc length $= r\theta$, where θ is in radians

$$\text{Sector area} = \frac{1}{2}r^2\theta, \text{ where } \theta \text{ is in radians}$$

Trigonometry

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$a^2 = b^2 + c^2 - 2bc\cos A$$

Statistics

$$\text{Mean} = \frac{\sum fx}{\sum f}$$

$$\text{Standard Deviation} = \sqrt{\frac{\sum fx^2}{\sum f} - \left(\frac{\sum fx}{\sum f}\right)^2}$$

Answer **all** the questions.

1 Calculate $\sqrt[3]{(-3.01)^2+2.8}$.

(a) Write down the first five digits on your calculator display.

Answer (a) _____ [1]

(b) Write your answer to part **(a)** correct to 3 decimal places.

Answer (b) _____ [1]

2 These are the first four terms of a sequence.

42 34 26 18

(a) Write down the eighth term in the sequence.

Answer (a) _____ [1]

(b) Write down an expression, in terms of n, for the nth term in the sequence.

Answer (b) _____ [1]

3 Given that $81 \div 27^{\frac{n}{3}} = 9$, find n.

Answer _____ [2]

For Examiner's Use

For Examiner's Use

4 (a) Two integers, 12 and x, are related such that their highest common factor is 6 and their lowest common multiple is 60.
Find the value of integer x.

Answer (a) $x =$ _____ [1]

(b) Andy bought an external hard drive with storage of 1×10^{12} bytes.
A 5-minute-long high definition video takes up 7.2×10^{9} bytes.

Assuming he continues to record all his videos in high definition, what would be the total duration that can be stored in the external hard drive?
Give your answer to the nearest minute.

Answer (b) _____ minutes

[1]

5 The angle, in degrees, of a quadrilateral $EFGH$ are represented by these expressions: Angle $E = 40 + 2x$, angle $F = 100 - x$, angle $G = 60 + 6x$ and angle $H = 70 + 2x$.

(a) Calculate the value of x.

Answer (a) _____ [2]

(b) What is the name of the quadrilateral?

Answer (b) _____ [1]

6 The value of 200 homes at Mount Ace estate is shown below.

Value of homes ($\$x$)	Number of homes
$200\,000 < x \leq 300\,000$	24
$300\,000 < x \leq 400\,000$	16
$400\,000 < x \leq 500\,000$	85
$500\,000 < x \leq 600\,000$	67
$600\,000 < x \leq 3\,000\,000$	8

The mean value for the homes at Mount Ace estate is \$505 500.
Explain if the mean value is a fair representation for the value of homes at Mount Ace estate. Give your reason.

Answer _____

_____

_____

_____ [2]

7 **(a)** Factorise completely $8y^2z - 18z + 4x^2y^2 - 9x^2$.

*Answer (a)*_____ [2]

(b) Simplify $(-ab^{-1})^3 \div \frac{1}{2}a^3b^{-2}$, expressing your answer in positive index form.

Answer (b) _____ [3]

8 $\xi = \{\text{integers } x : 1 \le x \le 20\}$
$P = \{x : \text{prime numbers}\}$
$Q = \{x : 1 + 3x < 18\}$

(a) List the elements in

(i) Q,

Answer(a)(i) _____ [1]

(ii) $P \cap Q$.

*Answer(a)(ii)*_____ [1]

(b) Show that $P' \cap Q \neq \phi$.

Answer (b) _____

_____ [1]

9 Two geometrically similar bottles A and B have base areas of 27 cm^2 and 75 cm^2 respectively.
Given that the capacity of bottle A is 0.21 litres, find the capacity of bottle B.

Answer _____ *l* [3]

10 A group of 15 students took a Science test and their results are represented in the stem-and-leaf diagram below.

Stem	Leaf
5	3 4 6 7
6	2 2 4 9 9
7	1 3 7
8	0 2 x

5 | 3 represents 53 marks

(a) Given that the range of the Science test results is 32, find the value of x.

Answer (*a*) x = _____ [1]

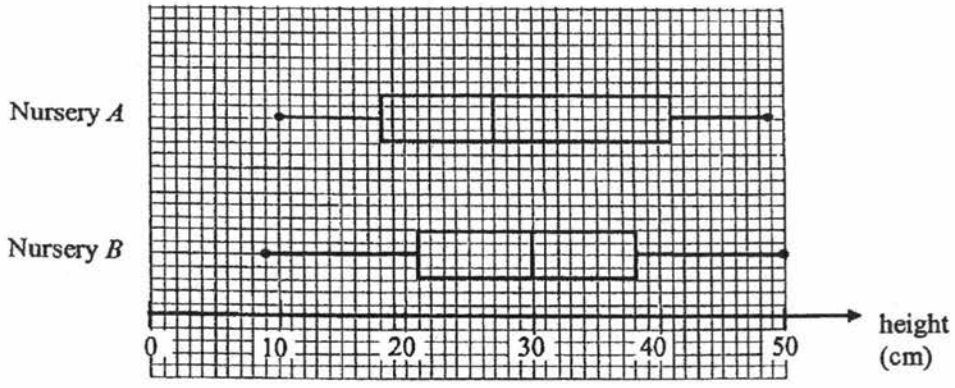
(b) The passing mark for the Science test is 55. A student from this group is chosen at random. Find the probability that this student failed the test.

Answer (*b*) _____ [1]

(c) Find the percentage of students who scored more than 75 marks.

Answer (*c*) _____ % [1]

11 The box plots below show the distribution of plants grown in two nurseries, *A* and *B*.



(a) Find the interquartile range for Nursery *A*.

Answer (a) _____ [1]

(b) For each of the statements below, write whether you agree or disagree. Give a reason for each answer, stating clearly which statistics you use to make your decision.

(i) On average, the plants in Nursery *A* grows taller than in Nursery *B*.

Answer _____ because _____

_____ [1]

(ii) A greater proportion of the plants grow above the height of 40 cm in Nursery *B* than do in Nursery *A*.

Answer _____ because _____

_____ [1]

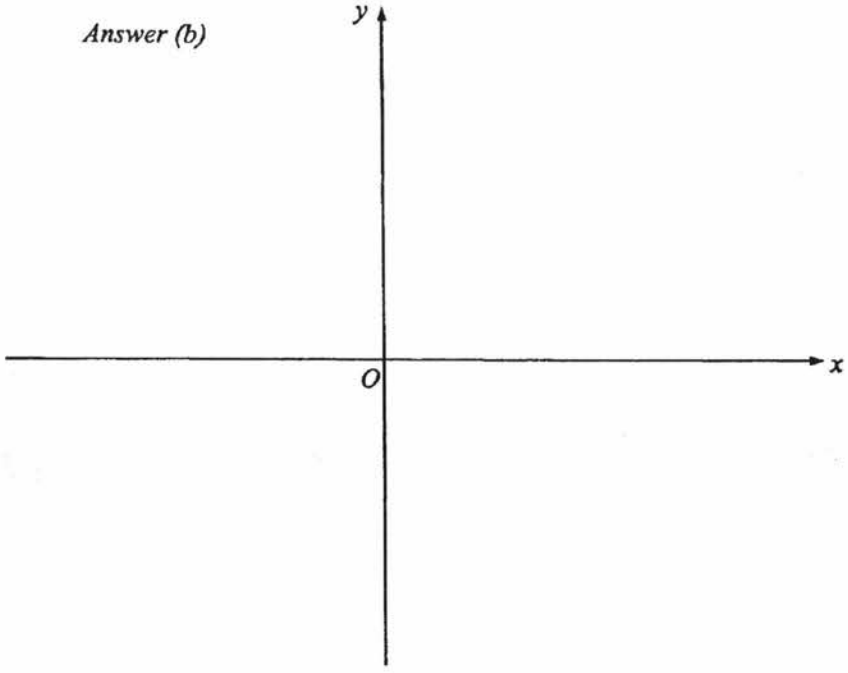
For Examiner's Use

12 (a) Express $x^2 - 6x + 4$ in the form $(x-a)^2 + b$.

Answer (a) _____ [1]

(b) Sketch the graph of $y = x^2 - 6x + 4$.

Answer (b) [2]



(c) The graph of $y = x^2 - 6x + 4$ is reflected in the y-axis. Write down the equation of the line of symmetry for the new graph.

Answer (c) _____ [1]

13 Mr Toh needs to tile his office floor which has an area of 60 square metres (sqm). Which company will offer a cheaper deal for Mr Toh? Justify your answers with calculations.

TIMBRE WORKS	TILE KING
$35 per sqm (for first 40 sqm) 30% discount thereafter	FLAT RATE $25 per sqm

Answer _____ [3]

14 Water is pumped into a cylindrical container at a constant rate such that x litres is pumped in t minutes. 144 litres of water is collected in the cylindrical container after 3 hours.

Find

(a) an equation for x in terms of t,

Answer (a) _____ [2]

(b) the time taken, in hours and minutes to fill a volume of 400 litres.

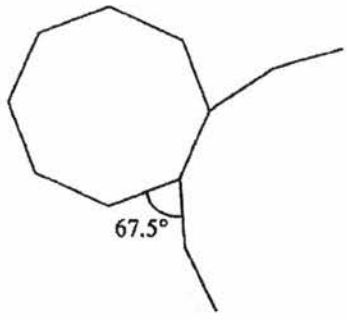
___ hours _____ minutes [2]

15 **(a)** Explain whether it is possible to form a regular polygon with an interior angle of 125°.

Answer (a) _____

_____ [2]

(b) The diagram shows a sketch of a n-sided regular polygon and a regular octagon. Calculate n.



Answer (b) n = _____ [3]

16 Bag A contains three balls numbered 2, 3 and 4 respectively.
Bag B contains four balls numbered 1, 3, 5 and 7 respectively.
A ball is taken at random from each bag and their respective numbers f and g are recorded.

(a) Complete the table to show the possible outcomes for the sum of the two numbers f and g, on the balls selected.

		f, number on ball from Bag A		
		2	3	4
g, number on ball from Bag B	1			
	3			
	5			
	7			

[1]

(b) Find the probability that

(i) $f+g<7$,

Answer (b)(i) _____ [1]

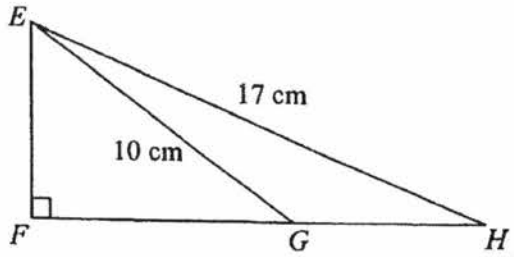
(ii) $f+g$ is an odd number,

Answer (b)(ii) _____ [1]

(iii) $f>g$.

Answer (b)(iii) _____ [1]

17 The figure shows triangle EFH where $EH = 17$ cm and $\angle EFH = 90°$
G is a point on FH such that $EG = 10$ cm.



(a) Given that $\sin \angle EGH = \frac{3}{5}$, find

(i) EF,

*Answer (a)(i)*_____ cm [1]

(ii) $\tan \angle EGH$.

*Answer (a)(ii)*_____ [2]

(b) Find the shortest distance from F to EH.

*Answer (b)*_____ cm [3]

(c) A circle C_1 is drawn passing through E, F and G.
A second circle C_2 is drawn passing through E, F and H.
Find the ratio of the circumference of C_1 to circumference of C_2,

Answer (c) _____ : _____ [1]

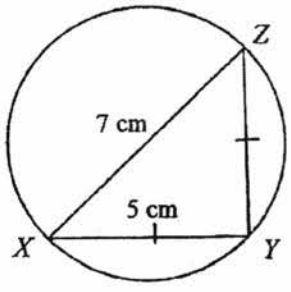
18 The mean, median and mode of the distribution of heights for 9 athletes are all equal to 165 cm.

Three of the athletes have a height of 165 cm and the tallest athlete is 170 cm.

Given that the heights of the athletes are integers, find the least possible height of the shortest athlete.

Answer _____ cm [3]

19 The diagram shows an isosceles triangle inscribed in a circle where $XZ = 7$ cm and $XY = YZ = 5$ cm. Determine whether XZ is a diameter of the circle.
Explain your answer.

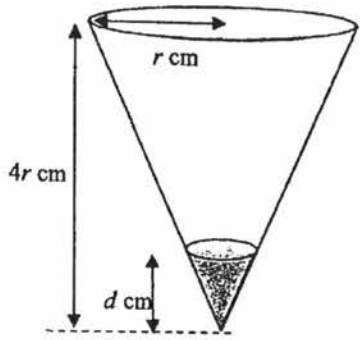


Answer _____

_____

_____ [2]

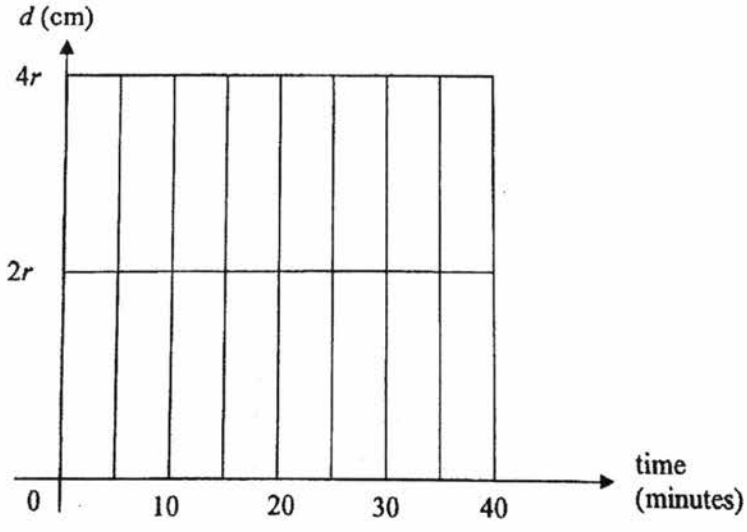
20 A container in the shape of an inverted cone has a top radius of r cm and a height of $4r$ cm. Water is poured into the container at a constant rate. It takes 40 minutes to fill the container completely with water.



(a) Calculate the time taken to fill the container to a height of $2r$ cm.

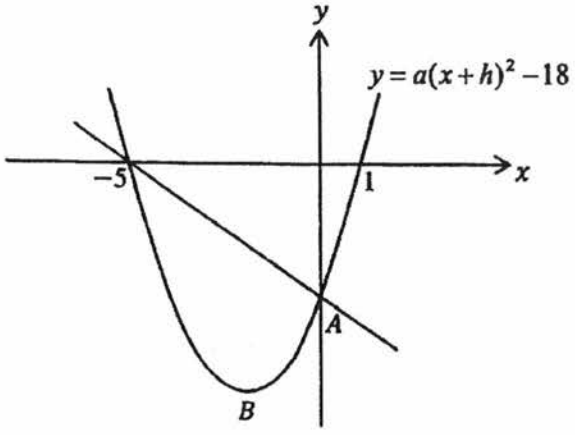
*Answer (a)*_____ minutes [2]

(b) A graph is drawn to show the relationship between the depth of the water, d cm, and the time taken, t minutes, as the container is filled. Complete the graph to represent how the depth of water changes with time.



[2]

21 The diagram below shows a curve of $y = a(x+h)^2 - 18$.
The curve cuts the x-axis at -5 and 1 and the y-axis at A.
B is the minimum point on the curve.



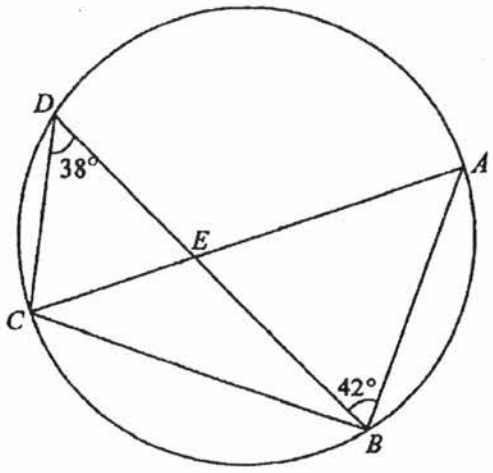
(a) Express the equation of the curve in the form of $y = a(x+h)^2 - 18$, where a and h are constants.

Answer (a) $y =$ _____ [3]

(b) A straight line cuts the curve at $x = -5$ and point A.
Find the equation of the straight line.

_____ [2]

22 The points A, B, C, and D lie on the circumference of a circle such that $\angle BDC = 38°$, $\angle ABD = 42°$ and $\angle ABC = 90°$. Chords AC and BD intersect at E.



(a) (i) Giving your reason, find angle ACD.

Answer (a)(i) _____ [1]

(ii) State whether EC is longer than ED. Give your reason clearly.

Answer (a)(ii) _____ [1]

(b) Describe where the centre of the circle is.

Answer (b) _____ [1]

23 The scale drawing shows the positions of two train stations, P and Q.
The scale is 1 cm to 10 km.

A third train station, R is 80 km from P on a bearing of 150°.
(a) Mark and label on the diagram the position of train station R. [1]

A train, T travels along a path which is equidistant from PR and RQ.
(b) Using ruler and compasses only, mark and label the path in which train T moves. [1]

(c) At a particular instant, the position of train T is such that it is equidistant from train stations P and Q. Using ruler and compasses only, mark and label the position of train T at that instant. [2]

(d) Train T approaches train station R at an average speed of 95 km/h. Calculate the time taken from its position in **(c)** to arrive at R. Give your answer in minutes.

Answer (d) _____ minutes [2]

END OF PAPER

TANJONG KATONG SECONDARY SCHOOL
Preliminary Examination 2017
Secondary 4

CANDIDATE NAME			
CLASS		INDEX NUMBER	

MATHEMATICS

4048/02

Paper 2

Wednesday 23 August 2017

2 hours 30 minutes

Additional Materials: Writing Paper
Graph Paper

READ THESE INSTRUCTIONS FIRST

Write your name, class and register number on all the work you hand in.
Write in dark blue or black pen.
You may use a pencil for any diagrams or graphs.
Do not use staples, paper clips, highlighters, glue or correction fluid.

Answer **all** questions.
If working is needed for any question it must be shown with the answer.
Omission of essential working will result in loss of marks.
You are expected to use a scientific calculator to evaluate explicit numerical expressions.
If the degree of accuracy is not specified in the question, and if the answer is not exact, give the answer to three significant figures. Give answers in degrees to one decimal place.
For π, use either your calculator value or 3.142, unless the question requires the answer in terms of π.

At the end of the examination, fasten all your work securely together.
The number of marks is given in brackets [ ] at the end of each question or part question.
The total of the marks for this paper is 100.

This document consists of **11** printed pages and 1 blank page.

1 A soccer club offers annual memberships for both adults and juniors.
The adult annual membership fee is $150.
Junior members need to pay 80% of the adult annual membership fee.

(a) Calculate the discount each junior member receives. [1]

If an adult member does not pay the membership fee by the due date, the club will charge a penalty of 5% per month until the fee is paid.
Simon paid the $150 membership fee exactly two months after the due date.

