Candidate's Name	Class	Register Number

BALESTIER HILL SECONDARY SCHOOL
PRELIMINARY EXAMINATION 2016
SECONDARY 4 NORMAL ACADEMIC

ADDITIONAL MATHEMATICS
PAPER 1

4044/01

Thursday **4 August** **1 hour 45 minutes**

Additional materials: Writing paper (5 sheets)

READ THESE INSTRUCTIONS FIRST

Write your name, class and register number on all the work you hand in.
Write in dark blue or black pen.
You may use a pencil for any diagrams or graphs.
Do not use staples, paper clips, highlighters, glue or correction fluid.

Answer all questions.
Write your answers on the separate Answer Paper provided.
Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.
The use of a scientific calculator is expected, where appropriate.
You are reminded of the need for clear presentation in your answers.

The number of marks is given in brackets [ ] at the end of each question or part question.
The total number of marks for this paper is 70.

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/ 70

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Mathematical Formulae

1. ALGEBRA

Quadratic Equation

For the equation $ax^2 + bx + c = 0$, $a \neq 0$,

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}.$$

Binomial expansion

$$(a+b)^n = a^n + \binom{n}{1}a^{n-1}b + \binom{n}{2}a^{n-2}b^2 + \ldots + \binom{n}{r}a^{n-r}b^r + \ldots + b^n,$$

where n is a positive integer and $\binom{n}{r} = \frac{n!}{(n-r)!r!} = \frac{n(n-1)\ldots(n-r+1)}{r!}$.

2. TRIGONOMETRY

Identities

$$\sin^2 A + \cos^2 A = 1$$
$$\sec^2 A = 1 + \tan^2 A$$
$$\operatorname{cosec}^2 A = 1 + \cot^2 A$$
$$\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$$
$$\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$$
$$\tan(A \pm B) = \frac{\tan A \pm \tan B}{1 \mp \tan A \tan B}$$
$$\sin 2A = 2\sin A \cos A$$
$$\cos 2A = \cos^2 A - \sin^2 A = 2\cos^2 A - 1 = 1 - 2\sin^2 A$$
$$\tan 2A = \frac{2\tan A}{1 - \tan^2 A}$$

Formulae for ΔABC

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}.$$
$$a^2 = b^2 + c^2 - 2bc\cos A.$$
$$\Delta = \frac{1}{2}ab\sin C.$$

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1 Find the range of values of x for which $(2x-1)(x+4)<18$. [3]

2(a) Express the principle values of the following in terms of π.

(i) $\cos^{-1}\left(\frac{\sqrt{3}}{2}\right)$, [1]

(ii) $\sin^{-1}\left(-\frac{1}{\sqrt{2}}\right)$. [1]

(b) State the range of values of x for which $\cos^{-1}x$ and $\sin^{-1}x$ are defined. [1]

3 The equation of a curve is given by $y=2x+\frac{9}{x^2}$. A point (x, y) moves along the curve in such a way that the x-coordinate is increasing at a constant rate of 0.08 units per second. Find the corresponding rate of change of the y-coordinate at the instant when $x=4$. [3]

4 Find the value of y for which $\frac{25}{5^{y+1}}=\sqrt{5^{y-2}}$. [4]

5 Given that $f(x)=(2x+1)^7x^3$, find the value of f '(2). [5]

6 It is given that the curve $y^2=kx$, where k is a constant, passes through the point $A(6, 3)$.

(i) Find the value of k. [1]

The points $B(2\frac{2}{3}, 2)$ and $C(2\frac{2}{3}, -2)$ also lie on the curve.

(ii) Sketch the graph, marking the points A, B and C clearly. [2]

(iii) Find the area of the triangle ABC. [2]

7 (a) Find the range of values of q for which $2x^2+qx-q$ is always positive. [3]

(b) Find the range of values of k for which the line $y=kx-2$ intersect the curve $y=\frac{3x-2}{1-2x}$ at two distinct points. [3]

8 In an AC circuit, the current I amperes is modeled by the following sine function $I=16\sin 3t$, where t is the time in milliseconds.

(i) State the amplitude of the current. [1]

(ii) Find the time at the first instant when the current is 10 amperes. [3]

(iii) Sketch the graph of I for the interval $0\le t\le\pi$. [3]

9 The function $f(x)=ax^3+x^2+bx-4$ has a factor of $x-2$ and a stationary value at $x=1$.

(i) Find the value of a and of b. [5]

(ii) Hence, factorise f(x) completely. [2]

10 (a) The first three terms in the expansion, in ascending powers of x, of $(3+4x^2)^8$ are $a+bx^2+cx^4$. Find the values of a, b and c. [3]

(b) The term independent of x in the expansion of $\left(x^3+\frac{p}{x^2}\right)^{10}$ is 210. Find the value of p where $p>0$. [5]

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11 (i) Prove the identity $\frac{1}{\cos x}-\frac{\cos x}{1+\sin x}=\tan x$. [4]

(ii) Hence, find all the angles between 0° and 360° for which

$$\frac{1}{\cos 2x}-\frac{\cos 2x}{1+\sin 2x}=\frac{5}{12}$$ [4]

12 (a) Find $\int \frac{1}{(7x+2)^8}\,dx$. [3]

(b) (i) Given that $y=\frac{2x^2}{3+2x}$, show that the first derivative is $\frac{4x(x+3)}{(3+2x)^2}$. [4]

(ii) Hence evaluate $\int_{-2}^{2}\frac{2x(x+3)}{(3+2x)^2}\,dx$. [4]

END OF PAPER

Solutions

1	$(2x-1)(x+4)<18$	
	$2x^2+8x-x-4<18$	M1
	$2x^2+7x-22<0$	
	$(2x+11)(x-2)<0$	M1
	$-5.5<x<2$	A1
2(a)(i)	$\frac{\pi}{6}$	B1
2(a)(ii)	$-\frac{\pi}{4}$	B1
2(b)	$-1\le x\le 1$	B1
3.	$y=2x+\frac{9}{x^2}$	
	$\frac{dy}{dx}=2-\frac{18}{x^3}$	M1
	When $x=4$, $\frac{dy}{dx}=\frac{55}{32}$	
	$\frac{dy}{dt}=\frac{dy}{dx}\times\frac{dx}{dt}$	
	$=\frac{55}{32}\times 0.08=\frac{11}{80}$	M1 A1
4.	$\frac{25}{5^{y+1}}=\sqrt{5^{y-2}}$	
	$\frac{5^2}{5^{y+1}}=5^{\frac{y-2}{2}}$	M1
	$5^{y-1}=5^{\frac{y-2}{2}}$	M1
	$1-y=\frac{y-2}{2}$	M1
	$y=1\frac{1}{3}$	A1
5.	$f(x)=(2x+1)^7x^3$	
	$f'(x)=3x^2(2x+1)^7+7x^3(2x+1)^6(2)$	M1 M1 M1
	$=x^2(2x+1)^6(20x+3)$	M1
	$f'(2)=2687500$	A1
6(i)	$y^2=kx$	

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	At $A(6, 3)$, $3^2 = 6k$ $k = 1\frac{1}{2}$	[B1]
6(ii)	[B1] Shape [B1] Points marked correctly	
6(iii)	Area of $\Delta ABC = \frac{1}{2}(4)\left(6-2\frac{2}{3}\right)$ $= 6\frac{2}{3}$ units^2	[M1] [A1]
7(a)	For $2x^2+qx-q$ to be always positive, $b^2-4ac<0$ $q^2-4(2)(-q)<0$ $q^2+8q<0$ $-8<q<0$	[M1] [M1] [A1]
7(b)	$y=\frac{3x-2}{1-2x}$ and $y=kx-2$ $\Rightarrow kx-2=\frac{3x-2}{1-2x}$ $(kx-2)(1-2x)=3x-2$ $2kx^2-kx-x=0$ Since line intersects curve at 2 distinct points, $b^2-4ac>0$ $(-k-1)^2>0$ $k>-1$ or $k<-1$	[M1] [M1] [A1]
8(i)	16	[B1]
8(ii)	$16\sin 3t=10$ $\sin 3t=\frac{5}{8}$ $\alpha=0.675$ $3t=0.675$ $t=0.225$ ms	[M1] [M1] [A1]
8(iii)	[B1] correct shape [B1] correct max and min value [B1] correct x intercepts	

9(i)	$f(x)=ax^3+x^2+bx-4$ $f(2)=0$ $a(2^3)+2^2+2b-4=0$ $4a+b=0$ --- (1) $f'(x)=3ax^2+2x+b$ $f'(1)=0$ $3a+2+b=0$ --- (2) Using substitution method, $a=2$, $b=-8$	[M1] [M1] [M1] [A1, A1]
9(ii)	$f(x)=2x^3+x^2-8x-4$ $=(x-2)(2x^2+5x+2)$ $=(x-2)(2x+1)(x+2)$	[M1] [A1]
10(a)	$(3+4x^2)^8 = a+bx^2+cx^4$ $a=6561$, $b=69984$, $c=326592$	[B1, B1, B1]
10(b)	$\left(x^3+\frac{p}{x^2}\right)^{10}$ $T_{r+1}=\binom{10}{r}(x^3)^{10-r}\left(\frac{p}{x^2}\right)^r$ $=\binom{10}{r}x^{30-5r}p^r$ For term independent of x, $30-5r=0$ $r=6$ $\binom{10}{6}p^6=210$ $210p^6=210$ $p=1$ since $p>0$	[M1] [M1] [M1] [M1] [A1]
11(i)	LHS $=\frac{1}{\cos x}-\frac{\cos x}{1+\sin x}$ $=\frac{1+\sin x-\cos^2 x}{\cos x(1+\sin x)}$ $=\frac{\sin^2 x+\sin x}{\cos x(1+\sin x)}$	[M1] [M1]

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	$= \dfrac{\sin x(\sin x+1)}{\cos x(1+\sin x)}$ $= \dfrac{\sin x}{\cos x}$ $= \tan x = \text{RHS}$	[M1] [A1]
11(ii)	$\dfrac{1}{\cos 2x} - \dfrac{\cos 2x}{1+\sin 2x} = \dfrac{5}{12}$ $\tan 2x = \dfrac{5}{12}$ $\alpha = 22.6°$ $2x = 22.6°, 202.6°, 382.6°, 562.6°$ $x = 11.3°, 101.3°, 191.3°, 281.3°$	[M1] [M1] [A1, A1]
12(a)	$\int \dfrac{1}{(7x+2)^8}\,dx = \int (7x+2)^{-8}\,dx$ $= \dfrac{(7x+2)^{-7}}{7(-7)} + C$ $= -\dfrac{1}{49(7x+2)^7} + C$	[M1, M1] [A1]
12(b)(i)	$y = \dfrac{2x^2}{3+2x}$ $\dfrac{dy}{dx} = \dfrac{(3+2x)(4x) - 2x^2(2)}{(3+2x)^2}$ $= \dfrac{4x^2+12x}{(3+2x)^2}$ $= \dfrac{4x(x+3)}{(3+2x)^2}$	[M1, M1] [M1] [A1]
12(b)(ii)	$\int_{-2}^{2} \dfrac{4x(x+3)}{(3+2x)^2}\,dx = \left[\dfrac{2x^2}{3+2x}\right]_{-2}^{2}$ $\dfrac{1}{2}\int_{-2}^{2} \dfrac{4x(x+3)}{(3+2x)^2}\,dx = \dfrac{1}{2}\left[\dfrac{8}{7} - \dfrac{8}{-1}\right]$ $\int_{-2}^{2} \dfrac{2x(x+3)}{(3+2x)^2}\,dx = \dfrac{1}{2}\left(\dfrac{64}{7}\right) = 4\dfrac{4}{7}$	[M1, M1] [M1, A1]

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Candidate's Name	Class	Register Number

BALESTIER HILL SECONDARY SCHOOL
PRELIMINARY EXAMINATION 2016
SECONDARY 4 NORMAL ACADEMIC

ADDITIONAL MATHEMATICS
PAPER 2

4044/02

Friday **12 August** **1 hour 45 minutes**

Additional materials: Writing paper (5 sheets)

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This paper consists of 5 printed pages, including this cover page.

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$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}.$$

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where n is a positive integer and $\binom{n}{r} = \frac{n!}{(n-r)!\,r!} = \frac{n(n-1)\ldots(n-r+1)}{r!}$.

2. TRIGONOMETRY

Identities

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$$\tan(A \pm B) = \frac{\tan A \pm \tan B}{1 \mp \tan A \tan B}$$
$$\sin 2A = 2\sin A \cos A$$
$$\cos 2A = \cos^2 A - \sin^2 A = 2\cos^2 A - 1 = 1 - 2\sin^2 A$$
$$\tan 2A = \frac{2\tan A}{1 - \tan^2 A}$$

Formulae for ΔABC

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}.$$
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$$\Delta = \frac{1}{2}ab\sin C.$$

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1. Given that $2x^4 + x^2 + x + 1 = (x^2 - 1)(Ax^2 + 3) + Bx + C$, find the values of A, B and C. Hence, find the remainder when 20 111 is divided by 99 by assigning a suitable value to x. [5]

2. Solve, for angles between 0 and 2π,

$$\cos 2x - 3\cos x = 1$$ [5]

3 A rectangular box has a square base of side $(3 + 2\sqrt{5})$ cm. Find its height in the form of $a + b\sqrt{5}$ if the volume is $(93 + 51\sqrt{5})$ cm³. [5]

4 Solve the equation $2^{2x+1} - 3(2^{x+1}) - 8 = 0$ [5]

5 Given that the roots of $2x^2 - 8x + 13 = 0$ are α and β,

(i) evaluate $\alpha^2 + \beta^2$, [3]

(ii) hence find the quadratic equation whose roots are $\frac{1}{\alpha^2} + \frac{1}{\beta^2}$. [3]

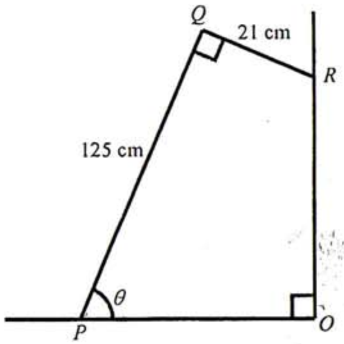
6 A curve for which $\frac{dy}{dx} = \frac{k}{x^2} - x$, where k is a constant, has a stationary point at $(3, -8\frac{1}{2})$.

(i) Find the value of k. [2]

(ii) Find the equation of the curve. [3]

(iii) Determine the nature of the turning point $(3, -8\frac{1}{2})$. [2]

7



The following figure shows a machining tool PQR leaning against a vertical wall OR. It is given that $QR = 21$ cm, $PQ = 125$ cm, $\angle PQR = \angle POR = 90°$. OP is the distance between the foot of the wall and the support of the machining tool, and it is such that $OP = 21\sin\theta + 125\cos\theta$.

(i) Express OP in the form $R\sin(\theta + \alpha)$, where $R > 0$ and α is an acute angle. [3]

Hence

(ii) state the greatest distance between the foot of the wall and the support of the tool and find the corresponding value of θ, [3]

(iii) find the value of θ when $OP = 100$ cm. [2]

8 The equation of a circle C is $x^2 + y^2 + 4x - 6y + 4 = 0$

(i) Find the radius of the circle and the coordinates of the centre of C. [4]

(ii) Explain why C touches the x-axis. [3]

(iii) Find the equation of another circle which is a reflection of C in the line $y = -1$. [2]

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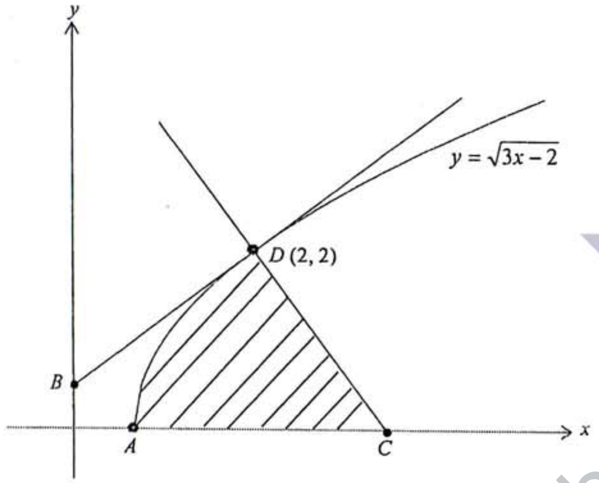
9 A closed cylindrical can of negligible thickness, radius r cm and height h cm, is to be constructed to hold a liquid of 350π cm^3. The cost is least if the cans have the smallest possible surface area.

(i) Express h in terms of r and show that the total surface area of the cylindrical can is $A = 2\pi r^2 + \frac{700\pi}{r}$. [3]

(ii) Given that r can vary, find the dimensions of the can (i.e. r and h) when the surface area is the least. [4]

(iii) The material used to manufacture the can cost \$0.05 per 1000 cm^2. Find the value of A when the cost is the least, and hence find the minimum cost of producing 1000 cans. [3]

10



The diagram shows part of the curve $y = \sqrt{3x-2}$, intersecting the x-axis at A. The tangent and normal to the point at $D(2, 2)$ cut the y-axis at B and x-axis at C respectively.

