Candidate Name:	Class	Index No.

FUHUA SECONDARY SCHOOL

Secondary Four Normal Academic

Preliminary Examination 2018

4NA

ADDITIONAL MATHEMATICS 4044/2
PAPER 2

Additional Materials:
Writing paper & Electronic calculator

DATE 17 Aug 2018
TIME 10 45 – 12 30
DURATION 1 h 45 min

INSTRUCTIONS TO CANDIDATES

Answer **all** questions.
Write your answers and working on the separate writing paper provided.
Write in dark blue or black pen.
You may use a soft pencil for any diagrams or graphs.

45

Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.

The number of marks is given in brackets [ ] at the end of each question or part question.
The total number of marks for this paper is **70**.
The use of an electronic calculator is expected, where appropriate.
You are reminded of the need for clear presentation in your answers.

PARENT'S SIGNATURE	FOR EXAMINER'S USE	
		/ 70

Setter: *Miss Winnifred Lim* *Vetter:* *Mr Chen Hong Ming*

This question paper consists of 6 printed pages including this page.

Mathematical Formulae

1. ALGEBRA

Quadratic Equation

For the equation $ax^2 + bx + c = 0$,

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Binomial expansion

$$(a+b)^n = a^n + \binom{n}{1}a^{n-1}b + \binom{n}{2}a^{n-2}b^2 + \ldots + \binom{n}{r}a^{n-r}b^r + \ldots + b^n$$

where n is a positive integer and $\binom{n}{r} = \frac{n!}{(n-r)!\,r!} = \frac{n(n-1)\ldots(n-r+1)}{r!}$

2. TRIGONOMETRY

Identities

$$\sin^2 A + \cos^2 A = 1$$
$$\sec^2 A = 1 + \tan^2 A$$
$$\operatorname{cosec}^2 A = 1 + \cot^2 A$$
$$\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$$
$$\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$$
$$\tan(A \pm B) = \frac{\tan A \pm \tan B}{1 \mp \tan A \tan B}$$
$$\sin 2A = 2 \sin A \cos A$$
$$\cos 2A = \cos^2 A - \sin^2 A = 2\cos^2 A - 1 = 1 - 2\sin^2 A$$
$$\tan 2A = \frac{2\tan A}{1 - \tan^2 A}$$

Formulae for ΔABC

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$a^2 = b^2 + c^2 - 2bc \cos A$$

$$\Delta = \frac{1}{2}ab \sin C$$

1 **Without the use of a calculator,** express $\cos 105°$ in the form $\frac{1}{4}\left(\sqrt{a}-\sqrt{b}\right)$, where a and b are integers to be determined. [3]

2 Angles G and H are in the same quadrant such that $\tan G=-\frac{1}{2}$ and $\cos H=\frac{5}{13}$.

Find the **exact** value of

(a) $\tan 2H$, [2]

(b) $\cos(G-H)$. [3]

3 The graphs of $y^2=2x$ and $y=mx^3$,where m is a constant, intersect at the point $A(2, 2)$.

(i) Find the value of m. [1]

(ii) Sketch, on the same diagram, the graphs of $y^2=2x$ and $y=mx^3$. [4]

4 **(i)** Prove the identity $\tan^2 A-\sin^2 A=\tan^2 A\sin^2 A$. [3]

(ii) Hence, solve $\tan^2 A-\sin^2 A=\frac{1}{2\cos^2 A}$ for $0°\le A\le 360°$. [4]

5 Given that the coefficient of x^2y^{10} in the expansion of $(x+ky^2)^7$ is 21504, find the value of k given that $k>0$. [4]

6 The roots of the quadratic equation $6x^2-4x+9=0$ are $(\alpha+1)$ and $(\beta+1)$.

(i) Show that $\alpha^2+\beta^2=-\frac{17}{9}$. [3]

(ii) Find a quadratic equation with roots $\frac{\alpha}{\beta}$ and $\frac{\beta}{\alpha}$. [3]

7 Given that $6\left(3^{2x-1}\right)-3\left(5^{x+2}\right)=5^x$, find the value of $\left(\frac{5}{9}\right)^x$. [4]

8 A curve has the equation $y=\frac{4x^2}{3x-5}$.

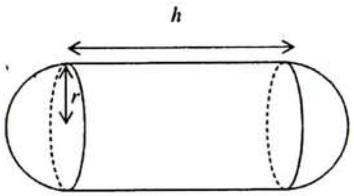
(i) Find the equation of the tangent to the curve at $x=2.5$. [3]

The point (p,q) ,where $p\neq 0$, is a stationary point on the curve. Find

(ii) the value of p and of q, [4]

(iii) the nature of the stationary point (p,q). [3]

9 The diagram shows a container made up of a cylinder of radius, r cm, and length, h cm, with hemispheres of radius r cm attached at each end of the cylinder.



It is given that the volume of the tank is 50π cm^3.

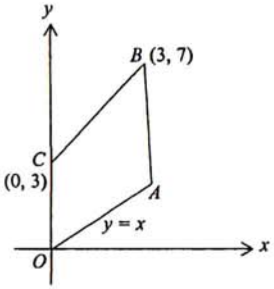
(i) Show that $h=\frac{50}{r^2}-\frac{4}{3}r$. [2]

(ii) Express the surface area, A, in terms of r and π. [2]

(iii) Given that r can vary, find the stationary value of A and determine whether it is a maximum or a minimum. [7]

10 Solutions to this question by accurate drawing will not be accepted.

In the diagram, the coordinates of B and C are (3, 7) and (0, 3) respectively. The line OA has the equation $y = x$.

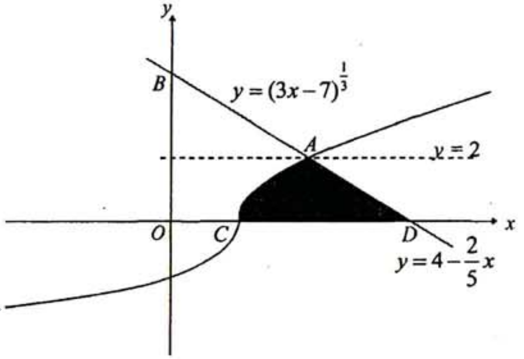


(a) Given that the perpendicular bisector of BC meets OA at A, find the coordinates of A. [5]

(b) A point D lies on BC such that the x – coordinate of D is p, $0 < p < 3$. Find, in terms of p, the area of triangle OBD. [3]

4

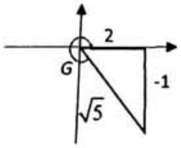
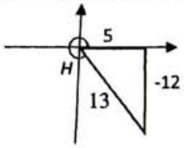
11

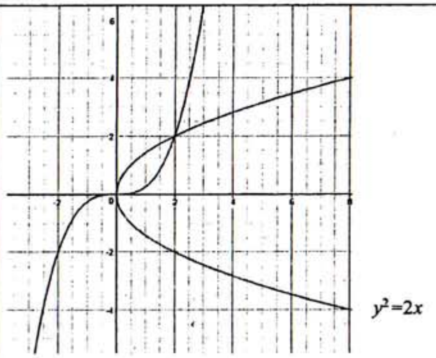


The diagram shows the line $y = 4 - \frac{2}{5}x$ intersecting the curve $y = (3x-7)^{\frac{1}{3}}$ at point A. Given that A lies on the line $y = 2$, find the area of the shaded region. [7]

~ End of Paper ~

Sec 4NA AMaths Prelim 2018 Paper 2 Answer Scheme

1		$\cos 105° = \cos(45+60)°$ $= \cos 45° \cos 60° - \sin 45° \sin 60°$ $= \left(\frac{\sqrt{2}}{2}\right)\left(\frac{1}{2}\right) - \left(\frac{\sqrt{2}}{2}\right)\left(\frac{\sqrt{3}}{2}\right)$ $= \frac{1}{4}\left(\sqrt{2} + \sqrt{6}\right)$ $a = 2, b = 6$	B1 B1 A1
2		 	
	(a)	$\tan 2H = \frac{2\left(\frac{12}{5}\right)}{1-\left(\frac{12}{5}\right)^2}$ $= \frac{-120}{119}$	B1 B1
	(b)	$\cos(G-H) = \cos G \cos H + \sin G \sin H$ $= \left(\frac{2}{\sqrt{5}}\right)\left(\frac{5}{13}\right) + \left(\frac{-1}{\sqrt{5}}\right)\left(\frac{-12}{13}\right)$ $= \frac{22}{13\sqrt{5}}$ $= \frac{22\sqrt{5}}{65}$	B1, B1 B1
3	a	At (2, 2), $2 = m(2)^3$ $m = \frac{1}{4}$	B1
	b		B2 B2

			
4	a	$\tan^2 A - \sin^2 A = \tan^2 A \sin^2 A$ LHS $= \frac{\sin^2 x}{\cos^2 x} - \sin^2 x$ $= \frac{\sin^2 x - \sin^2 x \cos^2 x}{\cos^2 x}$ $= \frac{\sin^2 x(1-\cos^2 x)}{\cos^2 x}$ $= \frac{\sin^2 x \sin^2 x}{\cos^2 x}$ $= \left(\frac{\sin^2 x}{\cos^2 x}\right)\left(\sin^2 x\right)$ $= \tan^2 x \sin^2 x$	M1 M1 A1
	b	$\tan^2 A - \sin^2 A = \frac{1}{2\cos^2 A}$ $\tan^2 A \sin^2 A = \frac{1}{2\cos^2 A}$ $\tan^2 A \sin^2 A \cos^2 A = \frac{1}{2}$ $\left(\frac{\sin^2 A}{\cos^2 A}\right)\left(\sin^2 A \cos^2 A\right) = \frac{1}{2}$ $\sin^4 A = \frac{1}{2}$ $\sin A = \pm\sqrt[4]{\frac{1}{2}}$ Basic angle = 57.234° $A = 57.2°,\ 122.8°,\ 237.2°,\ 302.8°$	B1 M1 M1 A1

5		General term $T_{r+1}=\binom{7}{r}(x)^{7-r}(ky^2)^r$	B1
		For x^2y^{10}, $r=5$	B1
		$T_{5+1}=\binom{7}{5}(x)^{7-5}(ky^2)^5$	
		$=21x^2y^{10}k^5$	
		$21504=21k^5$	M1
		$k=4$	A1
6	i	$6x^2-4x+9=0$	
		$\alpha+\beta+2=\frac{2}{3}$	
		$\alpha+\beta=-1\frac{1}{3}$	B1
		$\alpha\beta+\alpha+\beta+1=\frac{3}{2}$	
		$\alpha\beta-1\frac{1}{3}+1=\frac{3}{2}$	
		$\alpha\beta=1\frac{5}{6}$	B1
		$\alpha^2+\beta^2=(\alpha+\beta)^2-2\alpha\beta$	
		$=\left(-1\frac{1}{3}\right)^2-2\left(1\frac{5}{6}\right)$	B1
		$=-\frac{17}{9}$ (shown)	
	ii	When roots are $\frac{\alpha}{\beta}$ and $\frac{\beta}{\alpha}$	
		Sum of roots $=\frac{\alpha^2+\beta^2}{\alpha\beta}$	
		$=\frac{-\frac{17}{9}}{1\frac{5}{6}}=-\frac{34}{33}$	M1
		Product of roots = 1	M1
		Equation: $x^2+\frac{34}{33}x+1=0$	A1
		$33x^2+34x+33=0$	

7		$6(3^{2x-1})-3(5^{x+2})=5^x$	
		$6\left(\frac{3^{2x}}{3}\right)-3(25\times5^x)=5^x$	M1 M1
		$2(3^{2x})-75(5^x)=5^x$	
		$2(3^{2x})=76(5^x)$	M1
		$\frac{2}{76}=\frac{5^x}{3^{2x}}$	
		$\left(\frac{5}{9}\right)^x=\frac{2}{76}=\frac{1}{38}$	A1
8	i	$\frac{dy}{dx}=\frac{(8x)(3x-5)-(4x^2)(3)}{(3x-5)^2}$	M1
		$=\frac{12x^2-40x}{(3x-5)^2}$	
		At $x=2.5$, $\frac{dy}{dx}=-4$,	M1
		$y=10$	
		Sub into $y=mx+c$	
		$10=-4(2.5)+c$	
		$c=20$	
		Equation of tangent: $y=-4x+20$	A1
	ii	At stationary point, $\frac{dy}{dx}=0$	
		$\frac{12x^2-40x}{(3x-5)^2}=0$	M1
		$12x^2-40x=0$	
		$x(12x-40)=0$	
		$x=0$ (rej) or $x=3\frac{1}{3}$	M1
		When $x=3\frac{1}{3}$,	
		$y=\frac{4\left(\frac{10}{3}\right)^2}{3\left(\frac{10}{3}\right)-5}$	
		$y=\frac{80}{9}=8\frac{8}{9}$	

		Hence $p=3\frac{1}{3}$, $q=8\frac{8}{9}$	A1, A1
	iii	$\frac{dy}{dx}=\frac{12x^2-40x}{(3x-5)^2}$ $\frac{d^2y}{dx^2}=\frac{(24x-40)(3x-5)^2-(12x^2-40x)(2)(3x-5)(3)}{(3x-5)^4}$ At $x=3\frac{1}{3}$, $\frac{d^2y}{dx^2}=\frac{(24x-40)(3x-5)^2-(12x^2-40x)(2)(3x-5)(3)}{(3x-5)^4}$ $=\frac{8}{5}>0$ $\left(3\frac{1}{3},8\frac{8}{9}\right)$ is a minimum point.	M1 M1 A1
9	i	$50\pi=\frac{4}{3}\pi r^3+\pi r^2h$ $\pi r^2h=50\pi-\frac{4}{3}\pi r^3$ $h=\frac{50\pi-\frac{4}{3}\pi r^3}{\pi r^2}$ $h=\frac{50}{r^2}-\frac{4}{3}r$ (shown)	M1 A1
	ii	$A=4\pi r^2+2\pi rh$ $A=4\pi r^2+2\pi r\left(\frac{50}{r^2}-\frac{4}{3}r\right)$ $A=4\pi r^2+\frac{100\pi}{r}-\frac{8\pi r^2}{3}$ $A=\frac{4}{3}\pi r^2+\frac{100\pi}{r}$	M1 A1
	iii	$\frac{dA}{dr}=\frac{8}{3}\pi r-\frac{100\pi}{r^2}$ At stationary, $\frac{dA}{dr}=0$ $0=\frac{8}{3}\pi r-\frac{100\pi}{r^2}$ $\frac{8}{3}r=\frac{100}{r^2}$ $r^3=37.5$ $r=3.34716$ Stationary value of $A=\frac{4}{3}\pi(\sqrt[3]{37.5})^2+\frac{100\pi}{\sqrt[3]{37.5}}$ $=140.78$ $=141$ cm^2 (to 3 sf) $\frac{d^2A}{dr^2}=\frac{8}{3}\pi+\frac{200\pi}{r^3}$ When $r=\sqrt[3]{37.5}$, $\frac{d^2A}{dr^2}=\frac{8}{3}\pi+\frac{200\pi}{r^3}>0$ Hence, $A=141$ is a minimum.	M1 M1 A1 A1 M1 M1 A1
10	a	Gradient BC $=\frac{7-3}{3-0}=\frac{4}{3}$ Gradient of perpendicular bisector $=-\frac{3}{4}$ Midpoint BC $=\left(\frac{3}{2},\frac{7+3}{2}\right)=\left(\frac{3}{2},5\right)$ Sub $m=-\frac{3}{4}$ and $\left(\frac{3}{2},5\right)$ in $y=mx+c$, $5=\left(-\frac{3}{4}\right)\left(\frac{3}{2}\right)+c$ $c=\frac{49}{8}$ Equation of perpendicular bisector : $y=-\frac{3}{4}x+\frac{49}{8}$ When $y=x$, $x=-\frac{3}{4}x+\frac{49}{8}$ $x=\frac{7}{2}$ $A=\left(\frac{7}{2},\frac{7}{2}\right)$	M1 B1 A1 M1 A1
	b	Equation BC: $y=\frac{4}{3}x+3$	

Name:	Index Number:	Class:

52

HUA YI SECONDARY SCHOOL

4NA Preliminary Examination 2018 **4NA**

ADDITIONAL MATHEMATICS **4044/01**

Paper 1

1 August 2018

1 h 45 min

Additional materials: Answer Paper

READ THESE INSTRUCTIONS FIRST

Write your Name, Class and Index Number on all the work you hand in.
Write in dark blue or black pen.
You may use a soft pencil for any diagrams or graphs.
Do not use staples, paper clips, highlighters, glue or correction fluid.

Answer **all** questions.

Write your answers on separate Answer Paper provided.
Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.
The use of a scientific calculator is expected, where appropriate.
You are reminded of the need for clear presentation in your answers.

At the end of the examination, fasten all your work securely together.
The number of marks is given in brackets [ ] at the end of each question or part question.

The total number of marks for this paper is 70.

For Examiner's Use
70

This document consists of **5** printed pages including the cover page.

Turn Over

Mathematical Formulae

1. ALGEBRA

Quadratic Equation

For the equation $ax^2 + bx + c = 0$,

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Binomial Expansion

$$(a+b)^n = a^n + \binom{n}{1}a^{n-1}b + \binom{n}{2}a^{n-2}b^2 + \ldots + \binom{n}{r}a^{n-r}b^r + \ldots + b^n,$$

where n is a positive integer and $\binom{n}{r} = \frac{n!}{r!(n-r)!} = \frac{n(n-1)\ldots(n-r+1)}{r!}$

2. TRIGONOMETRY

Identities

$$\sin^2 A + \cos^2 A = 1$$
$$\sec^2 A = 1 + \tan^2 A$$
$$\operatorname{cosec}^2 A = 1 + \cot^2 A$$

$$\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$$
$$\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$$
$$\tan(A \pm B) = \frac{\tan A \pm B}{1 \mp \tan A \tan B}$$

$$\sin 2A = 2\sin A \cos A$$
$$\cos 2A = 2\cos^2 A - 1 = 1 - 2\sin^2 A = \cos^2 A - \sin^2 A$$
$$\tan 2A = \frac{2\tan A}{1 - \tan^2 A}$$

Formulae for ΔABC

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$
$$a^2 = b^2 + c^2 - 2bc\cos A$$
$$\Delta = \frac{1}{2}bc\sin A$$

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8 Solve $2\cos^2 x + \sin x + 1 = 0$ for $0° \le x \le 360°$. [5]

1 **(a)** Simplify the following, leaving your answer in positive index form. [2]

$x^4y \div (x^{-2}y^2)^2$

(b) Find the range of values of x for $28 - 13x - 6x^2 \leq 0$. [3]

2 **(a)** The polynomial $f(x)$ is given by $f(x) = x^3 - 7x - 6$.