(b) Calculate the penalty that Simon will be charged. [1]

The soccer club received a statement of the transactions in its saving account for the month of January 2017.

Date	Details	Deposit	Withdrawal	Balance
01 Jan 2017	Brought Forward			$63950.00
09 Jan 2017	Match Fees	$750.00		$64700.00
15 Jan 2017	Withdrawal		[redacted]	$42700.00
23 Jan 2017	Membership Fees	$3800.00		$46500.00
31 Jan 2017	Interest	$124.54		$46624.54

(c) (i) Calculate the withdrawal amount on 15 Jan 2017. [1]

(ii) Interest on the account is calculated on the minimum balance for the month and added to the account on the last day of the month.

What is the annual rate of interest for this account?
Write your answer, correct to one decimal place. [2]

(d) The soccer club plans to invest $120 000 in an account which pays compound interest at the rate of 2% per annum, compounded monthly.
Find the total amount that can be withdrawn at the end of 4 years. [2]

2 A toothpaste firm supplies tubes of toothpaste to 2 different stores.
The number of tubes of toothpaste supplied per delivery to each store, the sizes of the tubes and the number of deliveries made to each store over a year are shown below. [Turn over

		Number of tubes per delivery			Number of deliveries over a year
Size of tube		50 ml	75 ml	100 ml	
Name of store	Econ	400	300	400	2
	Prime	-	200	600	4

(i) Given that $\mathbf{T} = \begin{pmatrix} 400 & 300 & 400 \\ 0 & 200 & 600 \end{pmatrix}$, find the matrix product $\mathbf{S} = \mathbf{T}\begin{pmatrix} 50 \\ 75 \\ 100 \end{pmatrix}$. [1]

(ii) Describe what the elements in S represent. [1]

(iii) Write down two matrices such that the elements of their product under matrix multiplication would give the total number of tubes of toothpaste of each size supplied **by the firm** over a year. Find this product. [2]

---

3 (a) Solve the inequality $\frac{2p-1}{5} \leq \frac{3+p}{2}$. [2]

(b) Simplify $\frac{12x^2}{4y} \div \frac{6x^3}{y^4}$. [2]

(c) Simplify the expression $\frac{4w^2-36}{2w^2+7w+3}$. [3]

(d) (i) Express as a single fraction in its simplest form

$$\frac{2}{y+3} - \frac{3}{y-1}.$$ [2]

(ii) Solve the equation

$$\frac{2}{y+3} - \frac{3}{y-1} = 5.$$ [3]

4 (a) (i) Express 4536 as the product of its prime factors. [1]

(ii) Given that $\frac{4536}{k} = p^3$, where k and p are integers and p is as large as possible, find the values of k and of p. [1]

(iii) The lowest common multiple of two numbers is 4536.
The highest common factor of these two numbers is 126.
Both numbers are greater than 126.

Find the two numbers. [2]

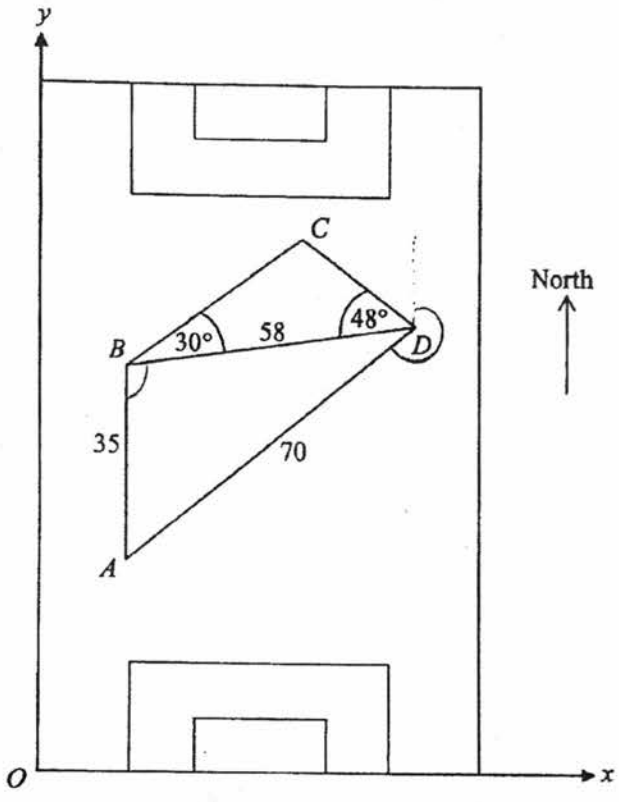
(b) When n is a positive integer, $2n + 3$ is an odd number.

(i) Write down an expression for the next odd number greater than $2n + 3$. [1]

(ii) Find and simplify an expression for the difference between the squares of these two odd numbers. [2]

(iii) Hence explain why the difference between the squares of two consecutive odd numbers is always a multiple of 8. [1]

5



(a) During a soccer match a ball is passed from A to B and then from B to D as shown in the diagram. B is due north of A.
$AB = 35$ m, $BD = 58$ m and $AD = 70$ m.

(i) Show that angle $DAB = 55.7°$. [1]

(ii) Find the bearing of A from D. [1]

(iii) Calculate the area of triangle DAB. [2]

(b) Another player is standing at C.
Angle $CBD = 30°$ and angle $BDC = 48°$.
Calculate the length CD. [2]

(c) The x- and y- axes are shown in the diagram.
$\overrightarrow{AD} = \begin{pmatrix} p \\ q \end{pmatrix}$, where p and q are measured in metres.

(i) Show that $p = 57.8$. [1]

(ii) Find the value of q. [2]

6 (a) A has coordinates $(-3, 5)$ and $\overrightarrow{AB}$ is given by $\begin{pmatrix} -7 \\ -4 \end{pmatrix}$.

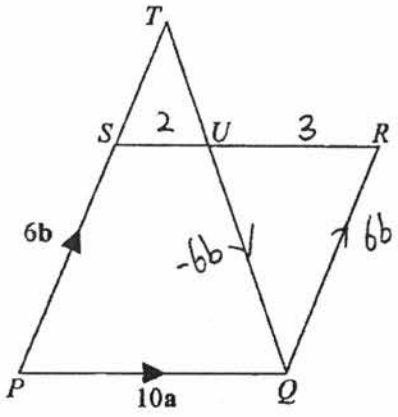
Find

(i) $|\overrightarrow{AB}|$, [1]

(ii) the position vector of B. [1]

(iii) Given that $\overrightarrow{CD}$ is parallel to $\overrightarrow{AB}$, and $\overrightarrow{CD} = \begin{pmatrix} k \\ 16 \end{pmatrix}$, find the value of k. [2]

(b)



$PQRS$ is a parallelogram.
$\overrightarrow{PS} = 6\mathbf{b}$ and $\overrightarrow{PQ} = 10\mathbf{a}$.

U is the point on SR such that $SU : SR = 2 : 5$.
When produced, PS and QU meet at T.

(i) Express each of the following, as simply as possible, in terms of **a** and/or **b**,

(a) $\overrightarrow{PR}$, [1]

(b) $\overrightarrow{SU}$, [1]

(c) $\overrightarrow{TU}$. [2]

(ii) Calculate the value of

(a) $\dfrac{\text{area of triangle } QRU}{\text{area of triangle } QUS}$, [1]

(b) $\dfrac{\text{area of triangle } SUT}{\text{area of triangle } PQT}$. [1]

7 **Answer the whole of this question on a sheet of graph paper.**

An **open** rectangular tank has a square base of side x metres.
The volume of the tank is 9 m^3.

(a) **(i)** Find an expression, in terms of x, for the height of the tank. [1]

(ii) Hence show that the total external surface area of the tank, A square metres, is given by

$$A = x^2 + \frac{36}{x}.$$ [1]

(b) The table below shows some values of x and the corresponding values of A.

x	2	2.5	3	4	5	6	7	8
A	22	20.7	21	25	32.2	42	54.1	p

(i) Find the value of p. [1]

(ii) Using a scale of 2 cm to represent 1 unit, draw a horizontal x-axis for $2 \le x \le 8$.
Using a scale of 2 cm to represent 10 m^2, draw a vertical A-axis for $20 \le A \le 80$.

On your axes, plot the points given in the table and join them with a smooth curve. [3]

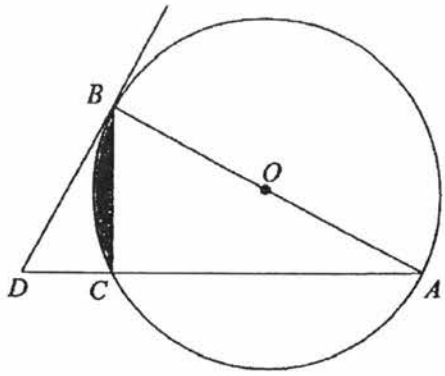
(iii) By drawing a tangent, find the gradient of the curve at the point where $x = 4$. [2]

(iv) Use your graph to find

(a) the value of x for which the surface area is 50 m^2. [1]

(b) the dimensions of the tank which has the least possible surface area. [2]

8 The diagram shows a circle, ABC, centre O.
BD is a tangent to the circle and it meets AC produced at D.



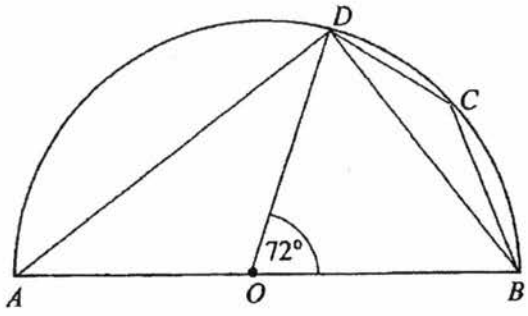
(a) Show that triangles ABD and BCD are similar. [2]

(b) Given that ratio area of triangle ABD : area of triangle $BCD = 4 : 1$ and the radius of the circle is 7.5 cm,

(i) show that angle $BAC = \frac{\pi}{6}$ radian, [2]

(ii) find the perimeter of the shaded region. [3]

(c) In the diagram, A, B, C and D are points on the circumference of a semi-circle, centre O.



(a) Calculate, stating your reasons clearly,

(i) angle DAB, [1]

(ii) angle ABD, [1]

(iii) reflex angle BCD. [2]

(b) Given that $OB = 3.5$ cm, find the area of the segment BCD. [3]

9 (a) The table shows the sizes of 50 pairs of ladies' shoes sold one day in a shoe shop.

Shoe sizes	5	6	7	7.5	8	8.5	9
Number of pairs of shoes sold	4	18	3	5	8	7	5

(i) Find the median shoe size. [1]

(ii) Find the modal shoe size. [1]

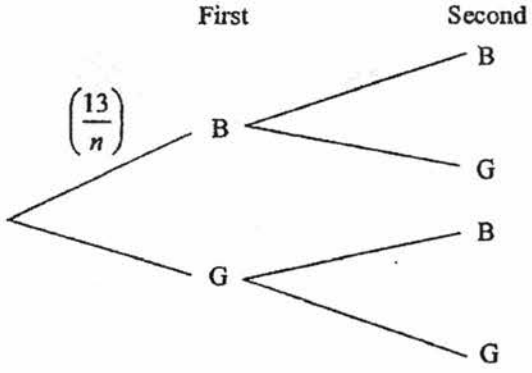
(iii) Explain which central measure would be the most appropriate and useful to the manager when she is ordering stock. [2]

(iv) Find the standard deviation of the shoe sizes. [1]

(v) The standard deviation of the shoe sizes of mens' shoes sold on the same day was 1.52.

Use this information to comment on one difference between the two distributions. [1]

(b) In a class of n students, 13 of them are boys and the rest are girls.
Two students are selected at random to represent the class at a conference.
The tree diagram shows the possible outcomes and their probabilities.



(i) Copy and complete the tree diagram. [2]

(ii) Find, as a single fraction in terms of n, the probability that

(a) the first student selected is a girl, [1]

(b) two boys are selected. [2]

(iii) The probability that two girls selected is $\frac{5}{18}$.

Find the total number of students in the class. [4]

10 Amos makes cookies.

The amount of dough needed to make one cookie is 8 grammes.

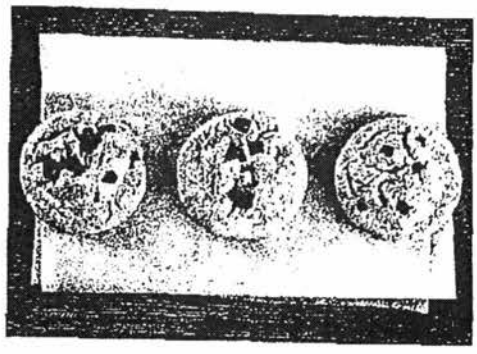
The density of the dough is 0.5333 g/cm^3.

(i) Find the volume of dough needed for each cookie. [1]

The dough is rolled into a sphere before baking.

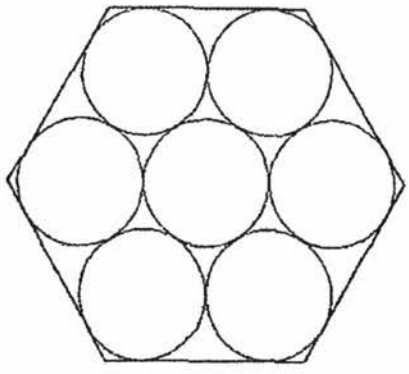
(ii) Calculate the radius of the sphere. [2]

When each cookie is baked, it forms a shape as shown.
The cookie can be modelled as a cylinder of radius 3 cm and a height of 0.7 cm.
The increase in volume is due to air trapped in the cookie.



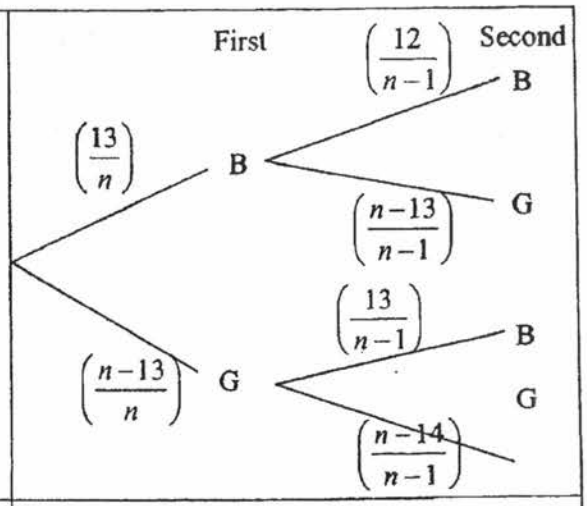
(iii) Calculate the volume of air trapped in the cookie. [2]

A regular hexagonal box is designed to hold 7 such cookies per layer, as shown.

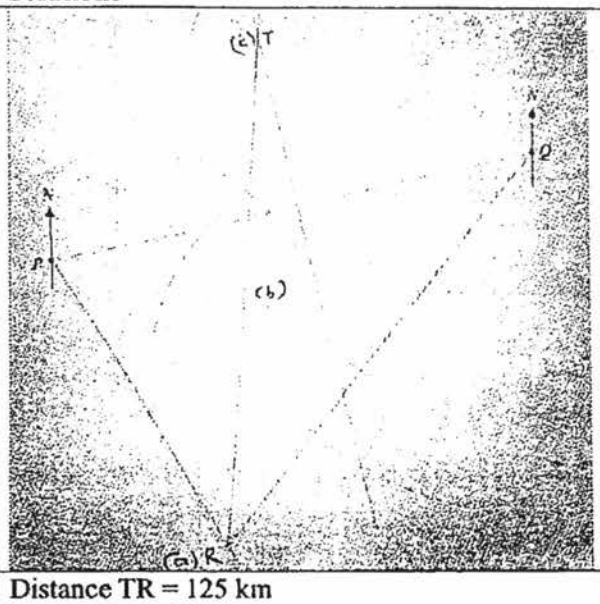


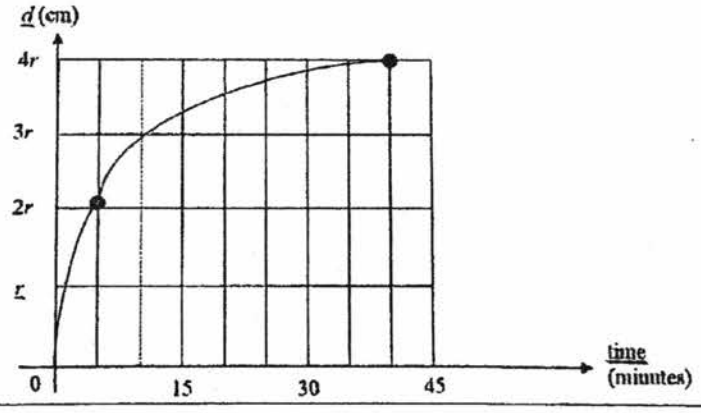
(iv) Find the volume of the box if it is to hold five layers of cookies. [5]

End of Paper

5	ai	$\cos D\hat{A}B = \dfrac{35^2 + 70^2 - 58^2}{2(35)(70)}$		bi	
	aii	235.7°		biia	$\dfrac{n-13}{n}$
	aiii	1011.97 m²		biib	$\dfrac{156}{n(n-1)}$
	b	$CD = 29.6$		biii	$n = 28$ or $n = 9$ (rej)
	ci	$\cos(90° - 55.7°) = \dfrac{p}{70}$	10	i	15.0 cm³
	cii	$q = 39.4$		ii	$r = 1.53$ cm
6	ai	8.06		iii	4.80 cm³
	aii	$\begin{pmatrix} -10 \\ 1 \end{pmatrix}$		iv	814.4745 cm³
	aiii	$k = 28$			
	bia	$\overrightarrow{PR} = 10\mathbf{a} + 6\mathbf{b}$			
	bib	$\overrightarrow{SU} = 4\mathbf{a}$			
	bic	$\overrightarrow{TU} = -4\mathbf{b} + 4\mathbf{a}$			
	biia	$\dfrac{3}{2}$			
	biib	$\dfrac{4}{25}$			

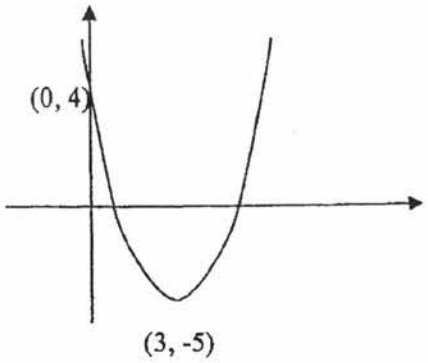
1	a	\$30	7	ai	$\frac{9}{x^2}$
	b	\$15		aii	$4x\left(\frac{9}{x^2}\right)$
	ci	\$22 000		bi	$p = 68.5$
	cii	3.5%		bii	All points correctly plotted Smooth curve drawn
	d	129985.79		biii	Draw tangent at $x = 4$ Grad = 6.38
2	i	$\begin{pmatrix} 82500 \\ 75000 \end{pmatrix}$		biva	$x = 6.8$
	ii	The element in S represent the total volume of toothpaste (in ml) supplied to Econ and Prime respectively.		bivb	Dimensions= 2.5 m × 2.5 m × 1.44 m
	iii	$\begin{pmatrix} 2 & 4 \end{pmatrix}\begin{pmatrix} 400 & 300 & 400 \\ 0 & 200 & 600 \end{pmatrix}$ $\begin{pmatrix} 800 & 1400 & 3200 \end{pmatrix}$	8	a	$\angle BCD = 90°$ (angles in semi-circle) $\angle ABD = 90°$ (tangent perpen. radius) $\therefore \angle ABC = \angle BCD$ $\angle BDC$ is common angle $\therefore \Delta ABD$ and ΔBCD are similar
3	a	$p \geq -17$		bi	$\frac{BD}{CD} = \frac{2}{1} \Rightarrow \frac{AB}{BC} = \frac{2}{1}$ Since radius = 7.5 cm AB =15 and BC = 7.5 cm $\sin B\hat{A}C = \frac{1}{2}$ $B\hat{A}C = \frac{\pi}{6}$ (shown)
	b	$\frac{y^3}{2x}$		bii	15.4 cm
	c	$\frac{4(w-3)}{2w+1}$		cai	$\angle DAB = 36°$ ($\angle$ at centre = 2 $\angle$ at circumference)
	di	$\frac{-y-11}{(y+3)(y-1)}$		caii	$\angle ABD = \frac{180-72}{2}$ (base $\angle$ of isos. Δ) $= 54°$
	dii	$y = 0.318$ or -2.52		caiii	216°
4	ai	$2^3 \times 3^4 \times 7$		cb	1.87 cm^2
	aii	$k = 21$ $p = 6$	9	a i	7.25
	aiii	504 and 1134		ii	6
	4bi	$2n + 5$		iii	Mode will be the most appropriate and useful as the manager can stock up more shoes of size 6.
	4bii	$8n + 16$		iv	1.25
	4biii	$8(n + 2)$ is a multiple of 8 for n is a positive integer		v	The shoe sizes of ladies are more consistent than the men's shoe sizes.