(i) Find the coordinates of A. [1]

(ii) Find the equation of the line CD. [3]

(iii) Find the coordinates of C. [1]

(iv) Find the area of the shaded region. [5]

END OF PAPER

Solutions

1	$2x^4 + x^2 + x + 1 = (x^2 - 1)(Ax^2 + 3) + Bx + C$	
	$A = 2, B = 1, C = 4$	[B1, B1, B1]
	$2x^4 + x^2 + x + 1 = (x^2 - 1)(2x^2 + 3) + x + C$	
	$2(10^4) + 10^2 + 10 + 1 = 20\ 111$	
	$\Rightarrow x = 10$	[M1]
	Hence, remainder = 10 + 4 = 14	[A1]
2	$\cos 2x - 3\cos x = 1$	
	$2\cos^2 x - 1 - 3\cos x = 1$	[M1]
	$2\cos^2 x - 3\cos x - 2 = 0$	
	$(\cos x - 2)(2\cos x + 1) = 0$	[M1]
	$\cos x = 2 \Rightarrow$ no solution	[A1]
	$\cos x = -\frac{1}{2}$	
	$\alpha = \frac{\pi}{3}$	[M1]
	$x = \frac{2\pi}{3}, \frac{4\pi}{3}$	[A1]
3	Height	
	$= \frac{93 + 51\sqrt{5}}{(3 + 2\sqrt{5})^2}$	[M1]
	$= \frac{93 + 51\sqrt{5}}{29 + 12\sqrt{5}}$	[M1]
	$= \frac{(93 + 51\sqrt{5})(29 - 12\sqrt{5})}{29^2 - (12\sqrt{5})^2}$	[M1]
	$= \frac{-363 + 363\sqrt{5}}{121}$	[M1]
	$= -3 + 3\sqrt{5}$	[A1]
4	$2^{2x+1} - 3(2^{x+1}) - 8 = 0$	
	$2^{2x} \times 2 - 3(2^x \times 2) - 8 = 0$	[M1]
	Let $u = 2^x$.	
	$2u^2 - 6u - 8 = 0$	[M1]

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	$(u-4)(u+1)=0$ $u=4$ or -1 $2^x=-1 \Rightarrow$ no solution $2^x=4 \Rightarrow x=2$	[M1] [A1] [A1]
5(i)	$2x^2-8x+13=0$ $\alpha+\beta=4;\ \ \alpha\beta=\frac{13}{2}$ $\alpha^2+\beta^2=(\alpha+\beta)^2-2\alpha\beta$ $=3$	[M1] [M1] [A1]
5(ii)	$\frac{1}{\alpha^2}+\frac{1}{\beta^2}=\frac{12}{169}$ $\frac{1}{\alpha^2}\times\frac{1}{\beta^2}=\frac{4}{169}$ Hence, $x^2-\frac{12}{169}x+\frac{4}{169}=0$	[M1] [M1] [A1]
6(i)	$\frac{dy}{dx}=\frac{k}{x^2}-x$ When $x=3$, $\frac{k}{3^2}-3=0$ $k=27$	[B1]
6(ii)	$\frac{dy}{dx}=\frac{27}{x^2}-x$ $y=\int\frac{27}{x^2}-x\ \mathrm{d}x$ $=\frac{27x^{-1}}{-1}-\frac{x^2}{2}+C$ $=-\frac{27}{x}-\frac{x^2}{2}+C$ At $(3,\ -8\frac{1}{2})$, $C=5$ Hence, $y=-\frac{27}{x}-\frac{x^2}{2}+5$	[M1] [M1] [A1]
6(iii)	$\frac{d^2y}{dx^2}=-\frac{54}{x^3}-1$	[M1]

	When $x=3$, $\frac{d^2y}{dx^2}=-3<0$ Hence, $(3,\ -8\frac{1}{2})$ is a maximum point.	[A1]
7(i)	$R=\sqrt{21^2+125^2}\ =\sqrt{16066}$ $\alpha=80.5°$ Hence, $\sqrt{16066}\sin(\theta+80.5°)$	[M1] [M1] [A1]
7(ii)	Greatest distance = 127 cm (3 sf) $\sin(\theta+80.5°)=1$ $\theta+80.5°=90°$ $\theta=9.5°$	[B1] [M1] [A1]
7(iii)	$21\sin\theta+125\cos\theta=100$ $\sqrt{16066}\sin(\theta+80.5°)=100$ $\sin(\theta+80.5°)=\frac{100}{\sqrt{16066}}$ $\alpha=52.1°$ $\theta+80.5°=52.1°$ (NA), $127.9°$ $\theta=47.4°$	[M1] [A1]
8(i)	$x^2+y^2+4x-6y+4=0$ $x^2+4x+2^2-2^2+y^2-6y+(-3)^2-(-3)^2+4=0$ $(x+2)^2+(y-3)^2=9$ Hence, centre = $(-2,\ 3)$, radius = 3	[M1, M1] [A1, A1]
8(ii)	When $y=0$, $x^2+4x+4=0$ $x=-2$ Since there is only 1 root, C touches the x-axis.	[M1] [M1] [A1]
8(iii)	New centre = $(-2,\ -5)$ Hence, $(x+2)^2+(y+5)^2=9$	[M1] [A1]
9(i)	$350\pi=\pi r^2h$ $h=\frac{350}{r^2}$ $A=2\pi r^2+2\pi rh$	[B1] [M1]

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	$= 2\pi r^2 + 2\pi r\left(\frac{350}{r^2}\right)$	
	$= 2\pi r^2 + \frac{700\pi}{r}$	[A1]
9(ii)	$\frac{dA}{dr} = 4\pi r - \frac{700\pi}{r^2}$	[M1]
	$4\pi r - \frac{700\pi}{r^2} = 0$	[M1]
	$r = 5.59$ cm	[A1]
	$h = 11.2$ cm	[A1]
9(iii)	$A = 590$ cm^2 (3 sf)	[B1]
	Cost $= \frac{589.739 \times 1000}{1000} \times 0.05$	[M1]
	$= \$29.49$	[A1]
10(i)	$A = (\frac{2}{3}, 0)$	[B1]
10(ii)	$\frac{dy}{dx} = \frac{1}{2}(3x-2)^{-\frac{1}{2}}(3)$	
	$= \frac{3}{2\sqrt{3x-2}}$	[M1]
	When $x = 2$, $\frac{dy}{dx} = \frac{3}{4}$	
	$\Rightarrow\ m_{CD} = -\frac{4}{3}$	[M1]
	Hence, CD: $y = -\frac{4}{3}x + \frac{14}{3}$	[A1]
10(iii)	$C = (3\frac{1}{2}, 0)$	[B1]
10(iv)	Area of shaded region	
	$= \int_{\frac{2}{3}}^{2} \sqrt{3x-2}\ \mathrm{d}x + \frac{1}{2}\left(\frac{3}{2}\right)(2)$	[M1, M1]
	$= \left[\frac{(3x-2)^{\frac{3}{2}}}{3\left(\frac{3}{2}\right)}\right]_{\frac{2}{3}}^{2} + 1.5$	[M1]

	$= \frac{16}{9} + 1.5$	[M1]
	$= 3\frac{5}{18}$ units^2	[A1]

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N Level Preliminary Examination 2016

Secondary 4 Normal Academic

ADDITIONAL MATHEMATICS 4044/01

Paper 1

02 August 2016
1000 – 1145
1 hour 45 minutes

Additional Materials: 8 pieces of writing paper

READ THESE INSTRUCTIONS FIRST

Write your name, class and register number on all the work you hand in.
Write in dark blue or black pen on both sides of the paper.
You may use a soft pencil for any diagrams or graphs.
Do not use staple, paper clips, highlighters, glue or correction fluid.

Answer all questions.
Write your answers on the separate answer paper provided.
Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.
The use of an approved calculator is expected, where appropriate.
You are reminded of the need for clear presentation in your answers.

At the end of the examination, fasten all your work securely together.
The number of marks is given in brackets [ ] at the end of each question or part question.
The total number of marks for this paper is 70.

This document consists of 5 printed pages.

Mathematical Formulae

1. ALGEBRA

Quadratic Equation

For the equation $ax^2 + bx + c = 0$,

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Binomial Expansion

$$(a+b)^n = a^n + \binom{n}{1} a^{n-1}b + \binom{n}{2} a^{n-2}b^2 + \cdots + \binom{n}{r} a^{n-r}b^r + \cdots + b^n,$$

where n is a positive integer and $\binom{n}{r} = \frac{n!}{(n-r)!\,r!} = \frac{n(n-1)\ldots(n-r+1)}{r!}$

2. Trigonometry

Identities

$$\sin^2 A + \cos^2 A = 1$$

$$\sec^2 A = 1 + \tan^2 A$$

$$\operatorname{cosec}^2 A = 1 + \cot^2 A$$

$$\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$$

$$\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$$

$$\tan(A \pm B) = \frac{\tan A \pm \tan B}{1 \mp \tan A \tan B}$$

$$\sin 2A = 2 \sin A \cos A$$

$$\cos 2A = \cos^2 A - \sin^2 A = 2\cos^2 A - 1 = 1 - 2\sin^2 A$$

$$\tan 2A = \frac{2\tan A}{1 - \tan^2 A}$$

Formulae for ΔABC

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$a^2 = b^2 + c^2 - 2bc\cos A$$

$$\Delta = \frac{1}{2}ab\sin C$$

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1 Find $\int \frac{4}{(2x-5)^3}\,dx$. [3]

2 Given that $2y = 4x^2 - 6x - 1$, determine the range of values of x for which $y < -1\frac{1}{2}$. [3]

3 (i) Differentiate $x\sqrt{2x+1}$ with respect to x and show that it reduces to $\frac{3x+1}{\sqrt{2x+1}}$. [2]

(ii) Hence, find $\int \frac{12x+4}{\sqrt{2x+1}}\,dx$. [2]

4 The carriage of a roller coaster reaches y metres above the ground when it is horizontally x metres away from the starting point, where $y = a\sin 2x + b$ for $0 \le x \le \pi$.
The carriage reaches a maximum height of 35 metres and a minimum height of 5 metres above the ground respectively.

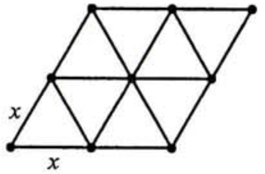
(i) Calculate the values of a and of b. [2]

(ii) With reference to (i), determine the **maximum height** of the carriage if

(a) only the value of a is halved. [1]

(b) only the value of b is doubled. [1]

5 The following rhombus is made up of eight equilateral triangles of side x cm.



(i) Show that the area of the rhombus, A cm², is given by the equation $A = 2\sqrt{3}x^2$. [2]

(ii) Given that A is increasing at a constant rate of $8\sqrt{3}$ cm²/s, calculate the value of the rate of increase of x when $x = 4$. [3]

6 (i) The points $P\left(-\frac{1}{2}, -\frac{1}{2}\right)$, $Q(2, 1)$ and $R\left(\frac{1}{2}, y\right)$ lie on the same Cartesian plane. If angle PQR is a right angle, find the value of y. [3]

(ii) Find the area of triangle PQR. [2]

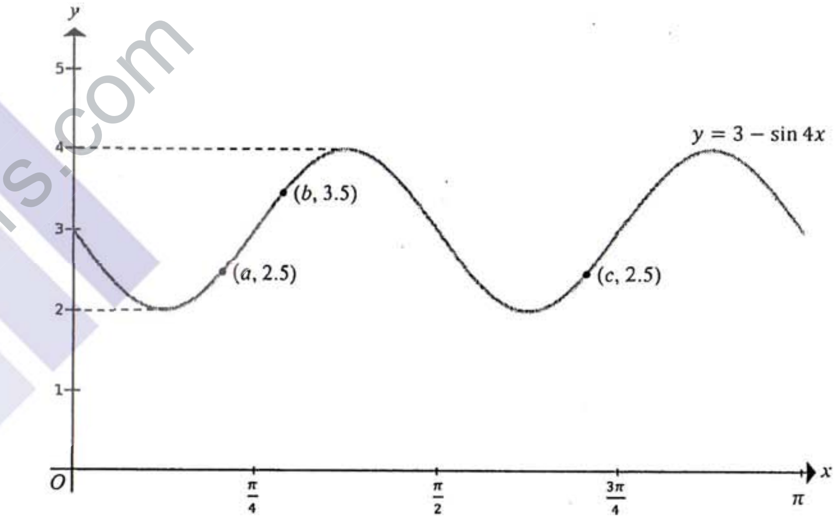
7 (i) Expand $(1+3x)(1-x)^6$ as far as the term in x^4. [3]

(ii) Hence, by using an appropriate substitution for x, evaluate 1.03×0.99^6. Give your answer correct to 5 decimal places. [2]

8 (i) Using the substitution $u = 8a^3$, or otherwise, express $64a^6 - 1$ as a product of two factors. [2]

(ii) Hence, by considering the sums and differences of cubes, express $64a^6 - 1$ as a product of four factors with integer coefficients. [3]

9



The diagram above shows the graph $y = 3 - \sin 4x$, for $0 \le x \le \pi$, passing through the points $(a, 2.5)$, $(b, 4.5)$ and $(c, 2.5)$, where a, b and c are constants.

(i) State the amplitude and period of the graph. [2]

(ii) Using the symmetry of the graph, or otherwise, and making a the subject; find an equation connecting

(a) a, c and π, [1]

(b) a, b and π. [2]

(iii) State the range of values of x, for $0 \le x \le \frac{\pi}{2}$, such that the graph is strictly increasing. [1]

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10 (i) Find, in radians, the two principal values of θ for which

$$5\tan^2\theta + 24\tan\theta - 5 = 0.$$ [4]

(ii) By using a suitable trigonometric identity, explain why the values of θ found in part (i) satisfy the equation $\tan 2\theta = \frac{5}{12}$. [2]

11 The equation of the tangent at the point $P(10, -10)$ on the parabola $y^2 = 10x$ is $2y + x + 10 = 0$.

(i) Find the equation of the normal to the parabola at P. [2]

The normal to the parabola at A intersects the parabola again at the point B.

(ii) Find the coordinates of B. [3]

(iii) Sketch the parabola and the normal to the parabola at A, clearly marking the points A and B on your sketch. [2]

12 The roots of the equation $x^2 - 6x + 2 = 0$ are α and β.

(i) State the value of $\alpha + \beta$ and of $\alpha\beta$. [2]

(ii) Hence, evaluate $\alpha^2 + \beta^2$. [1]

(iii) Find a quadratic equation with roots $\frac{1}{\alpha^2} - \beta^2$ and $\frac{1}{\beta^2} - \alpha^2$.