(i) Show that $x + 2$ is a factor of $f(x)$. [1]

(ii) Express $f(x)$ as a product of three linear factors. [3]

(b) Given that $3x^2 - 7x - 3 = Ax(x-2) + B(x-2) + C$ for all values of x, find the value of each of the constants, A, B and C. [4]

3 Given that $3x^2 + kx - 12 = 0$, where $k \neq 0$, show that there is no value of k for which the equation has equal roots. [3]

4 Prove that $\dfrac{\operatorname{cos}ec\theta}{\cos\theta} - \tan\theta = \cot\theta$. [3]

5 **(i)** Given that $\cos\phi = a + b$ and $\sin\phi = b - a$, show that $b^2 - a^2 = \frac{1}{2}\sin 2\phi$. [2]

(ii) Find $\cos\phi + \sin\phi$ and $\cos\phi - \sin\phi$. Hence, express $\dfrac{b}{a}$ in terms of $\tan\phi$. [4]

6 Given that $\cos A = -\dfrac{3}{5}$ and $\tan A = \dfrac{4}{3}$, evaluate the following and leave your answer in their simplest form, without the use of the calculators.

(i) $\cos 2A$ [2]

(ii) $\sin(90° - A)$

[2]

7 **(i)** Differentiate $(3x^4 + 4)^6$ with respect to x. [2]

(ii) Hence, evaluate $\int_0^1 x^3 (3x^4 + 4)^5 \, dx$. [3]

8 Solve $2\cos^2 x + \sin x + 1 = 0$ for $0° \leq x \leq 360°$. [5]

9 Differentiate with respect to x.

(i) $y = 2(2x^2 + 1)^2$ [2]

(ii) $y = \dfrac{3x^2 - x + 1}{x^2}$ [3]

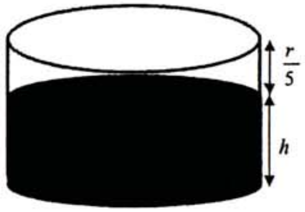
10 The coordinates of three points are $P = (7, -3)$, $Q = (4, -1)$ and $R = (1, 7)$. The perpendicular bisector of PR cuts the x-axis at S.

(i) Find the equation of the perpendicular bisector of PR. [4]

(ii) Find the distance of PS. [4]

(iii) Find the area of the quadrilateral $PQRS$. [2]

11 The diagram shows an open-topped cylinder of radius r cm. When the volume of the water in this cylinder is 80 cm^3, the height of the cylinder is h cm and the surface of the water is $\frac{r}{5}$ cm below the top of the cylinder.



(i) Show that the curve surface area, A cm^2, of the open-topped cylinder is given by $A = \dfrac{160}{r} + \dfrac{2\pi r^2}{5}$. [3]

(ii) Given that r varies, find the value of r for which A is stationary and the corresponding value of h. [4]

12 The equation of a circle is $2x^2+2y^2-16x-12y=22$.

(i) Express in the circle in the form $(x-a)^2+(y-b)^2=r^2$. [2]

(ii) Hence, find the centre and radius of the circle. [2]

(iii) The chord $y=x-2$ intersects the circle at two points P and Q, whose x-coordinates are $\frac{a+\sqrt{b}}{2}$ and $\frac{a-\sqrt{b}}{2}$ respectively. [5]

Find the value of a and of b.

Justify which point, P or Q is nearer to the origin.

END OF PAPER

HUA YI SECONDARY SCHOOL

4NA Preliminary Examination 2018 **4NA**

ADDITIONAL MATHEMATICS **4044/01**

Paper 1

1 August 2018

1 h 45 min

Additional materials: Answer Paper

MARKING SCHEME

55

1	(i)		Simplify the following, leaving your answer in positive index form $x^4y \div (x^{-2}y^2)^2$ $\frac{x^4y}{x^{-4}y^4}$ $M1$ $=x^8y^{-3}$ $=\frac{x^8}{y^3}$ $A1$	[2]
	(ii)		Find the range of values of x for $28-13x-6x^2 \le 0$. $6x^2+13x-28 \ge 0$ $(2x+7)(3x-4) \ge 0$ $M1$ $x \le -\frac{7}{2}$, $x \ge \frac{4}{3}$ $A2$	[3]
2	(i)		The polynomial $f(x)$ is given by $f(x)=x^3-7x-6$.	
		(a)	Show that $x+2$ is a factor of $f(x)$. $f(-2)=(-2)^3-7(-2)-6$ $B1$ $=0$	[1]
		(b)	Express $f(x)$ as a product of three linear factors. Any method : by inspection/long division/comparing coeff. $M1$ $(x+2)(Ax^2+Bx+C)$ $=(x+2)(x-3)(x+1)$ $A2$ (two other correct factors)	[3]
	(ii)		Given that $3x^2-7x-3=Ax(x-2)+B(x-2)+C$ for all values of x, find the value of each of the constants, A, B and C. Choose $x=2$, $C=-5$ $M1$ (Choosing appropriate values of x) Choose $x=0$, $B=-1$ $M1$	[4]

		Choose $x = 1$, $A = 3$ $M1$ $A = 3$, $B = -1$, $C = -5$ $A1$	
3	Given that $3x^2 + kx - 12 = 0$, where $k \neq 0$, show that there is no value of k for which the equation has equal roots. $b^2 - 4ac = k^2 + 144$ $M1$ Since $k^2 \geq 0$, $k^2 + 144 > 0$ $M1$ Since $b^2 - 4ac > 0$, equation will never have real and equal roots. Hence, no value of k. $A1$		[3]
4	Prove that $\frac{\cos ec\theta}{\cos\theta} - \tan\theta = \cot\theta$. $\frac{1}{\sin\theta\cos\theta} - \frac{\sin\theta}{\cos\theta}$ $M1$ $= \frac{1 - \sin^2\theta}{\sin\theta\cos\theta}$ $= \frac{\cos^2\theta}{\sin\theta\cos\theta}$ $M1$ $= \frac{\cos\theta}{\sin\theta}$ $A1$ $= \cot\theta$		[3]
5	Given that $\cos\phi = a + b$ and $\sin\phi = b - a$,		
	(i)	show that $b^2 - a^2 = \frac{1}{2}\sin 2\phi$, $b^2 - a^2$ $= (a+b)(b-a)$ $M1$ $= \cos\phi\sin\phi$ $= \frac{1}{2}(2\cos\phi\sin\phi)$ $A1$ $= \frac{1}{2}\sin 2\phi$	[2]

	(ii)	Find $\cos\phi + \sin\phi$ and $\cos\phi - \sin\phi$. Hence, express $\frac{b}{a}$ in terms of $\tan\phi$. $\cos\phi + \sin\phi = 2b$ $M1$ $\cos\phi - \sin\phi = 2a$ $M1$ $\frac{b}{a} = \frac{\frac{\sin\phi}{\cos\phi} + 1}{1 - \frac{\sin\phi}{\cos\phi}}$ $M1$ (*Divide each term by* $\cos\phi$) $= \frac{\tan\phi + 1}{1 - \tan\phi}$ $A1$	[4]
6	Given that $\cos A = -\frac{3}{5}$ and $\tan A = \frac{4}{3}$, evaluate the following and leave your answer in their simplest form, without the use of the calculators		
	(i)	$\cos 2A$ $2\cos^2 A - 1$ $= 2\left(-\frac{3}{5}\right)^2 - 1$ $M1$ $= -\frac{7}{25}$ $A1$	[2]
	(ii)	$\sin(90° - A)$ $\sin 90\cos A - \cos 90\sin A$ $M1$ $= \cos A$ $= -\frac{3}{5}$ $A1$	[2]
7	(i)	Differentiate $(3x^4 + 4)^6$ with respect to x. $6(3x^4 + 4)^5(12x^3)$ $M1$ $= 72x^3(3x^4 + 4)^5$ $A1$	[2]
	(ii)	Hence, evaluate $\int_0^1 x^3(3x^4 + 4)^5\,dx$.	[3]

		$\int_0^1 x^3(3x^4 + 2)^5\,dx$	

		$\frac{dy}{..} \quad x^2.(6x-1) - (3x^2 - x + 1)(2x)$ $M1$	

$\int_0^1 x^3\left(3x^4+2\right)^5 \mathrm{d}x$

$=\frac{1}{72}\left[\left(3x^4+4\right)^6\right]_0^1$ M1

$=\frac{1}{72}\left(7^6-4^6\right)$ M1

$=\frac{113553}{72}$ A1

8 Solve $2\cos^2 x+\sin x+1=0$ for $0^o \le x \le 360^o$. [5]

$2\left(1-\sin^2 x\right)+\sin x+1=0$ M1

$2-2\sin^2 x+\sin x+1=0$ M1

$(2\sin x-3)(\sin x+1)=0$ M1

$\sin x=\frac{3}{2}$ (NA) A1

$\sin x=-1,\quad x=270°$ A1

9 Differentiate with respect to *x*.

(i) $y=2\left(2x^2+1\right)^2$ [2]

$\frac{dy}{dx}=4\left(2x^2+1\right)(4x)$ M1

$=16x(\left(2x^2+1\right)$ A1

(ii) $y=\frac{3x^2-x+1}{x^2}$ [3]

$\frac{dy}{dx}=\frac{x^2.(6x-1)-\left(3x^2-x+1\right)(2x)}{\left(x^2\right)^2}$ M1

$=\frac{6x^3-x^2-6x^3+2x^2-2x}{x^4}$ M1

$=\frac{x^2-2x}{x^4}$

$=\frac{x-2}{x^3}$ A1

10 The coordinates of three points are $P=(7,-3)$, $Q=(4,-1)$ and $R=(1,7)$. The perpendicular bisector of *PR* cuts the *x*-axis at *S*.

(i) Find the equation of the perpendicular bisector of *PR*. [4]

Midpt = $P=(4,2)$ M1

Gradient of *PR* = $-\frac{5}{3}$

Gradient of perpendicular bisector = $\frac{3}{5}$ M1

$y-2=\frac{3}{5}(x-4)$ M1

$y=\frac{3}{5}x-\frac{2}{5}$ A1

(ii) Find the distance of *PS*. [4]

At $S,\ y=0,\quad x=\frac{2}{3}$ M1

$S=\left(\frac{2}{3},0\right)$ M1

$PS=\sqrt{(-3-0)^2+\left(7-\frac{2}{3}\right)^2}$ M1

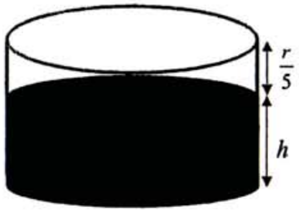
$=\frac{\sqrt{442}}{3}=7.01\,units$ A1

(iii) Find the area of the quadrilateral *PQRS*. [2]

$$\frac{1}{2}\begin{vmatrix} 7 & 1 & \frac{2}{3} & 4 & 7 \\ -3 & 7 & 0 & -1 & -3 \end{vmatrix} \qquad \text{M1}$$

$$= \frac{41}{3}\text{units}^2 \qquad \text{A1}$$

11 The diagram shows an open-topped cylinder of radius *r* cm. When the volume of the water in this cylinder is 80 cm^3, the height of the cylinder is *h* cm and the surface of the water is $\frac{r}{5}$ cm below the top of the cylinder.



(i) Show that the curve surface area, *A* cm^2, of the open-topped cylinder is given by $A = \frac{160}{r} + \frac{2\pi r^2}{5}$. [3]

$$\pi r^2 h = 80$$

$$h = \frac{80}{\pi r^2} \qquad M1$$

$$A = 2\pi r\left(h + \frac{r}{5}\right) \qquad M1$$

$$= 2\pi r\left(\frac{80}{\pi r^2} + \frac{r}{5}\right)$$

$$= \frac{160}{r} + \frac{2\pi r^2}{5} \qquad A1$$

(ii) Given that *r* varies, find the value of *r* for which *A* is stationary and the corresponding value of *h*. [4]

$$\frac{dA}{dr} = -160r^{-2} + \frac{4\pi r}{5} \qquad M1$$

$$For \quad stationary \quad A, \quad \frac{dA}{dr} = 0$$

$$-160r^{-2} + \frac{4\pi r}{5} = 0 \qquad M1$$

$$\frac{-800 + 4\pi r^3}{5r^2} = 0$$

$$r^3 = \frac{800}{4\pi}$$

$$= \sqrt[3]{\frac{800}{4\pi}} \qquad M1$$

$$h = \frac{80}{\sqrt[3]{\frac{800}{4\pi}}} = 1.60 \qquad A1$$

The equation of a circle is $2x^2 + 2y^2 - 16x - 12y = 22$.

12 (i) Express in the circle in the form $(x-a)^2 + (y-b)^2 = r^2$. [2]

$$2x^2 + 2y^2 - 16x - 12y = 22$$

$$(\div 2) \quad x^2 + y^2 - 8x - 6y = 11$$

$$(x-4)^2 + (y-3)^2 - 16 - 9 = 11$$

$$(x-4)^2 + (y-3)^2 = 36 \qquad A1\ (a \text{ and } b),\ A1\ (r)$$

(ii) Hence, find the centre and radius of the circle. [2]

(4, 3) B1

6 B1

(iii) The chord $y = x - 2$ intersects the circle at two points *P* and *Q*, whose *x*-coordinates are $\frac{a+\sqrt{b}}{2}$ and $\frac{a-\sqrt{b}}{2}$ respectively. [5]

Find the values of *a* and *b*.

Justify which points, *P* or *Q* is nearer to the origin.

$y = x - 2$......... (I)

$y = x - 2$......... (1)

$(x-4)^2 + (y-3)^2 = 36$ ....(2)

Solve eqns $M1$

$x^2 - 8x + 16 + x^2 - 10x + 25 = 36$

$2x^2 - 18x + 5 = 0$

$x = \frac{18 \pm \sqrt{284}}{4} = \frac{9 \pm \sqrt{71}}{2}$

$a = 9, b = 71$ $A2$

$P = \left(\frac{9+\sqrt{71}}{2}, \frac{5+\sqrt{71}}{2}\right)$

$Q = \left(\frac{9-\sqrt{71}}{2}, \frac{5-\sqrt{71}}{2}\right)$

$OP = \sqrt{\left(\frac{9+\sqrt{71}}{2}\right)^2 + \left(\frac{5+\sqrt{71}}{2}\right)^2}$ $M1$ *(show workings for the lengths)*

$OQ = \sqrt{\left(\frac{9-\sqrt{71}}{2}\right)^2 + \left(\frac{5-\sqrt{71}}{2}\right)^2}$

Since *OP > OQ, hence Q is nearer to the origin.* $A1$

END OF PAPER

Name:	Index Number:	Class:

HUA YI SECONDARY SCHOOL

4NA Preliminary Examination 2018 4NA

ADDITIONAL MATHEMATICS **4044/02**

Paper 2

7 August 2018

1 h 45 min

Additional materials: Answer Paper

READ THESE INSTRUCTIONS FIRST

Write your Name, Class and Index Number on all the work you hand in.
Write in dark blue or black pen.
You may use a soft pencil for any diagrams or graphs.
Do not use staples, paper clips, highlighters, glue or correction fluid.

Answer **all** questions.

Write your answers on separate Answer Paper provided.
Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.
The use of a scientific calculator is expected, where appropriate.
You are reminded of the need for clear presentation in your answers.

At the end of the examination, fasten all your work securely together.
The number of marks is given in brackets [ ] at the end of each question or part question.

The total number of marks for this paper is 70.

For Examiner's Use
70

This document consists of **5** printed pages including the cover page.

[Turn Over

Mathematical Formulae

1. ALGEBRA

Quadratic Equation

For the equation $ax^2 + bx + c = 0$,

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Binomial Expansion

$$(a+b)^n = a^n + \binom{n}{1}a^{n-1}b + \binom{n}{2}a^{n-2}b^2 + \ldots + \binom{n}{r}a^{n-r}b^r + \ldots + b^n,$$

where n is a positive integer and $\binom{n}{r} = \frac{n!}{r!(n-r)!} = \frac{n(n-1)\ldots(n-r+1)}{r!}$

2. TRIGONOMETRY

Identities

$$\sin^2 A + \cos^2 A = 1$$
$$\sec^2 A = 1 + \tan^2 A$$
$$\operatorname{cosec}^2 A = 1 + \cot^2 A$$

$$\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$$
$$\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$$
$$\tan(A \pm B) = \frac{\tan A \pm \tan B}{1 \mp \tan A \tan B}$$

$$\sin 2A = 2\sin A \cos A$$
$$\cos 2A = 2\cos^2 A - 1 = 1 - 2\sin^2 A = \cos^2 A - \sin^2 A$$
$$\tan 2A = \frac{2\tan A}{1 - \tan^2 A}$$

Formulae for ΔABC

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$
$$a^2 = b^2 + c^2 - 2bc\cos A$$
$$\Delta = \frac{1}{2}bc\sin A$$

1 The roots of the quadratic equation $3x^2 + 7x - 3 = 0$ are α and β.

(i) Show that $\alpha^2 + \beta^2 = \frac{67}{9}$. [3]

(ii) Find the value of $\alpha^3 + \beta^3$.. [2]

(iii) Find the equation whose roots are $\frac{1}{\alpha+1}$ and $\frac{1}{\beta+1}$. [3]

2 **(a)** Solve $x - 2\sqrt{x} = 3$. [3]

(b) Solve $26(5^x) - 5 = 5(25^x)$. [3]

(c) Given that $\frac{(3x)^r}{4}\left(\frac{1}{9x^2}\right)^{6-r} = \frac{A}{x^3}$ for all positive values of x, find the value of each of the constants r and A. [3]

3 The equation of a curve is $y = x^2 - 3x + 2$. Find

(i) $\frac{dy}{dx}$, [1]

(ii) the range of x for which the curve is an increasing function, [2]

(iii) the equation of the normal to the curve at $x = 3$. [4]

4 The gradient of a tangent of a curve C at the point $(3,\ 4)$ is given by $\frac{dy}{dx} = (3x-5)(x-1)$. Find the equation of the curve C. [5]

5 Express $6\cos\theta - 5\sin\theta$ in the form $R\cos(\theta+\alpha)$, where $R>0$ and $0<\alpha<\frac{\pi}{2}$. [2]

(i) Hence, or otherwise find all the angles in the range of $0<x<2\pi$ of the equation $6\cos\theta - 5\sin\theta = 1.6$. [4]

(ii) Find the minimum and maximum value of $(6\cos\theta - 5\sin\theta)^2$. [2]

6 Find, in ascending powers of x, the first three terms in the expansion of

(i) (a) $(1+2x)^6$ [2]

(b) $(2-x)^6$ [2]

(ii) Hence, find the coefficient of x^2 in the expansion $(2+3x-2x^2)^6$. [2]

7 **(i)** Write down the period and amplitude of $y = 2\sin 2x - 3$. [2]

(ii) Sketch the graph of $y = 2\sin 2x - 3$ for $0° \le x \le 360°$. [3]

(iii) Hence, find the number of solutions to $2\sin 2x = 1$ for $0° \le x \le 360°$. [1]

8 Integrate with respect to x

(i) $(2x-3)^8$ [2]

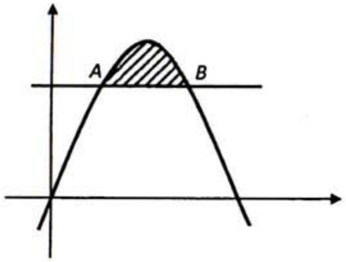
(ii) $\left(\frac{4}{2x+3}\right)^2$ [3]

9 **(a)** The length, W mm, of an elastic spring at time t seconds is given by $W = t^3 - 4t^2 + 16$. Find the value of t when the length is increasing at a rate of 3 mm/s. [3]

(b) The volume of a cube of sides x cm decreases at a rate of 0.5 cm^3/s. Find the rate of change of the side when the side is 4 cm. [3]

10 Prove that $\frac{2\sin(x-y)}{\cos(x-y)-\cos(x+y)} = \cot x - \cot y$. [3]

11 The diagram below shows part of the curve $y = 6x - x^2$ intersecting the line $y = 8$ at A and B.



(i) Find the coordinates of A and B. [3]

(ii) Find the area of the shaded region. [4]

END OF PAPER

Name:	Index Number:	Class:

1 The roots of the quadratic equation $3x^2 + 7x - 3 = 0$ are α and β.

Name:	Index Number:	Class:

HUA YI SECONDARY SCHOOL

4NA Preliminary Examination 2018 **4NA**

ADDITIONAL MATHEMATICS **4044/02**

Paper 2

7 August 2018

1 h 45 min

Additional materials: Answer Paper

MARKING SCHEME

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1 The roots of the quadratic equation $3x^2+7x-3=0$ are α and β.