Qn No.		Solutions
23	a	
	b	
	c	
	d	Distance TR = 125 km Time = 125 ÷ 95 = 78.9 minutes

Qn No.		Solutions
	b(i)	$\frac{5}{12}$
	b(ii)	$\frac{2}{3}$
	b(iii)	$\frac{1}{3}$
17	a(i)	EF = 6 cm
	a(ii)	$\tan \angle EGH = -\frac{6}{8} = -\frac{3}{4}$
	b	5.61*cm*
	c	10 : 17
18		Least possible height = 150 cm
19		$XY^2 + YZ^2 = 5^2 + 5^2 = 50$ $XZ^2 = 7^2 = 49$ Since $XY^2 + YZ^2 \neq XZ^2$, XZY is not a right-angled triangle. Hence, XZ is not a diameter (Angle in semicircle).
20	a	Time taken = $40 \div 2^3$ = 5 mins
	b	
21	a	$y = 2(x+2)^2 - 18$
	b	Eqn: $y = -2x - 10$
22	a(i)	$\angle ACD = 42°$ (angles in same segment)
	a(ii)	$\frac{EC}{\sin 38°} = \frac{ED}{\sin 42°}$ Since 42° > 38°, ED is longer than EC
	b	Given angle ABC = 90°, AC is a diameter of the circle (angle in semicircle) Centre is at midpoint of AC.

For Examiner's Use

For Examiner's Use

Qn No.		Solutions
	b	
	c	$x = -3$
13		Price for Timbre works $= (40)(35) + 0.7(35)(20)$ $= \$1890$ Price for Tile King $= 25(60)$ $= \$1500$ Tile king is cheaper
14	a	$x = \frac{144}{180}t$ $x = \frac{4}{5}t$
	b	$t = 8h\ 20\text{min}$
15	a	Ext angle of polygon = $180^\circ - 125^\circ = 55^\circ$ No of sides of polygon = $\frac{360}{55} = 6.545$ Since number of sides is not an integer, it is not possible to form a polygon with interior angle 125°.
	b	16 sides
16	a	

		f, number on ball from Bag *A*		
		2	3	4
g, number on ball from Bag *B*	1	***3***	***4***	***5***
	3	***5***	***6***	***7***
	5	***7***	***8***	***9***
	7	***9***	***10***	***11***

Answer Key:

Qn No.		Solutions
1	a	2.2804
	b	2.280
2	a	-14
	b	$-8n + 50$
3		$3^{4-n} = 3^2$ $n = 2$
4	a	$x = 2 \times 3 \times 5 = 30$
	b	694 min
5	a	$x = 10°$
	b	Kite
6		It is not a fair representation as - only 37.5% of the homes are valued above \$500,000 (majority of homes are valued less than \$505500) - the mean value is skewed by extreme values in the \$600,000 < x < \$3,000,000 group.
7	a	$(2z + x^2)(2y + 3)(2y - 3)$
	b	$\frac{-2}{b}$
8	a(i)	1, 2, 3, 4, 5
	a(ii)	2, 3, 5
	b	P' are not prime numbers. Since Q contains elements that are not prime, $P' \cap Q$ is not a null set. OR $P' \cap Q = \{1,4\}$ Hence, $P' \cap Q \neq \phi$
9		$V_{big} = 0.972l$
10	a	5
	b	$\frac{2}{15}$
	c	26.7%
11	a	23
	b(i)	Disagree because the median height in A is lesser than in B.
	b(ii)	Disagree because more than 25% of the plants in A grow to height greater than 40cm.
12	a	$(x-3)^2 - 5$

XINMIN SECONDARY SCHOOL
新民中学
SEKOLAH MENENGAH XINMIN
Mid-Year Examination 2017

CANDIDATE NAME

CLASS

INDEX NUMBER

MATHEMATICS 4048/1

Paper 1

9 May 2017

Secondary 4 Express / 5 Normal (Academic)

2 hours

Setter : Ms Pang Hui Chin
Vetter : Mrs Vivien Tay
Moderator: Mrs Sabrina Phang

Additional Materials: Nil

READ THESE INSTRUCTIONS FIRST

Write your name, register number and class on all the work you hand in.
Write in dark blue or black pen on both sides of the paper.
You may use an HB pencil for any diagrams or graphs.
Do not use staples, paper clips, glue or correction fluid.

Answer **all** questions.
If working is needed for any question it must be shown with the answer.
Omission of essential working will result in loss of marks.
The use of an approved scientific calculator is expected, where appropriate.
If the degree of accuracy is not specified in the question, and if the answer is not exact, give the answer to three significant figures. Give answers in degrees to one decimal place.
For π, use either your calculator value or 3.142, unless the question requires the answer in terms of π.

The number of marks is given in brackets [ ] at the end of each question or part question.
The total number of marks for this paper is **80.**

Errors	Qn No.	Errors	Qn No.
Accuracy		Simplification	
Brackets		Units	
Geometry		*Marks Awarded*	
Presentation		*Marks Penalised*	

For Examiner's Use

80

Parent's/Guardian's Signature:

Mathematical Formulae

Compound Interest

$$\text{Total amount} = P\left(1+\frac{r}{100}\right)^n$$

Mensuration

Curved surface area of a cone = πrl

Surface area of a sphere = $4\pi r^2$

$$\text{Volume of a cone} = \frac{1}{3}\pi r^2 h$$

$$\text{Volume of a sphere} = \frac{4}{3}\pi r^3$$

$$\text{Area of triangle } ABC = \frac{1}{2}ab\sin C$$

Arc length = $r\theta$, where θ is in radians

$$\text{Sector area} = \frac{1}{2}r^2\theta, \text{ where } \theta \text{ is in radians}$$

Trigonometry

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$a^2 = b^2 + c^2 - 2bc\cos A$$

Statistics

$$\text{Mean} = \frac{\sum fx}{\sum f}$$

$$\text{Standard deviation} = \sqrt{\frac{\sum fx^2}{\sum f} - \left(\frac{\sum fx}{\sum f}\right)^2}$$

Answer **all** the questions.

1 (a) Factorise completely $3ac - 7c - 18ab + 42b$.

Answer (a) ..... [1]

(b) If $9x^2 + 30x + k$ is a perfect square, state the value of k.

Answer (b) $k =$ ..... [1]

2 Solve the inequality $-2 \leq 2x - 7 < 19$.

er ..... [2]

3 Evaluate, giving your answer in standard form,

(a) $\dfrac{17.31+13.13}{4.041\times\sqrt{898.9}}$,

Answer (a) ..... [1]

(b) $2\left(7.8\times10^{-1}\right)+\left(3.9\times10^{2}\right)$.

Answer (b) ..... [1]

4 Given that x is an integer such that $-4 \le x \le 3$ and y is a prime number such that $0 < y \le 7$, find

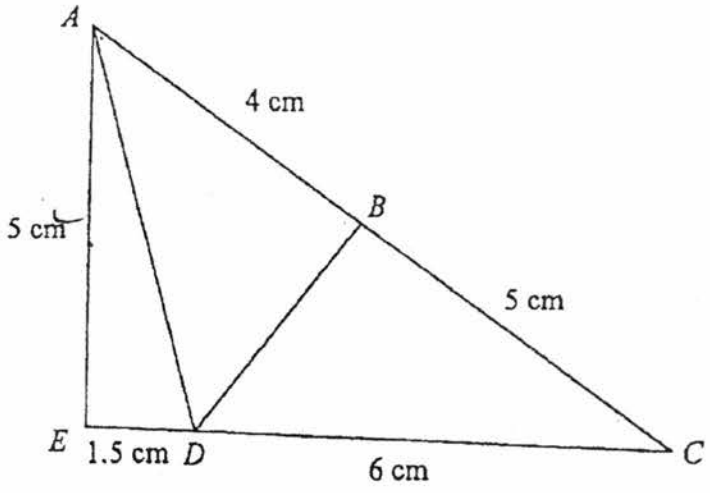
(a) the largest possible value of $\dfrac{x^2}{y}$,

Answer (a) ..... [1]

(b) the least possible value of $x^2 - y^2$.

Answer (b) ..... [1]

5 In the diagram, $AB = 4$ cm, $BC = 5$ cm, $CD = 6$ cm, $DE = 1.5$ cm and $AE = 5$ cm.



Show that triangles ACE and DCB are similar.

Answer In triangles ACE and DCB, ...

.....

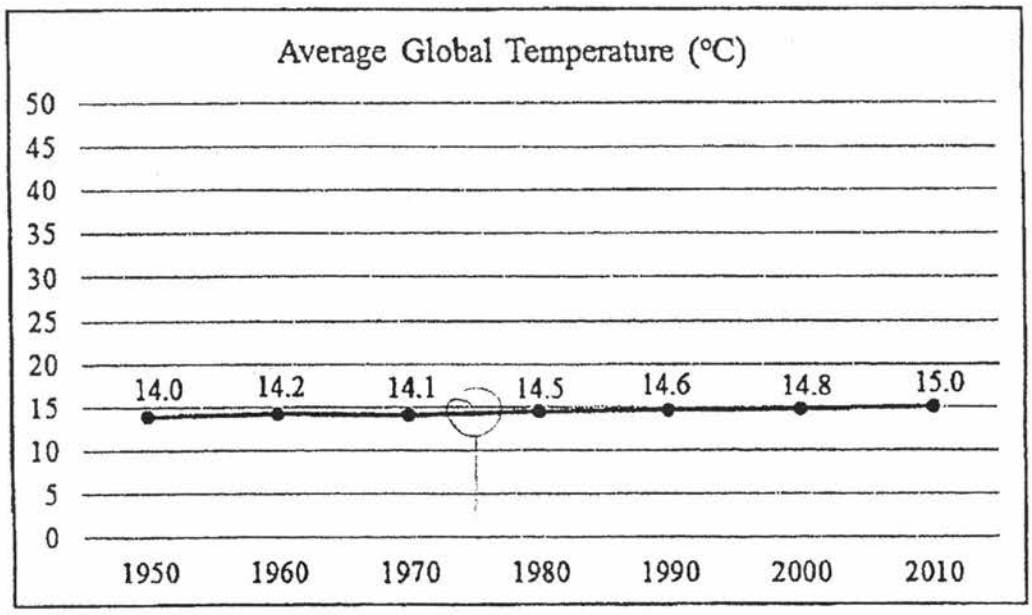
.....

..... [2]

6 Given that $\frac{1}{u}+\frac{1}{v}=\frac{1}{f}$, express v in terms of u and f.

..... [2]

7 An article in a newspaper reported the trend in the average global temperature from 1950 to 2010. The article contained the line graph shown below.



Can we determine the average global temperature in 1975 from the line graph? Explain your answer.

Answer .

[2]

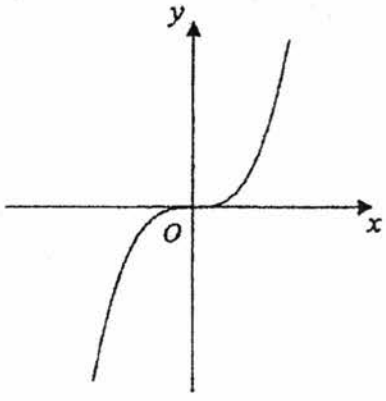
8 Solve $8^{3x-1} = 16$.

Answer $x =$ ..... [2]

9 The equations of the 2 graphs are in the form $y = x^n$.

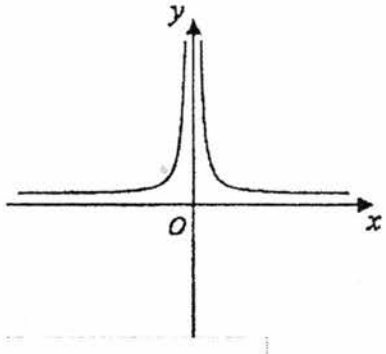
For each of the following, state a possible value of n.

(a)



Answer (a) $n =$ ..... [1]

(b)



Answer (b) $n =$ ..... [1]

10 Written as the product of its prime factors,

$$2160 = 2^4 \times 3^3 \times 5,$$
$$252 = 2^2 \times 3^2 \times 7.$$

(a) Find the smallest positive integer k such that $\frac{2160}{k}$ is a perfect cube.

Answer (a) $k =$ ..... [1]

(b) Write down the HCF of 252 and 2160 in index notation.

Answer (b) ..... [1]

11 The scale of a map is 2 cm : 0.4 km.

(a) Write this scale in the form 1 : n.

Answer (a) ..... : ..... [1]

(b) The actual area of a park is 4 km^2. Find the area, in square centimetres, of the park on the map.

..... cm^2 [2]

12 Solve the following simultaneous equations.

$$3x - 4y = 25$$
$$4x - 5y = 32$$

Answer $x =$ .....

$y =$ ..... [3]

13 In Singapore, Charlie pays $1.45 for 500 ml of bottled water.
When Charlie visited Japan, he paid ¥220 for 32 ounces of bottled water.

1 Singapore dollars = 77.96 Japanese Yen (¥)
1 ounce = 29.57 ml

Is bottled water cheaper in Singapore or in Japan?
You must show your calculations.

..... [3]

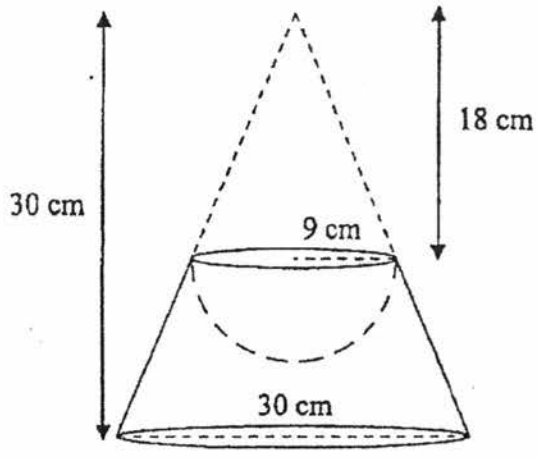
14 Simplify $\frac{1}{3-x}+\frac{3-x}{x^2-9}$.

Answer ..... [3]

15 Simplify $\left(\frac{25x^2y^0}{3x^0y^7}\right)^0 \times \left(\frac{3a}{2}\right)^{-3}$.

..... [3]

16 The diagram below shows a solid pet feeding bowl made from a truncated right circular cone with a hemispherical depression.



The truncated right circular cone is made by removing a cone with base radius 9 cm and and vertical height of 18 cm from a larger solid cone with a base diameter of 30 cm and a vertical height of 30 cm. The hemispherical depression has a radius of 9 cm.

The feeding bowl is to be made out of metal.
Calculate the volume of metal needed to make 10 of such feeding bowls, leaving your answer to the nearest whole number.

er ..... cm^3 [4]

17 Given that P is inversely proportional to $Q^2 + 1$ and that $P = 13$ when $Q = 1$,

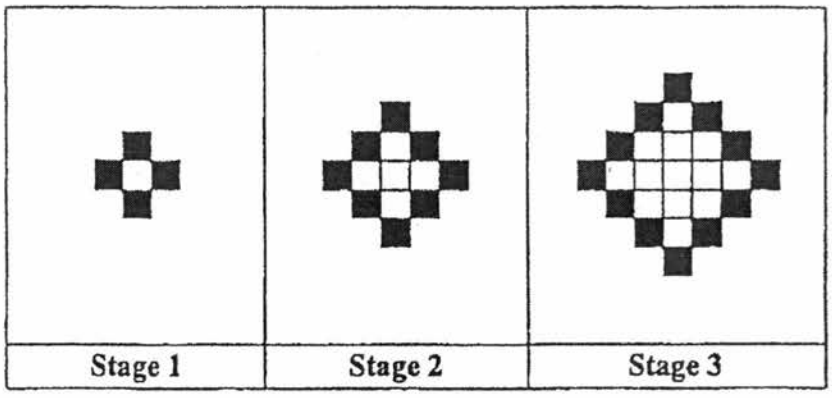
(a) express P in terms of Q,

Answer (a) ..... [2]

(b) find the values of Q when $P = 1$.

) Q = ... or ..... [2]

18 The diagram below shows a sequence of patterns made of squares of sides 1 unit each.



(a) Study the pattern and find the values of x and y.

Stage, n	Shaded area, S	Perimeter, P
1	4	12
2	8	20
3	12	28
4	x	y

Answer (a) $x =$ .....

$y =$ ..... [2]

(b) Express P in terms of n.