Give your answer in the form $ax^2 + bx + c = 0$ where a, b and c are integers. [5]

13 The function f(x) is defined by $f(x) = \frac{1}{3}x^3 + kx$, for all real values of x.

(i) Find the range of values of k for which f(x) has no stationary points. [2]

(ii) If $k = -4$,

(a) state the number of stationary points, [1]

(b) find the coordinates of the stationary points. [3]

(iii) If f(x) has only one stationary point,

(a) state the value(s) of k, [1]

(b) determine the nature of this stationary point. [2]

- End of Paper -

BBSS Sec 4 NA Additional Mathematics
Answer and Marking Scheme

Answers (Page 1)

Qn	Solutions	Marks
1	$\int \frac{4}{(2x-5)^3}\,dx$ $= 4\int (2x-5)^{-3}\,dx$ $= 4\left[\frac{(2x-5)^{-2}}{-2(2)}\right] + c$ $= -\frac{1}{(2x-5)^2} + c$	 M1 M1 A1
2	$2y = 4x^2 - 6x - 1$ $y = 2x^2 - 3x - \frac{1}{2}$ When $y < -1\frac{1}{2}$, $2x^2 - 3x - \frac{1}{2} < -1\frac{1}{2}$ $2x^2 - 3x + 1 < 0$ $(2x-1)(x-1) < 0$ $\therefore \frac{1}{2} < x < 1$	 M1 M1 A1
3i	$\frac{d}{dx}\left(x\sqrt{2x+1}\right)$ $= \frac{d}{dx}\left[x(2x+1)^{\frac{1}{2}}\right]$ $= x\left(\frac{1}{2}\right)(2x+1)^{-\frac{1}{2}}(2) + (2x+1)^{\frac{1}{2}}(1)$ $= x(2x+1)^{-\frac{1}{2}} + (2x+1)^{\frac{1}{2}}$ $= (2x+1)^{-\frac{1}{2}}(x + 2x + 1)$ $= \frac{3x+1}{\sqrt{2x+1}}$ (shown)	 M1 A1
3ii	$\int \frac{12x+4}{\sqrt{2x+1}}\,dx$ $= 4\int \frac{3x+1}{\sqrt{2x+1}}\,dx$ $= 4\left[x\sqrt{2x+1}\right] + c$ $= 4x\sqrt{2x+1} + c$	 M1 A1

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Answers (Page 2)

<table>
<tr><th>Qn</th><th colspan="2">Solutions</th><th>Marks</th></tr>
<tr><td>4(a)</td><td>EITHER
$y = a \sin 2x + b$

Max height achieved when sin 2x = 1
$35 = a(1) + b$
$35 = a + b$ (1)

Min height achieved when sin 2x = −1
$5 = a(-1) + b$
$5 = -a + b$ (2)

(1) + (2)
$40 = 2b$
$b = 20$

When $b = 20$,
$35 = a + 20$
$a = 15$</td><td>OR

Difference between max and min is 30 metres,
$a = \frac{30}{2}$
$a = 15$

$b = 5 + \frac{30}{2}$
$b = 20$</td><td>B1
B1</td></tr>
<tr><td>4(b)(i)</td><td colspan="2">(Maximum height would decrease by 7.5 m, from 35 m to) 27.5 m.</td><td>B1
allow ECF accept answer based on 4(a)</td></tr>
<tr><td>4(b)(ii)</td><td colspan="2">(Maximum height would increase by 20 m, from 35 m to) 55 m.</td><td>B1
allow ECF accept answer based on 4(a)</td></tr>
<tr><td></td><td colspan="2"></td><td></td></tr>
<tr><td>5(i)</td><td>EITHER
$A = 8\left[\frac{1}{2}(x)(x)\sin\left(\frac{\pi}{3}\right)\right]$
$A = 4x^2\left(\frac{\sqrt{3}}{2}\right)$
$\therefore A = 2\sqrt{3}x^2$</td><td>OR
$h = \sqrt{(2x)^2 - x^2}$
$h = \sqrt{3x^2}$
$h = \sqrt{3}x$
$A = 2x(\sqrt{3}x)$
$\therefore A = 2\sqrt{3}x^2$</td><td>M1
A1</td></tr>
<tr><td>5(ii)</td><td colspan="2">$\frac{dA}{dt} = \frac{dA}{dx} \times \frac{dx}{dt}$ $\left[\therefore \frac{dx}{dt} = \frac{dA}{dt} \div \frac{dA}{dx}\right]$

$\frac{dA}{dt} = 8\sqrt{3}$ (given in question)
$\frac{dA}{dx} = 4\sqrt{3}x$
$\frac{dx}{dt} = \frac{8\sqrt{3}}{4\sqrt{3}x}$
$\frac{dx}{dt} = \frac{2}{x}$

When $x = 4$,
$\frac{dx}{dt} = \frac{2}{4}$
$\frac{dx}{dt} = 0.5$ cm/s</td><td>M1
M1
A1</td></tr>
</table>

Answers (Page 3)

Qn	Solutions	Marks
6i	*PQR* is a right angle, Gradient *PQ* × gradient *QR* = −1 $\frac{1-\left(-\frac{1}{2}\right)}{2-\left(-\frac{1}{2}\right)} \times \frac{y-1}{\frac{1}{2}-2} = -1$ $\frac{3}{5} \times \frac{y-1}{-\frac{3}{2}} = -1$ $-\frac{2(y-1)}{3} = -\frac{5}{3}$ $2y - 2 = 5$ $y = 3\frac{1}{2}$	Accept other correct working steps M1 M1 A1
6ii	Area of triangle *PQR* $= \frac{1}{2}\begin{vmatrix} 2 & \frac{1}{2} & -\frac{1}{2} & 2 \\ 1 & \frac{7}{2} & -\frac{1}{2} & 1 \end{vmatrix}$ $= \frac{1}{2}\left[\left(7 - \frac{1}{4} - \frac{1}{2}\right) - \left(\frac{1}{2} - \frac{7}{4} - 1\right)\right]$ $= \frac{1}{2}\left(\frac{17}{2}\right)$ $= 4\frac{1}{4}$ units^2	M1 A1
7(i)	$(1 + 3x)(1 - x)^6$ $= (1 + 3x)[1 - 6x + 15x^2 - 20x^3 + 15x^4 + \cdots]$ Since we are only required to expand up to the term in x^4, $(1 + 3x)(1 - x)^6$ $\approx 1(1 - 6x + 15x^2 - 20x^3 + 15x^4) + 3x(1 - 6x + 15x^2 - 20x^3)$ $\approx 1 - 3x - 3x^2 + 25x^3 - 45x^4$	M1 M1 A1
7(ii)	1.03×0.99^6 $= [1 + 3(0.01)][1 - (0.01)]^6$ $\approx 1 - 3(0.01) - 3(0.01)^2 + 25(0.01)^3 - 45(0.01)^4$ ≈ 0.96972455 $= 0.96972$ Correct to 5 d.p.	M1 A1
8i	$64a^6 - 1$ $= (8a^3)^2 - (1)^2$ $= (u)^2 - (1)^2$ $= (8a^3 + 1)(8a^3 - 1)$	M1 A1
8ii	$64a^6 - 1$ $= (8a^3 + 1)(8a^3 - 1)$ $= [(2a)^3 + 1][(2a)^3 - 1]$ $= (2a + 1)[(2a)^2 - (2a) + 1]\ (2a - 1)[(2a)^2 + (2a) + 1]$ $= (2a + 1)(4a^2 - 2a + 1)(2a - 1)(4a^2 + 2a + 1)$	M1 M1 A1

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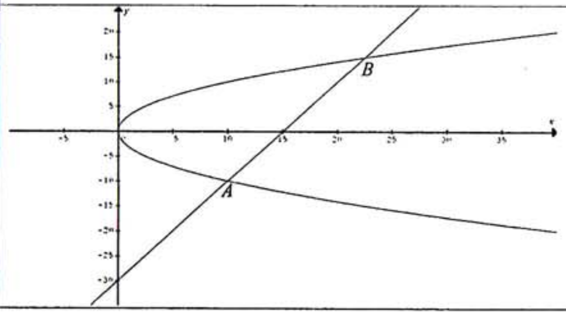
Answers (Page 4)

Answers (Page 4)

Qn	Solutions	Marks
9i	Amplitude: 1 Period: $\frac{\pi}{2}$	B1 B1
9iia	$c = a + \frac{\pi}{2}$ $a = c - \frac{\pi}{2}$	B1
9iib	$\frac{a+b}{2} = \frac{\pi}{4}$ $a + b = \frac{\pi}{2}$ $a = \frac{\pi}{2} - b$	M1 A1
9iii	$\frac{\pi}{8} < x < \frac{3\pi}{8}$	B1
10i	$5\tan^2\theta + 24\tan\theta - 5 = 0$ $(5\tan\theta - 1)(\tan\theta + 5) = 0$ $\tan\theta = \frac{1}{5}$ or $\tan\theta = -5$ $\theta = 0.1973955598$ or $\theta = -1.373400767$ $\therefore \theta = 0.197$ (3 sf) or $\theta = -1.37$ (3 sf)	M1 M1 A1 A1
10ii	$\tan 2\theta = \frac{2\tan\theta}{1-\tan^2\theta}$ $\frac{5}{12} = \frac{2\tan\theta}{1-\tan^2\theta}$ $5(1 - \tan^2\theta) = 12(2\tan\theta)$ $5 - 5\tan^2\theta = 24\tan\theta$ $5\tan^2\theta + 24\tan\theta - 5 = 0$ $\therefore$ The values in part (i) satisfy the equation $\tan 2\theta = \frac{5}{12}$.	M1 A1

Answers (Page 5)

Qn	Solutions	Marks
11i	Equation of tangent at $P(10, -10)$: $2y + x + 10 = 0$ $y = -\frac{1}{2}x - 5$ Gradient of normal = 2 Equation of normal at $P(10, -10)$: $y - (-10) = 2(x - 10)$ $y = 2x - 30$	M1 A1
11ii	$y^2 = 10x$ (1) $y = 2x - 30$ (2) Sub (2) into (1) $(2x - 30)^2 = 10x$ $4x^2 - 130x + 900 = 0$ $2x^2 - 65x + 450 = 0$ $(2x - 45)(x - 10) = 0$ $x = 22\frac{1}{2}$ or $x = 10$ (Reject) When $x = 22\frac{1}{2}$, $y = 2\left(22\frac{1}{2}\right) - 30$ $y = 15$ $\therefore B\left(22\frac{1}{2}, 15\right)$	M1 M1 A1
11iii		G1 Shape of curve and straight line G2 Points A and B labelled Condone if graphs are not labelled.

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Answers (Page 6)

Qn	Solutions	Marks
12i	$x^2-6x+2=0$ $\alpha+\beta=6$ $\alpha\beta=2$	 B1 B1
12ii	$\alpha^2+\beta^2=(\alpha+\beta)^2-2\alpha\beta$ $\alpha^2+\beta^2=(6)^2-2(2)$ $\alpha^2+\beta^2=32$	 B1
12iii	New roots $\frac{1}{\alpha^2}-\beta^2$ and $\frac{1}{\beta^2}-\alpha^2$	
	Sum of new roots $=\frac{1}{\alpha^2}-\beta^2+\frac{1}{\beta^2}-\alpha^2$ $=\frac{1}{\alpha^2}+\frac{1}{\beta^2}-\alpha^2-\beta^2$ $=\frac{\alpha^2+\beta^2}{(\alpha\beta)^2}-(\alpha^2+\beta^2)$	 M1
	$=\frac{32}{(2)^2}-32$ $=-24$	 A1
	Product of new roots $=\left(\frac{1}{\alpha^2}-\beta^2\right)\left(\frac{1}{\beta^2}-\alpha^2\right)$ $=\frac{1}{(\alpha\beta)^2}-1-1+(\alpha\beta)^2$	 M1
	$=\frac{1}{(2)^2}-1-1+(2)^2$ $=\frac{9}{4}$	 A1
	New equation $x^2-(-24)x+\left(\frac{9}{4}\right)=0$ $4x^2+96x+9=0$	 A1

Answers (Page 7)

<table>
<tr><th>Qn</th><th>Solutions</th><th>Marks</th></tr>
<tr><td>13a</td><td>$f(x)=\frac{1}{3}x^3+kx$
$f'(x)=x^2+k$
To find stationary points, we let $f'(x)=0$.
$x^2+k=0$
$x^2=-k$
Since there are no stationary points,
$\therefore k>0$</td><td>
M1

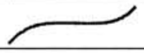
A1</td></tr>
<tr><td>13bi</td><td>If $k=-4$,
$f(x)=\frac{1}{3}x^3-4x$
$f'(x)=x^2-4$
To find stationary points, we let $f'(x)=0$.
$x^2-4=0$
$(x+2)(x-2)=0$
Hence there are 2 stationary points.</td><td>

B1</td></tr>
<tr><td>13bii</td><td>$(x+2)(x-2)=0$
$\therefore x=-2$ or $x=2$

When $x=-2$,
$f(-2)=\frac{1}{3}(-2)^3-4(-2)$
$f(-2)=5\frac{1}{3}$ $\therefore\left(-2,5\frac{1}{3}\right)$

When $x=2$,
$f(2)=\frac{1}{3}(2)^3-4(2)$
$f(2)=-5\frac{1}{3}$ $\therefore\left(2,-5\frac{1}{3}\right)$</td><td>

B1

B1</td></tr>
<tr><td>13ci</td><td>To have exactly one stationary point,
$k=0$</td><td>B1</td></tr>
<tr><td>13cii</td><td>
<table>
<tr><td>x</td><td>-0.1</td><td>0</td><td>0.1</td></tr>
<tr><td>$f'(x)$</td><td>+</td><td>0</td><td>+</td></tr>
<tr><td>gradient</td><td>/</td><td>---</td><td>/</td></tr>
<tr><td>shape of curve</td><td colspan="3"></td></tr>
</table>
There is a stationary point of inflexion at the point (0, 0).</td><td>Accept any reasonable working step

M1

A1</td></tr>
</table>

Bukit Batok Secondary School

Mathematical Formulae

1. ALGEBRA

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Bukit Batok Secondary School

Preliminary Examination 2016

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ADDITIONAL MATHEMATICS **4044/01**

Paper 2

03 August 2016

1000 – 1145

1 hour 45 minutes

Additional Materials: 8 pieces of writing paper

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The use of an approved calculator is expected, where appropriate.
You are reminded of the need for clear presentation in your answers.

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$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Binomial Expansion

$$(a+b)^n = a^n + \binom{n}{1}a^{n-1}b + \binom{n}{2}a^{n-2}b^2 + \cdots + \binom{n}{r}a^{n-r}b^r + \cdots + b^n,$$

where n is a positive integer and $\binom{n}{r} = \frac{n!}{(n-r)!\,r!} = \frac{n(n-1)\ldots(n-r+1)}{r!}$

2. Trigonometry

Identities

$$\sin^2 A + \cos^2 A = 1$$

$$\sec^2 A = 1 + \tan^2 A$$

$$\operatorname{cosec}^2 A = 1 + \cot^2 A$$

$$\sin(A \pm B) = \sin A\cos B \pm \cos A\sin B$$

$$\cos(A \pm B) = \cos A\cos B \mp \sin A\sin B$$

$$\tan(A \pm B) = \frac{\tan A \pm \tan B}{1 \mp \tan A\tan B}$$

$$\sin 2A = 2\sin A\cos A$$

$$\cos 2A = \cos^2 A - \sin^2 A = 2\cos^2 A - 1 = 1 - 2\sin^2 A$$

$$\tan 2A = \frac{2\tan A}{1 - \tan^2 A}$$

Formulae for ΔABC

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$a^2 = b^2 + c^2 - 2bc\cos A$$

$$\Delta = \frac{1}{2}ab\sin C$$

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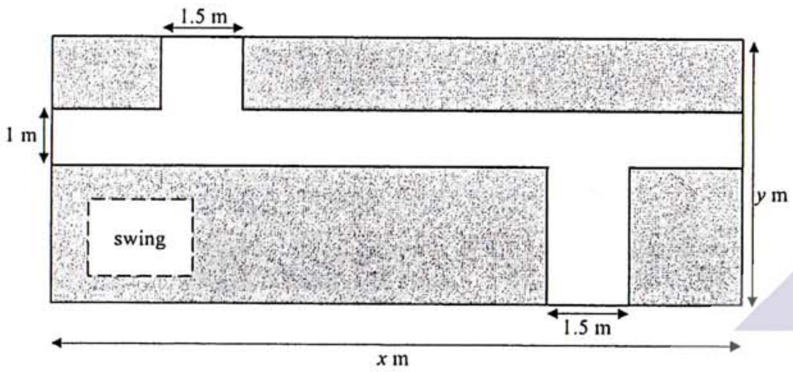
1 Find the gradient of the tangent to the curve $y = \frac{\sqrt{2x-1}}{3x^2-5}$ at the point where $x = 1$. [3]

2 (i) Write 9^{x-1} as a power of 3. [1]

(ii) Hence, or otherwise, find the value of x for which $9^{x-1} = 27 \times 3^{4x+5}$. [3]

3



The diagram (not drawn to scale) shows a rectangular courtyard x m long and y m wide located outside Raffles Place MRT Station. It contains a walking path and a swing. The shaded area represents the grass grown in the courtyard and the non-shaded area represents the walking path. The swing occupies an area of 4 m².