(i) State that $\alpha^2+\beta^2=\frac{67}{9}$.

$\alpha+\beta=-\frac{7}{3}$ M1

$\alpha\beta=-1$ M1

$\alpha^2+\beta^2=(\alpha+\beta)^2-2\alpha\beta$

$=\left(-\frac{7}{3}\right)^2-2(-1)$ M1

$=\frac{67}{9}$ (*shown*)

(ii) Find the value of $\alpha^3+\beta^3$.

$(\alpha+\beta)(\alpha^2-\alpha\beta+\beta^2)$

$=(\alpha+\beta)\left[(\alpha+\beta)^2-2\alpha\beta-\alpha\beta\right]$ M1

$=\left(-\frac{7}{3}\right)\left(\frac{67}{9}-(-1)\right)$ *from(i)*

$=-\frac{532}{27}$ A1

(iii) Find the equation whose roots are $\frac{1}{\alpha+1}$ and $\frac{1}{\beta+1}$.

$sum=\frac{1}{\alpha+1}+\frac{1}{\beta+1}$

$=\frac{\alpha+\beta+2}{\alpha\beta+\alpha+\beta+1}$

$=\frac{-\frac{7}{3}+2}{-1-\frac{7}{3}+1}$

$=\frac{1}{7}$ M1

$Pdt=\frac{1}{\alpha+1}\cdot\frac{1}{\beta+1}$

$=\frac{1}{(\alpha\beta+\alpha+\beta+1)}$

$=\frac{1}{-\frac{7}{3}}$

$=-\frac{3}{7}$ M1

		Eqn is $x^2-\frac{1}{7}x-\frac{3}{7}=0$ A1
2	(a)	Solve $x-2\sqrt{x}=3$. $x-3=2\sqrt{x}$ $x^2-6x+9=4x$ M1 $x^2-10x+9=0$ $(x-9)(x-1)=0$ $x=9$ *or* $x=1$ *(reject)* A2
	(b)	Solve $26(5^x)-5=5(25^x)$. *Let* $y=5^x$ $26y-5=5(y^2)$ M1 $5y^2-26y+5=0$ $(5y-1)(y-5)=0$ $5y-1=0$ $y=\frac{1}{5}$ $x=-1$ A1 $y-5=0$ $y=5$ $x=1$ A1
	(c)	.Given that $\frac{(3x)^r}{4}\left(\frac{1}{9x^2}\right)^{6-r}=\frac{A}{x^3}$ for all positive values of x, find the value of the constants r and A. $\frac{(3^r x^r)}{4}\left(\frac{1}{9}\right)^{6-r}\left(x^{-12+2r}\right)=\frac{A}{x^3}$ $\frac{(3^r)}{4}\left(\frac{1}{9}\right)^{6-r}\left(x^{-12+2r+r}\right)=\frac{A}{x^3}$ M1 *(group all x)* *Compare power of* $x, -12+3r=-3$ $r=3$ A1 *Sub r to* $\frac{(3^r)}{4}\left(\frac{1}{9}\right)^{6-r}$ $A=\frac{27}{2916}$ A1
3	The equation of a curve is $y=x^2-3x+2$. Find	
	(i)	$\frac{dy}{dx}$,

		$2x-3$ B1
	(ii)	the range of x for which the curve is an increasing function, $2x-3>0$ M1 $2x>3$ $x>1.5$ A1
	(iii)	the equation of the normal to the curve at $x=3$. *Gradient of normal* $=-\frac{1}{2(3)-3}$ M1 $=-\frac{1}{3}$ M1 $x=3, y=2$ $y-2=-\frac{1}{3}(x-2)$ M1 $y=-\frac{1}{3}x+3$ A1
4	The gradient of a tangent of a curve C at the point $(3, 4)$ is given by $\frac{dy}{dx}=(3x-5)(x-1)$. Find the equation of the curve C. $y=\int(3x-5)(x-1)\,dx$ $=\int 3x^2-8x+5\,dx$ M1 $=x^3-4x^2+5x+c$ M1 *Sub* $(3, 4)$ int*o the above* $4=3^3-4.(3)^2+5.(3)+c$ M1 $c=-2$ M1 $y=x^3-4x^2+5x-2$ A1	
5	Express $6\cos\theta-5\sin\theta$ in the form $R\cos(\theta+\alpha)$, where $R>0$ and $0<\alpha<\frac{\pi}{2}$. $\sqrt{6^2+5^2}\cos\left(\theta+\tan^{-1}\left(\frac{5}{6}\right)\right)$ $=\sqrt{61}\cos(\theta+0.69474)$ $=\sqrt{61}\cos(\theta+0.695)$ A2	
	(i)	Hence, or otherwise find all the angles in the range of $0<x<2\pi$ of the equation $6\cos\theta-5\sin\theta=1.6$.

		$\sqrt{61}\cos(\theta+0.69474)=1.6$ $\cos(\theta+0.69474)=\frac{1.6}{\sqrt{61}}$ M1

	(ii)	Sketch the graph of $y=2\sin 2x-3$ for 0° ≤ x ≤ 360°.

M1 – shape

$\sqrt{61}\cos(\theta+0.69474)=1.6$

$\cos(\theta+0.69474)=\frac{1.6}{\sqrt{61}}$ *M1*

$\theta+0.69474=1.36448,\ 2\pi-1.36448$ *M1*

$\theta=0.670,\ 4.22$ *A2*

(ii) Find the minimum and maximum value of $(6\cos\theta-5\sin\theta)^2$.

Min = 0 *B1*

Max = 61 *B1*

6 Find, in ascending powers of *x*, the first three terns in the expansion of

(i) (a) $(1+2x)^6$

$1+6(2x)+60x^2$

$=1+12x+60x^2$ *B2(−1mark for any wrong term)*

(b) $(2-x)^6$

$(2-x)^6$

$2^6+(6)(2)^5(-x)+15(2^4)(-x)^2$

$=64-192x+240x^2$ *B2(−1mark for any wrong term)*

(ii) Hence, find the coefficient of x^2 in the expansion $(2+3x-2x^2)^6$.

$12(-192)+240+60(64)$ *M1*

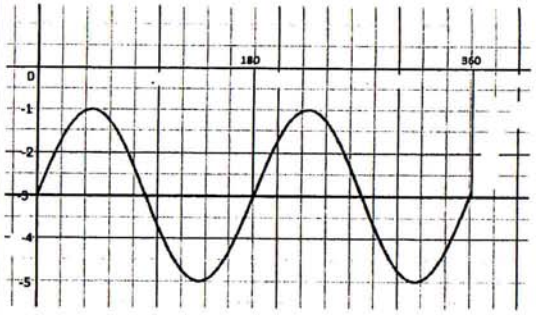
$=1776$ *A1*

7 (i) Write down the period and amplitude of $y=2\sin 2x-3$.

Period = 180^0 *B1*

Amplitude = 2 *B1*

(ii) Sketch the graph of $y=2\sin 2x-3$ for $0°\le x\le 360°$.



M1 – shape

M1 – max/ min pts

M1 – correct cycles

(iii) Hence, find the number of solutions to $2\sin 2x=1$ for $0°\le x\le 360°$.

4 solutions *B1*

8 Integrate with respect to *x*,

(i) $(2x-3)^8$

$\int(2x-3)^8$

$=\frac{(2x-3)^9}{9.(2)}+c$ *M1*

$=\frac{1}{18}(2x-3)^9+c$ *A1*

(ii) $\left(\frac{4}{2x+3}\right)^2$

$\int\left(\frac{4}{2x+3}\right)^2$

$=\int 16(2x+3)^{-2}+c$ *M1*

$=\frac{16(2x+3)^{-1}}{-1(2)}+c$ *M1*

$=-\frac{8}{(2x+3)}+c$ *A1*

9 **(i)** The length, W mm, of an elastic spring at time t seconds is given by .
$W = t^3 - 4t^2 + 16$. Find the value of t when the length is increasing at a rate of 3 mm/s.

$\frac{dW}{dt} = 3t^2 - 8t$ $\quad$ *M*1

$3 = 3t^2 - 8t$

$3t^2 - 8t - 3 = 0$ $\quad$ *M*1

$(3t+1)(t-3) = 0$

$t = 3 \quad t = -3(reject)$ $\quad$ *A*1

(ii) The volume of a cube of sides x cm decreases at a rate of 0.5 cm^3/s. Find the rate of change of the side when the side is 4 cm.

$V = x^3$

$\frac{dv}{dx} = 3x^2$ $\quad$ *M*1

$\frac{dv}{dt} = -0.5$

$\frac{dx}{dt} = \frac{dx}{dv} \times \frac{dv}{dt}$

$= \frac{1}{3(4)^2} \times -0.5$ $\quad$ *M*1

$= -\frac{1}{96}$ $\quad$ *A*1 (−1 *mark if no* − *ve sign*)

10 Prove that $\frac{2\sin(x-y)}{\cos(x-y)-\cos(x+y)} = \cot y - \cot x$.

$LHS = \frac{2\sin(x-y)}{\cos(x-y)-\cos(x+y)}$

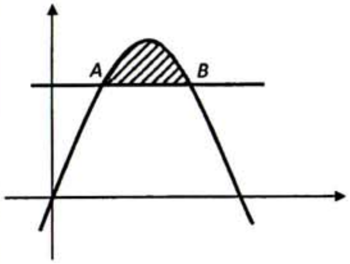
$= \frac{2\sin x\cos y - 2\cos x\sin y}{\cos x\cos y + \sin x\sin y - \cos x\cos y + \sin x\sin y}$ $\quad$ *M*1

$= \frac{2\sin x\cos y - 2\cos x\sin y}{2\sin x\sin y}$ $\quad$ *M*1

$= \frac{\sin x\cos y}{\sin x\sin y} - \frac{\cos x\sin y}{\sin x\sin y}$ $\quad$ *M*1

$= \cot y - \cot x$

11 The diagram below shows part of the curve $y = 6x - x^2$ intersecting the line $y = 8$ at A and B.



(i) Find the coordinates of A and B.

$y = 6x - x^2$

$sub\ y = 8$ $\quad$ *M*1

$8 = 6x - x^2$

$(x-2)(x-4) = 0$

$x = 2,\ x = 4$

$A = (2,8)$ $\quad$ *A*1

$B = (4,8)$ $\quad$ *A*1

(ii) Find the area of the shaded region.

Shaded region

$= \int_2^4 6x - x^2 dx - (8 \times 2)$ $\quad$ *M*1

$= \left[3x^2 - \frac{x^3}{3}\right]_2^4 - 16$ $\quad$ *M*1

$= \left[\frac{80}{3} - \frac{28}{3}\right] - 16$ $\quad$ *M*1

$= \frac{4}{3}\ units^2$ $\quad$ *A*1

END OF PAPER

Calculator Model:

4NA

Candidate Name : ____________________________ (   )   Class : ________

KRANJI SECONDARY SCHOOL
Preliminary Examination
Secondary 4 Normal Academic

ADDITIONAL MATHEMATICS [illegible]
Paper 1

Monday **30 July 2018** [illegible]

KRANJI SECONDARY KRANJI SECONDARY KRANJI SECONDARY KRANJI SECONDARY

READ THESE INSTRUCTIONS FIRST

An answer booklet will be provided with this question paper. You should follow the instructions on the front cover of the answer booklet. If you need additional answer paper ask the invigilator for a continuation booklet.

Answer **all** questions.
Write your answers on the separate answer booklet provided.
Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.
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The number of marks is given in brackets [ ] at the end of each question or part question.
The total number of marks for this paper is 70.

Set by : Ms Cynthia Wong

This question paper consists of **5** printed pages, including the cover page.

[Turn over

Mathematical Formulae

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Quadratic Equation

For the equation $ax^2 + bx + c = 0$,

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Binomial expansion

$$(a+b)^n = a^n + \binom{n}{1}a^{n-1}b + \binom{n}{2}a^{n-2}b^2 + \ldots + \binom{n}{r}a^{n-r}b^r + \ldots + b^n,$$

where n is a positive integer and $\binom{n}{r} = \frac{n!}{r!(n-r)!} = \frac{n(n-1)\ldots(n-r+1)}{r!}$

2. TRIGONOMETRY

Identities

$$\sin^2 A + \cos^2 A = 1$$
$$\sec^2 A = 1 + \tan^2 A$$
$$\operatorname{cosec}^2 A = 1 + \cot^2 A$$
$$\sin(A \pm B) = \sin A\cos B \pm \cos A\sin B$$
$$\cos(A \pm B) = \cos A\cos B \mp \sin A\sin B$$
$$\tan(A \pm B) = \frac{\tan A \pm \tan B}{1 \mp \tan A\tan B}$$
$$\sin 2A = 2\sin A\cos A$$
$$\cos 2A = \cos^2 A - \sin^2 A = 2\cos^2 A - 1 = 1 - 2\sin^2 A$$
$$\tan 2A = \frac{2\tan A}{1 - \tan^2 A}$$

Formulae for ΔABC

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$
$$a^2 = b^2 + c^2 - 2bc\cos A$$
$$\Delta = \frac{1}{2}ab\sin C$$

Answer **all** the questions.

1 When $10x^{40}-7x^{23}+ax^{4}$ is divided by $x+1$, the remainder is -3.
Determine the value of a. [2]

2 Find the first four terms in the expansion of $\left(1-\frac{x}{5}\right)^{8}$.
Hence, find the value of $(0.98)^{8}$, giving your answer correct to 4 decimal places. [4]

3 Given that $y=x+c$ is a tangent to the curve $y^{2}=kx$, where c and k are positive constants, prove that k is a multiple of c. [4]

4 The parabola $y^{2}=kx$, where k is a constant, passes through the point $A(-2,2)$.

(i) Find the value of k. [1]

The points $B(-8,4)$ and $C(-8,-4)$ also lie on the parabola.

(ii) Sketch the graph, marking the points A, B and C on your graph. [2]

(iii) Calculate the area of triangle ABC. [1]

5 It is given that $\int_{1}^{2} ax\,dx=6$, where a is a constant.

(i) Find the value of $\int_{3}^{6} ax\,dx$. [3]

(ii) Express $\int_{1}^{2}(ax+b)\,dx$ in terms of the constant b. [2]

6 Given that $\sqrt{a-b\sqrt{2}}=\frac{4}{1+\sqrt{2}}$ where a and b are integers, find, **without using a calculator**, the value of a and of b. [5]

7 The roots of the equation $2x^{2}-4x+8=0$ are α and β.

(i) Calculate the value of $\alpha^{2}+\beta^{2}$. [3]

(ii) Hence, or otherwise, find the equation whose roots are α^{3} and β^{3}. [3]

8 A circle has centre $C(-1,2)$ and radius 5.

(i) Given that the equation of the circle is $x^{2}+y^{2}+2gx+2fy+c=0$, find the value of each of the constants g, f and c. [3]

(ii) Show that the equation of the tangent to the circle at the point $P(3,5)$ is $4x+3y=27$. [3]

9 A solid rectangular block has a square base of side x cm and a height of y cm. The total surface area of the rectangular block is 90 cm^{2} and the total length of the 12 edges is 48 cm.

(i) Show that $x^{2}-8x+15=0$. [3]

(ii) Find the possible values of x and of y. [3]

10 **(a)** Find the set of values of x for which $4x^{2}-9>0$ and represent the solution set on the number line. [3]

(b) Find the range of values of m for which $x^{2}+mx+3m$ is always greater than 8 for all values of m. [3]

11 **(i)** Prove that $(\sec x - \tan x)(1+\operatorname{cosec}x) = \cot x$. [3]

(ii) Hence solve $(\sec x - \tan x)(1+\operatorname{cosec}x) = 2$ for $0° \le x \le 360°$. [3]

(iii) State the number of solutions of the equation $(\sec x - \tan x)(1+\operatorname{cosec}x) = 2$ in the range of $-360° \le x \le 720°$. [1]

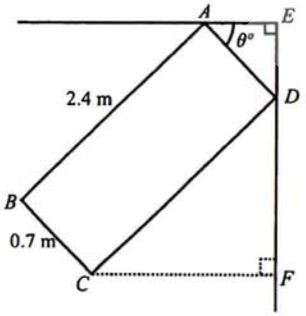
12 A curve has the equation $y = \dfrac{2x-4}{x+3}$ for $x > 0$.

(i) Obtain an expression for $\dfrac{dy}{dx}$ and hence explain why the curve has no turning points. [3]

(ii) Find the equation of the normal to the curve at the point where the curve crosses the x-axis. [4]

13 The diagram shows the plan of a rectangular table, in a corner of an office.

It is given that the table is 2.4 m by 0.7 m, $\angle AED = \angle CFD = 90°$ and that $\angle EAD = \theta°$.