Answer (b) ..... [1]

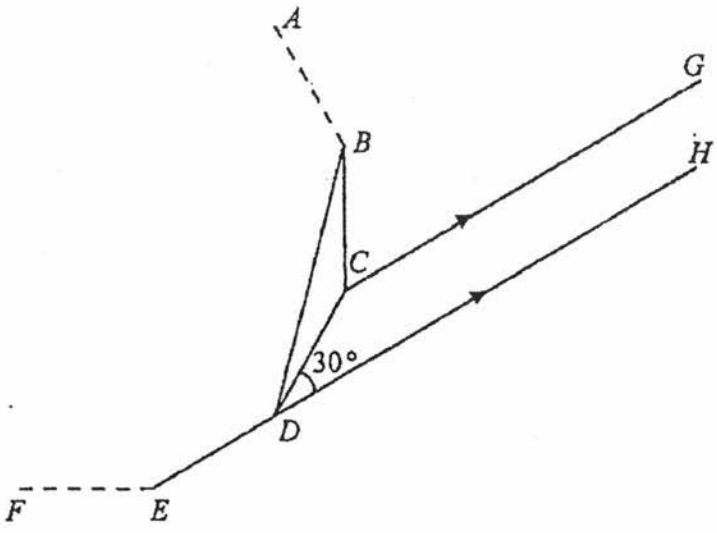
(c) Determine if the number 166 would appear in the P column, stating your reasons clearly.

Answer (c) .....

.....

.... [1]

19



In the diagram, $ABCDEF$ is an n-sided regular polygon with exterior angle $CDH = 30°$.
The lines CG and DH are parallel to each other.
Find

(a) the value of n,

Answer (a) $n =$ ..... [1]

(b) obtuse $\angle DCG$,

Answer (b) $\angle DCG =$ .....° [1]

(c) $\angle CBD$.

$\angle CBD =$ .....° [2]

20 $\xi = \{x : x \text{ is an integer such that } 40 \leq x \leq 50\}$

$A = \{x : x \text{ is a multiple of } 3\}$

$B = \{x : 2x + 5 < 99\}$

(a) Draw a Venn diagram to illustrate this information.

Answer (a)

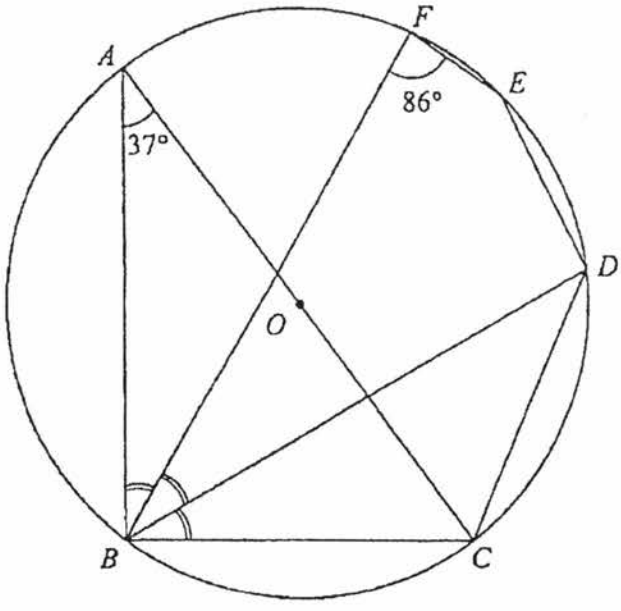
[2]

(b) List the elements of $A' \cap B'$ in set notation.

Answer (b) ..... [1]

(c) On your Venn diagram, shade the region which represents $A \cup B'$. [1]

21 In the diagram, A, B, C, D, E and F lie on a circle with centre O. AC is the diameter of the circle. $\angle ABF = \angle DBF = \angle CBD$.



If $\angle BAC = 37°$ and $\angle BFE = 86°$, find, giving reasons for each answer,

(a) $\angle ACB$,

Answer (a) $\angle ACB =$ ..... ° [2]

(b) $\angle DCA$,

Answer (b) $\angle DCA =$ ..... ° [1]

(c) $\angle FED$.

$\angle FED =$ ..... ° [1]

22 The staff of a company were asked about their monthly salary. The results are shown in the stem-and-leaf diagram.

1	010	050		
2	055	055	980	985
3	010	010	050	050
4	485	800	800	800
5	600	800	800	
6	750	750		
7				
8				
9				
10	999			

Key 3 | 010 means $3010

(a) Find the mean salary of the staff.

Answer (*a*) $ ..... [1]

(b) Find the median salary of the staff.

Answer (*b*) $ ..... [1]

(c) Does the mean or the median give a better representation of the salary of the staff in the company? Explain your answer.

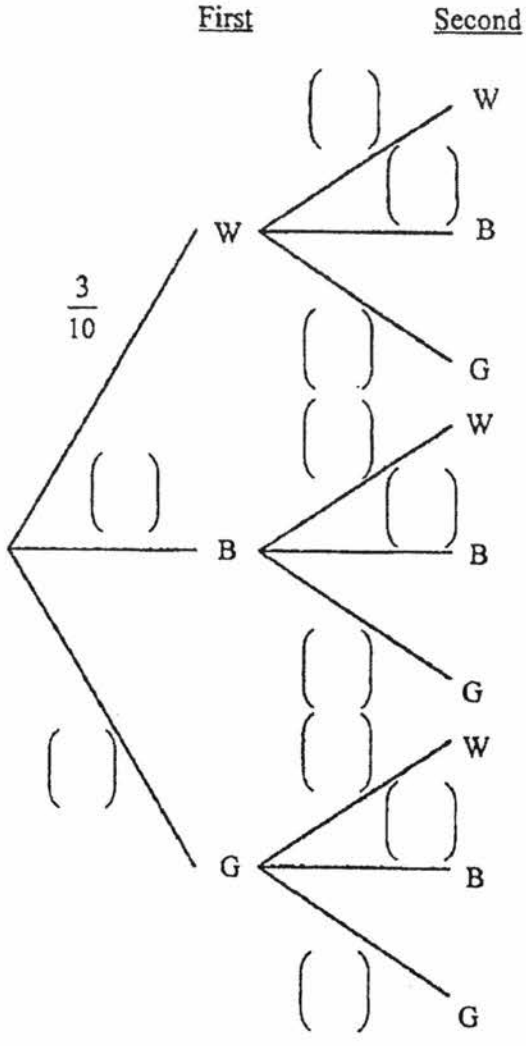
Answer (*c*) .....

.....

..... [2]

23 3 pairs of white socks, 2 pairs of black socks and 5 pairs of grey socks are mixed and placed in a drawer. On a particular day, Yan Xin woke up late. He randomly snatched two socks from the drawer, put them on and rushed to school.

(a) Complete the following tree diagram to show this information.



[2]

(b) Find, in its simplest form, the probability that Yan Xin has taken

(i) a pair of socks of the same colour,

..... [2]

23 (b) (ii) a pair of socks of different colours,

Answer (b)(ii) ..... [1]

Please turn over for Question 24

24 (a) By completing the square, express $x^2 - 6x + 5$ in the form $(x-a)^2 - b$.

Answer (*a*) ..... [2]

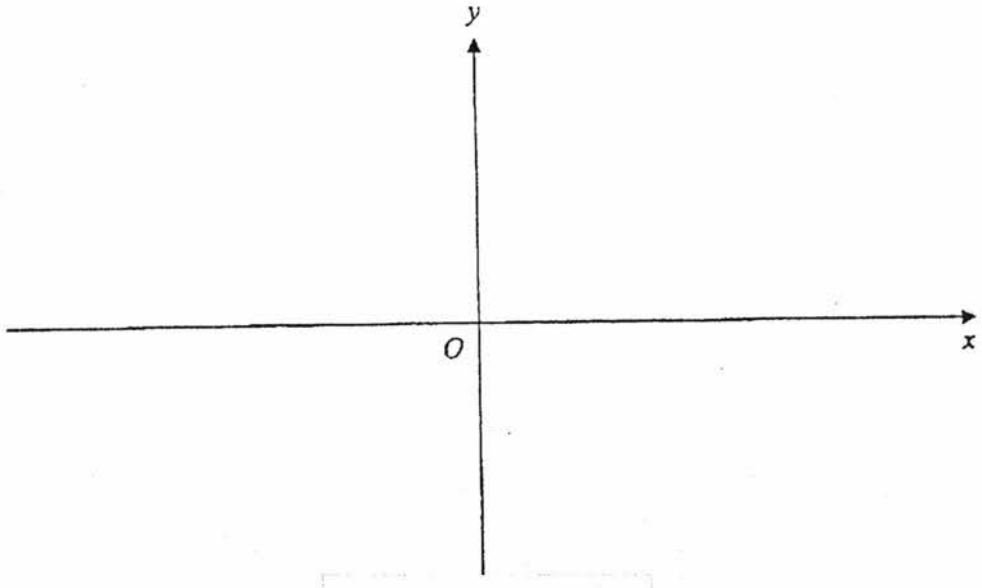
(b) Hence,

(i) solve the equation $x^2 - 6x + 5 = 0$,

Answer (*b*)(i) $x =$ ..... or ..... [2]

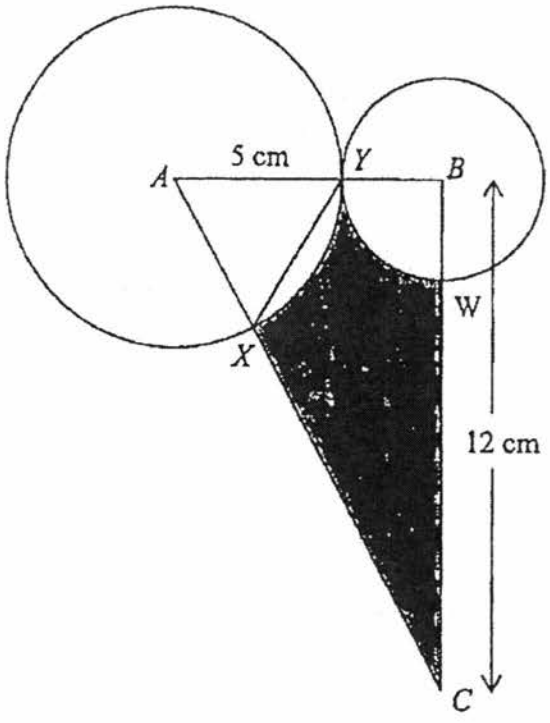
(ii) sketch the graph of $y = x^2 - 6x + 5$.

Answer (*b*)(ii)



[2]

25



In the diagram, ABC is a right-angled triangle such that two of its vertices A and B are the centres of two circles.

The minor arc length $WY = \frac{3\pi}{2}$ cm, $AY = 5$ cm and $BC = 12$ cm.

(a) Show that the length of BY is 3 cm.

Answer (a)

[1]

(b) Find the size of the angle XAY in radians.

Answer (b) $\angle XAY =$ ..... [2]

25 (c) Hence, find the area of the shaded region.

Answer (c) ..... cm^2 [3]

END OF PAPER

XINMIN SECONDARY SCHOOL
新民中学
SEKOLAH MENENGAH XINMIN
Mid-Year Examination 2017

CANDIDATE NAME

CLASS | | | | INDEX NUMBER | | |

MATHEMATICS

4048/2

Paper 2

2 May 2017

Secondary 4 Express / 5 Normal (Academic)

2 hours and 30 minutes

Setter : Mr Bennett Lim
Vetter : Mrs Vivien Tay
Moderator: Mrs Sabrina Phang

Additional Materials: Writing Paper; Graph Paper (1 sheet)

READ THESE INSTRUCTIONS FIRST

Write your name, register number and class on all the work you hand in.
Write in dark blue or black pen on both sides of the paper.
You may use an HB pencil for any diagrams or graphs.
Do not use staples, paper clips, glue or correction fluid.

Answer all questions.
If working is needed for any question it must be shown with the answer.
Omission of essential working will result in loss of marks.
The use of an approved scientific calculator is expected, where appropriate.
If the degree of accuracy is not specified in the question, and if the answer is not exact, give the answer to three significant figures. Give answers in degrees to one decimal place.
For π, use either your calculator value or 3.142, unless the question requires the answer in terms of π.

At the end of the examination, fasten all your work securely together.
The number of marks is given in brackets [ ] at the end of each question or part question.
The total number of marks for this paper is **100.**

Errors	Qn No.	Errors	Qn No.
Accuracy		Simplification	
Brackets		Units	
Geometry		Marks Awarded	
Presentation		Marks Penalised	

Parent's/Guardian's Signature:

For Examiner's Use
100

Mathematical Formulae

Compound Interest

$$\text{Total amount} = P\left(1+\frac{r}{100}\right)^n$$

Mensuration

$$\text{Curved surface area of a cone} = \pi rl$$

$$\text{Surface area of a sphere} = 4\pi r^2$$

$$\text{Volume of a cone} = \frac{1}{3}\pi r^2 h$$

$$\text{Volume of a sphere} = \frac{4}{3}\pi r^3$$

$$\text{Area of triangle } ABC = \frac{1}{2}ab\sin C$$

$$\text{Arc length} = r\theta, \text{ where } \theta \text{ is in radians}$$

$$\text{Sector area} = \frac{1}{2}r^2\theta, \text{ where } \theta \text{ is in radians}$$

Trigonometry

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$a^2 = b^2 + c^2 - 2bc\cos A$$

Statistics

$$\text{Mean} = \frac{\sum fx}{\sum f}$$

$$\text{Standard deviation} = \sqrt{\frac{\sum fx^2}{\sum f} - \left(\frac{\sum fx}{\sum f}\right)^2}$$

Answer **all** the questions.

1. Solve the equation $\frac{3}{x-5}-5=\frac{2x}{3-x}$. [4]

2. The Hangzhou-Changsa High-speed Railway runs at a speed of 350 km/h and covers a distance of 933 km between the two cities.

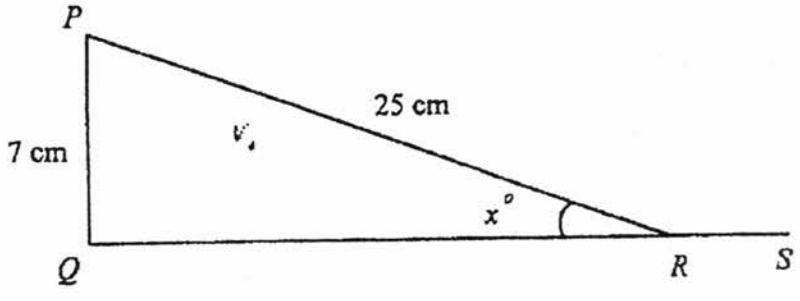
   (a) Find the speed of the train in m/s. [2]

   (b) Calculate the time taken for the train ride, giving your answer in hours and minutes, correct to the nearest minute. [2]

3. (a) On 12 September 2013, Tyler invested some money in a bank that pays simple interest at a rate of 3% per annum. He received \$573.75 in total interest on 12 December 2015. How much money did Tyler invest in the bank? [2]

   (b) Tyler also invested \$12 000 in another bank that pays compound interest at a rate of 2.25% per annum compounded half-yearly. How much money will Tyler get back at the end of 5 years? [2]

4.



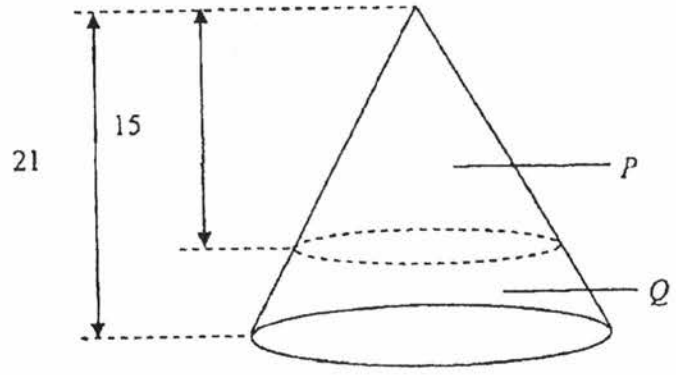
PQR is a right-angled triangle in which $\angle PRQ = x°$, $PQ = 7$ cm and $PR = 25$ cm. The point S lies on QR produced. Write down, as a fraction, the value of

(a) $\cos\angle PRS$, [2]

(b) $\tan(90-x)°$, [1]

(c) $\sin(180-x)°$. [1]

5. The following diagram shows an inverted solid cone that is cut up into 2 sections, P and Q, such that section P is a cone similar to the original cone. The height of cone P is 15 cm and the height of the original cone is 21 cm.



(a) If the curved surface area of cone P is 250 cm^2, calculate the curved surface area of the original cone. [2]

(b) Calculate the ratio of the volume of the original cone to the volume of cone P. [1]

(c) If the volume of section Q is v cm^3, calculate the volume of cone P in terms of v. [2]

---

6. The position vector of point A is $\begin{pmatrix} 2 \\ 5 \end{pmatrix}$ and $\overrightarrow{AB} = \begin{pmatrix} -3 \\ 4 \end{pmatrix}$.

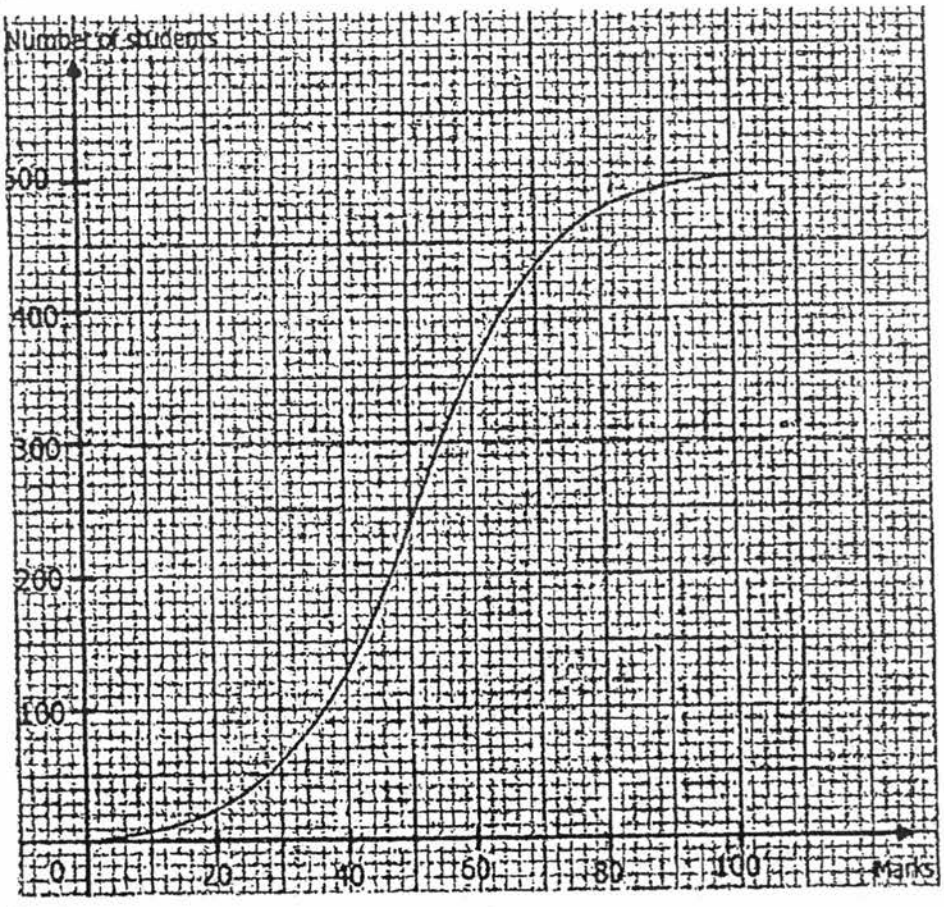
(a) Find $\left|\overrightarrow{AB}\right|$. [2]

(b) Find the coordinates of B. [2]

(c) Given that $\overrightarrow{CD}$ is parallel to $\overrightarrow{BA}$ and $\overrightarrow{CD} = \begin{pmatrix} k \\ 13.6 \end{pmatrix}$, find the value of k. [3]

---

7. The cumulative frequency curve below illustrates the marks obtained, out of 100, by 500 students in XMSS Mid-Year Examination.