(i) Given that the total area of grass in the courtyard is 249.5 m², show that $2xy - 2x - 3y = 504$. [3]

(ii) Given also that the perimeter of the courtyard is 70 m, calculate the dimensions of the courtyard. [4]

4 (i) Without using a calculator, express $\frac{3-\sqrt{2}}{1+\sqrt{2}}$ in the form $a + b\sqrt{2}$, where a and b are integers. [3]

(ii) Hence, or otherwise, find the values of the integers p and q for which $\frac{3-\sqrt{2}}{1+\sqrt{2}} = (p+\sqrt{2})(q-\sqrt{2})$. [5]

5 (a) Given that $y = \frac{x^3}{x^2+4}$, find $\frac{dy}{dx}$ and explain why y is an increasing function. [4]

(b) Given that $y = 4x(x-2)^3$, find $\frac{dy}{dx}$ and the set of values of x for which y is decreasing. [4]

6 Angles A and B are such that $2\sin(A-B) = 3\cos(A+B)$.

(i) Show that $\tan A = \frac{2\tan B+3}{3\tan B+2}$. [4]

(ii) Given also that $B = \frac{\pi}{6}$, find, without using a calculator, $\tan A$ in the form $a - b\sqrt{3}$, where a and b are rational numbers. [4]

7 The equation of a circle is $x^2 + y^2 - 6x - 8y = 0$.

(i) Find the coordinates of the centre of the circle and determine the length of the radius. [4]

(ii) The point $A(6, 8)$ lies on the circle. Find the coordinates of the point B such that AB is the diameter of the circle. [2]

(iii) The line $3x + 4y = 50$ meets the circle at a point C. Explain why this line is a tangent to the circle. [4]

BBSS Sec 4 NA Additional Mathematics
Answer and Marking Scheme

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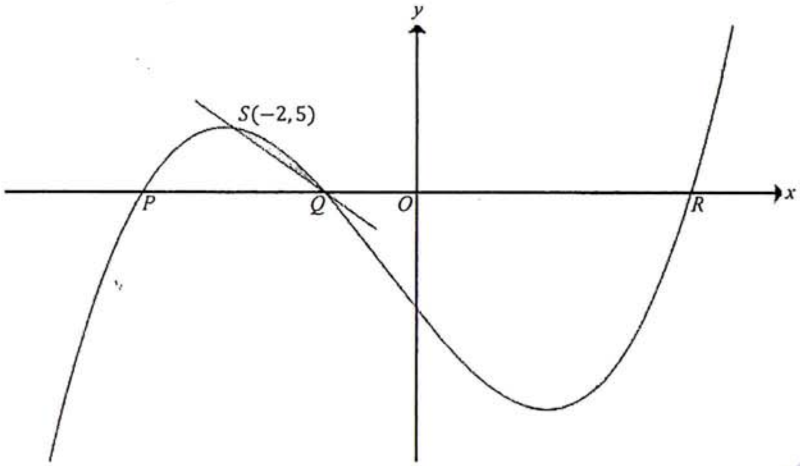
8



The diagram shows part of the curve $y = x^3 + ax^2 - 9x - 9$, where a is an integer.
Given that $x + 1$ is a factor of $x^3 + ax^2 - 9x - 9$,

(i) show that the value of a is 1, [2]

(ii) solve the equation $x^3 + x^2 - 9x - 9 = 0$ and hence state the coordinates of the points P, Q and R, where the curve meets the x-axis. [4]

A line through Q meets the curve again at the point $S(-2, 5)$.

(iii) Find the area of the shaded region between the curve and the line QS. [5]

9 (a) (i) Show that the equation $2\operatorname{cosec}^2 x + 7\cot x = 4\cot^2 x + 5$ may be written in the form $2\cot^2 x - 7\cot x + 3 = 0$. [1]

(ii) Hence solve the equation $2\operatorname{cosec}^2 x + 7\cot x = 4\cot^2 x + 5$, for $0° \leq x \leq 180°$. [4]

(b) (i) Express $4\cos\theta - 3\sin\theta$ in the form $R\cos(\theta + \alpha)$, where $R > 0$ and $0 < \alpha < \frac{\pi}{2}$ radians. [3]

(ii) The height, h metres, of a water wave at Sentosa Wave Pool is given by $h = 4\cos t - 3\sin t$, where t is the time in seconds after the start of the artificial wave generator.
After how many seconds does the wave reach a height of 4 metres again? [3]

- End of Paper -

BBSS Sec 4 NA Additional Mathematics
Answer and Marking Scheme

Answers (Page 1)

Qn	Solutions	Marks
1	$y = \frac{\sqrt{2x-1}}{3x^2-5} = \frac{(2x-1)^{\frac{1}{2}}}{3x^2-5}$	
	$\frac{dy}{dx} = \frac{(3x^2-5)\left[\frac{1}{2}(2x-1)^{-\frac{1}{2}}(2)\right]-(2x-1)^{\frac{1}{2}}(6x)}{(3x^2-5)^2}$	M1
	When $x = 1$,	
	$\frac{dy}{dx} = \frac{(3-5)\left[\frac{1}{2}(1)^{-\frac{1}{2}}(2)\right]-(1)^{\frac{1}{2}}(6)}{(3-5)^2}$	M1
	$\frac{dy}{dx} = \frac{-8}{4}$	
	$\frac{dy}{dx} = -2$	A1
2i	$9^{x-1} = 3^{2(x-1)}$	
	$= 3^{2x-2}$	B1
2ii	$9^{x-1} = 27 \times 3^{4x+5}$	
	$3^{2x-2} = 3^3 \times 3^{4x+5}$	M1
	$2x - 2 = 3 + 4x + 5$	M1
	$2x = -10$	
	$x = -5$	A1
3i	$xy - x(1) - y(1.5) + 1.5(1) - 4 = 249.5$	M1
	$xy - x - 1.5y = 252$	M1
	$2xy - 2x - 3y = 504$	A1
3ii	$2xy - 2x - 3y = 504$ (1)	
	$2x + 2y = 70$ (2)	
	$y = 35 - x$ (3)	M1
	Sub (3) into (1)	
	$2x(35 - x) - 2x - 3(35 - x) = 504$	
	$70x - 2x^2 - 2x - 105 + 3x = 504$	
	$2x^2 - 71x + 609 = 0$	
	$(2x - 29)(x - 21) = 0$	
	$\therefore x = 14.5$ or $x = 21$	A1
	When $x = 14.5$, $y = 35 - 14.5$ $y = 20.5$ (Reject, since $x > y$)	M1
	When $x = 21$, $y = 35 - 21$ $y = 14$	
	$\therefore$ Dimensions of the courtyard are 21 m × 14 m.	A1

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Answers (Page 2)

Qn	Solutions	Marks
4i	$\frac{3-\sqrt{2}}{1+\sqrt{2}} = \frac{3-\sqrt{2}}{1+\sqrt{2}} \times \frac{1-\sqrt{2}}{1-\sqrt{2}}$ $= \frac{3-\sqrt{2}-3\sqrt{2}+2}{1-2}$ $= \frac{5-4\sqrt{2}}{-1}$ $= -5 + 4\sqrt{2}$	M1 M1 A1
4ii	$\frac{3-\sqrt{2}}{1+\sqrt{2}} = (p+\sqrt{2})(q-\sqrt{2})$ $-5 + 4\sqrt{2} = (pq-2) + (q-p)\sqrt{2}$ By comparison, $pq - 2 = -5$ $pq = -3$ (1) $q - p = 4$ $p = q - 4$ (2) Sub (2) into (1), $q(q-4) = -3$ $q^2 - 4q + 3 = 0$ $(q-1)(q-3) = 0$ $\therefore q = 1$ or $q = 3$ When $q = 1$, $p = -3$ When $q = 3$, $p = -1$	M1 M1 A1 A1 A1
5a	$y = \frac{x^3}{x^2+4}$ $\frac{dy}{dx} = \frac{(x^2+4)(3x^2)-(x^3)(2x)}{(x^2+4)^2}$ $= \frac{3x^4+12x^2-2x^4}{(x^2+4)^2}$ $= \frac{x^4+12x^2}{(x^2+4)^2}$ $x^4 > 0$ and $12x^2 > 0, \therefore x^4 + 12x^2 > 0$. Also, $(x^2+4)^2 > 0$ Hence, $\frac{x^4+12x^2}{(x^2+4)^2} > 0$ and therefore y is an increasing function.	M1 A1 M1 A1
5b	$y = 4x(x-2)^3$ $\frac{dy}{dx} = 4x(3)(x-2)^2(1) + (x-2)^3(4)$ $\frac{dy}{dx} = (x-2)^2[12x + 4(x-2)]$ $\frac{dy}{dx} = (x-2)^2(12x + 4x - 8)$ $\frac{dy}{dx} = 8(2x-1)(x-2)^2$ For y to be decreasing, $\frac{dy}{dx} < 0$ $8(2x-1)(x-2)^2 < 0$ $(2x-1) < 0$ since $8(x-2)^2 > 0$ $\therefore x < \frac{1}{2}$	M1 A1 M1 A1

Answers (Page 3)

Qn	Solutions		Marks
6i	$2\sin(A-B) = 3\cos(A+B)$ $2(\sin A\cos B - \cos A\sin B) = 3(\cos A\cos B - \sin A\sin B)$ $2\sin A\cos B - 2\cos A\sin B = 3\cos A\cos B - 3\sin A\sin B$ Divide throughout by $\cos A\cos B$, $2\tan A - 2\tan B = 3 - 3\tan A\tan B$ $2\tan A + 3\tan A\tan B = 2\tan B + 3$ $\tan A(2 + 3\tan B) = 2\tan B + 3$ $\tan A = \frac{2\tan B+3}{3\tan B+2}$ (shown)		M1 M1 M1 A1
6ii	Either $\tan A = \frac{2\tan\left(\frac{\pi}{6}\right)+3}{3\tan\left(\frac{\pi}{6}\right)+2}$ $= \frac{2\left(\frac{\sqrt{3}}{3}\right)+3}{3\left(\frac{\sqrt{3}}{3}\right)+2}$ $= \frac{\frac{2\sqrt{3}+9}{3}}{\sqrt{3}+2}$ $= \frac{2\sqrt{3}+9}{3\sqrt{3}+6} \times \frac{3\sqrt{3}-6}{3\sqrt{3}-6}$ $= \frac{18-12\sqrt{3}+27\sqrt{3}-54}{27-36}$ $= \frac{-36+15\sqrt{3}}{-9}$ $= 4 - \frac{5}{3}\sqrt{3}$	Or $\tan A = \frac{2\tan\left(\frac{\pi}{6}\right)+3}{3\tan\left(\frac{\pi}{6}\right)+2}$ $= \frac{2\left(\frac{1}{\sqrt{3}}\right)+3}{3\left(\frac{1}{\sqrt{3}}\right)+2}$ $= \frac{\frac{2+3\sqrt{3}}{\sqrt{3}}}{\frac{3+2\sqrt{3}}{\sqrt{3}}}$ $= \frac{2+3\sqrt{3}}{3+2\sqrt{3}} \times \frac{3-2\sqrt{3}}{3-2\sqrt{3}}$ $= \frac{6-4\sqrt{3}+9\sqrt{3}-18}{9-12}$ $= \frac{5\sqrt{3}-12}{-3}$ $= 4 - \frac{5}{3}\sqrt{3}$	M1 M1 M1 A1

Answers (Page 4)

Qn	Solutions		Marks
	Either	OR	

Answers (Page 5)

Qn	Solutions	Marks

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Answers (Page 4)

<table>
<tr><th>Qn</th><th colspan="2">Solutions</th><th>Marks</th></tr>
<tr><td>7i</td><td>Either
$x^2+y^2-6x-8y=0$
$(x-3)^2+9+(y-4)^2+16=0$
$(x-3)^2+(y-4)^2=5^2$
Coordinates of the centre of circle
$=(3,4)$
Radius
$=5$ units</td><td>OR
$x^2+y^2-6x-8y=0$
$x^2+y^2+2(-3)x+2(-4)y=0$
$g=-3,\ f=-4$
Coordinates of the centre of circle
$=(3,4)$
Radius
$=\sqrt{(-3)^2+(-4)^2}$
$=5$ units</td><td>M1
M1
A1
A1</td></tr>
<tr><td>7ii</td><td colspan="2">Midpoint of AB = coordinates of the centre of circle.
Let the coordinates of $B=(x,y)$
$\left(\frac{x+6}{2},\ \frac{y+8}{2}\right)=(3,4)$
$x+6=6$
$x=0$
$y+8=8$
$y=0$
$\therefore B(0,0)$</td><td>M1
A1</td></tr>
<tr><td>7iii</td><td colspan="2">$x^2+y^2-6x-8y=0$ (1)
$3x+4y=50$ (2)
$y=\frac{50-3x}{4}$ (3)
Sub (3) into (1)
$x^2+\left(\frac{50-3x}{4}\right)^2-6x-8\left(\frac{50-3x}{4}\right)=0$
$x^2+\frac{1}{16}(2500-300x+9x^2)-6x-100+6x=0$
$16x^2+2500-300x+9x^2-1600=0$
$25x^2-300x+900=0$
$x^2-12x+36=0$
Discriminant, b^2-4ac
$=(-12)^2-4(1)(36)$
$=0$
$\therefore$ The line $3x+4y=50$ is a tangent to the circle at C.</td><td>M1
M1
A1
A1</td></tr>
</table>

Answers (Page 5)

Qn	Solutions	Marks
8i	Let $f(x)=x^3+ax^2-9x-9$ $f(-1)=(-1)^3+a(-1)^2-9(-1)-9$ $0=-1+a$ $\therefore a=1$	M1 A1
8ii	$x^3+x^2-9x-9=(x+1)(x^2+bx-9)$ Equating coefficients of x^2, $1=1+b$ $\therefore b=0$ $x^3+x^2-9x-9=(x+1)(x^2-9)$ $=(x+1)(x+3)(x-3)$ $x^3+x^2-9x-9=0$ $(x+3)(x+1)(x-3)=0$ $\therefore x=-3,\ -1$ or 3 $\therefore$ Coordinates: $P(-3,\ 0)$, $Q(-1,\ 0)$ and $R(3,\ 0)$	M1 M1 A1 A1
8iii	$Q(-1,\ 0)$, $S(-2,\ 5)$ Gradient $QS=\frac{5-0}{-2-(-1)}=-5$ Equation of line QS: $y-0=-5(x+1)$ $\therefore y=-5x-5$ Area of shaded region $=\int_{-2}^{-1}(x^3+x^2-9x-9)-(-5x-5)\,dx$ $=\int_{-2}^{-1}(x^3+x^2-4x-4)\,dx$ $=\left[\frac{x^4}{4}+\frac{x^3}{3}-2x^2-4x\right]_{-2}^{-1}$ $=\left(\frac{1}{4}-\frac{1}{3}-2+4\right)-\left(4-\frac{8}{3}-8+8\right)$ $=\frac{7}{12}$ units^2	Allow ECF M1 A1 M1 M1 A1

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Answers (Page 6)

Qn	Solutions	Marks
9ai	$2\operatorname{cosec}^2 x + 7\cot x = 4\cot^2 x + 5$ $2(1 + \cot^2 x) + 7\cot x = 4\cot^2 x + 5$ $2 + 2\cot^2 x + 7\cot x - 4\cot^2 x - 5 = 0$ $2\cot^2 x - 7\cot x + 3 = 0$	A1
9aii	$2\cot^2 x - 7\cot x + 3 = 0$ $(2\cot x - 1)(\cot x - 3) = 0$ $\cot x = \frac{1}{2}$ or $\cot x = 3$ $\tan x = 2$ or $\tan x = \frac{1}{3}$ $\alpha = 63.4349°$ or $\alpha = 18.4349°$ $x = 63.4349°$ or $x = 18.4349°$ $x = 63.4°$ or $x = 18.4°$ (1 dp)	M1 A1 A1 A1
9bi	$4\cos\theta - 3\sin\theta$ $R = \sqrt{4^2 + 3^2}$ $R = 5$ $\tan\alpha = \frac{3}{4}$ $\alpha = 0.6435$ rad $4\cos\theta - 3\sin\theta = 5\cos(\theta + 0.644)$	M1 M1 A1
9bii	$h = 4\cos t - 3\sin t$ $h = 5\cos(t + 0.6435)$ When $h = 4$, $4 = 5\cos(t + 0.6435)$ $\cos(t + 0.643) = \frac{4}{5}$ $\alpha = 0.6435$ $t + 0.6435 = 2\pi - 0.6435$ $t = 4.996185$ $t = 5.00$ seconds (3sf)	M1 M1 A1

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EAST VIEW SECONDARY SCHOOL
PRELIMINARY EXAMINATION 2016
SECONDARY FOUR NORMAL ACADEMIC

CANDIDATE NAME				
CLASS		INDEX NUMBER		

ADDITIONAL MATHEMATICS **4044/01**

Paper 1 **15 July 2016**

Total Marks: 70 **1 Hour 45 Minutes**

Additional Materials: Writing Paper (7 sheets)

READ THESE INSTRUCTIONS FIRST

Write your name, class and index number on all your answer sheets to be handed in.
Write in dark blue or black pen.
You may use a soft pencil for any diagrams, graphs or rough working.
Do not use staples, paper clips, highlighters, glue or correction fluid.

Answer **all** the questions.
Write your answers on the separate writing paper provided.
Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.
The use of a scientific calculator is expected, where appropriate.
You are reminded of the need for clear presentation in your answers.

At the end of the examination, fasten all your work securely together.
The number of marks is given in brackets [ ] at the end of each question or part question.

For Examiner's Use
70

This paper consists of <u>5</u> printed pages (including the cover page).

Setter: Miss Kong T. X.