(i) Show that $EF = 0.7\sin\theta + 2.4\cos\theta$ cm. [2]

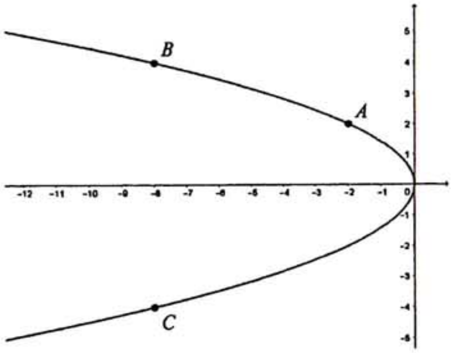
(ii) Express EF in the form $R\sin(\theta+\alpha)$ where $0° < \alpha < 90°$ and R is a positive constant. [3]

(iii) Find the value of θ when $EF = 2$ m. [3]

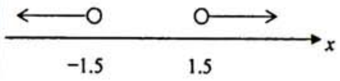
- End of Paper -

4NA AM Prelims Paper 1 (Marking Scheme)

1	Let $f(x)=10x^{40}-7x^{23}+ax^4$. Since $f(x)$ leaves a remainder of -3 when divided by $x+1$, $f(-1)=10(-1)^{40}-7(-1)^{23}+a(-1)^4=-3$ $10-(-7)+a=-3$ $a=-20$
2	$\left(1-\frac{x}{5}\right)^8$ $=1+\binom{8}{1}\left(-\frac{x}{5}\right)+\binom{8}{2}\left(-\frac{x}{5}\right)^2+\binom{8}{3}\left(-\frac{x}{5}\right)^3+...$ $=1-\frac{8}{5}x+\frac{28}{25}x^2-\frac{56}{125}x^3+...$ or $1-1.6x+1.12x^2-0.448x^2+...$ $\left(1-\frac{x}{5}\right)^8=(0.98)^8$ $1-\frac{x}{5}=0.98$, thus sub. $x=0.1$, $(0.98)^8=1-1.6(0.1)+1.12(0.1)^2-0.448(0.1)^3+...$ $=0.8508\,(4\text{ dp})$
3	$y=x+c$ -- (1) ; $y^2=kx$ -- (2) Sub. eq (1) into eq (2): $(x+c)^2=kx$ $x^2+2xc+c^2-kx=0$ $x^2+(2c-k)x+c^2=0$ Since the line is a tangent to the curve, $b^2-4ac=0$ $(2c-k)^2-4(1)(c^2)=0$ $(2c-k)^2=4c^2$ $(2c-k)=\pm 2c$ $2c-k=2c\ (NA)$ or $k=4c$ (thus k is a multiple of c)

4	**(i)**	$(2)^2=k(-2)$ $k=-2$
	(ii)	 **B1**: Parabolic curve ; **B1**: *A*, *B* and *C* correctly labelled on the curve
	(iii)	$\frac{1}{2}(8)(6)=24\text{ units}^2$
5	**(i)**	Method 1 $\int_1^2 ax\,dx=6$ $\left[\frac{ax^2}{2}\right]_1^2=6$ $\frac{a(2)^2}{2}-\frac{a(1)^2}{2}=6$ $\frac{3}{2}a=6$ $a=4$ $\int_3^6 ax\,dx=\left[2x^2\right]_3^6=54$ Method 2 $\int_3^6 ax\,dx=6\times 3^2=54$
	(ii)	$\int_1^2(ax+b)\,dx=\int_1^2 ax\,dx+\int_1^2 b\,dx$ $=6+[bx]_1^2$ $=6+b$

6		$\left(\sqrt{a-b\sqrt{2}}\right)^2=\left(\frac{4}{1+\sqrt{2}}\right)^2$ $a-b\sqrt{2}=\frac{16}{1+2\sqrt{2}+2}$ $=\frac{16}{3+2\sqrt{2}}\times\frac{3-2\sqrt{2}}{3-2\sqrt{2}}$ $=48-32\sqrt{2}$ $a=48$; $b=32$
7	(i)	$2x^2-4x+8=0$ Sum of roots , $\alpha+\beta=2$ Product of roots, $\alpha\beta=4$ $\alpha^2+\beta^2=(\alpha+\beta)^2-2\alpha\beta$ $=(2)^2-2(4)$ $=-4$
	(ii)	New sum of roots: $\alpha^3+\beta^3=(\alpha+\beta)(\alpha^2-\alpha\beta+\beta^2)$ $=(2)(-4-4)$ $=-16$ New product of roots: $\alpha^3\beta^3=4^3=64$ Thus new equation: $x^2+16x+64=0$
8	(i)	Equation of circle: $[x-(-1)]^2+(y-2)^2=25$ $(x+1)^2+(y-2)^2=25$ $x^2+2x+1+y^2-4y+4-25=0$ $x^2+y^2+2x-4y-20=0$ $g=1, f=-2, c=-20$

	(ii)	Gradient of $CP=\frac{5-2}{3-(-1)}=\frac{3}{4}$ Gradient of tangent to circle $=-\frac{4}{3}$ Equation of tangent to circle: $y-5=-\frac{4}{3}(x-3)$ $3y-15=-4x+12$ $4x+3y=27$
9	(i)	$2x^2+4xy=90$ -- (1) $8x+4y=48$ -- (2) Sub $y=12-2x$ into eq (1), $2x^2+4x(12-2x)=90$ $-6x^2+48x-90=0$ $x^2-8x+15=0$
	(ii)	$x^2-8x+15=0$ $(x-5)(x-3)=0$ $x=3$; $y=6$ $x=5$; $y=2$
10	(a)	$4x^2-9>0$ $(2x-3)(2x+3)>0$ $x<-\frac{3}{2}$ *or* $x>\frac{3}{2}$ 
	(b)	$x^2+mx+3m>8$ $x^2+mx+(3m-8)>0$ No real roots, i.e. $b^2-4ac<0$ $(m)^2-4(1)(3m-8)<0$ $m^2-12m+32<0$ $(m-8)(m-4)<0$ $4<m<8$

11	(i)	$(\sec x-\tan x)(1+\operatorname{cosec}x)$ $=\left(\frac{1}{\cos x}-\frac{\sin x}{\cos x}\right)\left(1+\frac{1}{\sin x}\right)$ $=\left(\frac{1-\sin x}{\cos x}\right)\left(\frac{1+\sin x}{\sin x}\right)$ $=\frac{1-\sin^2 x}{\sin x\cos x}$ $=\frac{\cos^2 x}{\sin x\cos x}=\frac{\cos x}{\sin x}=\cot x=RHS$(proven)
	(ii)	$(\sec x-\tan x)(1+\operatorname{cosec}x)=2$ $\cot x=2$ $\tan x=\frac{1}{2}$ basic $\angle$, $\alpha=26.5651°$ $x=26.6°\ ,\ 206.6°$ (1 dp)
	(iii)	1 cycle within 360° → 2 solutions $-360°\le\theta\le720°$ → 3 cycles → 6 solutions
12	(i)	$y=\frac{2x-4}{x+3}$ $\frac{dy}{dx}=\frac{2(x+3)-1(2x-4)}{(x+3)^2}=\frac{10}{(x+3)^2}$ Since $10>0$ and $(x+3)^2>0$, $\frac{dy}{dx}=\frac{10}{(x+3)^2}$
	(ii)	At x-axis, $y=0$: $\frac{2x-4}{x+3}=0$ $x=2$ $\frac{dy}{dx}=\frac{10}{(2+3)^2}=\frac{2}{5}$ Gradient of normal $=-\frac{5}{2}$ Equation of normal: $y-0=-\frac{5}{2}(x-2)$ $y+\frac{5}{2}x=5$

13	(i)	$ED=0.7\sin\theta$ / $DF=2.4\cos\theta$ $EF=0.7\sin\theta+2.4\cos\theta$
	(ii)	$0.7\sin\theta+2.4\cos\theta=R\sin(\theta+\alpha)$ $R=\sqrt{0.7^2+2.4^2}=2.5$ $\tan\alpha=\frac{2.4}{0.7}$ $\alpha=73.7398°$ $0.7\sin\theta+2.4\cos\theta=2.5\sin(\theta+73.7°)$
	(iii)	$EF=2.5\sin(\theta+73.740°)=2$ $\sin(\theta+73.740°)=\frac{2}{2.5}$ basic angle, $\alpha=53.130°$ $\theta+73.740°=53.130°, 180°-53.130°$ $\theta=-20.6°$ (rejected), 53.1° $\theta=53.1°$

73

Calculator Model:

4NA

Candidate Name : ______________________ (   )   Class : ______

KRANJI SECONDARY SCHOOL
Preliminary Examination
Secondary 4 Normal Academic

ADDITIONAL MATHEMATICS [illegible]
Paper 2

Wednesday **01 August 2018** [illegible]

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READ THESE INSTRUCTIONS FIRST

An answer booklet will be provided with this question paper. You should follow the instructions on the front cover of the answer booklet. If you need additional answer paper ask the invigilator for a continuation booklet.

Answer **all** questions.
Write your answers on the separate answer booklet provided.
Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.
The use of an approved scientific calculator is expected, where appropriate.
You are reminded of the need for clear presentation in your answers.

The number of marks is given in brackets [ ] at the end of each question or part question.
The total number of marks for this paper is 70.

Set by :   Ms Cynthia Wong

This question paper consists of **6** printed pages, including the cover page.

[Turn over

Mathematical Formulae

1. ALGEBRA

Quadratic Equation

For the equation $ax^2 + bx + c = 0$,

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Binomial expansion

$$(a+b)^n = a^n + \binom{n}{1}a^{n-1}b + \binom{n}{2}a^{n-2}b^2 + \ldots + \binom{n}{r}a^{n-r}b^r + \ldots + b^n,$$

where n is a positive integer and $\binom{n}{r} = \frac{n!}{r!(n-r)!} = \frac{n(n-1)\ldots(n-r+1)}{r!}$

2. TRIGONOMETRY

Identities

$$\sin^2 A + \cos^2 A = 1$$
$$\sec^2 A = 1 + \tan^2 A$$
$$\operatorname{cosec}^2 A = 1 + \cot^2 A$$
$$\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$$
$$\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$$
$$\tan(A \pm B) = \frac{\tan A \pm \tan B}{1 \mp \tan A \tan B}$$
$$\sin 2A = 2\sin A \cos A$$
$$\cos 2A = \cos^2 A - \sin^2 A = 2\cos^2 A - 1 = 1 - 2\sin^2 A$$
$$\tan 2A = \frac{2\tan A}{1 - \tan^2 A}$$

Formulae for ΔABC

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$
$$a^2 = b^2 + c^2 - 2bc\cos A$$
$$\Delta = \frac{1}{2}ab\sin C$$

Answer **all** the questions.

1 Giving your answer in radians as a multiple of π, state the principal value of

(i) $\cos^{-1}\left(\frac{\sqrt{3}}{2}\right)$, [1]

(ii) $\tan^{-1}(-1)$. [1]

2 The variable x and y are related by the equation $y = 13 - \frac{4}{x}$, where $x \neq 0$. Given that x increases at a rate of 0.25 unites per second, find the rate of increase of y when $y = 5$. [4]

3 **(i)** Prove that $(x-3)(x^2+3x+9) = x^3 - 27$. [1]

(ii) Hence find $\int \frac{x^3-27}{x-3}\,dx$, where $x \neq 3$. [3]

4 **(a)** Given that $2^{2x-1} = 5^{2-x}$., find the exact value of 20^x. [2]

(b) Find the value of a and of b if $\frac{5^n - 5^{n-1}}{2 \times 5^{2n}} = a \times 5^{bn}$. [3]

5 The equation of a curve is given by $y = x^5(x+2)^3$.

(i) Find the gradient of the curve at $x = -1$. [4]

(ii) Given that the tangent to the curve at $x = -1$ crosses the y-axis and passes through the point $(k, -3)$, find the value of k. [3]

6 **(a)** The acute angles A and B are such that $\sin(A-B) = \frac{3}{8}$ and $\cos A \sin B = \frac{1}{4}$.
Find the exact value of

(i) $\sin A \cos B$, [2]

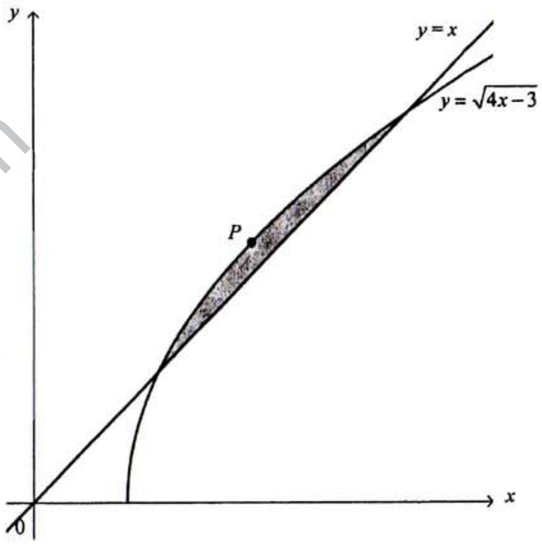
(ii) $\sin(A+B)$. [2]

(b) **Without using a calculator,** show that $\tan 15° = 2 - \sqrt{3}$. [4]

7. **(a)** Given that $x^3 + x^2 - 3 = Ax(x-1)(x+2) + Bx + C$ for all values of x, find the values of A, B and C.
Hence, state the remainder when $x^3 + x^2 - 3$ is divided by $x^2 - x$. [4]

(b) Given that $f(x) = 2x^3 - 13x^2 + 28x - 20$, showing all your workings, solve $f(x) = 0$. [4]

8.

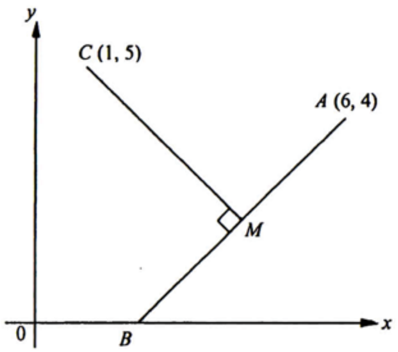


The diagram shows part of the curve $y = \sqrt{4x-3}$ intersecting the line $y = x$.
The point P lies on the curve and the tangent at P is parallel to the line $y = x$.

(i) Find the coordinates of point P. [4]

(ii) Find the area of the shaded region. [6]

9. **Solutions to this question by accurate drawing will not be accepted.**



In the diagram, CM is the perpendicular bisector of AB.
The point A is $(6, 4)$ and the point C is $(1, 5)$
AB is parallel to the line $y - x = 3$ and cuts the x-axis at point B.

(a) Find the coordinates of B. [2]

(b) Find the equation of CM. [3]

(c) It is given that a circle passes through the points A, C and M.

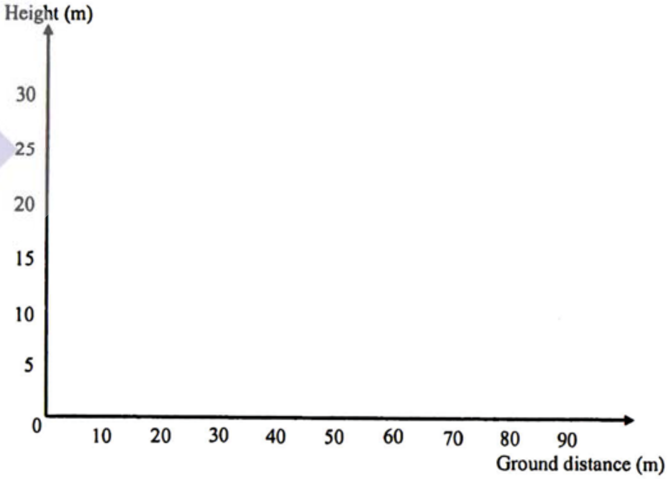
- **(i)** Explain why AC is the diameter of the circle. [1]
- **(ii)** Find the coordinates of the centre of the circle and find the exact radius of the circle. [3]
- **(iii)** State the equation of the circle. [2]

10. **(a)** The function that describes the position of a roller coaster K is given by $h = 5\sin\left(\frac{\pi x}{30}\right) + 20$, for $0 < x \le 100$, where h is the height above the ground in metres and x is the horizontal ground distance from the start point in metre.

- **(i)** Find the maximum height that the roller coaster K can reach and its corresponding ground distance from the start point. [3]
- **(ii)** Find the values of x when roller coaster K is 20 metres above the ground. [3]

(b) A new rollercoaster J is to be designed for the park.

The following graph shows the motion of rollercoaster J.



- **(i)** From the graph that shows the motion of roller coaster J, state its period and amplitude. [2]
- **(ii)** Form a trigonometric function of the form $h = a\sin(bx) + c$, where a, b and c are real numbers, that describes the motion of roller coaster J. [1]

(c) Which rollercoaster do you think provides a more thrilling ride experience? Justify your answer. [2]

- End of Paper -

4NA AM Prelims Paper 2 (Marking Scheme)

1	**(i)**	$\frac{\pi}{6}$
	(ii)	$-\frac{\pi}{4}$
2		When $y=5$, $y=13-\frac{4}{x}$ $5=13-\frac{4}{x}$ $x=\frac{1}{2}$ $\frac{dy}{dx}=\frac{4}{x^2}$ $\frac{dy}{dt}=\frac{dy}{dx}\times\frac{dx}{dt}$ $=\frac{4}{(0.5)^2}\times 0.25$ $=4$ units per second
3	**(i)**	$(x-3)(x^2+3x+9)$ $=x^3+3x^2+9x-3x^2-9x-27$ $=x^3-27$
	(ii)	$\int_{-1}^{2}\frac{(x-3)(x^2+3x+9)}{x-3}\,dx$ $=\int_{-1}^{2}(x^2+3x+9)\,dx$ $=\frac{x^3}{3}+\frac{3}{2}x^2+9x+c$
4	**(a)**	$2^{2x-1}=5^{2-x}$ $\frac{4^x}{2}=\frac{25}{5^x}$ $20^x=50$
	(b)	$\frac{5^n-5^{n-1}}{2\times 5^{2n}}=\frac{5^n\left(1-\frac{1}{5}\right)}{2\times 5^n\times 5^n}$ $=\frac{2}{5}\times 5^{-n}$ $a=\frac{2}{5}$; $b=-1$

5	**(i)**	$y=(x+2)^3x^5$ $\frac{dy}{dx}=3(x+2)^2x^5+(x+2)^3(5x^4)$ At $x=-1$, $\frac{dy}{dx}=3(-1+2)^2(-1)^5+(-1+2)^3\left[5(-1)^4\right]$ $=2$ Gradient of curve at $x=-1$ is 2.
	(ii)	At $x=-1$, $y=(-1+2)^3(-1)^5=-1$ Equation of tangent: $y-(-1)=2[x-(-1)]$ $y=2x+1$ At $y=-3$, $-3=2k+1$ $k=-2$
	(a) (i)	$\sin(A-B)=\sin A\cos B-\cos A\sin B$ $\frac{3}{8}=\sin A\cos B-\frac{1}{4}$ $\sin A\cos B=\frac{5}{8}$
	(a) (ii)	$\sin(A+B)$ $=\sin A\cos B+\cos A\sin B$ $=\frac{5}{8}+\frac{1}{4}$ $=\frac{7}{8}$
	(b)	$\tan 15°=\tan(60°-45°)$ $=\frac{\tan 60°-\tan 45°}{1+\tan 60°\tan 45°}$ $=\frac{\sqrt{3}-1}{1+\sqrt{3}}\times\frac{1-\sqrt{3}}{1-\sqrt{3}}$ $=\frac{\sqrt{3}-3-1+\sqrt{3}}{1-3}$ $=2-\sqrt{3}$ (shown)