(a) Find

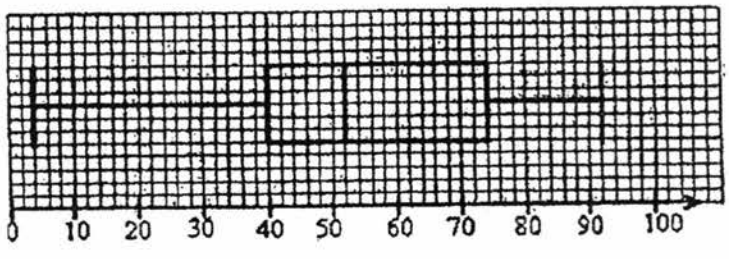
(i) the median mark, [1]

(ii) the interquartile range, [2]

(iii) the percentage of students who scored less than 50 marks. [2]

(b) Given that 15% of students scored a distinction, find the minimum marks students must score to get a distinction. [1]

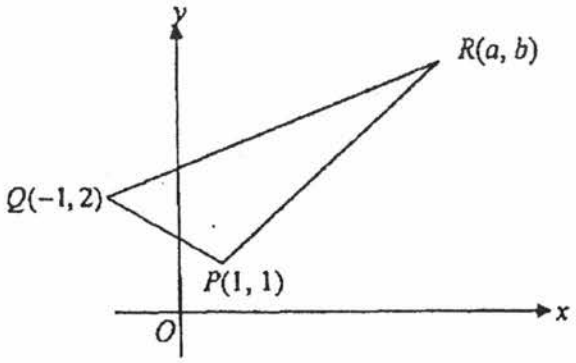
(c) The same 500 students sat for their Preliminary Examination. The box and whiskers diagram below illustrates the marks obtained.



(i) Which examination was more difficult? Give a reason for your answer. [1]

(ii) Which examination had more students scoring more than 70 marks? Explain your answer. [1]

8. The figure shows a triangle PQR with $P(1,1), Q(-1,2)$ and $R(a,b)$. The gradient of PQ, PR and QR are $-2n, 2n$ and n respectively.



Find

(a) the length of PQ, [2]

(b) the value of n, [2]

(c) the coordinates of R, [3]

(d) the equation of line QR. [2]

---

9. (a) It is given that $A = \begin{pmatrix} 2 & 2 \\ -4 & 6 \end{pmatrix}$ and $B = \begin{pmatrix} 1 & 5 \\ 0 & -1 \end{pmatrix}$.

Find

(i) matrix P if $P = B^2$, [1]

(ii) matrix Q if $A + 2Q = 2B$. [3]

(b) A tour agency records the weekly average number of tour packages to Japan and Korea sold in the months of May and June in 2016.
In May 2016, 25 Japan tour packages and 32 Korea tour packages were sold weekly. In June 2016, 30 Japan tour packages and 40 Korea tour packages were sold weekly. This information can be represented by the matrix

$$L = \begin{array}{c} \begin{matrix} \text{Japan} & \text{Korea} \end{matrix} \\ \begin{pmatrix} 25 & 32 \\ 30 & 40 \end{pmatrix} \begin{matrix} \text{May} \\ \text{June} \end{matrix} \end{array}$$

It is assumed that there are 4 weeks in each month.

(i) The prices of the Japan and Korea tour packages in 2016 were \$690 and \$900 respectively. Represent the prices of the tour packages by a 2×1 column matrix N. [1]

(ii) Evaluate the matrix $R = 4LN$. [2]

(iii) State what the elements of R represent. [1]

(iv) The tour agency decides to offer a discount on the tour packages bought in May and June 2017. The agency estimated a 30% increase and 60% increase in the sales of the Japan tour packages and Korea tour packages respectively compared to 2016.
By using matrix multiplication involving L, calculate the total estimated number of Japan and Korea tour packages sold weekly in May 2017 and June 2017 respec [2]

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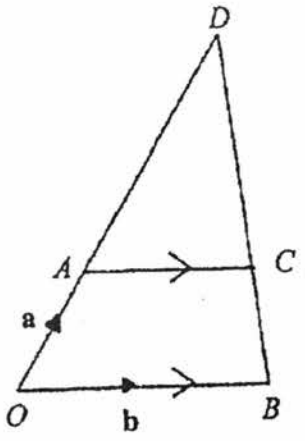
10. In the diagram, $OACB$ is a trapezium where AC is parallel to OB. The lines OA and BC are produced to the point D such that $\frac{OA}{AD} = \frac{1}{2}$.



(a) Given that $\overrightarrow{OA} = \mathbf{a}$ and $\overrightarrow{OB} = \mathbf{b}$, express, as simply as possible, in terms of $\mathbf{a}$ and/or $\mathbf{b}$,

(i) $\overrightarrow{BD}$, [1]

(ii) $\overrightarrow{OC}$. [2]

(b) Given that $\overrightarrow{OE} = 3\mathbf{a} + 2\mathbf{b}$,

(i) state the name of the quadrilateral $ODEB$, [1]

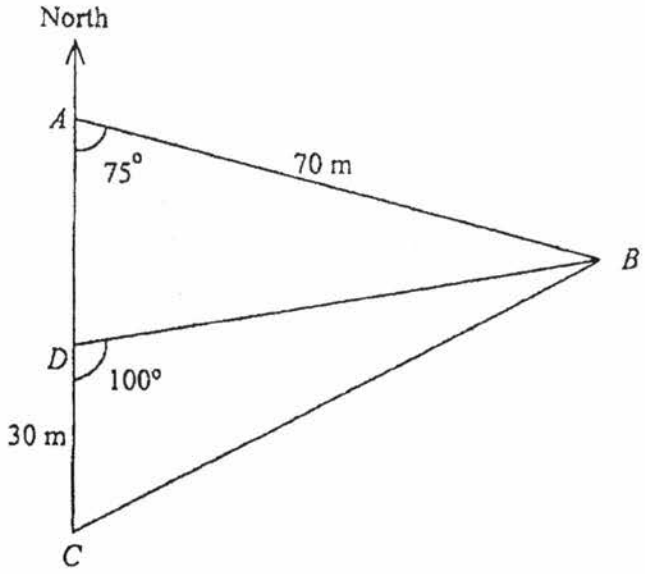
(ii) explain why O, C and E lie in a straight line. [2]

(c) Find

(i) $\frac{\text{area of } \Delta ADC}{\text{area of } \Delta ODB}$, [2]

(ii) $\frac{\text{area of } \Delta ODB}{\text{area of quadrilateral } ODEB}$. [3]

11. In a laser tag enclosure, A, B, C and D are points on level ground, with A due north of C and D. $\angle BAD = 75°$, $\angle BDC = 100°$, $AB = 70$ m and $CD = 30$ m.



(a) Show that the length of $BD = 68.66$ cm, correct to 2 decimal places. [2]

(b) Calculate

(i) the bearing of D from B, [1]

(ii) the length of CB, [2]

(iii) the area of ΔABD. [2]

In a game, Mario at point B ran along the path BA towards point A at a speed of 8 m/s. Sonic at the top of a 20-metre high guard tower at point D spotted Mario at point B.
He fired a shot that hit Mario when he was closest to the guard tower.
Assume that the time taken by the shot to hit the target from the time it was fired was negligible.

(c) Find

(i) the angle of depression of Mario from Sonic when the shot was fired, [3]

(ii) the time that elapsed from the instant Sonic spotted Mario at point B to the instant Sonic fired the shot. [2]

12. **Answer the whole of this question on a sheet of graph paper.**

The speed, v, in metres per second of a toy car on a race track propelled by a spring launcher is given by $v = 5 + 4t - t^2$, where t is the time in seconds. The table below shows the corresponding values of t and v.

t	0	1	1.5	2.5	4	5
v	5	8	8.75	8.75	5	0

(a) Draw the graph of $v = 5 + 4t - t^2$ for $0 \le t \le 5$. Use a scale of 2 cm to 1 s on the horizontal t-axis and 2 cm to 1 m/s on the vertical v-axis. [3]

(b) Use your graph to find the maximum speed reached by the car. [1]

(c) (i) By drawing a tangent, find the gradient of the graph at the point when $t = 3.5$ s. [2]

(ii) Use your answer to **c(i)** to explain what was happening to the car at $t = 3.5$ s. [1]

(d) (i) By adding a suitable line to your graph, solve $4t - t^2 - 2 = 0$. [4]

(ii) What do the solutions represent? [1]

---

13. Mr Mah is a motorcycle shop owner in Singapore who sells brand new motorcycles. He is interested in importing the brand new Kawasaki Z100SX motorcycle from Japan. The total costs to be incurred for importing the motorcycles to Singapore, include the amount payable to the manufacturer, shipping costs, government taxes and duty.

Information that Mr Mah needs is on the following page.
Mr Mah is interested in importing 20 motorcycles to sell.

(a) Calculate

(i) the cost of each motorcycle payable to the manufacturer, [1]

(ii) the shipping and insurance cost of each motorcycle. [2]

Mr Mah targets a profit of 15% of his total costs incurred.
Mr Mah needs to decide how much he should sell each motorcycle.

(b) Suggest a sensible selling price for each motorcycle.
Justify your proposed selling price with a **concluding statement**. [7]

Motorcycle Specifications	
Motorcycle Model	Kawasaki Z1000 SX
Year	2017
Weight	228 kg

Cost Payable to Manufacturer	
Price per Unit (S$)	S$18,250
Discount for purchases:	
> 9 units	2.5%
> 19 units	5.0%
> 29 units	7.5%

Shipping and Insurance Cost:	
Net weight (kg)	Cost (S$)
< 2,000	3,250
2,000 – 3,000	4,000
3,001 – 4,000	4,750
4,001 – 5,000	5,500
5,001 – 10,000	6,000
> 10,000	6,500

The following is extracted from the Singapore Land and Transport Authority (LTA[1]) website.

TAX STRUCTURE FOR MOTORCYCLES & SCOOTERS

Registration Fee[2] (RF)	S$140
Additional Registration Fee[3] (ARF)	Tiered Rate:

Vehicle OMV[4]	ARF Rate
First S$5,000	15%
Next S$5,000 (i.e. S$5,001 to S$10,000)	50%
Above S$10,000	100%

Excise Duty[5]	12% of OMV

[1] LTA is responsible for planning, operating, and maintaining Singapore's land transport infrastructure and systems.

[2,3] The RF and ARF are government taxes to be paid by the importer for the registration of the motorcycles for sale in Singapore.

[4] OMV (Open Market Value) – Refer [redacted] acturer of the motorcycle.

[5] It is a tax on the cost paid to the ma[redacted]

10) a) $k = 10$

b) $2^2 \times 3^2$

11 a) 1 : 20000

b) 100

12) $x = 3$, $y = -4$

13) Singapore

14) $\frac{2x}{(3-x)(x+3)}$

15) $\frac{8}{2a^3}$

16) 40150

17 a) $p = \frac{26}{q^2+1}$

b) 5 or -5

18 a) $x = 16$, $y = 36$

b) $p = 8n + 4$

19) a) $n = 12$

b) 150

c) 15°

20) b) $47, 49, 50

21) a) 53°

b) 60°

c) 150

22 a) $ 4241.95

b) 3767.50

23 b i) $\frac{33}{95}$

b) $\frac{56}{95}$

24 a) $(x-3)^2 - 4$

b i) 1 or 5

25 b) 0.983

c) 28.6

a) $(3a-7)(c-6b)$

b) $k=25$

2) $2\frac{1}{2} \leq x < 13$

3a) 2.57×10^{-1}

b) 3.9156×10^{2}

4a) ~~44~~ 8

b) -49

6) $v = \frac{uf}{u-f}$

7) NO because the line segments between the dots have no meaning

8) $x = \frac{7}{9}$

9a) $n = 3, 5 \ldots$
any odd positive integer > 1

b) $n = -2, -4 \ldots$
any even negative integer

Q8a) 2.24 b) n = 1/4

c) R(7, 4) d) 4y = x + 9

9a) i) $\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$ ii) $\begin{pmatrix} 0 & 4 \\ 1 & -4 \end{pmatrix}$

bi) $N = \begin{pmatrix} 690 \\ 900 \end{pmatrix}$ ii) $\begin{pmatrix} 184 & 200 \\ 226 & 800 \end{pmatrix}$

iii) $\begin{pmatrix} 83.7 \\ 103 \end{pmatrix}$

10ai) 3a − b

ii) a + 2/3 b

bi) Trapezium

ii) $\vec{OE} = 3\vec{OC}$

O, C, E collinear

ci) 4/9

ii) 1/3

11a) 68.66 b) 260° ii) 79.6 iii) 1020 m^2

e) 34.6° ii) 7.78 s

Q1) $x = 4$ or 7

2) a) 97.2 m/s b) 2h 40min

Q3 a) $8500 b) $13 420.43

4a) $-\frac{24}{25}$, b) $\tan(90 - x) = \frac{24}{7}$

c) $\sin(180 - x) = \frac{7}{25}$

5a) $490\,cm^2$

b) $\frac{343}{125}$

c) $\frac{125}{218} V$

6a) 5 units b) $(-1, 9)$ c) $m = -3.4$
$k = -10.2$

7a i) 51 (ii) 22 iii) 48%

b) 69

ci) mid-year is more difficult as median mark is lower

ii) Prelim exam. It has more than 25% of students scoring 74m or more higher than mid year. 25% scoring 62m or more

YUSOF ISHAK SECONDARY SCHOOL
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CANDIDATE NAME

CLASS | | | | INDEX NUMBER | | |

MATHEMATICS 4048 / 01
4 Express / 5 Normal (Academic)

Paper 1

16th August 2017

Candidates answer on the Question Paper

2 hours

READ THESE INSTRUCTIONS FIRST

Write your name, class and index number on all the work you hand in.
Write in dark blue or black pen.
You may use a HB pencil for any diagrams or graphs.
Do not use staples, paper clips, highlighters or correction fluid.

Answer all questions.
If working is needed for any question it must be shown with the answer.
Omission of essential working will result in loss of marks.
The use of an approved scientific calculator is expected, where appropriate.
If the degree of accuracy is not specified in the question, and if the answer is not exact, give the answer to three significant figures. Give answers in degrees to one decimal place.
For π, use either your calculator value or 3.142, unless the question requires the answer in terms of π.

At the end of the examination, fasten all your work securely together.
The number of marks is given in brackets [ ] at the end of each question or part question.

The total number of marks for this paper is 80.

For Examiner's Use

This document consists of 17 printed pages.

Setter: Mr Eric Koh

[Turn over

Answer **all** the questions.

1. Evaluate the following, leaving your answer correct to four significant figures.

$$\frac{-3.3^2 \times \sqrt{2^3}}{\left[1-8\left(7+7^{-1}\right)\right]^2} \times \sin\frac{\pi}{3}$$

Answer ..... [1]

2. The value of a house decreased by 14.3% between 2000 and 2016.
In 2000 the house was valued at $850 000.
Find its value in 2016.

Answer $..... [2]

3. A container is unloaded by 6 men in 24 minutes.
Given that all the men work at the same rate, find how long it would take 9 men to unload the same container.

Answer ..... minutes [2]

4. A car manufacturer states that a particular car
   - Uses 5 litres of fuel in travelling 100 km
   - produces 115 grams of CO_2 for each kilometer travelled.

   Use this information to calculate the mass of CO_2 produced by 1 litre of fuel.
   Give your answer in kilograms.

Answer ..... kg [2]

5. **(a)** Factorise completely $50p^2 - 72q^2$.

   (b) Solve the equation $\frac{x-2}{4} - \frac{x+1}{3} = 1$.

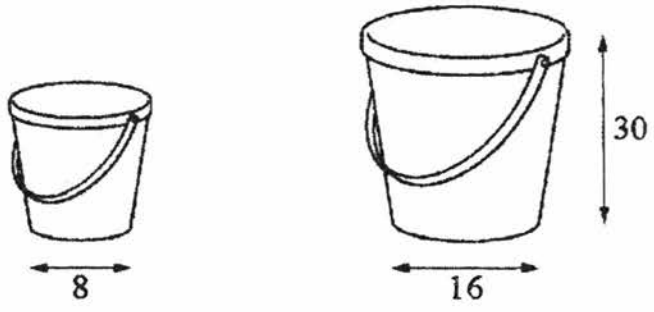
   (c) $T = 2\pi\sqrt{\frac{h}{g}}$. Make h the subject of the formula.

Answer (a) ..... [2]

Answer (b) ..... [2]

Answer (c) ..... [2]

6. Similar buckets are available in two sizes.
The larger bucket has height 30 cm and base diameter 16 cm.
The small bucket has base diameter 8 cm.



(a) Find the height of the small bucket.

(b) Given that the small bucket has volume 850 cm^3, find the volume of the large bucket.

Answer (a) ..... cm [1]

Answer (b) ..... cm^3 [2]

7. The temperature inside a greenhouse is $p^{\circ}C$, and outside it is $-q^{\circ}C$, where p and q are positive integers.
Write down an expression for
(a) the difference between the two temperatures,

(b) the mean of the two temperatures.

Answer (a) .....°C [1]

Answer (b) .....°C [1]

8. Green Line trains run every 10 minutes.
Red Line trains run every 20 minutes.
Purple Line trains run every 35 minutes.
One train from each Line leaves the city centre at 09 00.
After how many minutes will trains from all three Lines next leave the city centre in the same time?

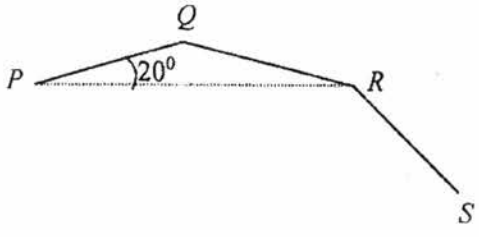
Answer ..... minutes [2]

9. PQ, QR and RS are adjacent sides of a regular polygon. Given that $\angle RPQ = 20°$, calculate

(a) the exterior angle of the polygon,

(b) the number of sides of the polygon,

(c) $\angle PRS$.