Mathematical Formulae

1. ALGEBRA

Quadratic Equation

For the equation $ax^2 + bx + c = 0$,

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}.$$

Binomial Expansion

$$(a+b)^n = a^n + \binom{n}{1}a^{n-1}b + \binom{n}{2}a^{n-2}b^2 + \ldots + \binom{n}{r}a^{n-r}b^r + \ldots + b^n,$$

where n is a positive integer and $\binom{n}{r} = \frac{n!}{r!(n-r)!} = \frac{n(n-1)\ldots(n-r+1)}{r!}$

2. TRIGONOMETRY

Identities

$$\sin^2 A + \cos^2 A = 1$$
$$\sec^2 A = 1 + \tan^2 A$$
$$\operatorname{cosec}^2 A = 1 + \cot^2 A$$
$$\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$$
$$\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$$
$$\tan(A \pm B) = \frac{\tan A \pm \tan B}{1 \mp \tan A \tan B}$$
$$\sin 2A = 2\sin A \cos A$$
$$\cos 2A = \cos^2 A - \sin^2 A = 2\cos^2 A - 1 = 1 - 2\sin^2 A$$
$$\tan 2A = \frac{2\tan A}{1 - \tan^2 A}$$

Formulae for ΔABC

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$
$$a^2 = b^2 + c^2 - 2bc\cos A.$$
$$\Delta = \frac{1}{2}bc\sin A.$$

---

1 The function f is defined by $f(x) = ax^3 + 2ax^2 - 15x + b$, where a and b are constants. It is given that $f(x)$ is divisible by $x + 3$ but leaves a remainder of -12 when divided by $x - 1$.

---

6 Given that $\tan A = -\frac{3}{4}$ and $\cos B = -\frac{5}{13}$ such that both A and B lie in the same quadrant.

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1 The function f is defined by $f(x) = ax^3 + 2ax^2 - 15x + b$, where a and b are constants. It is given that f(x) is divisible by $x + 3$ but leaves a remainder of -12 when divided by $x - 1$.

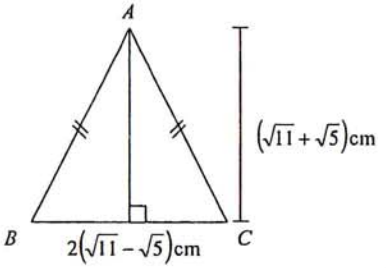
(i) Find the values of a and b. [4]

(ii) Solve $f(x) = 0$. [4]

2 (i) Find $\int \left(2x^2 - \frac{4}{x^3} + 3\right) dx$. [2]

(ii) Hence, evaluate $\int_1^3 \left(2x^2 - \frac{4}{x^3} + 3\right) dx$. [2]

3



In the diagram, $AB = AC$, $AD = (\sqrt{11} + \sqrt{5})$cm, $BC = 2(\sqrt{11} - \sqrt{5})$cm and $\angle ADC = 90°$.

Find,

(i) the length of AC in in the form $m\sqrt{n}$, where m and n are integers. [3]

(ii) Express $\tan \angle ACB$ in the form $a + b\sqrt{5}$ where a and b are integers. [4]

4 (a) Given the equation $9x^2 - 6mx + m^2 = 0$.

(i) Show that the equation has real and equal roots for all values of m. [2]

(ii) Find the value of x when $m = 2$. [1]

(b) Given that $Ax^3 - 5x^2 + 2x + 2 = (x+1)(x-2)(2x-B) + Cx + D$ for all values of x, where A, B, C and D are constants, find the values of A, B, C and D. [5]

5 (i) Find the equation of the tangent to the curve $y = x^2 + 4x - 5$ at the point $(1, 0)$. [3]

(ii) The term independent of x in the expansion of $\left(px - \frac{1}{2x^2}\right)^9$ is -672.

Find the value of p, where $p > 0$. [3]

6 Given that $\tan A = -\frac{3}{4}$ and $\cos B = -\frac{5}{13}$ such that both A and B lie in the same quadrant.

Find the exact value of each of the following without the use of a calculator.

(i) $\sin(A + B)$, [3]

(ii) $\cos(B - 30°)$. [2]

7 A circle, C_1 passes through the x- axis at two points (7, 0) and (-3, 0). Given that the line $y + 5x = 11$ passes through the centre of C_1.

(i) Find the centre of C_1. [2]

(ii) Hence, find the equation of C_1. [2]

8 A voltage V (in volts) varies with time t (in milliseconds) such that $V = 9\sin t + 2\cos t$, where $t \geq 0$.

(i) Express V in the form $R\cos(t - \alpha)$, where R>0 and $0 < \alpha < \frac{\pi}{2}$. [3]

(ii) Find the maximum voltage and state the time needed for the voltage to reach its first maximum value. [2]

(iii) Find the first instant, t_1 when $V = 1$. [2]

9 (a) Express $y = 4x^2 - 16x + 9$ in the form $y = a(x-h)^2 + k$ where a, h and k are constants. Hence, state the minimum value of y. [4]

(b) The area, A cm², of a quadrant is given by $A = \frac{\pi}{8}r^2$ where r cm is the radius of the quadrant. If the area of the quadrant is increasing at a constant rate of 9π cm²/s, find the value of r when the radius is increasing at 0.5 cm/s. [4]

10 (i) Given that $y = \frac{4x}{x+3}$, find $\frac{dy}{dx}$. [2]

(ii) Hence find $\int \left(\frac{2}{x+3}\right)^2 dx$. [2]

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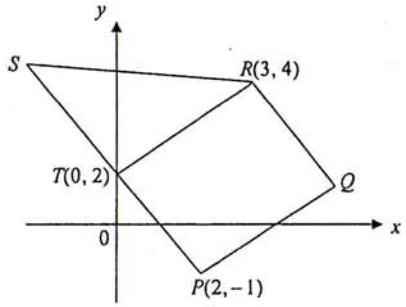
11 Solutions to this question by accurate drawing will not be accepted.

The diagram shows a trapezium $PQRS$ where the coordinates of P, T and R are $(2,-1)$, $(0,2)$ and $(3,4)$ respectively. T is a point on the y-axis such that $PQRT$ is a parallelogram.



(i) Show that the coordinates of Q are $(5, 1)$. [2]

(ii) Find the equation of line TQ. [2]

(iii) Find the coordinates of S, given that $ST : TP$ is $4 : 5$. [2]

(iv) Stating your reason clearly, conclude if RS is parallel to the x-axis. [1]

(v) Find the area of parallelogram $PQRT$. [2]

End of Paper

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EAST VIEW SECONDARY SCHOOL
PRELIMINARY EXAMINATION 2016
SECONDARY FOUR NORMAL ACADEMIC

CANDIDATE NAME				
CLASS		**INDEX NUMBER**		

ADDITIONAL MATHEMATICS

4044/01

Paper 1

XX Jul 2016

Total Marks: 70

1 Hour 45 Minutes

Additional Materials: Writing Paper (7 sheets)

READ THESE INSTRUCTIONS FIRST

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At the end of the examination, fasten all your work securely together.
The number of marks is given in brackets [ ] at the end of each question or part question.

For Examiner's Use

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Setter: Miss Kong T. X.

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1 The function f is defined by $f(x)=ax^3+2ax^2-15x+b$, where a and b are constants. It is given that f(x) is divisible by $x+3$ but leaves a remainder of -12 when divided by $x-1$.

(i) Find the values of a and b. [4]

(ii) Solve $f(x)=0$. [4]

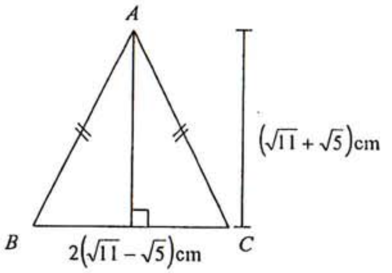
Qn	Solution	MS	Marker's Report
1(i)	$f(-3)=0$ $-27a+18a+45+b=0$ $-9a+b=-45------(1)$	[M1]	
	$f(1)=-12$ $a+2a-15+b=-12$ $3a+b=3-------(2)$	[M1]	
	Solve (1)&(2) simultaneous equation $a=4, b=-9$	[A2]	
1(ii)	By long division, $(x+3)(4x^2-4x-3)=0$ $(x+3)(2x+1)(2x-3)=0$ $x=-3,-0.5,1.5$	[M1] [M1] [M1] [A1]	

2 (i) Find $\int\left(2x^2-\frac{4}{x^3}+3\right)dx$. [2]

(ii) Hence, evaluate $\int_1^3\left(2x^2-\frac{4}{x^3}+3\right)dx$. [2]

Qn	Solution	MS	Marker's Report
2(i)	$\frac{2x^3}{3}+\frac{2}{x^2}+3x+C$	[B2]	
2(ii)	$\int_1^3\left(2x^2-\frac{4}{x^3}+3\right)dx=\left[\frac{2x^3}{3}+\frac{2}{x^2}+3x\right]_1^3$		
	$=\left(18+\frac{2}{9}+9\right)-\left(\frac{2}{3}+2+3\right)$	[M1]	
	$=21\frac{5}{9}$	[A1]	

3



In the diagram, $AB=AC$, $AD=\left(\sqrt{11}+\sqrt{5}\right)$cm, $BC=2\left(\sqrt{11}-\sqrt{5}\right)$cm and $\angle ADC=90°$. Find,

(i) the length of AC in in the form $m\sqrt{n}$, where m and n are integers. [3]

(ii) Express $\tan\angle ACB$ in the form $a+b\sqrt{5}$ where a and b are integers. [4]

Qn	Solution	MS	Marker's Report
3(i)	$AC^2=\left(\sqrt{11}-\sqrt{5}\right)^2+\left(\sqrt{11}+\sqrt{5}\right)^2$	[M1]	
	$=\left(11-2\sqrt{11}\sqrt{5}+5\right)+\left(11+2\sqrt{11}\sqrt{5}+5\right)$ $=\left(16-2\sqrt{55}\right)+\left(16+2\sqrt{55}\right)$		
	$=32\text{ cm}^2$	[M1]	
	$AC=4\sqrt{2}$ cm	[A1]	
3(ii)	$\tan\angle ACB=\frac{\sqrt{11}+\sqrt{5}}{\sqrt{11}-\sqrt{5}}$	[M1]	
	$=\frac{\sqrt{11}+\sqrt{5}}{\sqrt{11}-\sqrt{5}}\times\frac{\sqrt{11}+\sqrt{5}}{\sqrt{11}+\sqrt{5}}$	[M1]	
	$=\frac{11+2\sqrt{55}+5}{11-5}$	[M1]	
	$=\frac{8}{3}+\frac{\sqrt{5}}{6}$	[A1]	

4 (a) Given the equation $9x^2-6mx+m^2=0$.

(i) Show that the equation has real and equal roots for all values of m. [2]

5 (i) Find the equation of the tangent to the curve $y=x^2+4x-5$ at the point (1, 0). [3]

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4 (a) Given the equation $9x^2 - 6mx + m^2 = 0$.

(i) Show that the equation has real and equal roots for all values of m. [2]

(ii) Find the value of x when $m = 2$. [1]

(b) Given that $Ax^3 - 5x^2 + 2x + 2 = (x+1)(x-2)(2x-B) + Cx + D$ for all values of x, where A, B, C and D are constants, find the values of A, B, C and D. [5]

Qn	Solution	MS	Marker's Report
4(a)(i)	$9x^2 - 6mx + m^2 = 0$ $b^2 - 4ac$ $= (-6m)^2 - 4(9)(m^2)$ $= 36m^2 - 36m^2$ $= 0$ Since $b^2 - 4ac = 0$, the roots are real and equal.	[M1] [A1]	
4(a)(ii)	$9x^2 - 12x + 4 = 0$ $(3x-2)^2 = 0$ $x = \frac{2}{3}$	[B1]	
4(b)	By comparing coefficient $A = 2$ $B = 3$ $C = 3$ $D = -4$	[M1] [A1] [A1] [A1] [A1]	

5 (i) Find the equation of the tangent to the curve $y = x^2 + 4x - 5$ at the point $(1, 0)$. [3]

(ii) The term independent of x in the expansion of $\left(px - \frac{1}{2x^2}\right)^9$ is -672.

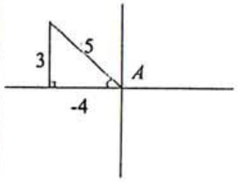
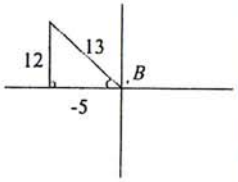
Find the value of p, where $p > 0$. [3]

Qn	Solution	MS	Marker's Report
5(i)	$y = x^2 + 4x - 5$ $\frac{dy}{dx} = 2x + 4$ When $x = 1$, $y = 0$ $\frac{dy}{dx} = 6$ At $(1,0)$, $0 = 6(1) + c$ $c = -6$ Equation of tangent: $y = 6x - 6$		
5(ii)	$T_{r+1} = {}^9C_r p^{9-r}\left(-\frac{1}{2}\right)^r x^{9-3r}$ When r = 3, $p = \pm 2$ Since $p > 0$, $p = 2$	[M1] [M1] [A1]	

6 Given that $\tan A = -\frac{3}{4}$ and $\cos B = -\frac{5}{13}$ such that both A and B lie in the same quadrant. Find the exact value of each of the following without the use of a calculator.

(i) $\sin(A+B)$, [3]

(ii) $\cos(B - 30°)$. [2]

Qn	Solution	MS	Marker's Report
6(i)	$\tan A = -\frac{3}{4}, \cos B = -\frac{5}{13}$  		

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	$\sin(A+B)=\left(\frac{3}{5}\right)\left(-\frac{5}{13}\right)+\left(-\frac{4}{5}\right)\left(\frac{12}{13}\right)$ $=-\frac{63}{65}$		
6(ii)	$\cos(B-30°)=\cos B\cos 30+\sin B\sin 30$ $=\left(-\frac{5}{13}\right)\left(\frac{\sqrt{3}}{2}\right)+\left(\frac{12}{13}\right)\left(\frac{1}{2}\right)$ $=\frac{12-5\sqrt{3}}{26}$	[M1] [A1] for either sin 30° or cos 30° [A1]	

7 A circle, C_1 passes through the x- axis at two points (7, 0) and (-3, 0). Given that the line $y+5x=11$ passes through the centre of C_1.

(i) Find the centre of C_1. [2]

(ii) Hence, find the equation of C_1. [2]

Qn	Solution	MS	Marker's Report
7(i)	$\frac{-3+7}{2}=2$ Perpendicular bisector of the chord formed by (7, 0) and (-3, 0) is $x=2$ Solving $x=2$ and $y+5x=11$ simultaneously, $y+5(2)=11$ $y=1$ Centre of the circle is (2, 1)	[M1] [A1]	
7(ii)	Radius $=\sqrt{(7-2)^2+(0-1)^2}$ $=\sqrt{26}$ Equation of circle is $(x-2)^2+(y-1)^2=26$ Or $x^2+y^2-4x-2x-21=0$	[M1] [A1]	

8 A voltage V (in volts) varies with time t (in milliseconds) such that $V=9\sin t+2\cos t$, where $t\geq 0$.

(i) Express V in the form $R\cos(t-\alpha)$, where $R>0$ and $0<\alpha<\frac{\pi}{2}$. [3]

(ii) Find the maximum voltage and state the time needed for the voltage to reach its first maximum value. [2]

(iii) Find the first instant, t_1 when $V=1$. [2]

Qn	Solution	MS	Marker's Report
8(i)	9 sin t + 2 cos t = 2 cos t + 9 sin t Comparing 2 cos t + 9 sin t with a cos t + b sin t, $a=2$ and $b=9$. From the right-angled triangle, $R=\sqrt{2^2+9^2}$ $=\sqrt{85}$ and $\tan\alpha=\frac{9}{2}$ $\alpha\approx 1.352$ Hence 9 sin t + 2 cos t = $\sqrt{85}$ cos (t − 1.35).	[M1] [M1] [A1]	
8(ii)	9 sin t + 2 cos t = $\sqrt{85}$ cos (t − 1.35) $-1\leq$ cos (t − 1.35) ≤ 1 $-\sqrt{85}\leq\sqrt{85}$ cos (t − 1.35) $\leq\sqrt{85}$ Hence 9 sin t + 2 cos t has a maximum value of $\sqrt{85}$ when cos (t − 1.35) = 1, that is, when t − 1.35 = 0 t = 1.35ms	[B1] [M1] [A1]	
8(iii)	9 sin t + 2 cos t = 1 $\sqrt{85}$ cos (t − 1.35) = 1 cos (t − 1.35) = $\frac{\sqrt{85}}{85}$ basic angle ≈ 1.462 t − 1.35 = 1.462 t ≈ 2.812 Hence, first instance when V=1 is t= 2.81ms.	[M1] [A1]	

9 (a) Express $y=4x^2-16x+9$ in the form $y=a\,(x-h)^2+k$ where a, h and k are constants. Hence, state the minimum value of y. [4]

10 (i) Given that $y=\frac{4x}{x+3}$, find $\frac{dy}{dx}$. [2]

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9 (a) Express $y = 4x^2 - 16x + 9$ in the form $y = a(x-h)^2 + k$ where a, h and k are constants. Hence, state the minimum value of y. [4]

(b) The area, A cm², of a quadrant is given by $A = \frac{\pi}{8}r^2$ where r cm is the radius of the quadrant. If the area of the quadrant is increasing at a constant rate of 9π cm²/s, find the value of r when the radius is increasing at 0.5 cm/s. [4]

Qn	Solution	MS	Marker's Report
9(a)	$y = 4x^2 - 16x + 9$ $= 4\left(x^2 - 4x + \frac{9}{4}\right)$ $= 4\left[(x-2)^2 - 4 + \frac{9}{4}\right]$ $= 4\left[(x-2)^2 - \frac{7}{4}\right]$ $= 4(x-2)^2 - 7$ Min value: −7	Correct a,b,c 1m each [B1]	
9(b)	$\frac{dA}{dr} = \frac{\pi}{4}r$ Given $\frac{dA}{dt} = 9\pi$, $\frac{dr}{dt} = 0.5$ $\frac{dA}{dr} = \frac{dA}{dt} \times \frac{dt}{dr}$ $\frac{\pi r}{4} = 9\pi \times \frac{1}{0.5}$ $r = 72$	[M1] [M1] [M1] [A1]	

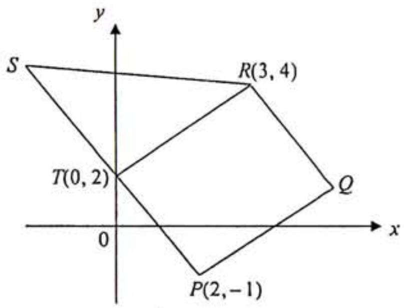
10 (i) Given that $y = \frac{4x}{x+3}$, find $\frac{dy}{dx}$. [2]

(ii) Hence find $\int \left(\frac{2}{x+3}\right)^2 dx$. [2]

Qn	Solution	MS	Marker's Report
10(i)	$\frac{dy}{dx} = \frac{(x+3)(4) - 4x(1)}{(x+3)^2}$ $= \frac{12}{(x+3)^2}$	[M1] [A1]	
10(ii)	$\int \left(\frac{2}{x+3}\right)^2 dx = \int \frac{4}{(x+3)^2} dx$ $= \frac{1}{3}\int \frac{12}{(x+3)^2} dx$ $= \frac{1}{3}\left(\frac{4x}{x+3}\right)$ $= \frac{4x}{3(x+3)}$	[M1] [A1]	

11 **Solutions to this question by accurate drawing will not be accepted.**

The diagram shows a trapezium $PQRS$ where the coordinates of P, T and R are $(2, -1)$, $(0, 2)$ and $(3, 4)$ respectively. T is a point on the y-axis such that $PQRT$ is a parallelogram.