7.	(a)	$x^3+x^2-3=Ax(x-1)(x+2)+Bx+C$ Sub. $x=0$: $\quad C=-3$ Sub. $x=1$: $\quad 1+1-3=A(0)+B-3$ $B=2$ Sub. $x=-1$: $(-1)^3+(-1)^2-3=A(-1)(-2)(1)-2-3$ $-3=2A-5$ $A=1$ When divided by x^2-x, Remainder $=Bx+C \quad =2x-3$
	(b)	$f(x)=2x^3-13x^2+28x+7=27$ $\therefore \quad g(x)=2x^3-13x^2+28x-20=0$ When $g(x)$ is divided by $(x-2)$: $g(2)=0$ $\therefore (x-2)$ is a factor of $g(x)$. $\begin{array}{r} 2x^2-9x+10 \\ x-2\overline{)2x^3-13x^2+28x-20} \\ \underline{2x^3-4x^2} \\ -9x^2+28x \\ \underline{-9x^2+18x} \\ 10x-20 \\ \underline{10x-20} \\ 0 \end{array}$ $g(x)=(x-2)(2x^2-9x+10)$ $=(x-2)(x-2)(2x-5)$ $g(x)=0 \rightarrow \underline{x=2 \text{ or } x=2.5}$

8	(i)	$y=(4x-3)^{\frac{1}{2}}$ $\frac{dy}{dx}=\frac{1}{2}(4x-3)^{-\frac{1}{2}}(4)=\frac{2}{\sqrt{4x-3}}$ Since tangent at P is parallel to line $y=x$, $\frac{dy}{dx}=1$ $\frac{2}{\sqrt{4x-3}}=1$ $2=\sqrt{4x-3}$ $4=4x-3$ $x=\frac{7}{4}$ At $x=\frac{7}{4}$, $y=\sqrt{4\left(\frac{7}{4}\right)-3}=2$ Coordinate of point P = $\left(\frac{7}{4},2\right)$
	(ii)	Solving $y=\sqrt{4x-3}$ and $y=x$ $x=\sqrt{4x-3}$ $x^2=4x-3$ $x^2-4x+3=0$ $(x-3)(x-1)=0$ $x=1 \text{ or } x=3$ Area of shaded region $=\int_1^3 \sqrt{4x-3}\,dx-\frac{1}{2}(1+3)(2)$ $=\left[\frac{(4x-3)^{\frac{3}{2}}}{4\left(\frac{3}{2}\right)}\right]_1^3-4$ $=\frac{1}{6}\left[(4x-3)^{\frac{3}{2}}\right]_1^3-4$ $=\frac{1}{6}\left(9^{\frac{3}{2}}-1\right)-4$ $=\frac{13}{3}-4$ $=\frac{1}{3}$ units^2

9	(a)	Gradient. of *AB* = Gradient of line $y-x=3$ = 1 Let the point *B* be $(x_B, 0)$: Gradient of *AB* = $\frac{4-0}{6-x_B}=1$ $4=6-x_B$ $x_B=2$ *B* is (2, 0)
	(b)	Gradient of *CM* = −1 Equation of *CM*: $y-5=-1(x-1)$ $y+x=6$
	(c) (i)	*AC* is the diameter (right angle in a semicircle)
	(ii)	Centre = midpoint of AC $=\left(\frac{1+6}{2},\frac{5+4}{2}\right)=(3.5, 4.5)$ Radius of circle = $\frac{1}{2}\sqrt{(1-6)^2+(5-4)^2}=\underline{\frac{1}{2}\sqrt{26}}$
	(iii)	Equation of circle: $(x-3.5)^2+(y-4.5)^2=\left(\frac{1}{2}\sqrt{26}\right)^2$ $(x-3.5)^2+(y-4.5)^2=6.5$

10	(a)	(i) $h=5\sin\left(\frac{\pi x}{30}\right)+20$ max h = $h=5(+1)+20=25\,m$ occurs when $\sin\left(\frac{\pi x}{30}\right)=1$ $\frac{\pi x}{30}=\frac{\pi}{2}$ $x=15$ m
		(ii) $h=5\sin\left(\frac{\pi x}{40}\right)+20=20$ $\sin\left(\frac{\pi x}{30}\right)=0$ $\frac{\pi x}{30}=0,\pi,2\pi$ $x=0$, 30m, 60m
	(b)	(i) Period = 40 m ; Amplitude = 20 m
		(ii) 40 m − 1 cycle 1 m − $\frac{1}{40}$ cycle 2π − $\frac{2\pi}{40}$ cycle → $b=\frac{2\pi}{40}$ $h=10\sin\left(\frac{2\pi x}{40}\right)+20$
	(c)	**K** : Drop = $25-15=10$ m Vs **J**: $30-10=20$ m **J** is more thrilling as the drop is greater. Or **K**: 1 cycle → 60 m Vs **J**: 40m **J** is more thrilling as it undergoes more cycles per ride.

NORTH VISTA SECONDARY SCHOOL

PRELIMINARY EXAMINATION 2018

70

NAME: ______________________________( ) **CLASS:** _________

SUBJECT: ADDITIONAL MATHEMATICS **DATE: 7 AUGUST 2018**

LEVEL/STREAM: SECONDARY 4 NORMAL (ACADEMIC) **TIME: 1 HOUR 45 MINUTES**

CODE : 4044/01

INSTRUCTIONS TO CANDIDATES

Write your name, register number and class on all the work you hand in.
Write in dark blue or black pen on both sides of the paper.
You may use a HB pencil for any diagrams or graphs.
Do not use staples, paper clips, highlighters, glue or correction fluid.

Answer **all** the questions.
Write your answers on the separate answer paper provided.
Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.
The use of an approved scientific calculator is expected, where appropriate.
You are reminded of the need for clear presentation in your answers.

At the end of the examination, fasten all your work securely together.
The number of marks is given in brackets [ ] at the end of each question or part question.
The total number of marks for this paper is **70**.

This question paper consists of 5 printed pages.

[Turn over

Mathematical Formulae

1. ALGEBRA

Quadratic Equation

For the equation $ax^2+bx+c=0$,

$$x=\frac{-b\pm\sqrt{b^2-4ac}}{2a}$$

Binomial expansion

$$(a+b)^n=a^n+\binom{n}{1}a^{n-1}b+\binom{n}{2}a^{n-2}b^2+\ldots+\binom{n}{r}a^{n-r}b^r+\ldots+b^n,$$

where n is a positive integers and $\binom{n}{r}=\frac{n!}{(n-r)!r!}=\frac{n(n-1)\ldots(n-r+1)}{r!}$

2. TRIGONOMETRY

Identities

$$\sin^2 A+\cos^2 A=1$$

$$\sec^2 A=1+\tan^2 A.$$

$$\operatorname{cosec}^2 A=1+\cot^2 A$$

$$\sin(A\pm B)=\sin A\cos B\pm\cos A\sin B$$

$$\cos(A\pm B)=\cos A\cos B\mp\sin A\sin B$$

$$\tan(A\pm B)=\frac{\tan A\pm\tan B}{1\mp\tan A\tan B}$$

$$\sin 2A=2\sin A\cos A$$

$$\cos 2A=\cos^2 A-\sin^2 A=2\cos^2 A-1=1-2\sin^2 A$$

$$\tan 2A=\frac{2\tan A}{1-\tan^2 A}$$

Formulae for ΔABC

$$\frac{a}{\sin A}=\frac{b}{\sin B}=\frac{c}{\sin C}$$

$$a^2=b^2+c^2-2bc\cos A$$

$$\Delta=\frac{1}{2}ab\sin C$$

1 Express $\frac{5}{3-\sqrt{2}}+\left(4+\sqrt{2}\right)^2$ in the form $a+b\sqrt{2}$, stating the value of a and of b. [4]

2 For the graph of $y=2\sin 3x+4$, where x is in radians, state

(i) the amplitude, [1]

(ii) the period. [1]

3 Solve $\sin 2x+\cos x=0$ for $0° \le x \le 360°$. [3]

4 Find the set of values of x for which $x(5x-2)>2(x^2-6x+4)$. [3]

5 Using the substitution $u=3^x$, solve $3(3^x)-28=-9(3^{-x})$. [5]

6 The equation of a curve is $y=4x^3-9x^2+6x-9$.

(i) Find the x-coordinate of the point(s) at which the tangent to the curve is a horizontal line. [3]

(ii) Using differentiation, determine the nature of the stationary point(s) on the curve. [3]

7 The variables y and t are such that

$$\frac{dy}{dt}=\frac{1}{8}\left(\frac{1}{2}t-3\right)^2.$$

Given that $y=-3\frac{3}{4}$ when $t=0$, find y when $t=2$. [5]

8 **(i)** Given that $y=\frac{x^4}{3+x}$ for $x>0$, find $\frac{dy}{dx}$ and explain why y is an increasing function. [4]

(ii) Given that $y=(3x+2)(4x-3)^3$, find $\frac{dy}{dx}$ and the set of values for which y is decreasing. [4]

9 **(i)** Differentiate $\sqrt{4x^3+2}$ with respect to x. [2]

(ii) Hence find the value of

$$\int_2^3 \frac{3x^2}{\sqrt{4x^3+2}}\,dx.$$ [4]

10 The function f(x) is defined by $f(x)=2x^3+ax^2+bx+6$ for all real x. It is given that $x-2$ is a factor of f(x) and that when f(x) is divided by $x+2$, the remainder is 20.

(i) Find the value of each of the constants a and b. [4]

(ii) Express f(x) as the product of linear factors. [3]

11 Water leaks from a container at a rate of k cm³/s. The volume, V cm³, of the water in the container, when the height of the water is h cm, is given by

$$V=\frac{2\pi h^3}{7}+4\pi.$$

Find an expression for the rate of change of the height of water in the container, in terms of h and k. [4]

12 The line $3x-y=5$ is a tangent to a circle at the point $A(1,-2)$.

(i) Find the equation of the normal to the circle at A. [3]

The equation of another normal to the circle is $y=2x-18$.

(ii) Find the equation of the circle. [5]

13 **(a)** Find, in radians, the two principal values of A for which

$$5\tan^2 A + 9\tan A = 2.$$ [3]

(b) **(i)** Express $4\sin\theta - 3\cos\theta$ in the form $R\sin(\theta - \alpha)$ where $R > 0$ and $0 < \alpha < \frac{\pi}{2}$ radians. [3]

(ii) The height, h centimetres, of a wave of water in a tank being used for a Physics experiment is given by

$$h = 4\sin t - 3\cos t$$

where t is the time in seconds after the start of the experiment. After how many seconds does the wave first reach a height of 1.5 cm? [3]

END OF PAPER

[Turn Over

4NA AM Prelim P1 2018

1	$\frac{5}{3-\sqrt{2}}+\left(4+\sqrt{2}\right)^2$	
	$=\frac{5}{3-\sqrt{2}}\left(\frac{3+\sqrt{2}}{3+\sqrt{2}}\right)+\left(16+8\sqrt{2}+2\right)$	M1(rationalise) M1 (expansion)
	$=\frac{15+5\sqrt{2}}{7}+\left(16+8\sqrt{2}+2\right)$ $=20\frac{1}{7}+8\frac{5}{7}\sqrt{2}$	A1,A1
	$a=20\frac{1}{7}$, $b=8\frac{5}{7}$	
2	(i) 2	B1
	(ii) $\frac{2\pi}{3}$	B1
3	$\sin 2x+\cos x=0$	
	$2\sin x\cos x+\cos x=0$	M1
	$\cos x(2\sin x+1)=0$	M1
	$x=90,210,270,330$	A1
4	$5x^2-2x>2x^2-12x+8$	
	$3x^2+10x-8>0$	M1
	$(3x-2)(x+4)>0$	M1
	$x<-4, x>\frac{2}{3}$	A1
5	$3u-28=\frac{-9}{u}$	M1
	$3u^2-28u+9=0$	
	$(3u-1)(u-9)=0$	M1
	$u=\frac{1}{3}$ or $u=9$	M1
	$x=-1$ or $x=2$	A2
6i	$\frac{dy}{dx}=12x^2-18x+6$	M1
	$12x^2-18x+6=0$	
	$\left(2x-1\right)\left(x-1\right)=0$	M1
	$x=\frac{1}{2}$ or $x=1$	A1
ii	$\frac{d^2y}{dx^2}=24x-18$	M1
	When $x=\frac{1}{2}$, $\frac{d^2y}{dx^2}=-6<0(\text{max})$	A1
	When $x=1$, $\frac{d^2y}{dx^2}=6>0(\text{min})$	A1
7	$y=\int\frac{1}{8}\left(\frac{1}{2}t-3\right)^2 dt$	
	$=\frac{\frac{1}{8}\left(\frac{1}{2}t-3\right)^3}{3(\frac{1}{2})}+c$	M2
	$=\frac{1}{12}\left(\frac{1}{2}t-3\right)^3+c$	
	$c=-1\frac{1}{2}$	M1
	$y=\frac{1}{12}\left(\frac{1}{2}t-3\right)^3-1\frac{1}{2}$	M1
	When t=2, $y=-2\frac{1}{6}$	A1
8i	$\frac{dy}{dx}=\frac{(3+x)(4x^3)-x^4}{(3+x)^2}$	M2
	$=\frac{12x^3+3x^4}{(3+x)^2}$	M1
	Since $x>0, 12x^3+3x^4>0, (3+x)^2>0, \frac{dy}{dx}>0$	
	Therefore increasing function	A1
ii	$\frac{dy}{dx}=(3x+2)[3(4x-3)^2(4)]+(4x-3)^3)(3)$	M2
	$=(4x-3)^2(48x+15)=3(16x+5)(4x-3)^2$	M1
	For decreasing function, $\frac{dy}{dx}<0$ Since $(4x-3)^2>0$ therefore	
	$48x+15<0$ $x<-\frac{5}{16}$	A1

9i	$\frac{dy}{dx}=\frac{1}{2}\left(4x^3+2\right)^{-\frac{1}{2}}(12x^2)$	M1
	$=\frac{6x^2}{\sqrt{4x^3+3}}$	A1
ii	$\frac{1}{2}\int_2^3\frac{6x^2}{\sqrt{4x^3+2}}dx$	M1
	$=\frac{1}{2}[\sqrt{4x^3+2}]_2^3$	M1
	$=\frac{1}{2}[\sqrt{4(3)^3+2}-\sqrt{4(2)^3+2}]$	M1
	$=2.33$	A1
10i	$f(2)=16+4a+2b+6=0$ $4a+2b=-22$	M1
	$f(-2)=16+4a-2b+6=20$ $4a-2b=30$	M1
	$a=1, b=-13$	A2
ii	Long Division	M1
	$f(x)=(x-2)(2x^2+5x-3)$	M1
	$=(x-2)(2x-1)(x+3)$	A1
11	$\frac{dV}{dh}=\frac{6\pi h^2}{7}$	M1
	$\frac{dV}{dt}=\frac{dV}{dh}\times\frac{dh}{dt}$	M1
	$-k=\frac{6\pi h^2}{7}\times\frac{dh}{dt}$	M1
	$\frac{dh}{dt}=\frac{-7k}{6\pi h^2}cm/s$	A1
12i	$m_{\perp}=-\frac{1}{3}$	M1
	$y=-\frac{1}{3}x+c$ $-2=-\frac{1}{3}+c$ $c=-1\frac{2}{3}$	M1
	$y=-\frac{1}{3}x-1\frac{2}{3}$ or $3y+x=-5$	A1

ii	$2x-18=-\frac{1}{3}x-1\frac{2}{3}$	M1
	$x=7$	M1
	Centre (7,-4)	M1
	Radius $=\sqrt{(7-1)^2+(-4-(-2))^2}=\sqrt{40}$	M1
	$(x-7)^2+(y+4)^2=40$	A1
13a	$(5\tan A-1)(\tan A+2)=0$	M1
	$\tan A=\frac{1}{5}$ or $\tan A=-2$	
	$A=-1.11, 0.197$	A2
bi	$R\sin\alpha=3$ $R\cos\alpha=4$	M1
	$\tan\alpha=\frac{3}{4}$ $\alpha=0.6435$	M1
	$R=5$	
	$5\sin(\theta-0.644)$	A1
ii	$\sin(\theta-0.6435)=\frac{1.5}{5}$	M1
	$\alpha=0.3046$	M1
	$\theta-0.6435=0.3046$	
	$\theta=0.948$	A1

NORTH VISTA SECONDARY SCHOOL

PRELIMINARY EXAMINATION 2018

70

NAME: ______________________________( ) **CLASS:** ________

SUBJECT: ADDITIONAL MATHEMATICS **DATE: 13 AUGUST 2018**

LEVEL/STREAM: SECONDARY 4 NORMAL (ACADEMIC) **TIME: 1 HOUR 45 MINUTES**

CODE: 4044/02

INSTRUCTIONS TO CANDIDATES

Write your name, register number and class on all the work you hand in.
Write in dark blue or black pen on both sides of the paper.
You may use a HB pencil for any diagrams or graphs.
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[Turn over

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$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Binomial expansion

$$(a+b)^n = a^n + \binom{n}{1}a^{n-1}b + \binom{n}{2}a^{n-2}b^2 + \ldots + \binom{n}{r}a^{n-r}b^r + \ldots + b^n,$$

where n is a positive integers and $\binom{n}{r} = \frac{n!}{(n-r)!r!} = \frac{n(n-1)\ldots(n-r+1)}{r!}$

2. TRIGONOMETRY

Identities

$$\sin^2 A + \cos^2 A = 1$$

$$\sec^2 A = 1 + \tan^2 A.$$

$$\operatorname{cosec}^2 A = 1 + \cot^2 A$$

$$\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$$

$$\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$$

$$\tan(A \pm B) = \frac{\tan A \pm \tan B}{1 \mp \tan A \tan B}$$

$$\sin 2A = 2\sin A\cos A$$

$$\cos 2A = \cos^2 A - \sin^2 A = 2\cos^2 A - 1 = 1 - 2\sin^2 A$$

$$\tan 2A = \frac{2\tan A}{1 - \tan^2 A}$$

Formulae for ΔABC

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$a^2 = b^2 + c^2 - 2bc\cos A$$

$$\Delta = \frac{1}{2}ab\sin C$$

1 **(i)** Sketch, on the same axes, the graphs of $y^2=-4x$ and $y=\frac{1}{3}x$. [2]

(ii) Find the coordinates of the points of intersection of these graphs. [3]

2 Without using a diagram, determine whether the curve $y=4x^2-9x+2$ and the line $y=2x-5$ have 0, 1 or 2 points of intersection. [4]

3 The roots of the quadratic equation $2x^2-7x-4=0$ are α and β.

(i) Show that $\alpha^2+\beta^2=\frac{65}{4}$. [3]

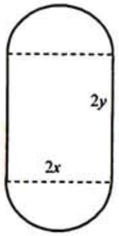
(ii) Hence or otherwise, find the equation whose roots are α^2+1 and β^2+1. [4]

4 The equation of a curve is $y=\frac{4x-3}{\sqrt{2x+5}}$.

(i) Find the value of $\frac{\mathrm{d}y}{\mathrm{d}x}$ at the point on the curve where $x=2$. [4]

(ii) Find the equation of the tangent to the curve at $x=2$. [3]

5



The diagram shows a window which is formed by a rectangle with sides $2x$ metres and $2y$ metres and two semicircles. The perimeter of the window is 15 metres.