Answer (a) ..... [1]

(b) ..... [1]

(c) $\angle PRS$ = ... [1]

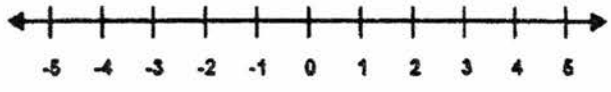
10. P is directly proportional to Q^2.

If Q is increased by 200%, find the percentage increase of P.

Answer .....% [2]

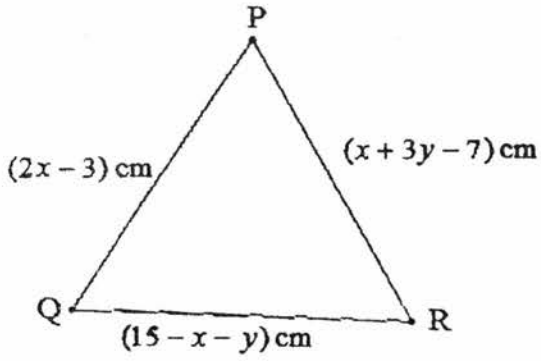
11. Solve the inequalities $\frac{10x+8}{3}+2<5+4x<8$.

Show your solution on the number line below.



[3]

12. The diagram shows an equilateral triangle PQR with $PQ = (2x - 3)$ cm, $QR = (15 - x - y)$ cm and $PR = (x + 3y - 7)$ cm.



(a) Using the information shown in the diagram, write down and simplify two simultaneous equations in x and y.

(b) Solve these equations to find the value of x and the value of y.

Answer (a) .....

.....[2]

(b) x = ... y = ... [2]

13. The information shows the common injuries children suffer in the United States of America (USA) in 2013.

(a) Explain **one way** in which the information is misleading.

Answer .....

.....

..... [2]

(b) Suggest **one recommendation** to overcome the misleading information provided.

Answer .....

.....

..... [1]

14. A map is drawn to a scale of 1 : 50 000.

(a) An airport runway is represented by a line of length 5.8 cm on the map. Calculate, in km, the actual length of the runway.

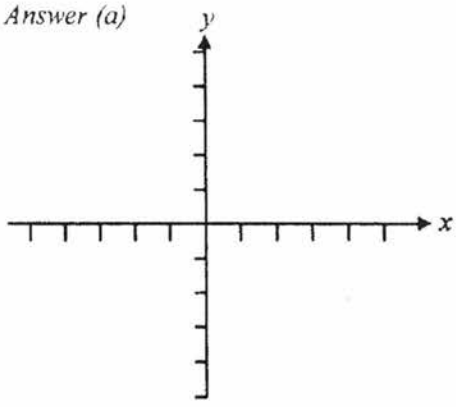
(b) The actual area of the airport is 6.5 km^2. Calculate, in square centimetres, the area on the map which represents the airport.

Answer (a) ..... km [1]

(b) ..... cm^2 [2]

15. **(a)** Sketch the graph of $y = (1 - x)(x - 3)$.

Answer (a)



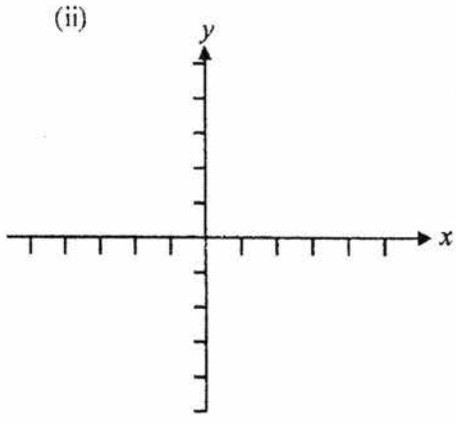
[2]

(b) **(i)** Express $x^2 - 4x + 5$ in the form $(x - a)^2 + b$.

(ii) Sketch the graph of $y = x^2 - 4x + 5$.

Answer (b)(i) $x^2 - 4x + 5 =$ ..... [1]

(ii)



[2]

16. A company produces three types of soft drinks in 2 different sizes.

The following matrices shows the weekly production, in thousands of litres and the cost per litre in cents, for producing soft drinks of any flavour in 2 different sizes.

$$\begin{array}{c} \\ \text{Regular} \\ \text{Large} \end{array} \begin{array}{c} \begin{array}{ccc} \text{Raspberry} & \text{Orange} & \text{Lemon} \end{array} \\ \begin{pmatrix} 15 & 26 & 18 \\ 14 & 24 & 16 \end{pmatrix} \end{array} \qquad \begin{array}{c} \\ \text{Cost} \end{array} \begin{array}{c} \begin{array}{cc} \text{Regular} & \text{Large} \end{array} \\ \begin{pmatrix} 45 & 60 \end{pmatrix} \end{array}$$

(a) Find $\begin{pmatrix} 45 & 60 \end{pmatrix}\begin{pmatrix} 15 & 26 & 18 \\ 14 & 24 & 16 \end{pmatrix}$.

Answer (a) ..... [2]

(b) Explain what your answer to **(a)** represents.

Answer (b) .....

..... [1]

17.

$\varepsilon = \{x : x \text{ is an integer and } 0 < x \leq 15\}$

$A = \{x : x \text{ is a prime number}\}$

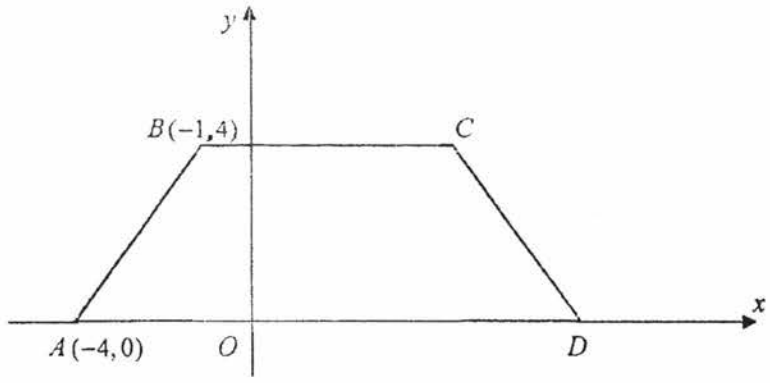
$B = \{x : x \text{ is an integer divisible by } 3\}$

Draw a Venn diagram to illustrate this information, showing elements in each set clearly.

Answer ε

[2]

18. $ABCD$ is a trapezium in which $BC = 8$ units. A is the point $(-4, 0)$ and B is the point $(-1, 4)$. The area of the trapezium is 50 square units.



(a) Calculate the length of AB.

Answer (a)..... [1]

(b) Find the coordinates of C.

Answer (b) (..... , ...) [1]

(c) Find the coordinates of D.

Answer (c) (..... , ...) [2]

(d) Write down the value of $\cos \angle ABC$.

Answer (d) $\cos \angle ABC =$..... [1]

19. A production line produces loaves of bread with a mass of 500 grams each.
Two separate production lines, P and Q, were operated and 10 loaves were taken as samples from each line which had the following masses:

Line P 502, 487, 488, 490, 507, 500, 498, 491, 505, 490

Line Q 510, 501, 482, 489, 496, 506, 478, 489, 503, 492

(a) Find the mean mass of the products from both lines.

Answer (a) Line P..... [1]

Line Q ..... [1]

(b) Find the standard deviation of the product mass from both lines.

Answer (b) Line P..... [1]

Line Q ..... [1]

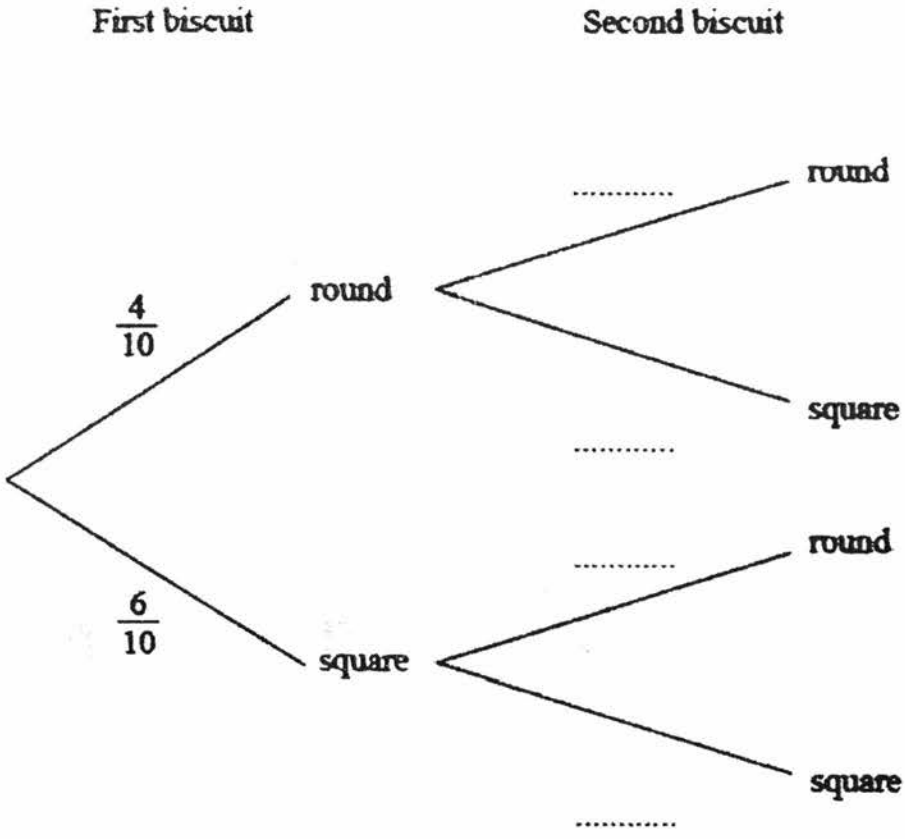
(c) If a loaf from each line is picked at random and each weighs 480 grams and 485 grams respectively, which line did the lighter loaf likely to come from?
Justify your decision with explanation.

Answer .....

.....

..... [2]

20. On a plate there are ten biscuits.
Four of the biscuits are round and six of the biscuits are square.
Joe chooses a biscuit at random from the plate and eats it.
He then chooses another biscuit at random from the plate.
The tree diagram shows the possible outcomes and some of the probabilities.



(a) Complete the tree diagram. [2]

(b) Calculate the probability that Joe chooses

(i) two round biscuits,

(ii) one round biscuit and one square biscuit.

Answer (b)(i) ..... [1]

(b)(ii) ..... [2]

21. (a) Simplify the expression $(3x^2y)^3 \times (5x^{-3}y^4)^{-1}$, giving your answer in positive index notation.

Answer (a) ..... [2]

(b) Solve $\left(\frac{1}{8}\right)^{-\frac{2}{3}} \times 32^{\frac{3}{5}} = 2^{p-2} \div 2^2$.

Answer (b) p =..... [2]

(c) Express the number 0.0040589 in standard form.

Answer (c) ..... [1]

22.

(a) Given that $p = \begin{pmatrix} 3 \\ 4 \end{pmatrix}$ and $q = \begin{pmatrix} m \\ 2 \end{pmatrix}$, find

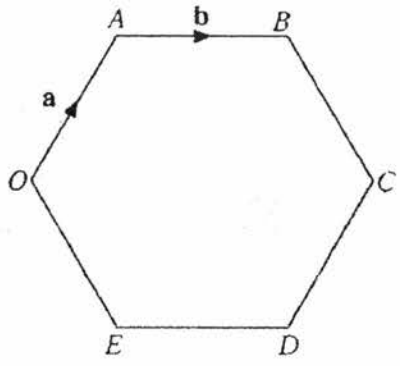
(i) $|p|$,

(ii) the value of m such that $p + q$ is parallel to the y-axis.

Answer (a)(i) .....units [1]

Answer (a)(ii)..... [1]

(b) In the diagram, $OABCDE$ is a regular hexagon. $\overrightarrow{OA} = \mathbf{a}$, $\overrightarrow{AB} = \mathbf{b}$.



(I) Express the following vectors, as simply as possible, in terms of **a** and **b**.

(i) $\vec{OC}$,

(ii) $\vec{BC}$,

(iii) $\vec{AD}$.

(II) What type of quadrilateral is $ABCD$? Justify your answer using vectors.

Answer (b)(I)(i) ..... [1]

(ii) ..... [1]

(iii) ..... [1]

Answer (II)..... [1]

23. All construction lines must be clearly shown.

(a) Construct, and label clearly, the quadrilateral $ABCD$ in which $AB = BC = CD$, $\angle ABC = 70°$ and $\angle BAD = 100°$.
The line AB has been drawn for you. [2]

(b) On the quadrilateral, construct
(i) the bisector of angle ABC, [1]

(ii) the perpendicular bisector of the line BC. [1]

(c) The two bisectors in **(b)** intersect at the point P. Measure and write down the length of BP, in cm, correct to 1 decimal place.

Answer (c) ..... [1]

End of Paper

YUSOF ISHAK SECONDARY SCHOOL
PRELIMINARY EXAMINATION 2017

CANDIDATE NAME

CLASS

INDEX NUMBER

Mathematics
4 Express / 5 Normal Academic

4048/02

Paper 2

18 August 2017

2 hours 30 minutes

Additional Materials: Answer paper
Graph Paper (1 sheet)

READ THESE INSTRUCTIONS FIRST

Write in dark blue or black pen.
You may use an HB pencil for any diagrams or graphs.
Do not use staples, paper clips, glue or correction fluid.

Answer all questions.
If working is needed for any question, it must be shown with the answer.
Omission of essential working will result in loss of marks.
You are expected to use a scientific calculator to evaluate explicit numerical expressions.
If the degree of accuracy is not specified in the question, and if the answer is not exact, give the answer to three significant figures. Give answers in degrees to one decimal place.
For π, use either your calculator value or 3.142, unless the question requires the answer in terms of π.

The number of marks is given in brackets [ ] at the end of each question or part question.
The total number of marks for this paper is **100**.

Mathematical Formulae

Compound interest

$$\text{Total amount} = P\left(1+\frac{r}{100}\right)^n$$

Mensuration

$$\text{Curved surface area of cone} = \pi rl$$

$$\text{Surface area of a sphere} = 4\pi r^2$$

$$\text{Volume of a cone} = \frac{1}{3}\pi r^2 h$$

$$\text{Volume of a sphere} = \frac{4}{3}\pi r^3$$

$$\text{Area of triangle } ABC = \frac{1}{2}ab\sin C$$

$$\text{Arc length} = r\theta, \text{ where is } \theta \text{ in radians}$$

$$\text{Sector area} = \frac{1}{2}r^2\theta, \text{ where } \theta \text{ is in radians}$$

Trigonometry

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$a^2 = b^2 + c^2 - 2bc\cos A$$

Statistics

$$\text{Mean} = \frac{\Sigma fx}{\Sigma f}$$

$$\text{Standard deviation} = \sqrt{\frac{\Sigma fx^2}{\Sigma f} - \left(\frac{\Sigma fx}{\Sigma f}\right)^2}$$

[3]

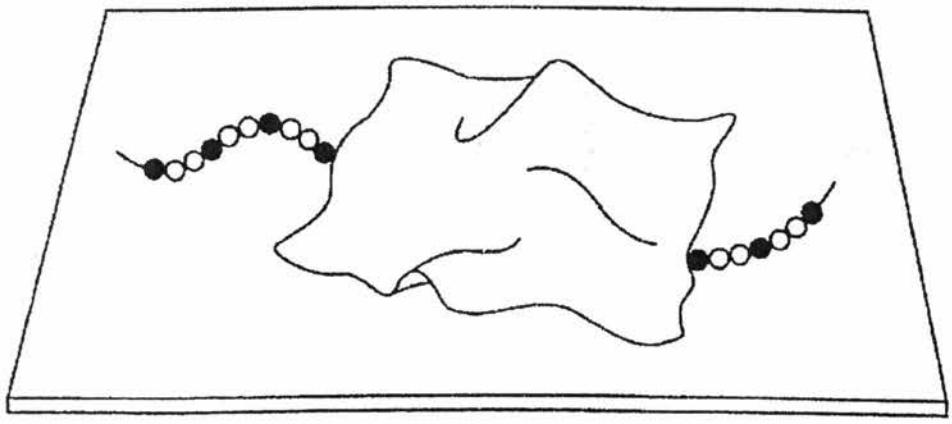
1. (a) Solve the equation $(1+4x)^2 = 81$. [2]

(b) Express as a single fraction in its simplest form $\frac{1}{2x+3} + \frac{3}{2x-1}$. [2]

(c) Find the integers x such that $2x+1 < 9 < 3x+1$. [2]

(d) Factorise completely $a^2 + 9b^2 - 6ab - 2a + 6b$. [2]

2. (a) A string of beads on a table is partly covered by a piece of cloth as shown.
There are 2 white beads between every 2 black beads.
Altogether, there are 14 black beads.
John guessed that the number of white beads was 28.
Do you agree? Justify your decision with calculations. [3]



(b) It is given that $3b = 4a$ and $2c = 5a$.

(i) Find $a:b:c$. [2]

(ii) If $a+b+c = 10$, find b. [3]

3. John bought x light bulbs for \$25.

(a) Write down an expression in terms of x for the price, in dollars, he had paid for each light bulb.

(b) He wanted to sell each light bulb at a profit of 50 cents. [1]

Show that his selling price for each light bulb was $\$\frac{50+x}{2x}$. [1]

(c) John managed to sell 8 light bulbs at this price. Write down an expression, in terms of x, for

(i) the total amount of money, in dollars, he had received for selling the 8 light bulbs. [1]

(ii) the number of light bulbs that remained unsold. [1]

(d) John sold the remaining light bulbs at \$2 each.
Write down an expression in terms of x for the total amount of money, in dollars, he had received from selling these light bulbs. [1]

(e) John received \$46 altogether.
Form an equation in x and show that it reduces to $x^2 - 29x + 100 = 0$. [3]

(f) Hence or otherwise, find the number of light bulbs John had bought. [3]

[5]

4.