(i) Show that the coordinates of Q are (5, 1). [2]

(ii) Find the equation of line TQ. [2]

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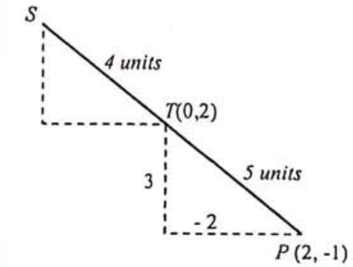
(iii) Find the coordinates of S, given that $ST : TP$ is 4 : 5. [2]

(iv) Stating your reason clearly, conclude if RS is parallel to the x-axis. [1]

(v) Find the area of parallelogram $PQRT$. [2]

Qn	Solution	MS	Marker's Report
11(i)	midpoint of PR $=\left(\frac{3+2}{2},\frac{4-1}{2}\right)$ $=\left(\frac{5}{2},\frac{3}{2}\right)$ Let Q be (x, y) $\left(\frac{0+x}{2},\frac{2+y}{2}\right)=\left(\frac{5}{2},\frac{3}{2}\right)$ $\frac{x}{2}=\frac{5}{2}$ and $\frac{2+y}{2}=\frac{3}{2}$ $x=5$ and $y=1$ $\therefore$ coordinates of $Q(5, 1)$ (shown)	[M1] [A1]	
11(ii)	gradient of TQ $=\frac{1-2}{5-0}$ $=-\frac{1}{5}$ equation of line TQ is $y=-\frac{1}{5}x+2$	[M1] [A1]	
11(iii)	 $S=\left(\left[0+\frac{4}{5}(-2)\right],\left[2+\frac{4}{5}(3)\right]\right)$ $=\left(-1\frac{3}{5},4\frac{2}{5}\right)$	[M1] [A1]	
11(iv)	No because y-coordinate of S is not 4, so SR is not horizontal OR y-coordinate of R is not $4\frac{2}{5}$, so SR is not horizontal OR gradient of $SR=\frac{4\frac{2}{5}-4}{-1\frac{3}{5}-3}$ $=-\frac{2}{23}$ $\neq 0$	[B1]	
11(v)	area of parallelogram $PQRT$ $=\frac{1}{2}\begin{vmatrix}3 & 5 & 2 & 0 & 3\\ 4 & 1 & -1 & 2 & 4\end{vmatrix}$ $=\frac{1}{2}\left\|\left[(3-5+4+0)-(6+0+2+20)\right]\right\|$ $=\frac{1}{2}\left\|\left[2-(28)\right]\right\|$ $=\frac{1}{2}\left\|-26\right\|$ $=13$ units^2	[M1] [A1]	

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	y-coordinate of S is not 4, so SR is not horizontal OR		

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ADDITIONAL MATHEMATICS 4044/02

Paper 2 21 July 2016

Total Marks: 70 1 Hour 45 Minutes

Additional Materials: Writing Paper (7 sheets)

READ THESE INSTRUCTIONS FIRST

Write your name, class and index number on all your answer sheets to be handed in.
Write in dark blue or black pen.
You may use a soft pencil for any diagrams, graphs or rough working.
Do not use staples, paper clips, highlighters, glue or correction fluid.

Answer **all** the questions.
Write your answers on the separate writing paper provided.
Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.
The use of a scientific calculator is expected, where appropriate.
You are reminded of the need for clear presentation in your answers.

At the end of the examination, fasten all your work securely together.
The number of marks is given in brackets [ ] at the end of each question or part question.

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Mathematical Formulae

1. ALGEBRA

Quadratic Equation

For the equation $ax^2 + bx + c = 0$,

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}.$$

Binomial Expansion

$$(a+b)^n = a^n + \binom{n}{1}a^{n-1}b + \binom{n}{2}a^{n-2}b^2 + \ldots + \binom{n}{r}a^{n-r}b^r + \ldots + b^n,$$

where n is a positive integer and $\binom{n}{r} = \frac{n!}{r!(n-r)!} = \frac{n(n-1)\ldots(n-r+1)}{r!}$

2. TRIGONOMETRY

Identities

$$\sin^2 A + \cos^2 A = 1$$
$$\sec^2 A = 1 + \tan^2 A$$
$$\operatorname{cosec}^2 A = 1 + \cot^2 A$$
$$\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$$
$$\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$$
$$\tan(A \pm B) = \frac{\tan A \pm \tan B}{1 \mp \tan A \tan B}$$
$$\sin 2A = 2 \sin A \cos A$$
$$\cos 2A = \cos^2 A - \sin^2 A = 2\cos^2 A - 1 = 1 - 2\sin^2 A$$
$$\tan 2A = \frac{2\tan A}{1 - \tan^2 A}$$

Formulae for ΔABC

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$
$$a^2 = b^2 + c^2 - 2bc\cos A.$$
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1 (i) Find the range of values of m for which the line $y = mx - 2$ intersects the curve $y = x^2 + 3x + 2$ at two distinct points. [3]

(ii) Find the set of values for which $(2x - 9)(x + 6) \leq -34$. [3]

2 (i) Show that the equation $\cos^2 x - \cos x = 2\sin^2 x$ can be written in the form

$$3\cos^2 x - \cos x - 2 = 0.$$ [1]

(ii) Hence solve the equation $\cos^2 x - \cos x = 2\sin^2 x$, for $0° \leq x \leq 360°$. [4]

3 (i) Evaluate $\dfrac{4 \times 2^{n-2}}{2^n - 2^{n-1}}$. [2]

(ii) By using the substitution $u = 3^x$ or otherwise, find the values of x such that

$$9^x - 12(3^x) + 27 = 0$$ [4]

4 (i) The first three terms in the expansion of $(2 + ax)^n$, where $a < 0$, in ascending powers of x, are $256 + px + 16128x^2 + ...$
Find the value of a, of p and of n [4]

(ii) Hence find the x^2 term in the expansion of $(1 + 2x)^2(2 + ax)^n$. [2]

5 The equation of a circle is $x^2 + y^2 + 4x - 8y + 15 = 0$.

(i) Find the radius and the coordinates of the centre of the circle. [2]

(ii) The circle is reflected in the y-axis. Find the equation of the new circle. [2]

6 The temperature, T °C, in a greenhouse at time t hours after midnight on a certain day is modelled by the formula $T = 25 + 2\sin(\frac{\pi t}{12})$ for $0 \leq t \leq 24$.

(i) Calculate the temperature, T °C, of the greenhouse when $t = 6$. [1]

(ii) Explain why this model suggests that the range of temperature, T °C, of the greenhouse is $23 \leq T \leq 27$. [2]

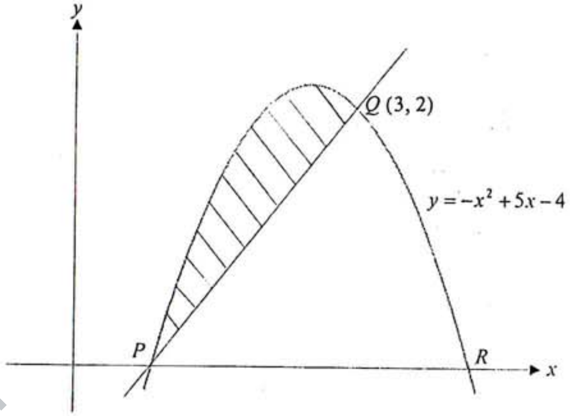
(iii) Tomato plants grow best when $T \leq 24$. Find the length of time for which the temperature of the greenhouse is suitable for the growth of tomato plants. [4]

7 The equation of a curve is $y = \frac{2}{3}x^3 - x^2 - 4x + 5$.

(i) Find the coordinates of the stationary points on the curve. [4]

(ii) Determine the nature of each of these stationary points. [3]

8



The diagram shows part of the curve $y = -x^2 + 5x - 4$. The normal to the curve at the point $Q(3, 2)$ meets the x axis at the point P. The curve $y = -x^2 + 5x - 4$ cuts the x-axis at points P and R. Find

(i) the coordinates of P and R, [3]

(ii) the area of the shaded region. [4]

9 (i) Find the coordinates of the points where the line $y = \frac{1}{2}x$ cuts the curve $y^2 = -4x$. [4]

(ii) Sketch the graphs of $y = \frac{1}{2}x$ and $y^2 = -4x$ on the same axes. Label your graphs and the points of intersection clearly. [3]

10 The roots of the equation $5x^2 - 2x + 4 = 0$ are α and β.

(i) State the value of $\alpha + \beta$ and $\alpha\beta$. [2]

(ii) Find the quadratic equation in x whose roots are α^2 and β^2. [4]

(iii) Find the value of $\alpha^3 + \beta^3$. [2]

11 (a) Show that $\dfrac{1 + \cos 2A + \cos A}{\sin 2A + \sin A} = \cot A$. [3]

(b) Given that $\dfrac{d^2y}{dx^2} = 4x - 3$ and the curve has a stationary point at $A\left(1, \dfrac{1}{3}\right)$, find the equation of the curve. [4]

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Mathematical Formulae

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CANDIDATE NAME				
CLASS		INDEX NUMBER		

ADDITIONAL MATHEMATICS 4044/02

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$$\sin(\mathrm{A} \pm \mathrm{B}) = \sin A \cos B \pm \cos A \sin B$$
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1 (i) Find the range of values of m for which the line $y = mx - 2$ intersects the curve $y = x^2 + 3x + 2$ at two distinct points. [3]

(ii) Find the set of values for which $(2x-9)(x+6) \le -34$. [3]

Qn	Solution	MS	Marker's Report
1(i)	$mx-2=x^2+3x+2$ $x^2+(3-m)x+4=0$ $a=1,\ b=3-m,\ c=4$ $(3-m)^2-4(1)(4)>0$ $m^2-6m-7>0$ $(m-7)(m+1)>0$ $m<-1,\ m>7$	[M1] [M1] [A1]	
1(ii)	$2x^2+12x-9x-54+34 \le 0$ $2x^2+3x-20 \le 0$ $(2x-5)(x+4) \le 0$ $-4 \le x \le \frac{5}{2}$	[M1] [M1] [A1]	

2 (i) Show that the equation $\cos^2 x - \cos x = 2\sin^2 x$ can be written in the form [1]

$$3\cos^2 x - \cos x - 2 = 0.$$

(ii) Hence solve the equation $\cos^2 x - \cos x = 2\sin^2 x$, for $0° \le x \le 360°$. [4]

Qn	Solution	MS	Marker's Report
2(i)	$\cos^2 x-\cos x=2\sin^2 x$ $\cos^2 x-\cos x=2(1-\cos^2 x)$ $\cos^2 x-\cos x=2-2\cos^2 x$ $3\cos^2 x-\cos x-2=0$ (shown)	[M1] [A1]	
2(ii)	$\cos^2 x-\cos x=2\sin^2 x$ $3\cos^2 x-\cos x-2=0$ $(3\cos x+2)(\cos x-1)=0$ $\cos x=-\frac{2}{3}$ or $\cos x=1$ Basic $x=48.19°$ $x=0°$ or $360°$ $x=180°-48.19°=131.81°$ or $x=180°+48.19°=228.19°$ Ans: $x=0°, 131.8°, 228.2°$ or $360°$ *deduct if extra or short of angles*	[M1] [A1] [A2]	

3 (i) Evaluate $\dfrac{4\times 2^{n-2}}{2^n - 2^{n-1}}$. [2]

(ii) By using the substitution $u = 3^x$ or otherwise, find the values of x such that [4]

$$9^x - 12(3^x) + 27 = 0$$

Qn	Solution	MS	Marker's Report
3(i)	$\dfrac{4\times 2^{n-2}}{2^n-2^{n-1}}$ $=\dfrac{2^n}{2^n(1-2^{-1})}$ $=\dfrac{1}{(1-\frac{1}{2})}$ $=2$	[M1] [A1]	
3(ii)	$u=3^x$ $9^x-12(3^x)+27=0$ $u^2-12u+27=0$ $(u-3)(u-9)=0$ $u=3$ or $u=9$ $3^x=3$ or $3^x=9$ $x=1$ or $x=2$	[M1] [M1] [A2]	

4 (i) The first three terms in the expansion of $(2+ax)^n$, where $a<0$, in ascending powers of x, are $256+px+16128x^2+\ldots$

Find the value of a, of p and of n [4]

(ii) Hence find the x^2 term in the expansion of $(1+2x)^2(2+ax)^n$. [2]

Qn	Solution	MS	Marker's Report
4(i)	$(2+ax)^n=2^n+\binom{n}{1}2^{n-1}(ax)+\binom{n}{2}2^{n-2}(ax)^2+\ldots$ $=2^n+na2^{n-1}x+\frac{n(n-1)}{2}a^2 2^{n-2}x^2+\ldots$ $=2^n+na2^{n-1}x+n(n-1)a^2 2^{n-3}x^2+\ldots$ Comparing with $256+px+16128x^2+\ldots$ $2^n=256 \Rightarrow n=8$ $8(8-1)a^2 2^{8-3}=16128 \Rightarrow a^2=9 \Rightarrow a=-3$ (a is negative) $p=(8)(-3)2^{8-1}=-3072$	[M1] [A1] [A1] [A1]	

4(ii)	$(1+2x)^2(2+ax)^n=(1+4x+4x^2)(256-3072x+16128x^2+\ldots)$ x^2 term is $16128x^2+4x(-3072x)+4x^2(256)=4864x^2$	[M1] [A1]	

6 The temperature, T °C, in a greenhouse at time t hours after midnight on a certain day is modelled by the formula $T=25+2\sin(\frac{\pi t}{\ldots})$ for $0 \le t \le 24$.

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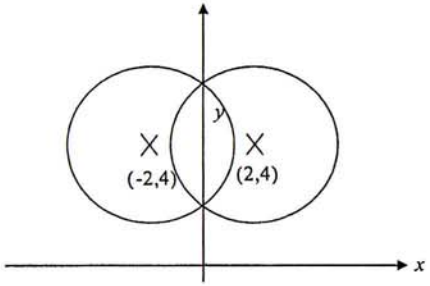
	deduct if extra or short of angles		

4(ii)	$(1+2x)^2(2+ax)^n=(1+4x+4x^2)(256-3072x+16128x^2+\ldots)$ x^2 term is $16128x^2+4x(-3072x)+4x^2(256)=4864x^2$	[M1] [A1]	

5 The equation of a circle is $x^2+y^2+4x-8y+15=0$.

(i) Find the radius and the coordinates of the centre of the circle. [2]

(ii) The circle is reflected in the y-axis. Find the equation of the new circle. [2]

Qn	Solution	MS	Marker's Report
5(i)	$x^2+y^2+4x-8y+15=0$ $2g=4 \qquad 2f=-8$ $g=2 \qquad f=-4$ Centre: $(-2,4)$ Radius $=\sqrt{(2)^2+(-4)^2-15}$ $=\sqrt{5}$ units	[M1] [A1] [A1]	
5(ii)	After reflection: Centre $(2,4)$ Radius $=\sqrt{5}$ $\therefore$ Equation of new circle: $(x-2)^2+(y-4)^2=5$ 	[M1] [A1]	

	negative) $p=(8)(-3)2^{8-1}=-3072$	[A1]	

6 The temperature, T °C, in a greenhouse at time t hours after midnight on a certain day is modelled by the formula $T=25+2\sin(\frac{\pi t}{12})$ for $0\le t\le 24$.