(i) Find an expression for y in terms of x. [2]

(ii) Show that the area, A square metres, of the window is given by

$$A=15x-\pi x^2.$$ [2]

Given that x and y vary, find

(iii) the value of x for which A is a maximum, [4]

(iv) the maximum value of A. [1]

[Turn Over

6 The line $2x-5y=10$ meets the x-axis at the point A and the y-axis at the point B.

(i) Write down the coordinates of A and of B. [2]

The perpendicular bisector of AB meets the line $y=3x$ at the point C. Find

(ii) the coordinates of C, [5]

(iii) the area of the quadrilateral $OBAC$, where O is the origin. [3]

7 **(a)** **(i)** Calculate the coefficient of x^3 in the expansion of $(4+3x)^7$. [2]

(ii) Hence, calculate the coefficient of x^{10} in the expansion of $(2+3x^7)^2(4+3x)^7$. [3]

(b) (i) Expand fully $\left(1-\frac{3}{x^3}\right)^3$. [3]

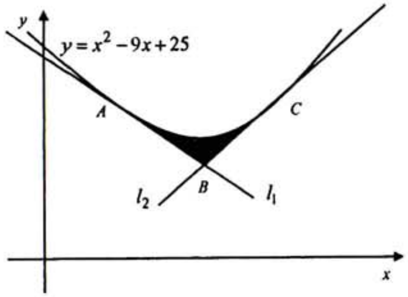
(ii) Hence find $\int\left(1-\frac{3}{x^3}\right)^3\mathrm{d}x$. [3]

8 **(i)** Prove that $\sin 2A-\cos 2A\tan A=\tan A$. [3]

(ii) Solve for values of x between 0 and π radians,

$$\cos(2x+0.5)=0.3.$$ [4]

9



The diagram shows part of the curve $y=x^2-9x+25$ and two lines l_1 and l_2. The lines l_1 and l_2 are tangents to the curve at the points A and C and have gradient -3 and 3 respectively.

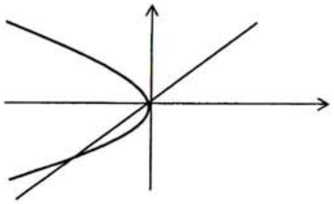
(i) Find the coordinates of each of the points A and C. [4]

The lines l_1 and l_2 intersect at the point B, which has a y-coordinate of $\frac{5}{2}$.

(ii) Find the area of the shaded region ABC. [6]

END OF PAPER

4NA AM Prelim P2 2018

1(i)		B1 B1
(ii)	$\left(\frac{1}{3}x\right)^2=-4x$	
	$x\left(\frac{1}{9}x+4\right)=0$	M1
	$x=0,x=-36$	
	$(0,0)$ and $(-36,-12)$	A2
2	$4x^2-9x+2=2x-5$	M1
	$4x^2-11x+7=0$	
	$b^2-4ac=(-11)^2-4(4)(7)=9$	M1,M1
	Since $b^2-4ac>0$ therefore there are 2 points of intersection	A1
3i	$\alpha+\beta=\frac{7}{2}$	M1
	$\alpha\beta=-2$	M1
	$\alpha^2+\beta^2=\left(\frac{7}{2}\right)^2-2(-2)=16.25$	A1
ii	$\alpha^2+1+\beta^2+1=\frac{65}{4}+2=18\frac{1}{4}$	M1
	$\left(\alpha^2+1\right)\left(\beta^2+1\right)=\alpha^2\beta^2+\alpha^2+\beta^2+1$	
	$=(-2)^2+16.25+1=21.25$	M2
	$x^2-18.25x+21.25=0$	A1
4i	$\frac{dy}{dx}=\frac{\sqrt{2x+5}(4)-(4x-3)[\frac{1}{2}(2x+5)^{-\frac{1}{2}}(2)]}{(2x+5)}$	M2
	$=\frac{4x+23}{(2x+5)^{\frac{3}{2}}}$	M1

	When x=2, $\frac{dy}{dx}=1\frac{4}{27}$	A1
ii	$y=1\frac{4}{27}x+c$	
	When $x=2, y=1\frac{2}{3}$	M1
	$1\frac{2}{3}=1\frac{4}{27}(2)+c$ $c=-\frac{17}{27}$	M1
	$y=1\frac{4}{27}x-\frac{17}{27}$ or $27y=31x-17$	A1
5i	$2y+2y+2\pi x=15$	M1
	$y=\frac{15-2\pi x}{4}$	A1
ii	Area $=4x\left(\frac{15-2\pi x}{4}\right)+\pi x^2$	M1
	$=15x-2\pi x^2+\pi x^2$	M1
	$=15x-\pi x^2$	
iii	$\frac{dA}{dx}=15-2\pi x$	M1
	$15-2\pi x=0$	M1
	$x=\frac{15}{2\pi}$	A1
	$\frac{d^2A}{dx^2}=-2\pi<0$	A1
	Therefore A is a maximum	
iv	$A=17.9cm^2$	B1
6i	A(5,0) B(0,-2)	B2
ii	$m_{AB}=\frac{2}{5}$	
	$m_{\perp}=-\frac{5}{2}$	M1
	$M_{AB}=(2.5,-1)$	M1
	Equation of perpendicular bisector: $y=-\frac{5}{2}x+5\frac{1}{4}$	M1
	$3x=-\frac{5}{2}x+5\frac{1}{4}$	M1
	$x=\frac{21}{22}$	

80

	$C(\frac{21}{22},2\frac{19}{22})$	A1
iii	Area = $\frac{1}{2}\left\vert\begin{matrix}0 & 0 & 5 & \frac{21}{22} & 0\\ 0 & -2 & 0 & 2\frac{19}{22} & 0\end{matrix}\right\vert$	M1
	$\frac{1}{2}(14\frac{7}{22}+10)=12\frac{7}{44}units^2$	M1,A1
7ai	$\binom{7}{3}(4)^4(3)^3=241920$	M1,A1
ii	$\left(4+12x^7+9x^{14}\right)(4+3x)^7$	M1
	Coeff of $x^{10}=241920(12)=2903040$	M1,A1
bi	$1+3(-\frac{3}{x^3})+\binom{3}{2}(-\frac{3}{x^3})^2+\binom{3}{3}(-\frac{3}{x^3})^3$	M2
	$=1-\frac{9}{x^3}+\frac{27}{x^6}-\frac{27}{x^9}$	A1
ii	$\int 1-\frac{9}{x^3}+\frac{27}{x^6}-\frac{27}{x^9}dx$ $=x-\frac{9x^{-2}}{-2}+\frac{27x^{-5}}{-5}-\frac{27x^{-8}}{-8}+c$	M2
	$=x+\frac{9}{2x^2}-\frac{27}{5x^5}+\frac{27}{8x^8}+c$	A1
8i	$\frac{dy}{dx}=2x-9$	M1
	$2x-9=3 \quad 2x-9=-3$ $x=6 \quad x=3$	M1
	$C(6,7) \quad A(3,7)$	A2
ii	Area under curve = $[\frac{x^3}{3}-\frac{9x^2}{2}+25x]_3^{4.5}$	M1
	$=51\frac{3}{4}-43\frac{1}{2}=8\frac{1}{4}$	M1
	Area of trapezium= $\frac{1}{2}(7+2.5)(1.5)=7\frac{1}{8}$	M1,M1
	Shaded region = $2(8.25-7\frac{1}{8})=2.25units^2$	M1,A1
9i	$2\sin A\cos A-(2\cos^2 A-1)\tan A$	B2
	$=2\sin A\cos A-2\cos^2 A\tan A+\tan A$	
	$=2\sin A\cos A-2\cos^2 A\left(\frac{\sin A}{\cos A}\right)+\tan A$	B1
ii	$\cos(2x+0.5)=0.3$	
	$\alpha=1.266$	M1
	$2x+0.5=1.266,2\pi-1.266$	M1
	$x=0.383,2.26$	A2

Name		Index Number		Class	

ST. ANTHONY'S CANOSSIAN SECONDARY SCHOOL
Preliminary Examination 2018
Secondary 4 Normal Academic

Additional Mathematics **4044/01**

Paper 1
13 August 2018

Setter: Ms Joyce Frances Liu
1 hour 45 minutes

Additional Materials: Answer Paper

ST ANTHONY'S CANOSSIAN SECONDARY SCHOOL ST ANTHONY'S CANOSSIAN SECONDARY SCHOOL ST ANTHONY'S CANOSSIAN SECONDARY SCHOOL
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ST ANTHONY'S CANOSSIAN SECONDARY SCHOOL ST ANTHONY'S CANOSSIAN SECONDARY SCHOOL ST ANTHONY'S CANOSSIAN SECONDARY SCHOOL

READ THESE INSTRUCTIONS FIRST

Write your name, index number and class on all the work you hand in.
Write in dark blue or black pen on both sides of the paper.
You may use HB pencil for any diagrams or graphs.
Do not use staples, paper clips, glue or correction fluid.

Answer **all** questions.
Write your answers on the separate Answer Paper provided.
Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.
The use of an approved scientific calculator is expected, where appropriate.
You are reminded of the need for clear presentation in your answers and working.

At the end of the examination, fasten all your work securely together.
The number of marks is given in brackets [ ] at the end of each question or part question.
The total number of marks for this paper is 70.

70

Parent's Signature:

This document consists of **4** printed pages.

[Turn over

Mathematical Formulae

1. ALGEBRA

Quadratic Equation

For the equation $ax^2+bx+c=0$,

$$x=\frac{-b\pm\sqrt{b^2-4ac}}{2a}$$

Binomial Expansion

$$(a+b)^n=a^n+\binom{n}{1}a^{n-1}b+\binom{n}{2}a^{n-2}b^2+\ldots+\binom{n}{r}a^{n-r}b^r+\ldots+b^n,$$

where n is a positive integer and $\binom{n}{r}=\frac{n!}{(n-r)!r!}=\frac{n(n-1)\ldots(n-r+1)}{r!}$

2. TRIGONOMETRY

Identities

$$\sin^2 A+\cos^2 A=1$$

$$\sec^2 A=1+\tan^2 A$$

$$\operatorname{cosec}^2 A=1+\cot^2 A$$

$$\sin(A\pm B)=\sin A\cos B\pm\cos A\sin B$$

$$\cos(A\pm B)=\cos A\cos B\mp\sin A\sin B$$

$$\tan(A\pm B)=\frac{\tan A\pm\tan B}{1\mp\tan A\tan B}$$

$$\sin 2A=2\sin A\cos A$$

$$\cos 2A=\cos^2 A-\sin^2 A=2\cos^2 A-1=1-2\sin^2 A$$

$$\tan 2A=\frac{2\tan A}{1-\tan^2 A}$$

Formulae for ΔABC

$$\frac{a}{\sin A}=\frac{b}{\sin B}=\frac{c}{\sin C}.$$

$$a^2=b^2+c^2-2bc\cos A.$$

$$\Delta=\frac{1}{2}bc\sin A$$

1 Solve $2\cos^2 x + 5\sin x = -1$ for $540° \le x \le 720°$. [3]

2 The volume of a cuboid is $\left(4\sqrt{2} - 3\sqrt{3}\right)$ m^3 and it has a square base with side $\left(\sqrt{3} - \sqrt{2}\right)$ m. Find the height of the cuboid, expressing your answer in the form $\left(a\sqrt{2} + b\sqrt{3}\right)$ m, where a and b are integers. [4]

3 Using the substitution $y = 2^{\frac{x}{2}}$, solve $2^{x+1} - 15\sqrt{2^x} = 8$. [4]

4 Find the set of values of k for which $(k+3)x^2 + 4x + k$ is always negative. [5]

5 A straight line with gradient $\frac{3}{2}p$ cuts the y-axis at (0, –1). Find the set of values of p for which the line intersects the curve $y = 2x^2 + 5x + 1$ at two distinct points. [5]

6 **(i)** Find the coordinates of the point on the curve $y = x^3 - 6x^2 + 12x - 4$ where the tangent to the curve is parallel to the x-axis. [3]

(ii) Determine the nature of the stationary point on the curve. [2]

7 The gradient of a curve is given by $\frac{\mathrm{d}y}{\mathrm{d}x} = \frac{4}{\sqrt{2x+3}}$. The curve passes through the points (3, 8) and (11, m), find the value of m. [6]

8 **(i)** Differentiate $\frac{1}{5x^2 - 4}$ with respect to x. [2]

(ii) Hence evaluate $\int_1^3 \left(x - \frac{10x}{\left(5x^2 - 4\right)^2} \right) \mathrm{d}x$. [4]

9 The function f is defined by $\mathrm{f}(x) = 2x^3 + ax^2 - 5x + b$, where a and b are constants. It is given that $2x - 1$ is a factor of f(x) and when f(x) is divided by $x - 1$, the remainder is –3.

(i) Find the value of a and of b. [4]

(ii) Solve the equation $\mathrm{f}(x) = 0$, giving each root correct to two decimal places where appropriate. [3]

10 When the depth of water in a container is x cm, the volume of water is V cm^3, where $V = \frac{3}{16}\pi x^3$. Water is flowing into the container at a constant rate of 16 cm^3/s.

(i) Find, in terms of π, the volume of water in the container when the depth of water in the container is 2 cm. [1]

(ii) Calculate the rate of increase of the depth of water in the container when $x = 8$. [4]

(iii) Find the depth of water when the rate of increase of water in the container is 0.2 cm/s. [2]

11 The points A(–3, 7) and B(4, 8) lie on a circle C_1 and the line $3x + 4y = 19$ passes through the centre of the circle.

(i) Find the equation of C_1. [7]

(ii) The circle C_1 is reflected about x-axis to give the circle C_2. Find the equation of C_2. [2]

12 **(a)** Sketch the graph of $y = 2\cos 3x - 1$ for $0° \le x \le 180°$. [3]

(b) The displacement, x cm, of a load attached to a spring in a physics experiment is given by $x = \sin t - \sqrt{2}\cos t$, where t is the time in seconds after the load is released.

(i) Express $\sin t - \sqrt{2}\cos t$ in the form $R\sin(t - \alpha)$, where R is positive and α is an acute angle measured in radians. [3]

(ii) Find the time, in seconds, at which the displacement first reach 1.2 cm. [3]

End of Paper

Marking Scheme for 4NA Additional Mathematics Paper 1 (Preliminary Examination 2018)

Qn	Worked Solution	Marks Allocation	Total	Remarks
1.	$2\cos^2 x + 5\sin x = -1$, $540° \le x \le 720°$ $2(1-\sin^2 x) + 5\sin x = -1$ $2\sin^2 x - 5\sin x - 3 = 0$ $(\sin x - 3)(2\sin x + 1) = 0$ $\sin x = -\frac{1}{2}$ or $\sin x = 3$ (rejected) Basic angle $= \sin^{-1}\frac{1}{2}$ $= 30°$ Since $\sin x < 0$, x lies in the 3rd or 4th quadrant. $x = 540° + 30°,\ 720° - 30°$ $= 570°, 690°$	M1 M1 A1	3	Using $\cos^2 x + \sin^2 x = 1$. Basic trigonometric equation.
2	Area of square base $= \left(\sqrt{3}-\sqrt{2}\right)^2$ $= 3 - 2\sqrt{6} + 2$ $= \left(5 - 2\sqrt{6}\right)$ m^2 Height of cuboid $= \frac{4\sqrt{2}-3\sqrt{3}}{5-2\sqrt{6}}$ $= \frac{4\sqrt{2}-3\sqrt{3}}{5-2\sqrt{6}}\left(\frac{5+2\sqrt{6}}{5+2\sqrt{6}}\right)$ $= \frac{20\sqrt{2}+8\sqrt{12}-15\sqrt{3}-6\sqrt{18}}{25-4(6)}$ $= 20\sqrt{2}+8\sqrt{4\times3}-15\sqrt{3}-6\sqrt{9\times2}$ $= 20\sqrt{2}+16\sqrt{3}-15\sqrt{3}-18\sqrt{2}$ $= \left(2\sqrt{2}+\sqrt{3}\right)$ m	M1 √M1 √M1 A1	4	No M1 if use volume to divide by length. Rationalising denominator. M1 even if denominator is length.

3	$2^{x+1} - 15\sqrt{2^x} = 8$ $2\left(2^{\frac{x}{2}}\right)^2 - 15\left(2^{\frac{x}{2}}\right) = 8$ Let $y = 2^{\frac{x}{2}}$, $2y^2 - 15y - 8 = 0$ $(2y+1)(y-8) = 0$ $y = 8$ or $y = -\frac{1}{2}$ $2^{\frac{x}{2}} = 8$ or $2^{\frac{x}{2}} = -\frac{1}{2}$ (rej) $2^{\frac{x}{2}} = 2^3$ $\frac{x}{2} = 3$ $\therefore x = 6$	M1 M1 M1 A1	4	
4	When $y = (k+3)x^2 + 4x + k$, Coefficient of $x^2 = k+3$ For curve to be a maximum curve, $k+3<0$ $\Rightarrow k < -3$ When $(k+3)x^2 + 4x + k = 0$, Discriminant $= 4^2 - 4(k+3)(k)$ $= -4k^2 - 12k + 16$ For equation to have no real roots, $-4k^2 - 12k + 16 < 0$ $k^2 + 3k - 4 > 0$ $(k-1)(k+4) > 0$ $\Rightarrow k < -4$ or $k > 1$ $\therefore k < -4$	A1 M1 √M1 A1 A1	5	Their discriminant < 0.