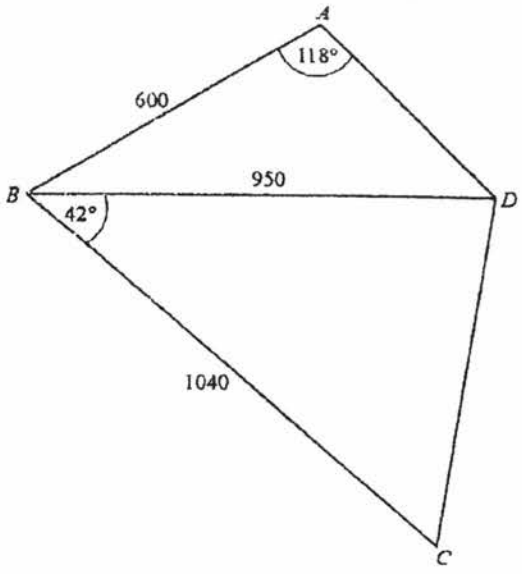


Figure 1

Figure 1 shows the quadrilateral $ABCD$. Quadrilateral $ABCD$ represent a level enclosed area for the rabbits with a path BD.
$AB = 600$ m, $BC = 1040$ m, $BD = 950$ m and $\angle CBD = 42^\circ$ and $\angle BAD = 118^\circ$.

(a) Calculate

(i) $\angle ABD$, [4]

(ii) the length of CD, [4]

(iii) the shortest distance from C to BD. [2]

(b) An eagle is flying directly above the path BD at a height of 500 m.
Calculate the greatest angle of depression of the point C as seen by the eagle. [2]

[6]

5. P, Q, R, S and T are the different shaped blocks of ice stored in the refrigerated enclosed room.

(a) At 10 p.m. on Monday the cooling system failed, and the blocks started to melt.
At the end of each 24 hour period, the volume of each block was 14% less than its volume at the start of that period.

(i) Block P has a volume of 7500 cm^3 at 10 p.m. on Monday.
Calculate its volume at 10 p.m. on Wednesday. [2]

(ii) Block Q had a volume of 6490 cm^3 at 10 p.m. on Tuesday.
Calculate the volume at 10 p.m. on the previous day. [2]

(iii) **Showing your working clearly,** find on which day the volume of R was half its volume at 10 p.m. on Monday. [2]

(b) At 10 p.m. on Monday, Block S was a hemisphere with radius 18 cm.
Calculate

(i) its volume, [2]

(ii) its **total** surface area. [2]

(c) As block T melted, its shape was always **geometrically similar** to its original shape.
It had a volume of 5000 cm^3 when its height was 12 cm.
Calculate its height when its volume was 1080 cm^3. [2]

6. Figure 2A shows the cross-section of an underground train tunnel.

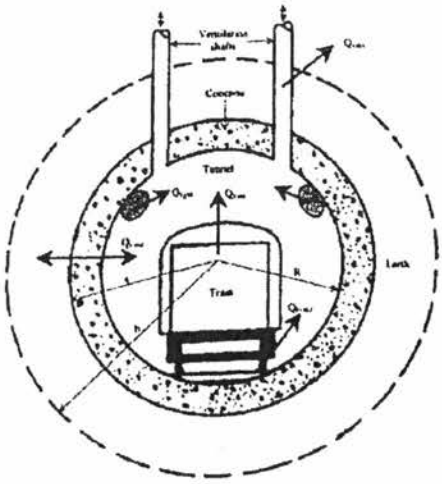


Figure 2A

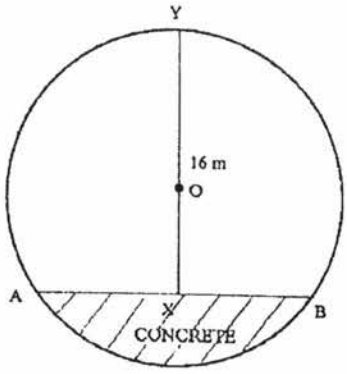


Figure 2B

With reference to Figure 2B.
AB represents the horizontal track surface, where the shaded region beneath it is covered with concrete.
Arc AYB represents the metal ceiling of the tunnel.
O is the centre of the circle with radius r metres.
X is the midpoint of AB and its vertically below Y.
Given that $AB = XY = 16\text{m}$.

(a) Calculate

(i) the value of r. [3]

(ii) $\angle AOX$, [1]

(iii) the volume of concrete used for the tunnel, given the tunnel is 900 m long. [3]

(b) A similar model of the tunnel is made. The radius of the model's cross-section is 5 cm.
Calculate the curved surface area of the model's ceiling. [3]

(c) A 130 metre long train travelling at a speed of 50 km/h entered the tunnel.
Calculate the time, in minutes and seconds, needed for the train to completely travel out of the tunnel. [2]

[8]

7.

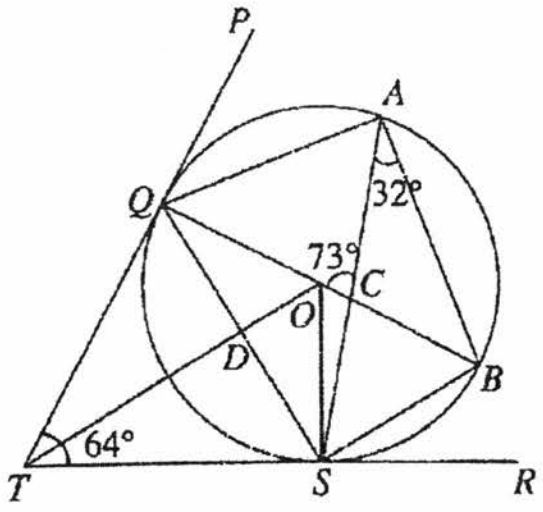


Figure 3

Figure 3 shows the circle $ABSQ$.
$ABSQ$ has centre O. TQP and TSR are tangents to the circle.
$\angle QTS = 64°$, $\angle SAB = 32°$ and $\angle ACQ = 73°$.

(a) Joseph commented that there are **at least** three right angles in Figure 3.
Justify his comment with workings and reasons. [3]

(b) Calculate

(i) $\angle SQB$, [1]

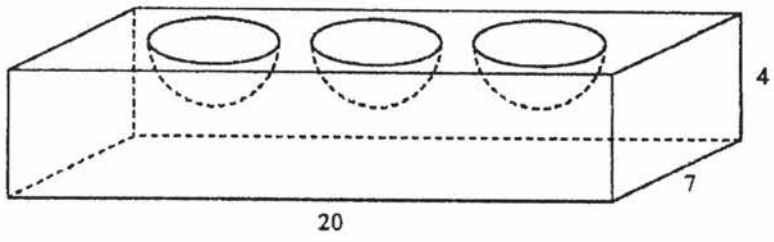
(ii) $\angle TOQ$, [2]

(iii) $\angle ABQ$, [2]

(iv) $\angle BSR$. [2]

[9]

8. A wooden cuboid has length 20 cm, width 7 cm and height 4 cm.
Three **hemisphere**, each of radius 1.5 cm, are hollowed out of the top of the cuboid, to leave the block as shown in the diagram.



(a) Calculate the volume of wood in the block. [2]

(b) The four vertical sides are painted pink.
Calculate the total area that is painted pink. [1]

(c) The inside of each **hemispherical** hollow is painted white.
The flat part of the top of the block is painted green.
Calculate the total area that is painted

(i) white, [1]

(ii) green. [1]

[10]

9. Answer the whole of this question on a sheet of graph paper.

The variables x and y are connected by the equation $y = 4x + \frac{60}{x} - 30$.

Some corresponding values of x and y are given in the following table.

x	1.5	2	2.5	3	4	5	7	8
y	16	a	4	2	1	b	6.6	9.5

(a) Calculate the values of a and b. [1]

(b) Using the scales of 2 cm to represent 1 unit of x and 1 cm to represent 1 unit of y, draw the graph of $y = 4x + \frac{60}{x} - 30$ for the range $1.5 \leq x \leq 8$. [3]

(c) From your graph, find

(i) the least value of y, [1]

(ii) the range of values of x for which $y = 4x + \frac{60}{x} - 30 < 8$. [2]

(d) Find, by drawing a tangent, the gradient of the curve when $x = 5$. [2]

(e) By drawing a suitable straight line on the same axes, find the solutions of the equation $3x^2 + 60 - 30x = 0$. [3]

10. All employees in Singapore have a compulsory savings known as the Central Provident Fund (CPF).
Each worker is required to save a certain percentage of what he earns each month with the CPF and the employer contributes another percentage of his salary to his CPF account.
The total CPF contribution is then kept into 3 accounts in the proportion as shown in the table below.

Contribution rates from 1 January 2016 for private sector and public sector non-pensionable employees being:

- Singapore Citizen
- SPR* from the third year of obtaining SPR status
- SPR during the first two years of obtaining SPR status but who has jointly applied with employer to contribute at full employer-full employee rates

*SPR (Permanent Resident)

Employee's age (years)	Contribution Rates from 1 Jan 2016 (for monthly wages ≥ $750)		
	By Employer (% of wage)	By Employee (% of wage)	Total (% of wage)
55 and below	17	20	37
Above 55 to 60	13	13	26
Above 60 to 65	9	7.5	16.5
Above 65	7.5	5	12.5

Figure 4A

Allocation rates from 1 January 2016 for private sector and public sector non-pensionable employees

Employee's age (years)	Allocation Rates from 1 Jan 2016 (for monthly wages ≥ $750)		
	Ordinary Account (% of wage)	Special Account (% of wage)	Medisave Account (% of wage)
35 and below	23	6	8
Above 35 to 45	21	7	9
Above 45 to 50	19	8	10
Above 50 to 55	15	11.5	10.5
Above 55 to 60	12	3.5	10.5
Above 60 to 65	3.5	2.5	10.5
Above 65	1	1	10.5

Figure 4B

In October 2016, Mr Ong who is 38 years old, earns $3000 a month, while his wife, who is 34 years old, earns $2000 a month.

(a) Calculate Mr Ong's contribution and his employer's contribution to his CPF account monthly. [2]

Both Mr Ong and his wife have just paid the 10% downpayment for their HDB flat which costs $400 000. They intend to pay the rest over a period of 20 years.

(b) Calculate how much they will have to pay per month for the 20 years. [2]

For a part of the amount they have to pay, the Ongs will use the money from both their Ordinary Accounts, and they will borrow the balance from a bank.

(c) Show that the amount from both their Ordinary Accounts to be used for the monthly payment of the flat is $1090. [2]

(d) Calculate the amount of money they have to borrow from the bank over the period of 20 years. [1]

The Ongs have to pay a simple interest rate of 1.48% for Year 1 and 1.58% thereafter.

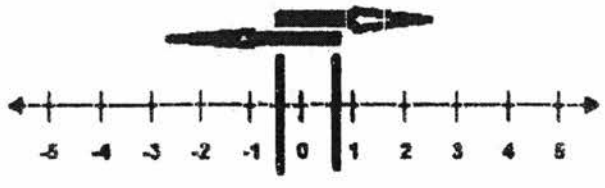
(e) Calculate the total amount they have to pay the bank after 20 years. [3]

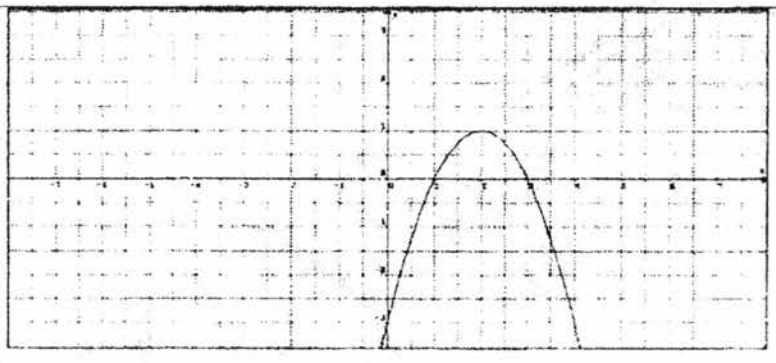
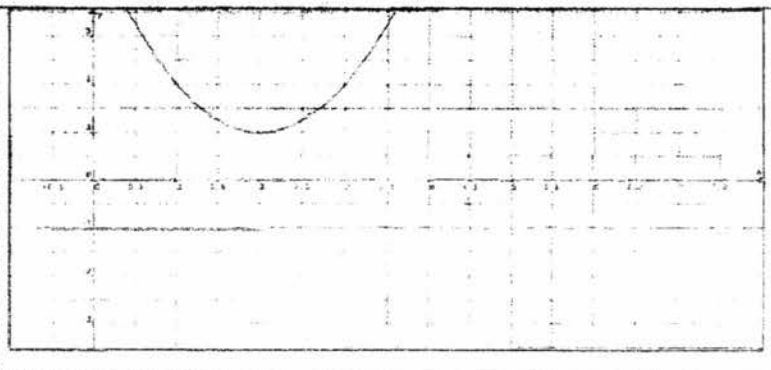
-End of Paper-

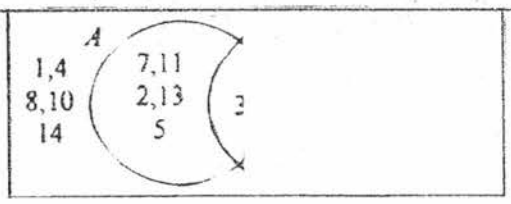
YUSOF ISHAK SECONDARY SCHOOL
PRELIMINARY EXAMINATION 2017
MATHEMATICS PAPER 1
SEC 4E/5N

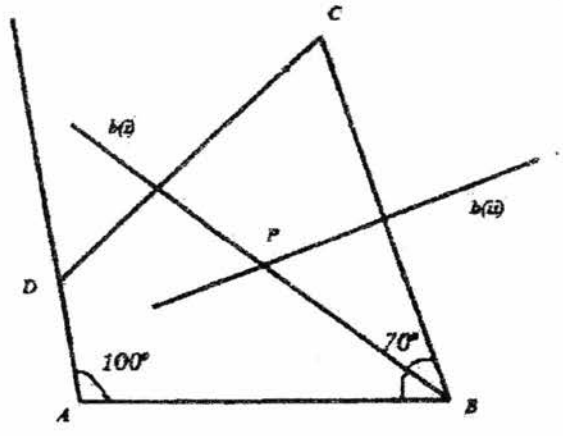
MARKING SCHEME

1	$\frac{-3.3^2 \times \sqrt{2^3}}{\left[1-8\left(7+7^{-1}\right)\right]^2} \times \sin\frac{\pi}{3} = -0.0084628$ [$\sin\frac{\pi}{3}$, radian mode] $= -0.008463$ (4 sig. figures)	Do not accept -0.0001785935 (Degree mode) B1 [1]
2	$\$850000 \times (100-14.3)\%$ $= \$728450$	M1 A1 [2]
3	24×6 16 minutes	M1 A1 [2]
4	1 litre = 20 km 20 km will emit $115 \times 20 = 2300$ grams of CO_2 2.3 kg	M1 A1 [2]
5(a)	$50p^2 - 72q^2$ $2(25p^2 - 36q^2)$ $2(5p-6q)(5p+6q)$	M1 A1 [2]
5(b)	$\frac{x-2}{4} - \frac{x+1}{3} = 1$ $\frac{3(x-2)}{12} - \frac{4(x+1)}{12} = 1$ $\frac{3x-6-4x-4}{12} = 1$ $-x-10 = 12$ $x = -22$	M1 A1 [2]
5(c)	$T = 2\pi\sqrt{\frac{h}{g}}$ $\left(\frac{T}{2\pi}\right)^2 = \frac{h}{g}$ $h = g\left(\frac{T}{2\pi}\right)^2$ or $h = \frac{gT^2}{4\pi^2}$	M1 A1 [2]
6(a)	As the two buckets are similar $\therefore \frac{\text{Height of small bucket}}{\text{Height of large bucket}} = \frac{8}{16}$ $\frac{\text{Height of small bucket}}{30} = \frac{8}{16}$ Height of small bucket $= \frac{8}{16} \times 30 = 15$ cm	B1 [1]
6(b)	$\frac{\text{Volume of large bucket}}{\text{Volum of small bucket}} = \left(\frac{16}{8}\right)^3$ $\frac{\text{Volume of large bucket}}{850} = (2)^3$ Volume of large bucket $= 8 \times 850 = 6800\ \text{cm}^3$	M1 A1 [2]

7(a)	$p+q$	B1 [1]
7(b)	$\frac{1}{2}(p-q)$	B1 [1]
8	LCM of 10, 20, 35 $= 5\times 2\times 2\times 7$ $= 140$ After 140 minutes	M1 A1 [2]
9(a)	40°	B1 [1]
9(b)	9	B1 [1]
9(c)	120°	B1 [1]
10	$P \alpha Q^2$ $P = kQ^2$ where k is a constant New $P_{NEW} = k(3Q)^2$ Percentage increase $= \frac{k(9Q^2 - Q^2)}{kQ^2}\times 100\% = 800\%$	 M1 A1 [2]
11	$\frac{10x+8}{3} + 2 < 5 + 4x < 8$ $\frac{10x+8}{3} + 2 < 5 + 4x$ and $5 + 4x < 8$ $10x + 8 + 6 < 15 + 12x$ and $4x < 3$ $2x > -1$ and $x < \frac{3}{4}$ $x > -\frac{1}{2}$ $\therefore -\frac{1}{2} < x < \frac{3}{4}$  	M1 A1 A1 [3]
12(a)	$2x - 3 = x + 3y - 7$ or $2x - 3 = 15 - x - y$ or $x - 3y = -4$ $3x + y = 18$ $x + 3y - 7 = 15 - x - y$ $2x + 4y = 22$ (Any two of the equations) $x + 2y = 11$	M1 A1 [2]
12(b)	$x = 5$ $y = 3$	M1 A1 [2]

13(a)	The information did not specify the total of number of children surveyed/population. OR The information did not specify the information was obtained in one hositpal/all hositpals in the USA. OR BIG HEADLINE makes you think that 5.3% of children get spinal cord injuries...a pretty scary statistic for parents.	Any 1 with explanation B2 [2] To explain why is this important to mention the population of the children surveyed.
13(b)	For the record, the real figure should be based on the number of injuries per year out of a population of certain number in that country	B1 [1]
14(a)	1 : 50 000 1 cm represent 0.5 km 5.8 cm represent $0.5 \times 5.8 = 2.9$ km	B1 [1]
14(b)	1 cm^2 represent 0.5×0.5 km^2 $\frac{6.5}{0.25} = 26\,cm^2$	M1 A1 [2]
15(a)		
15(b)(i)	$x^2 - 4x + 5 = (x-2)^2 + 1$	B1 [1]
15(b)(ii)		B1 – turning point B1 – y intercept [2]
16(a)	(1515 2610 1770)	M1 A1 [2]
16(b)	The total weekly costs for Raspberry, Orange and Lemon drinks are \$15.15, \$26.10 and \$17.70 respectively	B1 [1]