(i) Calculate the temperature, T °C, of the greenhouse when $t=6$. [1]

(ii) Explain why this model suggests that the range of temperature, T °C, of the greenhouse is $23\le T\le 27$. [2]

(iii) Tomato plants grow best when $T\le 24$. Find the length of time for which the temperature of the greenhouse is suitable for the growth of tomato plants. [4]

Qn	Solution	MS	Marker's Report
6(i)	When $t=6$, $T=25+2\sin\left(\frac{6\pi}{12}\right)$ $T=27$	[B1]	
6(ii)	Minimum temperature occurs when $\sin\left(\frac{\pi t}{12}\right)=-1$ $\therefore T=25-2=23$ Maximum temperature occurs when $\sin\left(\frac{\pi t}{12}\right)=1$ $\therefore T=25+2=27$ Hence range of temperature is $23\le T\le 27$	[M1] [M1]	
6(iii)	$25+2\sin\left(\frac{\pi t}{12}\right)=24$ $2\sin\left(\frac{\pi t}{12}\right)=-1$ $\sin\left(\frac{\pi t}{12}\right)=-\frac{1}{2}$ $\frac{\pi t}{12}=\pi+\frac{\pi}{6},\ 2\pi-\frac{\pi}{6}$ $t=14,\ 22$ Duration = 8 hours	[M1] [M1] [M1] [A1]	

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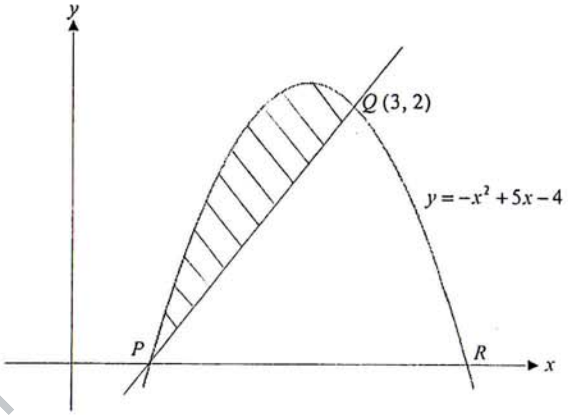
7 The equation of a curve is $y=\frac{2}{3}x^3-x^2-4x+5$.

(i) Find the coordinates of the stationary points on the curve. [4]

(ii) Determine the nature of each of these stationary points. [3]

Qn	Solution	MS	Marker's Report
7(i)	$y=\frac{2}{3}x^3-x^2-4x+5$		
	$\frac{dy}{dx}=2x^2-2x-4$	[M1]	
	$\frac{dy}{dx}=0$		
	$2x^2-2x-4=0$		
	$x^2-x-2=0$		
	$(x-2)(x+1)=0$		
	$x=2,-1$	[M1]	
	When $x=-1$, $y=\frac{22}{3}$	[A1]	
	When $x=2$, $y=-\frac{5}{3}$		
	$\therefore\left(-1,\frac{22}{3}\right)$ and $\left(2,-\frac{5}{3}\right)$	[A1]	
7(ii)	$\frac{d^2y}{dx^2}=4x-2$	[M1]	
	At $x=-1$,		
	$\frac{d^2y}{dx^2}=4(-1)-2<0$	[A1]	
	Hence, $\left(-1,\frac{22}{3}\right)$ is a max. point.		
	At $x=2$,		
	$\frac{d^2y}{dx^2}=8-2$		
	$=6<0$	[A1]	
	Hence, $\left(2,-\frac{5}{3}\right)$ is a min. point.		

8



The diagram shows part of the curve $y=-x^2+5x-4$. The normal to the curve at the point $Q(3, 2)$ meets the x axis at the point P. The curve $y=-x^2+5x-4$ cuts the x-axis at points P and R.

Find

(i) the coordinates of P and R, [3]

(ii) the area of the shaded region. [4]

Qn	Solution	MS	Marker's Report
8(i)	$-x^2+5x-4=0$ $(x-1)(x-4)=0$ $x=1, x=4$ $\therefore P(1,0)$ $\therefore R(4,0)$		
8(ii)	area under graph $y=-x^2+5x-4$ $\int_1^3(-x^2+5x-4)dx=\left[-\frac{1}{3}x^3+\frac{5}{2}x^2-4x\right]_1^3$ $=(-9+\frac{45}{2}-12)-(-\frac{1}{3}+\frac{5}{2}-4)$ $=3\frac{1}{3}units^2$ area of shaded region $=3\frac{1}{3}-\frac{1}{2}(2\times2)=1\frac{1}{3}units^2$		

9 (i) Find the coordinates of the points where the line $y=\frac{1}{2}x$ cuts the curve [4]

10 The roots of the equation $5x^2-2x+4=0$ are α and β.

(i) State the value of $\alpha+\beta$ and $\alpha\beta$. [2]

(ii) Find the quadratic equation in x whose roots are α^2 and β^2. [4]

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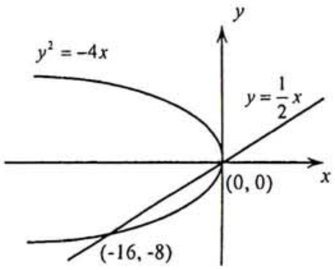
9 (i) Find the coordinates of the points where the line $y=\frac{1}{2}x$ cuts the curve $y^2=-4x$. [4]

(ii) Sketch the graphs of $y=\frac{1}{2}x$ and $y^2=-4x$ on the same axes. Label your graphs and the points of intersection clearly. [3]

Qn	Solution	MS	Marker's Report
9(i)	$y^2=-4x$		
	$\left(\frac{1}{2}x\right)^2=-4x$	[M1]	
	$x^2=-16x$	[M1]	
	$x(x+16)=0$		
	$x=0 \quad x=-16$		
	$y=0 \quad y=-8$		
	$(0\,,0)$	[A1]	
	$(-16\,,-8)$	[A1]	
9(ii)	 [G1 each for graph and label] [G1 for intersection point and label]		

10 The roots of the equation $5x^2-2x+4=0$ are α and β.

(i) State the value of $\alpha+\beta$ and $\alpha\beta$. [2]

(ii) Find the quadratic equation in x whose roots are α^2 and β^2. [4]

(iii) Find the value of $\alpha^3+\beta^3$. [2]

Qn	Solution	MS	Marker's Report
10(i)	$x^2-\frac{2}{5}x+\frac{4}{5}=0$		
	$\alpha+\beta=\frac{2}{5}, \alpha\beta=\frac{4}{5}$	[B2]	
10(ii)	$\alpha^2+\beta^2=(\alpha+\beta)^2-2\alpha\beta$		
	$\alpha^2+\beta^2=\left(\frac{2}{5}\right)^2-2\left(\frac{4}{5}\right)$	[M1]	
	$\alpha^2+\beta^2=-\frac{36}{25}$	[A1]	
	$\alpha^2\beta^2=(\alpha\beta)^2$		
	$=\left(\frac{4}{5}\right)^2$		
	$=\frac{16}{25}$	[B1]	
	$x^2-\left(-\frac{36}{25}\right)x+\frac{16}{25}=0$		
	$25x^2+36x+16=0$	[A1]	
10(iii)	$\alpha^3+\beta^3$		
	$=(\alpha+\beta)(\alpha^2-\alpha\beta+\beta^2)$		
	$=\left(\frac{2}{5}\right)\left(-\frac{36}{25}-\frac{4}{5}\right)$	[M1]	
	$=-\frac{112}{125}$	[A1]	

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11 (a) Show that $\dfrac{1+\cos 2A+\cos A}{\sin 2A+\sin A}=\cot A$. [3]

(b) Given that $\dfrac{d^2y}{dx^2}=4x-3$ and the curve has a stationary point at $A\left(1,\dfrac{1}{3}\right)$, find the equation of the curve. [4]

Qn	Solution	MS	Marker's Report
11(i)	$\text{LHS}=\dfrac{1+\cos 2A+\cos A}{\sin 2A+\sin A}$ $=\dfrac{1+2\cos^2 A-1+\cos A}{2\sin A\cos A+\sin A}$ $=\dfrac{2\cos A(\cos A+1)}{2\sin A(\cos A+1)}$ $=\cot A$ $=RHS$	 [M1] [M1] [A1]	
11(ii)	$\dfrac{d^2y}{dx^2}=4x-3$ $\dfrac{dy}{dx}=2x^2-3x+c$ $0=2-3+c$ $c=1$ $\dfrac{dy}{dx}=2x^2-3x+1$ $y=\dfrac{2}{3}x^3-\dfrac{3}{2}x^2+x+c_1$ $\dfrac{1}{3}=\dfrac{2}{3}-\dfrac{3}{2}+1+c_1$ $c_1=\dfrac{1}{6}$ $y=\dfrac{2}{3}x^3-\dfrac{3}{2}x^2+x+\dfrac{1}{6}$	 [M1] [M1] [M1] [A1]	

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End of Paper

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ADDITIONAL MATHEMATICS 4044/1
PAPER 1

Additional Materials:
Writing paper & Electronic calculator

DATE 17 Aug 2016
TIME 1005 – 1150
DURATION 1 h 45 min

INSTRUCTIONS TO CANDIDATES

55

Answer **all** questions.
Write your answers and working on the separate writing paper provided.
Write in dark blue or black pen.
You may use a soft pencil for any diagrams or graphs.

Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.

The number of marks is given in brackets [ ] at the end of each question or part question.
The total number of marks for this paper is **70**.
The use of an electronic calculator is expected, where appropriate.
You are reminded of the need for clear presentation in your answers.

PARENT'S SIGNATURE	FOR EXAMINER'S USE	
		/ 70

Setter: *Mr Chen Hong Ming* *Vetter:* *Ms Winnifred Lim*

This question paper consists of 5 printed pages including this page.

Mathematical Formulae

1. ALGEBRA

Quadratic Equation

For the equation $ax^2 + bx + c = 0$,

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Binomial expansion

$$(a+b)^n = a^n + \binom{n}{1}a^{n-1}b + \binom{n}{2}a^{n-2}b^2 + \ldots + \binom{n}{r}a^{n-r}b^r + \ldots + b^n$$

where n is a positive integer and $\binom{n}{r} = \frac{n!}{(n-r)!r!} = \frac{n(n-1)\ldots(n-r+1)}{r!}$

2. TRIGONOMETRY

Identities

$$\sin^2 A + \cos^2 A = 1$$
$$\sec^2 A = 1 + \tan^2 A$$
$$\operatorname{cosec}^2 A = 1 + \cot^2 A$$
$$\sin(A \pm B) = \sin A\cos B \pm \cos A\sin B$$
$$\cos(A \pm B) = \cos A\cos B \mp \sin A\sin B$$
$$\tan(A \pm B) = \frac{\tan A \pm \tan B}{1 \mp \tan A\tan B}$$
$$\sin 2A = 2\sin A\cos A$$
$$\cos 2A = \cos^2 A - \sin^2 A = 2\cos^2 A - 1 = 1 - 2\sin^2 A$$
$$\tan 2A = \frac{2\tan A}{1 - \tan^2 A}$$

Formulae for ΔABC

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$
$$a^2 = b^2 + c^2 - 2bc\cos A$$
$$\Delta = \frac{1}{2}ab\sin C$$

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1 (i) State the values of x for which $\cos^{-1} x$ is defined. [1]

(ii) Express the principal value of $\tan^{-1}\left(-\frac{\sqrt{3}-1}{\sqrt{3}+1}\right)$ in terms of π. [1]

(iii) Given that P denotes the principal value of $\cos^{-1} x$, state the maximum value of P. [1]

2 Find the set of values of k for which the equation $y = 8x^2 - 3kx + 18$ lies strictly above the x-axis. [3]

3 The volume of water in a container, V cm^3, varies with the depth of water, h cm. Given that $V = \frac{2}{3}\pi h^3$ and that water is poured into the container at 90cm^3/s, find the rate of increase of the depth of water when $h = 1.5$cm. [4]

4 Given that $\tan A = p$, $p > 0$, and angle A is reflex, express in terms of p,

(i) $\cot 2A$, [1]

(ii) $\cos A$, [1]

(iii) $\sin 2A$. [2]

5 Integrate with respect to x

(i) $\frac{\sqrt{x}-x}{x}$, [1]

(ii) $\sqrt[3]{(1-4x)^2}$. [3]

6 (i) Find, in ascending powers of x, the first 4 terms of $(2-3x)^7$. [2]

(ii) Hence, determine the coefficient of x^3 in the expansion $\left(1+\frac{x}{2}\right)^2(2-3x)^7$. [2]

7 By using the substitution $p = 2^x$, or otherwise, find the values of x such that

$$4^{x+1} + 2^{x+3} = 2^x + 2$$

[5]

8 The line $y + 2x = 3$ intersects the curve $y^2 = 8x$ at points A and B.

(i) Find the coordinates of points A and B. [3]

(ii) Sketch, on the same axis, the graphs of $y + 2x = 3$ and $y^2 = 8x$, labelling clearly points A and B. [3]

9 Show that $\frac{3^{2n+1} - 3^{2n-1}}{9^{n+1} - 9^{n-1}} = \frac{3}{10}$. [3]

10 (i) Differentiate $x^3\sqrt{3x^2+1}$, leaving your answer in the form $\frac{Ax^4 + Bx^2}{\sqrt{3x^2+1}}$. [3]

(ii) Hence, evaluate $\int_0^4 \frac{x^2(4x^2+1)}{7\sqrt{3x^2+1}}dx$. [3]

11 Given the equation, $f(x) = \frac{1+x^2}{2x^2-3}, x > 0$.

(i) Find $f'(x)$. [3]

(ii) Find the equation of the tangent to the curve, $y = f(x)$ at the point $x = 2$. [3]

(iii) Show that f is a decreasing function for $x > 0$. [2]

12 The equation of a circle is $x^2 + y^2 - 12x + 6y + 11 = 0$

(i) Find the radius and coordinates of the centre of the circle. [3]

(ii) Show that the circle does not cut the y-axis. [1]

(iii) Show that the point $A(3, 2)$ lies on the circle. [2]

(iv) Find the equation of the tangent to the circle at A. [3]

(v) Find the equation of the tangent to the circle that is parallel to the tangent at A. [2]

13 An underwater locator beacon is installed in the aircraft's flight recorder for retrieval

No	Solution	Remarks

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13 An underwater locator beacon is installed in the aircraft's flight recorder for retrieval and use in the investigation of aeronautical accidents. A flight recorder's beacon emits a regular ultrasonic pulse that is described by the equation

$$P = 45 \sin 40t + 35 \cos 40t + 100$$

where t (measured in degrees) is the time in milliseconds after the crash and P is the intensity of the pulse, measured in decibels (dB).

(i) Express P in the form $A\sin(40t + B) + C$. [3]

(ii) State the maximum intensity of the pulse and the time when it first occurs after the crash. [2]

For the flight recorder to be detected, the intensity of the pulse must be above 120dB.