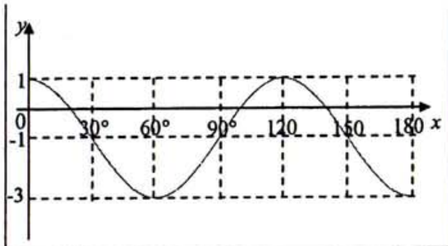
<table>
<tr><td>5</td><td>$y=\frac{3}{2}px-1$ -----(1)
$y=2x^2+5x+1$ -----(2)
Subst. (1) into (2): $\frac{3}{2}px-1=2x^2+5x+1$
$2x^2+\left(5-\frac{3}{2}p\right)x+2=0$
Discriminant $=\left(5-\frac{3}{2}p\right)^2-4(2)(2)$
$=\frac{9}{4}p^2-15p+9$
For line to intersect curve at 2 distinct points,
$\frac{9}{4}p^2-15p+9>0$
$3p^2-20p+12>0$
$(p-6)(3p-2)>0$
$\therefore p<\frac{2}{3}$ or $p>6$</td><td>M1
√M1
√M1
√M1
A1</td><td>5</td><td>Their discriminant > 0.</td></tr>
<tr><td>6(i)</td><td>$\frac{dy}{dx}=3x^2-12x+12$
When $\frac{dy}{dx}=0$, $3x^2-12x+12=0$
$x^2-4x+4=0$
$(x-2)^2=0$
$x=2$
When $x=2$, $y=(2)^3-6(2)^2+12(2)-4$
$=4$
$\therefore$ (2, 4)</td><td>M1
√M1
A1</td><td rowspan="2">5</td><td>Their dy/dx = 0.</td></tr>
<tr><td>6(ii)</td><td><table><tr><td>x</td><td>1.9</td><td>2</td><td>2.1</td></tr><tr><td>Sign of $\frac{dy}{dx}$</td><td>$3(1.9)^2-12(1.9)+12>0$</td><td>0</td><td>$3(2.1)^2-12(2.1)+12>0$</td></tr><tr><td>Sketch of tangent</td><td>╱</td><td>—</td><td>╱</td></tr></table>$\therefore$ (2, 4) is a stationary point of inflexion.</td><td>√M1
A1</td><td>1st derivative test using their dy/dx and their point found in (i).</td></tr>
<tr><td></td><td></td><td></td><td></td><td></td></tr>
</table>

7.	$y=\int\frac{4}{\sqrt{2x+3}}\,dx$ $=\int 4(2x+3)^{-\frac{1}{2}}\,dx$ $=4\left[\frac{(2x+3)^{\frac{1}{2}}}{2\left(\frac{1}{2}\right)}\right]+c$ $=4\sqrt{2x+3}+c$ Since (3, 8) lies on the curve, $8=4\sqrt{2(3)+3}+c$ $c=-4$ $\Rightarrow$ Eq. of curve is $y=4\sqrt{2x+3}-4$. Since (11, m) lies on the curve, $\therefore m=4\sqrt{2(11)+3}-4$ $=16$	M1 M1 √M1 A1 √M1 A1	6	Knowing that y is the integration of dy/dx. Subst. (3, 8) into their equation of curve to find c. Subst. (11, m) into their equation of curve.
8(i)	$\frac{d}{dx}\left(\frac{1}{5x^2-4}\right)=\frac{d}{dx}(5x^2-4)^{-1}$ $=-(5x^2-4)^{-2}(10x)$ $=-\frac{10x}{(5x^2-4)^2}$	M1 A1	6	
8(ii)	$\int_1^3-\frac{10x}{(5x^2-4)^2}\,dx=\left[\frac{1}{5x^2-4}\right]_1^3$ $=\left[\frac{1}{5(3)^2-4}\right]-\left[\frac{1}{5(1)^2-4}\right]$ $=-\frac{40}{41}$ $\int_1^3\left(x-\frac{10x}{(5x^2-4)^2}\right)dx$ $=\int_1^3 x\,dx+\int_1^3-\frac{10x}{(5x^2-4)^2}\,dx$ $=\left[\frac{x^2}{2}\right]_1^3+\left(-\frac{40}{41}\right)$ $=\frac{(3)^2}{2}-\frac{(1)^2}{2}+\left(-\frac{40}{41}\right)$ $=3\frac{1}{41}$	√M1 √M1 M1 A1		Separating into two terms using addition rule. Correct integration of x.

9(i)	Since $f\left(\frac{1}{2}\right)=0$, $2\left(\frac{1}{2}\right)^3+a\left(\frac{1}{2}\right)^2-5\left(\frac{1}{2}\right)+b=0$ $\frac{1}{4}a+b-\frac{9}{4}=0$ $a=9-4b$ -----(1) Since $f(1)=-3$, $2(1)^3+a(1)^2-5(1)+b=-3$ $a+b=0$ -----(2) Subst. (1) into (2): $(9-4b)+b=0$ $-3b=-9$ $b=3$ Subst. $b=3$ into (1): $a=9-4(3)$ $=-3$ $\therefore a=-3,\ b=3$	M1 M1 A1 A1	7	
9(ii)	$f(x)=2x^3-3x^2-5x+3$ $\overline{x^2-x-3}$ $2x-1\,\overline{)\,2x^3-3x^2-5x+3}$ $\underline{-(2x^3-x^2)}$ $-2x^2-5x+3$ $\underline{-(-2x^2+x)}$ $-6x+3$ $\underline{-(-6x+3)}$ 0 $f(x)=(2x-1)(x^2-x-3)$ Since $f(x)=0$, $(2x-1)(x^2-x-3)=0$ $2x-1=0$ or $x^2-x-3=0$ $x=0.5$ or $x=\frac{-(-1)\pm\sqrt{(-1)^2-4(1)(-3)}}{2(1)}$ $=2.30$ or -1.30 (to 2 d.p.) $\therefore x=0.5$ or $x=2.30$ or $x=-1.30$	√M1 √M1 A1		Long division using their value of *a* and *b*.

10(i)	When $x=2$, $V=\frac{3}{16}\pi(2)^3$ $=\frac{3}{2}\pi$ $\therefore$ Volume of water $=\frac{3}{2}\pi$ cm^3	A1	7	
10(ii)	$\frac{dV}{dx}=\frac{3}{16}\pi(3x^2)$ $=\frac{9}{16}\pi x^2$ $\frac{dV}{dt}=16$ $\frac{dV}{dt}=\frac{dV}{dx}\times\frac{dx}{dt}$ When $x=8$, $16=\frac{9}{16}\pi(8)^2\times\frac{dx}{dt}$ $\frac{dx}{dt}=\frac{16}{36\pi}$ $=0.14147\ldots$ $\therefore$ Rate of increase = 0.141 cm/s (to 3 s.f.)	M1 √M1, √M1 A1		M1 for substituting $x=8$ into their d*V*/d*x*. M1 for chain rule with d*V*/d*t* = 16 substituted in.
10(iii)	When $\frac{dx}{dt}=0.2$, $16=\frac{9}{16}\pi x^2\times0.2$ $16=\frac{9}{80}\pi x^2$ $x^2=16+\frac{9}{80}\pi$ $=45.270739\ldots$ $x=6.72835\ldots$ $\therefore$ Depth of water = 6.73 cm (to 3 s.f.)	√M1 A1		M1 for substituting d*V*/d*t* = 16, d*x*/d*t* = 0.2 and their d*V*/d*x* into the chain rule.

11(i)	Gradient of $AB = \frac{7-8}{-3-4}$ $= \frac{1}{7}$ Gradient of perpendicular bisector of AB $= -1 \div \frac{1}{7}$ $= -7$ Midpoint of $AB = \left(\frac{-3+4}{2}, \frac{7+8}{2}\right)$ $= \left(\frac{1}{2}, 7\frac{1}{2}\right)$ $y - 7\frac{1}{2} = -7\left(x - \frac{1}{2}\right)$ $\Rightarrow$ Eq. of perpendicular bisector of AB is $y = -7x + 11$.-----(1) $3x + 4y = 19$ -----(2) Subst. (1) into (2): $3x + 4(-7x + 11) = 19$ $3x - 28x + 44 = 19$ $-25x = -25$ $x = 1$ Subst. $x = 1$ into (1): $y = -7(1) + 11$ $= 4$ $\Rightarrow$ Centre of circle is (1, 4). Radius $= \sqrt{(4-1)^2 + (8-4)^2}$ $= 5$ units $(x-1)^2 + (y-4)^2 = 5^2$ $\therefore (x-1)^2 + (y-4)^2 = 25$	√M1 M1 A1 √M1 A1 A1 A1	9	-1 divide by their gradient of *AB*. Attempt to solve simultaneous equations of their equation of perpendicular bisector of *AB* and the line $3x + 4y = 19$.
11(ii)	Centre of $C_2 = (1, -4)$ $(x-1)^2 + (y-(-4))^2 = 5^2$ $\therefore (x-1)^2 + (y+4)^2 = 25$	√M1 A1		Based on their centre of C_1.

12(a)		B3	9	B1 – Correct shape (period) B1 – Correct intervals at *x*-axis + Label. B1 – Correct intervals at *y*-axis.
12(b)(i)	$R = \sqrt{1^2 + \left(\sqrt{2}\right)^2}$ $= \sqrt{3}$ $\alpha = \tan^{-1}\left(\frac{\sqrt{2}}{1}\right)$ $= 0.955316\ldots$rad $\therefore \sin t - \sqrt{2}\cos t = \sqrt{3}\sin(t - 0.955)$ (to 3 s.f.)	M1 M1 A1		
12(b)(ii)	When $x = 1.2$, $\sqrt{3}\sin(t - 0.955316\ldots) = 1.2$ $\sin(t - 0.955316\ldots) = 0.69282\ldots$ Basic angle $= 0.76539\ldots$ rad Since $\sin(t - 0.955316\ldots) > 0$, $t - 0.955316\ldots$ lies in the 1st or 2nd quadrant. Since $t \geq 0$, $t - 0.955316\ldots \geq -0.955316\ldots$ $t - 0.955316\ldots = 0.76539\ldots$ $t = 1.720706\ldots$ Required time = 1.72 seconds (to 3 s.f.)	√M1 √M1 A1		Equating their *x* to 1.2. Basic trigo equation.

Name		Index Number		Class	

ST. ANTHONY'S CANOSSIAN SECONDARY SCHOOL
Preliminary Examination 2018
Secondary 4 Normal Academic

Additional Mathematics **4044/02**

Paper 2
20 August 2018

Setter: Ms Joyce Frances Liu
1 hour 45 minutes

Additional Materials: Answer Paper

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where n is a positive integer and $\binom{n}{r}=\frac{n!}{(n-r)!r!}=\frac{n(n-1)\ldots(n-r+1)}{r!}$

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$$\cos 2A=\cos^2 A-\sin^2 A=2\cos^2 A-1=1-2\sin^2 A$$

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Formulae for ΔABC

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$$a^2=b^2+c^2-2bc\cos A.$$

$$\Delta=\frac{1}{2}bc\sin A$$

1 **(i)** Sketch, on the same axes, the graphs of $y = 2x^3$ and $y^2 = -4x$. [2]

(ii) Find the coordinates of the points of intersection of these graphs. [3]

2 The rate, in milligram per second, at which a chemical substance reacts with another substance is given as $t^2 - mt + 2m$, where t is the time in seconds after the start of the experiment and m is a constant greater than 4. Show that it is possible for the chemical reaction to take place at a rate of 5 milligrams per second for all real values of m. [5]

3 The equation of a curve is $y = \frac{3x+2}{\sqrt{x+1}}$.

(i) Find the value of $\frac{dy}{dx}$ at the point on the curve where $x = 3$. [4]

(ii) Find the equation of the normal to the curve at $x = 3$. [3]

4 The seventh term in the expansion of $\left(x^2 - \frac{1}{2x^3}\right)^n$ is independent of x.

(i) Show that $n = 15$. [4]

(ii) Hence, find the coefficient of x^{25} in the expansion of $\left(x^2 - \frac{1}{2x^3}\right)^n$. [3]

5 The roots of the quadratic equation $x^2 - 6x - 8 = 0$ are $2\alpha + 1$ and $2\beta + 1$.

(i) Find the value of $\alpha + \beta$ and $\alpha\beta$. [4]

(ii) Hence, or otherwise, find the quadratic equation, with integer coefficients, whose roots are α^2 and β^2. [4]

6 **Solutions to this question by accurate drawing will not be accepted.**

ABCD is a rhombus in which the coordinates of the points *A*, *B* and *C* are (0, –3), (–1, 2) and (4, 1) respectively.

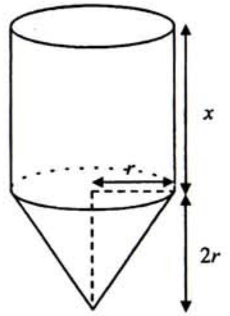
(i) Find the coordinates of the point *D*. [3]

(ii) Calculate the area of the rhombus *ABCD*. [2]

The line *AB* meets the *x*-axis at *M*.

(iii) Calculate the area of triangle *OAM*, where *O* is the origin. [4]

7



The diagram shows a solid which is formed by a circular cylinder of height x cm and radius r cm fixed to a cone of height $2r$ cm and radius r cm. The volume of the solid is 60 cm^3.

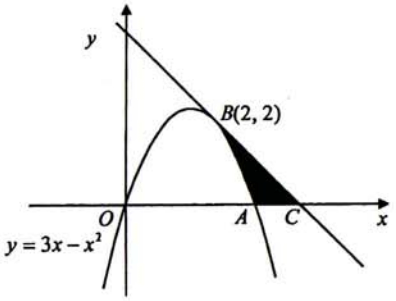
(i) Express x in terms of π and r. [2]

(ii) Show that the total surface area, A cm^2, of the solid is given by

$$A = \frac{120}{r} + \left(\sqrt{5} - \frac{1}{3}\right)\pi r^2.$$ [3]

(iii) Given that x and r can vary, find the value of r which gives the minimum value of A. [4]

8



The diagram shows part of the curve $y = 3x - x^2$ which crosses the x-axis at the origin O and a point A.

(i) Find the coordinates of A. [2]

The tangent to the curve at $B(2, 2)$ cuts the x-axis at C.

(ii) Find the equation of the tangent to the curve at B. [3]

(iii) Find the area of the shaded region ABC. [5]

9 **(a)** Prove that $\tan A\left(2\cos\frac{A}{2} - \sec\frac{A}{2}\right) = 2\sin\frac{A}{2}$. [5]

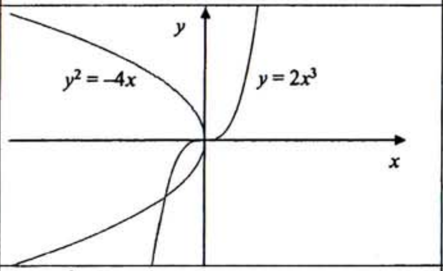
(b) P and Q are angles in the same quadrant such that $\sin P = -\frac{4}{5}$ and $\tan Q = \frac{12}{5}$. Without using calculator,

(i) find the value of $\tan(P+Q)$, [2]

(ii) show that $\cos\frac{Q}{2}$ can be written in the form $\frac{a\sqrt{13}}{13}$, where a is an integer. [3]

End of Paper

Marking Scheme for 4NA Additional Mathematics Paper 2 (Preliminary Examination 2018)

Qn	Worked Solution	Marks Allocation	Total	Remarks
1(i)		B2	5	B1 for $y^2 = -4x$. B1 for $y = 2x^3$.
1(ii)	$y=2x^3$ -----(1) $y^2=-4x$ -----(2) Subst. (1) into (2): $(2x^3)^2=-4x$ $4x^6=-4x$ $4x^6+4x=0$ $4x(x^5+1)=0$ $4x=0$ or $x^5+1=0$ $x=0$ or $x^5=-1$ $x=0$ or $x=-1$ Subst. $x=0$ into (1): $y=2(0)^3$ $=0$ Subst. $x=-1$ into (1): $y=2(-1)^3$ $=-2$ $\therefore$ (0, 0) and (–1, –2)	M1 A1, A1		
2	When rate = 5 milligram per second, $t^2-mt+2m=5$ $t^2-mt+2m-5=0$ Discriminant $=(-m)^2-4(1)(2m-5)$ $=m^2-8m+20$ $=(m-4)^2+20-(-4)^2$ $=(m-4)^2+4$ Since $m>4$, $(m-4)^2>0$ $(m-4)^2+4>4$ $(m-4)^2+4>0$ $\therefore$ Since discriminant > 0, it is possible for the chemical reaction to take place at a rate of 5 milligrams per second for all real values of m.	M1 √M1 M1 M1 A1	5	Finding discriminant from their equation.

3(i)	$y=\dfrac{3x+2}{\sqrt{x+1}}$ $\dfrac{dy}{dx}=\dfrac{[3](x+1)^{\frac{1}{2}}-(3x+2)\left[\frac{1}{2}(x+1)^{-\frac{1}{2}}(1)\right]}{\left(\sqrt{x+1}\right)^2}$ $=\dfrac{3(x+1)^{\frac{1}{2}}-\frac{1}{2}(3x+2)(x+1)^{-\frac{1}{2}}}{x+1}$ $=\dfrac{\frac{1}{2}(x+1)^{-\frac{1}{2}}\left[6(x+1)-(3x+2)\right]}{x+1}$ $=\dfrac{6x+6-3x-2}{2(x+1)^{\frac{3}{2}}}$ $=\dfrac{3x+4}{2\sqrt{(x+1)^3}}$ When $x=3$, $\dfrac{dy}{dx}=\dfrac{3(3)+4}{2\sqrt{((3)+1)^3}}$ $=\dfrac{13}{16}$	M1 M1 A1 A1	7	Factorise out $(x+1)^{0.5}$.
3(ii)	Gradient of normal $=-1\div\dfrac{13}{16}$ $=-\dfrac{16}{13}$ When $x=3$, $y=\dfrac{3(3)+2}{\sqrt{(3)+1}}$ $=5\dfrac{1}{2}$ $\Rightarrow\left(3,\ 5\dfrac{1}{2}\right)$ $y-5\dfrac{1}{2}=-\dfrac{16}{13}(x-3)$ $\therefore y=-\dfrac{16}{13}x+9\dfrac{5}{26}$	√M1 M1 A1		Finding gradient of normal from their dy/dx.

4(i)	$T_{r+1}=\binom{n}{r}(x^2)^{n-r}\left(-\frac{1}{2x^3}\right)^r$ $=\binom{n}{r}(x^2)^{n-r}\left(-\frac{1}{2}\right)^r(x^{-3})^r$ Power of $x=2(n-r)+(-3r)$ $=2n-5r$ At T_7, $r=6$ and Power of $x=0$, $2n-5(6)=0$ $2n=30$ $\therefore n=15$ (Shown)	M1 √M1 √M1 A1	7	Power of x from their general term. Subst. $r=6$ and equating their power of x to 0.
4(ii)	$T_{r+1}=\binom{15}{r}(x^2)^{15-r}\left(-\frac{1}{2}\right)^r(x^{-3})^r$ Power of $x=2(15-r)+(-3r)$ $=30-5r$ For term in x^{25}, Power of $x=25$, $25=30-5r$ $5r=5$ $r=1$ $\therefore$ Coefficient of $x^{25}=\binom{15}{1}\left(-\frac{1}{2}\right)^1$ $=-7\frac{1}{2}$	√M1 √M1 A1		Equating their power of x to 25. Subst. their r to find the coefficient.

5(i)	$(2\alpha+1)+(2\beta+1)=-\frac{(-6)}{1}$ $2\alpha+2\beta+2=6$ $2(\alpha+\beta)=4$ $\therefore \alpha+\beta=2$ $(2\alpha+1)(2\beta+1)=\frac{-8}{1}$ $4\alpha\beta+2\alpha+2\beta+1=-8$ $4(\alpha\beta)+2(\alpha+\beta)=-9$ $4(\alpha\beta)+2(2)=-9$ $4(\alpha\beta)=-13$ $\therefore \alpha\beta=-3\frac{1}{4}$	M1 A1 M1 A1	8	
5(ii)	Sum of new roots $=\alpha^2+\beta^2$ $=(\alpha+\beta)^2-2\alpha\beta$ $=(2)^2-2\left(-3\frac{1}{4}\right)$ $=\frac{21}{2}$ Product of new roots $=(\alpha^2)(\beta^2)$ $=(\alpha\beta)^2$ $=\left(-3\frac{1}{4}\right)^2$ $=\frac{169}{16}$ $x^2-\frac{21}{2}x+\frac{169}{16}=0$ $\therefore 16x^2-168x+169=0$	√M1 √M1 √M1 A1		Using their α + β and αβ. Using their αβ. Using their sum and product of new roots.