17		B2 B1 (one number wrong)
18(a)	Length of $AB = \sqrt{4^2 + 3^2} = 5 units$	B1 [1]
18(b)	$C(7,4)$	B1 [1]
18(c)	$50 = \frac{1}{2}(8+x)\times 4 \Rightarrow x = 17$ $D(13, 0)$	M1 A1 [2]
18(d)	$\cos \angle ABC = -\frac{3}{5}$	B1 [1]
19(a)	Mean mass of Line P = 495.8 g Mean mass of Line Q = 494.6 g	B1 B1 [2]
19(b)	Standard deviation of Line P = 7.07 g Standard deviation of Line Q = 9.92 g	B1 B1 [2]
19(c)	The lighter loaf is likely to come from Q where **the mean is lower**. The mass of line Q's products **are also more varied from their mean** value and hence, **a higher chance of being lighter**.	B1 B1 [2]
20(a)	$\frac{3}{9}, \frac{6}{9}, \frac{4}{9}, \frac{5}{9}$ oe	B1 for all three correct [1]
20(b)(i)	$\frac{12}{90}$	FT from their tree diagram 1FT [1]
20(b)(ii)	$\frac{48}{90}$	FT from their tree diagram. B1 for $\frac{24}{90}$ oe FT seen Or M1 for $\left(\frac{4}{10}\times\frac{6}{9}\right)+\left(\frac{6}{10}\times\frac{4}{9}\right)$ oe FT 2FT [2]

21(a)	$(3x^2y)^3 \times (5x^{-3}y^4)^{-1}$ $= 27x^6y^3 \times \frac{1}{5}x^3y^{-4}$ $= \frac{27x^9}{5y}$	M1 A1 [2]
21(b)	$\left(\frac{1}{8}\right)^{-\frac{2}{3}} \times 32^{\frac{3}{5}} = 2^{p-2} \div 2^2$ $4 \times 8 = 2^{p-4}$ $2^5 = 2^{p-4}$ $p = 9$	M1 A1 [2]
21 (c)	$0.0040589 = 4.0589 \times 10^{-3}$	B1 [1]
22(a)(i)	$\lvert p \rvert = \sqrt{(3)^2 + (4)^2}$ $\lvert p \rvert = 5$ units	B1 [1]
22(a)(ii)	$m = -3$	B1 [1]
22(I)(b)(i)	$\overrightarrow{OC} = 2\overrightarrow{AB} = 2\mathbf{b}$	B1 [1]
22(b)(ii)	$\overrightarrow{BC} = \overrightarrow{BA} + \overrightarrow{AO} + \overrightarrow{OC}$ $= -\mathbf{b} - \mathbf{a} + 2\mathbf{b}$ $= \mathbf{b} - \mathbf{a}$	B1 [1]
22(b)(iii)	$\overrightarrow{AD} = \overrightarrow{AB} + \overrightarrow{BC} + \overrightarrow{CD}$ $= \mathbf{b} + \mathbf{b} - \mathbf{a} - \mathbf{a}$ $= 2\mathbf{b} - 2\mathbf{a}$	B1[1]
22(b)(II)	Since $\overrightarrow{AD} = 2\overrightarrow{BC}$ $AD \parallel BC$ $ABCD$ is a trapezium	B1 [1]
23(a) **(b)(i)** **(b)(ii)**		(a)[2] (b)(i) [1] (b)(ii) [1] 2 possible location of point D. But no effect on the answer.
23(c)	$BP = 3.9\text{ cm} \pm 0.1\text{cm}$	B1 [1]

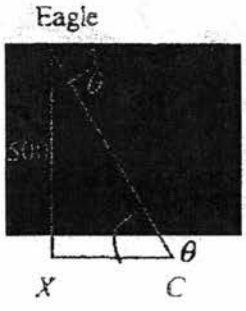
Yusof Ishak Secondary School
Preliminary Examination 2017
Mathematics Paper 2

Marking Scheme

1 (a)	$(1+4x)^2 = 81$ $1+4x = \pm\sqrt{81}$ $1+4x = 9$ or $1+4x = -9$ $4x = 8$ or $4x = -10$ $x = 2$ or $x = -2.5$	M1 A1 [2]
1 (b)	$\frac{1}{2x+3} + \frac{3}{2x-1}$ $= \frac{1(2x-1)+3(2x+3)}{(2x+3)(2x-1)}$ $= \frac{2x-1+6x+9}{(2x+3)(2x-1)}$ $= \frac{8x+8}{(2x+3)(2x-1)}$	M1 A1 [2]
1 (c)	$2x+1<9<3x+1$ $2x+1<9$ and $9<3x+1$ $\Rightarrow 2x<8$ and $3x>8$ $\Rightarrow x<4$ and $x>\frac{8}{3}$ x=3	M1 A1 [2]
1 (d)	$a^2+9b^2-6ab-2a+6b$ $=(a^2+9b^2-6ab)-2a+6b$ $=(a-3b)^2-2(a-3b)$ $=(a-3b)(a-3b-2)$	M1 A1 [2]

2(a)	Number of sets of 2 white beads and 1 black bead $14-1=13$ Total number of white beads $13\times 2=26$  Disagree Students must be able to explain and show how they obtained the answer	B3 [3]
2(b)(i)	$3b=4a \Rightarrow \frac{a}{b}=\frac{3}{4} \Rightarrow a:b=3:4$ $2c=5a \Rightarrow \frac{a}{c}=\frac{2}{5} \Rightarrow a:c=2:5$ $\therefore a:b:c=6:8:15$	B1 B1 [2]
2(b)(ii)	Let $a=6k, b=8k, c=15k$ $6k+8k+15k=10$ $k=\frac{10}{29}$ $\therefore b=\frac{80}{29}$	M1 A1 A1 [3]

3 (a)	x bulbs cost \$25 1 bulb cost $\$\frac{25}{x}$	B1 [1]
3 (b)	Selling price for each light bulb = $\$\frac{25}{x} + \$0.50$ $= \$\frac{25 + \$0.50x}{x}$ $= \$\frac{2(25 + 0.5x)}{2x}$ $= \$\frac{50 + x}{2x}$	B1 [1]
3 (c)(i)	Total amount = $\$\frac{50 + x}{2x} \times 8$ $= \$\frac{4(50 + x)}{x}$	B1 [1]
3(c)(ii)	Number of unsold light bulbs = $x - 8$	B1 [1]
3(d)	Total amount = \$2 × $(x - 8)$ = \$2$(x - 8)$	B1 [1]
3(e)	$\frac{4(50 + x)}{x} + 2(x - 8) = 46$ $\frac{200 + 4x}{x} + 2x - 16 = 46$ $\frac{200 + 4x + 2x^2 - 16x}{x} = 46$ $2x^2 - 12x + 200 = 46x$ $2x^2 - 58x + 200 = 0$ $x^2 - 29x + 100 = 0$ (Shown)	M1 A1 A1 [3]
3(f)	$x = \frac{-(-29) \pm \sqrt{(-29)^2 - 4(1)(100)}}{2(1)}$ $x = \frac{29 \pm \sqrt{441}}{2}$ $x = \frac{29 \pm 21}{2}$ $x = 25$ or $x = 4$ The number of light bulbs cannot be less than 8. $\therefore x = 4$ is not applicable The number of light bulbs, $x = 25$.	M1 A1 A1 [3]

4(a)(i)	In ΔABD, Using Sine Rule, $\frac{\sin 118^\circ}{950} = \frac{\sin \angle ADB}{600}$ $\Rightarrow \sin \angle ADB = \frac{600 \times \sin 118^\circ}{950}$ $\angle ADB = 33.89^\circ$ $\angle ADB = 33.9^\circ$ (1 decimal place) $\angle ABD = 180^\circ - 118^\circ - 33.9^\circ$ $= 28.1^\circ$	M1 A1 A1 A1 [4]
4(a)(ii)	In ΔBCD, Using Cosine Rule, $CD = \sqrt{950^2 + 1040^2 - 2(950)(1040)\cos 42^\circ}$ $CD = 718.1$ $CD = 718$ m (3 sig. figures)	M2 A1 A1 [4]
4(a)(iii)	Let the required distance be h. Area of $\Delta BCD = \frac{1}{2} \times 950 \times 1040 \times \sin 42^\circ$ $\frac{1}{2} \times 950 \times 1040 \times \sin 42^\circ = \frac{1}{2} \times 950 \times h$ $h = 1040 \times \sin 42^\circ$ $h = 695.9$ $h = 696$ m (3 sig. figures)	 M1 A1 [2]
4(b)	The greatest angle of depression occurs when the eagle is directly above the point on BD such that it is nearest to C.  $\tan\theta = \frac{500}{695.9}$ $\theta = 35.7^\circ$ Greatest angle of depression is 35.7° (1 decimal place)	 M1 A1 [2]

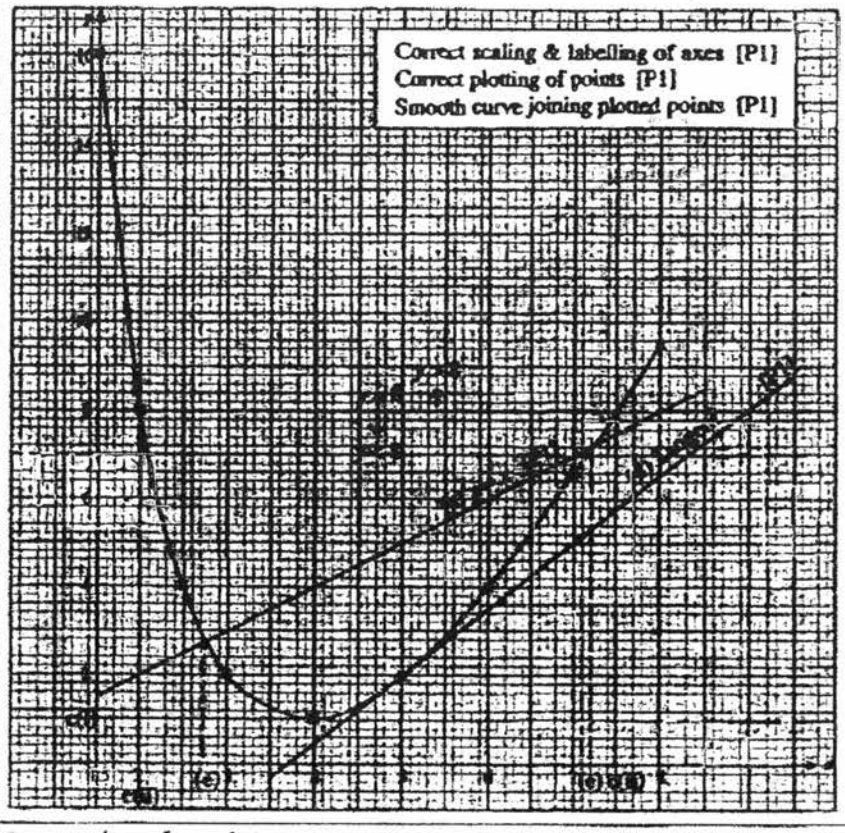
5(a)(i)	On Monday, volume = 7500 cm^3. On Tuesday, volume = 86% of 7500 $= \frac{86}{100} \times 7500 = 6450$ On Wednesday, volume = 86% of 6450 $= \frac{86}{100} \times 6450 = 5547$ $= 5547$ cm^3 (3 sig. figures)	M1 A1 [2]
5(a)(ii)	Let x be the actual volume of Block Q, the volume of Block Q has been reduced as 86% of its actual volume on Tuesday. $\Rightarrow 86\%$ of $x = 6450$ $x = 6450 \times \frac{100}{86} = 7500$ Actual volume of Block Q on Monday = 7500cm^3 (3 sig. figures)	M1 A1 [2]
5(a)(iii)	Let v be the volume of Block R on Monday On Tuesday, volume = $\frac{86}{100}v = 0.86v$ On Wednesday, volume = $\frac{86}{100}(0.86v) = 0.7396v$ On Thursday, volume = $\frac{86}{100}(0.7396v) = 0.6361v$ On Friday, volume = $\frac{86}{100}(0.6361v) = 0.547v$ On Saturday, volume = $\frac{86}{100}(0.547v) = 0.470v$ Volume reduces to half on Saturday.	M1 A1 [2]
5 (b)(i)	Volume of hemisphere = $\frac{1}{2}\left(\frac{4}{3}\pi r^3\right)$ Volume of $S = \frac{1}{2}\left(\frac{4}{3}\pi(18)^3\right)$ $= \frac{2}{3} \times 3.142 \times 5832$ $= 12216.1$ $= 12200$ cm^3 (3 sig. figures)	M1 A1 [2]
5(b)(ii)	Total surface area of solid hemisphere S $= \frac{1}{2}\left(4\pi r^2\right) + \pi r^2$ $= \frac{1}{2}\left(4\pi(18)^2\right) + \pi(18)^2$ $= 2036016 + 1018008$ $= 3054024$ $= 3050$ cm^2 (3 sig. figures)	M1 A1 [2]
5(c)	$\frac{\text{Volume before}}{\text{Volume after}} = \left(\frac{\text{height before}}{\text{height after}}\right)^3$ $\left(\frac{5000}{1080}\right) = \left(\frac{12}{h}\right)^3$	

	$\frac{125}{27} = \left(\frac{12}{h}\right)^3$ $\left(\frac{5}{3}\right)^3 = \left(\frac{12}{h}\right)^3$ $\frac{5}{3} = \frac{12}{h}$ $5h = 36$ $h = 7.2$ cm	M1 A1 [2]

6(a)(i)	$OA^2 = OX^2 + AX^2$ $r^2 = (16-r)^2 + 8^2$ $r^2 = 256 - 32r + r^2 + 8^2$ $32r = 320$ $r = 10$	M1 A1 A1 [3]
6(a)(ii)	$\sin \angle AOX = \frac{AX}{OA}$ $= \frac{8}{10}$ $\angle AOX = \sin^{-1}\left(\frac{8}{10}\right)$ $\angle AOX = 53.1^\circ$ (1 decimal place)	B1 [1]
6(a)(iii)	$\angle AOB = 2(53.1^\circ)$ Shaded region $= \frac{2(53.1^\circ)}{360^\circ} \times \pi \times 10^2 - \frac{1}{2} \times 10 \times 10 \sin 2(53.1^\circ)$ $=44.74\ m^2$ ∴ Volume of concrete used $= 44.74 \times 900$ $= 40\,266\ m^3$ $= 40\,300\ m^3$ (correct to 3 sig. figures)	M1 A1 A1 [3]
6 (b)	Length of the model tunnel $= \frac{900}{10} \times 5$ $= 450$ m Reflex $\angle AOB = 360^\circ - 2(53.1)^\circ = 253.74^\circ$ Curved surface area $= \frac{253.74^\circ}{360^\circ} \times 2\pi \times 5 \times 450$ $= 9965.6\ cm^2$ $= 9970\ cm^2$ (3 sig. figures)	B1 M1 A1 [3]
6 (c)	Total distance the train has to travel = 900+130=1030m. Time taken $= \frac{1030}{50000} \times 60 = 1.236$ minutes 1 minute 14 seconds	M1 A1 [2]

7(a)	$\angle BSQ = 90°$ (rt. $\angle$ in a semicircle) $\angle BAQ = 90°$ (rt. $\angle$ in a semicircle) $\angle OST$ or $\angle OQT = 90°$ (tangent perp. radius at point of contact)	B1 B1 B1 [3]
7(b)(i)	$\angle SQB = 32°$ ($\angle$s in the same segment)	B1 [1]
7(b)(ii)	$\angle OTQ = \frac{64°}{2} = 32°$ (OT bisects $\angle QTS$) $\angle TOQ = 180 - 32 - 90 = 58°$ ($\angle$ sum of Δ)	M1 A1 [2]
7(b)(iii)	$\angle ACB = 180° - 73° = 107°$ (adj. $\angle$s on a str. line) $\angle ABQ = 180 - 32 - 107 = 41°$ ($\angle$ sum of Δ)	M1 A1 [2]
7(b)(iv)	$QT = ST$ (tangents drawn to circle form ext. point are equal) $\angle DST = \frac{180° - 64°}{2}$ (base $\angle$s of isosceles Δ) $= 58°$ $\angle BSR = 180° - 90° - 58°$ (adj. $\angle$s on a str. line) $= 32°$ OR $\angle BSR = 32°$ (alternate segment theorem)	M1 A1 [2] B1, B1 [2]

8(a)	Volume of wood in the block = $(20 \times 7 \times 4) - 3\left(\frac{1}{2} \times \frac{4}{3} \times \pi \times 1.5^3\right)$ $= 560 - 21.2085$ $= 538.7915$ $= 539\ cm^3$ (3 significant figures)	M1 A1 [2]
8(b)	Total area that is painted pink = $2(7 \times 4) + 2(20 \times 4) = 56 + 160$ $= 216\ cm^2$	B1 [1]
8(c)(i)	Total area that is painted white = $2 \times \pi \times (1.5)^2 \times 3$ $= 42.417$ $= 42.4\ cm^2$ (3 significant figures)	B1 [1]
8(c)(ii)	Total area that is painted green = $(20 \times 7) - 3(\pi \times 1.5^2)$ $= 118.7915$ $= 119\ cm^2$	B1 [1]

9(a)	$a = y = 4(2) + \frac{60}{2} - 30$ $= 8 + 30 - 30$ $= 8$ $b = y = 4(5) + \frac{60}{5} - 30$ $= 20 + 12 - 30$ $= 2$	1 wrong B0 B1 [1]
8(b)		P1 P1 P1 [3]
8(c)(i)	Least value of $y = 1$	[B1]
8(c)(ii)	For $4x + \frac{60}{x} - 30 < 8$ $2 < x < 7.5$	M1 A1 [2]
8(d)	At $x = 5$, using 2 points on the tangent, (7, 5) and (5, 2) Gradient of tangent $= \frac{5-2}{7-5} = \frac{3}{2}$ or 1.5	M1 A1 [2]
8(e)	$3x^2 + 60 - 30x = 0$ $3x + \frac{60}{x} - 30 = 0$ $4x + \frac{60}{x} - 30 = x$ The solution is the intersection of the graphs	M1

	$y = 4x + \frac{60}{x} - 30$ and $y = x$ *i.e.* when $x = 2.75$ or $x = 7.2$	P1 A1 [3]

10(a)	Mr Ong's monthly contribution $= \frac{20}{100} \times \$3000 = \$600$ His employer's monthly contribution $= \frac{17}{100} \times \$3000 = \$510$	B1 B1 [2]
10(b)	They have to pay $\frac{90}{100} \times \$400000 = \$360000$ over 20 years Each month, they have to pay $\frac{\$360000}{20 \times 12} = \$1500$	M1 A1 [2]
10(c)	Amount to be used for monthly payment $= \left(\frac{21}{100} \times \$3000\right) + \left(\frac{23}{100} \times \$2000\right) = \$1090$ (Shown)	M1 A1 [2]
10(d)	They have to borrow $(\$1500 - \$1090) \times 20 \times 12 = \$98400$	B1 [1]
10(e)	They have to pay $\left(\frac{1.48}{100} \times 98400 \times 1\right) = \$1456.32$ Year 1 Interest $\left(\frac{1.58}{100} \times 98400 \times 19\right)$ Year 2 onwards $\$29539.68$ $\$98400 + \$145632 + \$2953968$ $= \$129396$	 M1 M1 [A1]