6(i)	Midpoint of $AC=\left(\frac{0+4}{2},\frac{-3+1}{2}\right)$ $=(2,-1)$ Let the coordinates of D be (x_1, y_1). $\left(\frac{x_1+(-1)}{2},\frac{y_1+2}{2}\right)=(2,-1)$ $\Rightarrow \frac{x_1-1}{2}=2$ $x_1=5$ $\Rightarrow \frac{y_1+2}{2}=-1$ $y_1=-4$ $\therefore D(5,-4)$	M1 √M1 A1	9	Equating their midpoint of AC to the midpoint of BD.
6(ii)	Area of $ABCD$ $=\frac{1}{2}\begin{vmatrix}0 & 5 & 4 & -1 & 0\\ -3 & -4 & 1 & 2 & -3\end{vmatrix}$ $=\frac{1}{2}\{[0+5+8+3]-[(-15)+(-16)+(-1)+0]\}$ $=24$ units^2	√M1 A1		Using their coordinates of D.
6(iii)	Gradient of $AB=\frac{2-(-3)}{-1-0}$ $=-5$ $y-(-3)=-5(x-0)$ $\Rightarrow$ Eq. of AB is $y=-5x-3$. At x-axis, $y=0$, $0=-5x-3$ $5x=-3$ $x=-\frac{3}{5}$ $\Rightarrow M\left(-\frac{3}{5},0\right)$ $\therefore$ Area of $\Delta OAM=\frac{1}{2}\times\frac{3}{5}\times 3$ $=\frac{9}{10}$ units^2	M1 A1 A1 A1		
7(i)	$\frac{1}{3}\pi r^2(2r)+\pi r^2(x)=60$ $\frac{2}{3}\pi r^3+\pi r^2(x)=60$ $\pi r^2(x)=60-\frac{2}{3}\pi r^3$ $\therefore x=\frac{60}{\pi r^2}-\frac{2}{3}r$ -----(1)	M1 A1	9	

7(ii)	Slant height of cone $=\sqrt{r^2+(2r)^2}$ $=\sqrt{5}r$ cm $A=\pi r\left(\sqrt{5}r\right)+2\pi r(x)+\pi r^2$ $=\left(\sqrt{5}+1\right)\pi r^2+2\pi rx$ -----(2) Subst. (1) into (2): $A=\left(\sqrt{5}+1\right)\pi r^2+2\pi r\left(\frac{60}{\pi r^2}-\frac{2}{3}r\right)$ $=\left(\sqrt{5}+1\right)\pi r^2+\frac{120}{r}-\frac{4}{3}\pi r^2$ $=\left(\sqrt{5}+1-\frac{4}{3}\right)\pi r^2+\frac{120}{r}$ $=\frac{120}{r}+\left(\sqrt{5}-\frac{1}{3}\right)\pi r^2$ (Shown)	M1 M1 A1		
7(iii)	$A=120r^{-1}+\left(\sqrt{5}-\frac{1}{3}\right)\pi r^2$ $\frac{dA}{dr}=120\left(-r^{-2}\right)+\left(\sqrt{5}-\frac{1}{3}\right)\pi(2r)$ $=-\frac{120}{r^2}+2\left(\sqrt{5}-\frac{1}{3}\right)\pi r$ When $\frac{dA}{dr}=0$, $-\frac{120}{r^2}+2\left(\sqrt{5}-\frac{1}{3}\right)\pi r=0$ $2\left(\sqrt{5}-\frac{1}{3}\right)\pi r=\frac{120}{r^2}$ $r^3=120\div\left[2\left(\sqrt{5}-\frac{1}{3}\right)\pi\right]$ $=10.03744\ldots$ $r=2.15712\ldots$ $\left.\begin{array}{l}\frac{d^2A}{dr^2}=-120\left(-2r^{-3}\right)+2\left(\sqrt{5}-\frac{1}{3}\right)\pi\\ =\frac{240}{r^3}+2\left(\sqrt{5}-\frac{1}{3}\right)\pi\\ \text{When } r=2.15712\ldots,\\ \frac{d^2A}{dr^2}=\frac{240}{(2.15712..)^3}+2\left(\sqrt{5}-\frac{1}{3}\right)\pi\\ =35.8657\ldots>0\\ \Rightarrow A \text{ is a minimum when } r=2.15712\ldots\end{array}\right\}$ $\therefore$ Required value of $r=2.16$ (to 3 s.f.)	M1 √M1 M1 A1		Equating their dA/dr to 0.

Calculator Model:		

Name	Index Number	Class

WOODGROVE SECONDARY SCHOOL
N LEVEL PRELIMINARY EXAMINATION 2018

WOODGROVE SECONDARY

LEVEL & STREAM : SECONDARY 4 NORMAL (ACADEMIC)
SUBJECT (CODE) : ADDITIONAL MATHEMATICS (4044)
PAPER NO : 02
DATE / DAY : 13 AUG 2018 / MONDAY
DURATION : 1 HOUR 45 MINUTES

READ THESE INSTRUCTIONS FIRST

Write your class, index number and name on the work you hand in.
Write in dark blue or black pen on both sides of the paper.
You may use a pencil for any diagrams or graphs.
Do not use staples, paper clips, glue or correction fluid.

Answer **all** the questions.
Write your answers on the separate writing paper provided.
Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.
The use of an approved scientific calculator is expected, where appropriate.
You are reminded of the need for clear presentation in your answers.

At the end of the examination, fasten all your work securely together.
The number of marks in given in brackets [ ] at the end of each question or part question.
The total number of marks for this paper is 70.

DO NOT TURN OVER THE QUESTION PAPER UNTIL YOU ARE TOLD TO DO SO.

This document consists of 5 printed pages including this cover page.
Setter : Ms Nicole Ng

Mathematical Formulae

1. ALGEBRA

Quadratic Equation

For the equation $ax^2 + bx + c = 0$,

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Binomial Expansion

$$(a+b)^n = a^n + \binom{n}{1}a^{n-1}b + \binom{n}{2}a^{n-2}b^2 + \ldots + \binom{n}{r}a^{n-r}b^r + \ldots + b^n,$$

where n is a positive integer and $\binom{n}{r} = \frac{n!}{r!(n-r)!} = \frac{n(n-1)\ldots(n-r+1)}{r!}$

2. TRIGONOMETRY

Identities

$$\sin^2 A + \cos^2 A = 1$$
$$\sec^2 A = 1 + \tan^2 A$$
$$\operatorname{cosec}^2 A = 1 + \cot^2 A$$
$$\sin(A \pm B) = \sin A\cos B \pm \cos A\sin B$$
$$\cos(A \pm B) = \cos A\cos B \mp \sin A\sin B$$
$$\tan(A \pm B) = \frac{\tan A \pm \tan B}{1 \mp \tan A\tan B}$$
$$\sin 2A = 2\sin A\cos A$$
$$\cos 2A = \cos^2 A - \sin^2 A = 2\cos^2 A - 1 = 1 - 2\sin^2 A$$
$$\tan 2A = \frac{2\tan A}{1 - \tan^2 A}$$

Formulae for $\Delta\ ABC$

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$
$$a^2 = b^2 + c^2 - 2bc\cos A$$
$$\Delta = \frac{1}{2}bc\sin A$$

1 Given that $\frac{9^{n+2}-3^{2n+2}}{2^6}=2^a3^b$, where a and b are integers, find the value of a and express b in terms of n. [3]

2 Find the range of values of c for which the graph of $y=cx^2+3x+c$ lies entirely below the x-axis. [4]

3 Integrate the following with respect to x.

(i) $(3x^2-4)(2+5x)$ [3]

(ii) $\frac{1+x^5-\sqrt{x}}{x^3}$ [3]

4 The gradient of the tangent to the curve $y=ax^3+bx^2+3$ at the point $(1,-3)$ is -10. Calculate the values of the constant a and b. [5]

5 Given that the ninth term in the expansion of $\left(2x^2+\frac{1}{x}\right)^n$ is independent of x, find the value of

(i) n, [3]

(ii) the ninth term. [2]

6 Solve the equation $2x^3-9x^2+11x-2=0$, giving answers correct to two decimal places where appropriate. [6]

7 **(a)** By using an appropriate substitution, or otherwise, solve $5(2^{x+1})=2^{2x}+16$. [3]

(b) Solve $\frac{2}{\cos^2 x}+\tan^2 x=3$ for $0° < x < 360°$. [4]

8 A Norman window shown in Figure 1, has the shape of a rectangle with height h cm, surmounted by a semicircle, with radius r cm, as shown in Figure 2. The perimeter of the window is 150 cm.

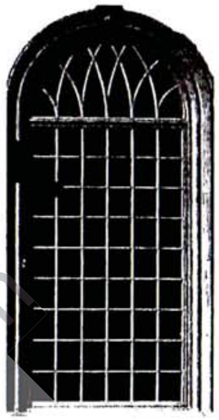


Figure 1

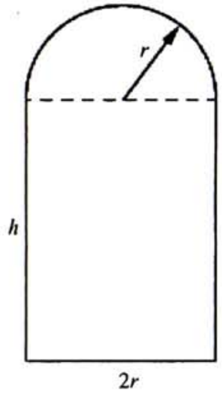


Figure 2

(i) Show that the area of the window, A cm², can be represented by the expression,

$$A=150r-2r^2-\frac{\pi r^2}{2}.$$ [3]

(ii) Determine the area of the window, so that the maximum amount of light can be admitted. [4]

9 The curve $y=ax^3+bx+11$ has a minimum point at $(2,-5)$.

(i) Show that $a=1$ and $b=-12$. [4]

(ii) Find the other stationary point of y and determine its nature. [4]

10 Given that $\tan\theta = -\frac{3}{4}$ and that θ is an obtuse angle, find the exact values of the following without finding the value of θ.

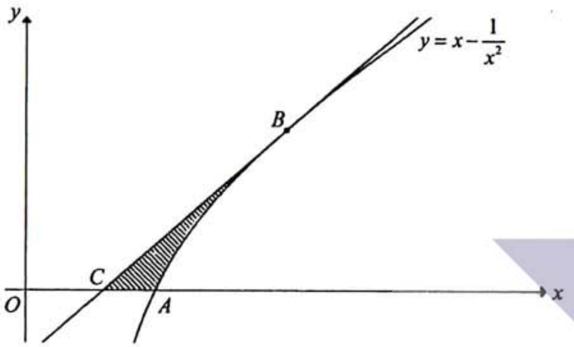
(i) $\sin 2\theta$ [2]

(ii) $\cot(180° - \theta)$ [2]

(iii) $\cos(\theta + 30°)$ [2]

(iv) $\cos\frac{\theta}{2}$ [3]

11



The diagram above shows part of the curve of $y = x - \frac{1}{x^2}$ which intersects the x-axis at point A. The tangent to the curve at point $B\left(2,\ 1\frac{3}{4}\right)$ intersects the x-axis at point C. Find

(i) the equation of the tangent, [4]

(ii) the coordinates of points A and C, [2]

(iii) the shaded area ABC. [4]

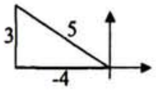
END OF PAPER

Woodgrove Secondary School
Mathematics Department
2018 4NA Additional Mathematics Prelim Paper 02

Setter: Ms Nicole Ng

No.	Solution	Marks	Remarks
1	$\frac{3^{2n+4}-3^{2n+2}}{2^6}$	M1	
	$=\frac{3^{2n+2}(9-1)}{2^6}$		
	$3^{2n+2}2^{-3}$	M1	
	$a=-3, b=2n+2$	A1	
2	$9-4c^2<0 \quad and \quad c<0$	M1,M1	
	$4c^2-9>0$		
	$(2c+3)(2c-3)>0$		
	$c<-\frac{3}{2} \;\; OR \;\; c>\frac{3}{2}$	M1	
	$\therefore c<-\frac{3}{2}$	A1	
3(i)	$\int(3x^2-4)(2+5x)\ dx$		
	$=\int 15x^3+6x^2-20x-8\ dx$	M1	
	$=\frac{15x^4}{4}+2x^3-10x^2-8x+c$	A2	A1 if there is 1 mistake.
3(ii)	$\int\frac{1+x^5-\sqrt{x}}{x^3}\ dx$		
	$=\int x^{-3}+x^2-x^{-\frac{5}{2}}\ dx$	M1	
	$=\frac{x^{-2}}{-2}+\frac{x^3}{3}-\frac{x^{-\frac{3}{2}}}{-\frac{3}{2}}+c$	M1	
	$=-\frac{1}{2x^2}+\frac{x^3}{3}+\frac{2}{3\sqrt{x^3}}+c$	A1	
4	$\frac{dy}{dx}=3ax^2+2bx$	M1	
	$-10=3a+2b....(1)$	M1	
	$-3=a+b+3$	M1	
	$a=-6-b.........(2)$		
	Sub (2) into (1):		
	$-10=3(-6-b)+2b$		
	$-10=-18-b$		
	$b=-8, a=2$	A1,A1	
5(i)	$T_9=\binom{n}{8}(2x^2)^{n-8}(x^{-1})^8$	M1	
	$x^{2n-16-8}=x^0$	M1	
	$2n=24$	A1	
	$n=12$		
5(ii)	$T_9=\binom{12}{8}(2)^4$	M1	
	$=495(16)$		
	$=7920$	A1	
6	Let $f(x)=2x^3-9x^2+11x-2$		
	$(x-2), f(2)=0$	M1	
	$2x^2-5x+1$		
	$x-2\overline{)2x^3-9x^2+11x-2}$	M1	Accept alternative method of comparing coefficients
	$\underline{-)2x^3-4x^2}$		
	$-5x^2+11x$		
	$\underline{-)-5x^2+10x}$		
	$x-2$		
	$\underline{-)x-2}$		
	$\underline{0}$		
	$\therefore f(x)=(x-2)(2x^2-5x+1)$	M1	
	$f(x)=0$		
	$x=2 \quad OR \quad 2x^2-5x+1=0$	A1	For $x=2$
	$x=\frac{5\pm\sqrt{(-5)^2-4(2)(1)}}{2(2)}$	M1	
	$=\frac{5\pm\sqrt{17}}{4}$		
	$=2.28, 0.22$	A1	For both values

7(a)	Let $y = 2^z$. $5(2y) = y^2 + 16$ $y^2 - 10y + 16 = 0$ $(y-2)(y-8) = 0$ $y = 2$ *OR* $y = 8$ $z = 1$ *OR* $z = 3$	M1 M1 A1	
7(b)	$2\sec^2 x + \tan^2 x = 3$ $2(\tan^2 x + 1) + \tan^2 x = 3$ $3\tan^2 x = 1$ $\tan x = \pm\sqrt{\frac{1}{3}}$ $\alpha = 30°$ $x = 30°, 150°, 210°, 330°$	M1 M1 M1 A1	
8(i)	$P = 2h + 2r + \pi r$ $2h = 150 - 2r - \pi r$ $h = 75 - r - \frac{\pi r}{2}$ $A = 2rh + \frac{\pi r^2}{2}$ $A = 2r(75 - r - \frac{\pi r}{2}) + \frac{\pi r^2}{2}$ $= 150r - 2r^2 - \frac{\pi r^2}{2}$ *(shown)*	M1 M1 $\left.\right\}$A1	
8(ii)	$\frac{dA}{dr} = 150 - 4r - \pi r$ $\frac{dA}{dr} = 0$ $r = \frac{150}{4+\pi}$ $= 21.004 \approx 21.0\,\text{cm}$ $A = 150(21.004) - 2(21.004)^2 - \frac{\pi(21.004)^2}{2}$ $= 1575 \approx 1580\,\text{cm}^2$	M1 M1 M1 A1	

9(i)	Subs $(2, -5)$, $-5 = 8a + 2b + 11$ $4a + b + 8 = 0$.....................(1) $\frac{dy}{dx} = 3ax^2 + b$ $0 = 12a + b$ $b = -12a$..............................(2) Subs (2) into (1): $a = 1, b = -12$	M1 M1 M1 A1	
9(ii)	$\frac{dy}{dx} = 3x^2 - 12$ $3x^2 - 12 = 0$ $x = \pm 2$ Subs $x = -2, y = 27$ $\therefore$ stationary point is $(-2, 27)$. $\frac{d^2y}{dx^2} = 6x$ Subs $x = -2, \frac{d^2y}{dx^2} < 0$ $\therefore (-2, 27)$ is a maximum point.	M1 A1 $\left.\right\}$M1 A1	Accept first derivative test
10(i)	$\sin 2\theta = 2\sin\theta\cos\theta$ $= 2 \times \frac{3}{5} \times \left(-\frac{4}{5}\right)$ $= -\frac{24}{25}$ 	M1 A1	
10(ii)	$\cot(180° - \theta) = \frac{1}{\tan(180° - \theta)}$ $= \frac{1}{-\tan\theta}$ $= \frac{4}{3}$	M1 A1	
10(iii)	$\cos(\theta + 30°) = \cos\theta\cos 30° - \sin\theta\sin 30°$ $= -\frac{4}{5}\left(\frac{\sqrt{3}}{2}\right) - \frac{3}{5}\left(\frac{1}{2}\right)$ $= \frac{-4\sqrt{3} - 3}{10}$ *OR* $-\frac{4\sqrt{3} + 3}{10}$	M1 A1	

10(iv)	$\cos\theta = 2\cos^2\frac{\theta}{2} - 1$ $-\frac{4}{5} = 2\cos^2\frac{\theta}{2} - 1$ $\cos^2\frac{\theta}{2} = \frac{1}{10}$ $\cos\frac{\theta}{2} = \frac{1}{\sqrt{10}}\ OR\ -\frac{1}{\sqrt{10}}(rejected)$	M1 M1 A1	
11(i)	$\frac{dy}{dx} = 1 + \frac{2}{x^3}$ At point B, $m = \frac{5}{4}$ Subs $B\left(2,\ 1\frac{3}{4}\right), \frac{7}{4} = \frac{5}{4}(2) + c$ $c = -\frac{3}{4}$ Equation of tangent: $y = \frac{5}{4}x - \frac{3}{4} \quad OR \quad 4y = 5x - 3$	M1 M1 M1 A1	
11(ii)	$A(1, 0), C\left(\frac{3}{5}, 0\right)$	B2	
11(iii)	$ABC = \frac{1}{2}\left(2 - \frac{3}{5}\right)\left(1\frac{3}{4}\right) - \int_1^2 x - x^{-2}\ dx$ $= \frac{1}{2}\left(1\frac{2}{5}\right)\left(1\frac{3}{4}\right) - \left[\frac{x^2}{2} - \frac{x^{-1}}{-1}\right]_1^2$ $= 1\frac{9}{40} - \left[\frac{x^2}{2} + \frac{1}{x}\right]_1^2$ $= 1\frac{9}{40} - 1$ $= \frac{9}{40}$ sq units	M2 M1 M1 A1	M1 for finding each area For correct integration For subtracting area under graph from triangle