(iii) Calculate the duration within the first 9 milliseconds when the flight recorder was detectable. [4]

- End of Paper -

No	Solution	Remarks
1i	$\cos^{-1} x$ is defined for $-1 \le x \le 1$	B1
1ii	Principal value of $\tan^{-1}\left(-\frac{\sqrt{3}-1}{\sqrt{3}+1}\right) = -\frac{\pi}{12}$	B1
1iii	Maximum principal value of $\cos^{-1} x = 180°$	B1
2	$y = 8x^2 - 3kx + 18$ $b^2 - 4ac < 0$ $9k^2 - 4(8)(18) < 0$ $k^2 - 64 < 0$ $(k-8)(k+8) < 0$ *sketch* $-8 < k < 8$	M1 M1 A1
3	$V = \frac{2}{3}\pi h^3$ $\frac{dV}{dh} = 2\pi h^2$ $\frac{dV}{dh} = 2\pi(1.5)^2 = \frac{9}{2}\pi$ $\frac{dV}{dt} = 90 cm^3 / s$ $\frac{dV}{dt} = \frac{dV}{dh} \times \frac{dh}{dt}$ $90 = \frac{9}{2}\pi \times \frac{dh}{dt}$ $\frac{dh}{dt} = \frac{20}{\pi} cm / s = 6.36619\, cm / s \approx 6.37 cm / s$	M1 M1 M1 A1
4i	$\tan A = p$ $\cot 2A = \frac{1}{\tan 2A} = \frac{1-\tan^2 A}{2\tan A} = \frac{1-p^2}{2p}$	B1
4ii	$\cos A = -\frac{1}{\sqrt{1+p^2}}$	B1
4iii	$\sin 2A = 2\sin A\cos A$ $= 2 \times \frac{-p}{\sqrt{1+p^2}} \times \frac{-1}{\sqrt{1+p^2}}$ $= \frac{2p}{1+p^2}$	M1 A1
5i	$\int \frac{\sqrt{x} - x}{x} dx$ $= \int x^{-\frac{1}{2}} - 1 dx$ $= 2x^{\frac{1}{2}} - x + c$	B1

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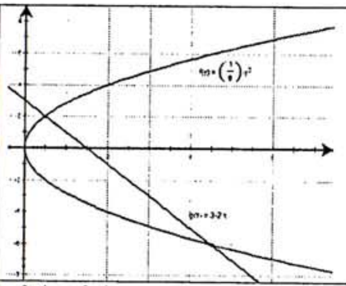
5ii	$\int \sqrt[3]{(1-4x)^2}\,dx$ $= \int (1-4x)^{\frac{2}{3}}dx$ $= \frac{3(1-4x)^{\frac{5}{3}}}{5(-4)} + c$ $= -\frac{3(1-4x)^{\frac{5}{3}}}{20} + c$	B3
6i	$(2-3x)^7$ $= 2^7 - 7(2^6)(3x) + 21(2^5)(3x)^2 - 35(2^4)(3x)^3 + \dots$ $= 128 - 1344x + 6048x^2 - 15120x^3 + \dots$	B2
6ii	$\left(1+\frac{x}{2}\right)^2 (2-3x)^7$ $= \left(1 + x + \frac{x^2}{4}\right)(128 - 1344x + 6048x^2 - 15120x^3 + ..)$ $= \dots - 15120x^3 + 6048x^3 - 336x^3 + \dots$ $= \dots - 9408x^3 + \dots$ *Coefficien t* $= -9408$	M1 A1
7	$4^{x+1} + 2^{x+3} = 2^x + 2$ $2^{2x+2} + 8(2^x) = 2^x + 2$ $4(2^x)^2 + 7(2^x) - 2 = 0$ $4p^2 + 7p - 2 = 0$ $(4p-1)(p+2) = 0$ $2^x = -2(rej)$ $2^x = \frac{1}{4} = 2^{-2}$ $x = -2$	M1 M1 M1 A1 A1
8i	$y + 2x = 3$ $y = 3 - 2x$ $y^2 = 8x$ $(3-2x)^2 = 8x$ $9 - 12x + 4x^2 = 8x$ $4x^2 - 20x + 9 = 0$ $(2x-9)(2x-1) = 0$ $x = \frac{1}{2}, \frac{9}{2}$ $y = 2, -6$ $(\frac{1}{2}, 2)$ & $(\frac{9}{2}, -6)$	M1 A2

8ii		**B1** (line) **B2** (Parabola)
9i	$\frac{3^{2n+1} - 3^{2n-1}}{9^{n+1} - 9^{n-1}}$ $= \frac{3^{2n+1} - 3^{2n-1}}{3^{2n+2} - 3^{2n-2}}$ $= \frac{3^{2n}(3 - 3^{-1})}{3^{2n}(3^2 - 3^{-2})}$ $= \frac{\left(\frac{8}{3}\right)}{\left(\frac{80}{9}\right)} = \frac{3}{10}$	M1 M1 A1
10i	$\frac{d}{dx} x^3 \sqrt{3x^2+1}$ $= 3x^2\sqrt{3x^2+1} + x^3 \times \frac{1}{2} \times \frac{6x}{\sqrt{3x^2+1}}$ $= \frac{3x^2(3x^2+1)}{\sqrt{3x^2+1}} + \frac{3x^4}{\sqrt{3x^2+1}}$ $= \frac{12x^4 + 3x^2}{\sqrt{3x^2+1}}$	M2 A1
10ii	$\frac{d}{dx} x^3\sqrt{3x^2+1} = \frac{12x^4+3x^2}{\sqrt{3x^2+1}}$ $\frac{d}{dx} x^3\sqrt{3x^2+1} = \frac{3x^2(4x^2+1)}{\sqrt{3x^2+1}}$ $\frac{d}{dx}\frac{1}{3(7)} x^3\sqrt{3x^2+1} = \frac{x^2(4x^2+1)}{7\sqrt{3x^2+1}}$ $\int_0^4 \frac{x^2(4x^2+1)}{7\sqrt{3x^2+1}}dx$ $= \left[\frac{1}{3(7)} x^3\sqrt{3x^2+1}\right]_0^4$ $= \left(\frac{1}{3(7)} \times 4^3\sqrt{3(4)^2+1}\right) - \left(\frac{1}{3(7)} \times 0^3\sqrt{3(0)^2+1}\right)$ $= \frac{64}{3}$	M1 M1 A1

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11i	$f'(x)$	

	$9 + y^2 - 36 + 6y + 11 = 0$	M1

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	$=\frac{64}{3}$	A1

11i	$f'(x)$ $=\frac{d}{dx}\left(\frac{1+x^2}{2x^2-3}\right)$ $=\frac{(2x^2-3)(2x)-(1+x^2)(4x)}{(2x^2-3)^2}$ $=\frac{4x^3-6x-4x-4x^3}{(2x^2-3)^2}$ $=\frac{-10x}{(2x^2-3)^2}$	M2 A1
11ii	When x = 2 $f(2)=\frac{1+2^2}{2(2)^2-3}=\frac{5}{5}=1$ $f'(2)=\frac{-10(2)}{[2(2)^2-3]^2}=\frac{-20}{25}=-\frac{4}{5}$ Subst (2, 1), $m=-\frac{4}{5}$ in $y=mx+c$ $1=\left(-\frac{4}{5}\right)(2)+c$ $c=\frac{13}{5}$ Equation of Tangent: $y=-\frac{4}{5}x+\frac{13}{5}$	M1 M1 A1
11iii	$\because x>0$ $-10x<0$ $(2x^2-3)^2>0\forall x$ $\therefore \frac{-10x}{(2x^2-3)^2}<0$ $\therefore \frac{dy}{dx}<0$ Hence it will be a decreasing function.	B1 B1
12i	$x^2+y^2-12x+6y+11=0$ $(x-6)^2+(y+3)^2=36+9-11=34$ Centre = (6, -3) Radius = $\sqrt{34}\approx 5.83095$ units	B3
12ii	The x-coordinate of the centre is 6 which is greater than the radius of 5.83 hence, the circle will not touch the y-axis.	B1
12iii	When $x=3$, $9+y^2-36+6y+11=0$ $y^2+6y-16=0$ $(y-2)(y+8)=0$ $y=2,-8$ Therefore (3, 2) lies on the circle.	M1 A1
12iv	Gradient of radius from centre to A $=\frac{2-(-3)}{3-6}=-\frac{5}{3}$ Gradient of tangent $=\frac{3}{5}$ Subst (3, 2), $m=\frac{3}{5}$ in $y=mx+c$ $2=\frac{3}{5}(3)+c$ $c=\frac{1}{5}$ $y=\frac{3}{5}x+\frac{1}{5}$ $5y=3x+1$	M1 M1 A1
12v	Let B(x,y) $\left(\frac{x+3}{2},\frac{y+2}{2}\right)=(6,-3)$ $x=9, y=-8$ Subst (9, -8), $m=\frac{3}{5}$ in $y=mx+c$ $-8=\frac{3}{5}(9)+c$ $c=-\frac{67}{5}$ $y=\frac{3}{5}x-\frac{67}{5}$ $5y=3x-67$	M1 A1
13i	$P=45\sin 40t+35\cos 40t+100$ $=\sqrt{45^2+35^2}\sin(40t+\tan^{-1}\frac{35}{45})+100$ $=5\sqrt{130}\sin(40t+37.8749)+100$ $A=5\sqrt{130}=57.008\approx 57.0$ $B=37.9°$ $C=100$	B3
13ii	Maximum intensity = $5\sqrt{130}+100=157dB$ $40t+37.8749=90$ $t=1.303125ms$	B1

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	It occurs when t = 1.30 milliseconds	**B1**
13iii	$P = 5\sqrt{130}\ \sin(40t + 37.8749) + 100$ $P > 120$ $5\sqrt{130}\ \sin(40t + 37.8749) > 20$ $\sin(40t + 37.8749) > 0.350823$ $0 \le t \le 9$ $0 \le 40t \le 360$ $37.8749 \le 40t + 37.8749 \le 397.8749$ $B.A, \alpha = 20.5376$ $40t + 37.8749 = 20.5376\,(rej), 180 - 20.5376, 360 + 20.5376$ $40t + 37.8749 = 159.4623, 380.5376$ $40t = 121.5873, 342.6626$ $t = 3.03968, 8.5665$ *Duration* $= (9 - 8.566) + 3.039$ $= 3.4731$ $\approx 3.47\ ms$	**M1** **M1** **M1** **A1**

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Candidate Name:	Class	Index No.

FUHUA SECONDARY SCHOOL

Secondary Four Normal Academic

Preliminary Examination 2016

4NA

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ADDITIONAL MATHEMATICS 4044/2
PAPER 2

Additional Materials:
Writing paper & Electronic calculator

DATE	**18 Aug 2016**
TIME	**0750 – 0935**
DURATION	**1 h 45 min**

INSTRUCTIONS TO CANDIDATES

Answer **all** questions.
Write your answers and working on the separate writing paper provided.
Write in dark blue or black pen.
You may use a soft pencil for any diagrams or graphs.

Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.

The number of marks is given in brackets [ ] at the end of each question or part question.
The total number of marks for this paper is **70**.
The use of an electronic calculator is expected, where appropriate.
You are reminded of the need for clear presentation in your answers.

PARENT'S SIGNATURE	**FOR EXAMINER'S USE**	
		/ 70

Setter: *Mr Chen Hong Ming* *Vetter:* *Ms Winnifred Lim*

This question paper consists of 5 printed pages including this page.

Mathematical Formulae

1. ALGEBRA

Quadratic Equation

For the equation $ax^2 + bx + c = 0$,

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Binomial expansion

$$(a+b)^n = a^n + \binom{n}{1}a^{n-1}b + \binom{n}{2}a^{n-2}b^2 + \ldots + \binom{n}{r}a^{n-r}b^r + \ldots + b^n$$

where n is a positive integer and $\binom{n}{r} = \frac{n!}{(n-r)!r!} = \frac{n(n-1)\ldots(n-r+1)}{r!}$

2. TRIGONOMETRY

Identities

$$\sin^2 A + \cos^2 A = 1$$
$$\sec^2 A = 1 + \tan^2 A$$
$$\operatorname{cosec}^2 A = 1 + \cot^2 A$$
$$\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$$
$$\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$$
$$\tan(A \pm B) = \frac{\tan A \pm \tan B}{1 \mp \tan A \tan B}$$
$$\sin 2A = 2\sin A \cos A$$
$$\cos 2A = \cos^2 A - \sin^2 A = 2\cos^2 A - 1 = 1 - 2\sin^2 A$$
$$\tan 2A = \frac{2\tan A}{1 - \tan^2 A}$$

Formulae for ΔABC

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$
$$a^2 = b^2 + c^2 - 2bc \cos A$$
$$\Delta = \frac{1}{2}ab \sin C$$

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1 It is given that $2(x-2)=\sqrt{3}(x-3)$. **Without using a calculator,** find x in the form of $a+b\sqrt{3}$. [3]

2 Divide $x^4+x^3-5x^2+13x-6$ by x^2+3x-2. [3]

3 The first three terms in the binomial expansion of $\left(2+\frac{x}{p}\right)^n$ are $256+256x+qx^2$.
Find the values of n, p and q. [3]

4 The function f is given by $f(x)=23+8x+x^2-2x^3$.

(i) Find the range of values of x for which f is a decreasing function. [4]

(ii) Represent this set of values on a number line. [1]

5 (i) Show that $(\tan\theta+\sec\theta)^2=\frac{1+\sin\theta}{1-\sin\theta}$. [3]

(ii) Hence, solve the equation $(\tan\theta+\sec\theta)^2=2$ for $0° < x < 360°$. [3]

6 It is given that $f''(x)=12x-4$ and that $f'(2)=11$ and $f(-1)=-7$. Find $f(x)$ in terms of x. [5]

7 The roots of the equation $3x^2+5x-4=0$ are α and β.

(i) Evaluate $\alpha^2+\beta^2$. [3]

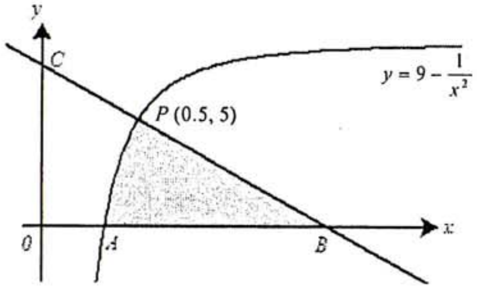
(ii) Find, in the form $ax^2+bx+c=0$, where a, b and c are integers, the equation whose roots are $\alpha^2-\frac{1}{\beta}$ and $\beta^2-\frac{1}{\alpha}$. [3]

8 The polynomial f is given by $f(x)=2x^3+ax^2+bx+6=0$.

(i) Given that $x+2$ is a factor of $f(x)$ and $f(x)$ gives a remainder of 7.5 when divided by $2x+1$, find the value of a and of b. [4]

(ii) Hence, show that $f(x)=0$ has only one real root. [3]

9



The diagram shows the graph $y=9-\frac{1}{x^2}$ which crosses the x-axis at A.

(i) Find the coordinate of A. [1]

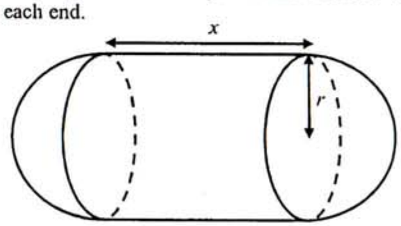
(ii) Find $\frac{dy}{dx}$. [1]

The normal to the curve at P (0.5, 5) cuts the x-axis at B and the y-axis at C.

(iii) Find the coordinates of B. [4]

(iv) Find the area of the shaded region. [4]

10 Matherone is a new drug developed by ChenCorp© to enhance memory and problem solving skills. This drug comes in a capsule which consist of a cylinder capped by 2 hemispheres on each end.



Given that the cylinder has a length of x mm and a radius of r mm and that the total volume of the capsule is 150mm^3,

(i) form an equation for x in terms of r. [1]

(ii) Hence, show that the surface area of the capsule, A, is given by

$$A=\frac{4}{3}\pi r^2+\frac{300}{r}$$ [3]

For the drug to be most effective, the capsule must be designed with the least possible surface area to delay the absorption over a length of time.

(iii) Find the least surface area possible for the drug capsule. [5]
(Show your working clearly that your answer is indeed the least area)

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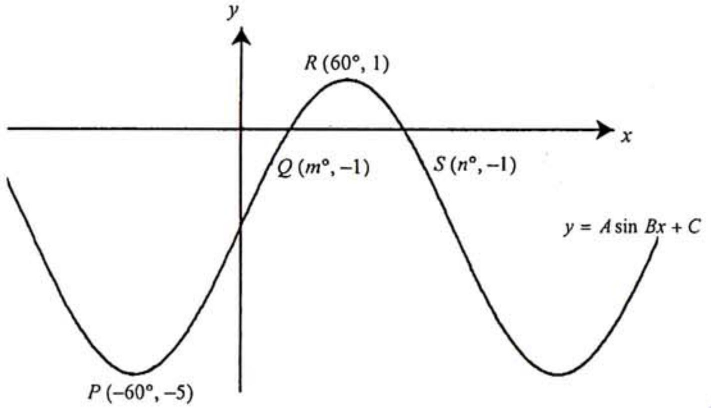
11 (a) The curve below shows the graph $y = A\sin Bx + C$. P, Q, R and S are 4 points that lie on the curve such that R is a maximum point and P is a minimum point.



(i) Find the value of A, B and C. [3]

(ii) Form an equation connecting m and n. [1]

(b) Solve, for angles between 0 and 2π,

(i) $2\cos 2x = 2\sin x - 1$, [5]

(ii) $\sin(y + 4) = 3\cos y$. [4]

- End of Paper -

(iii) Find the least surface area possible for the drug capsule. [5]
(Show your working clearly that your answer is indeed the least area)

No	Solution	Remarks
1	$2(x-2) = \sqrt{3}(x-3)$	
	$2x - 4 = x\sqrt{3} - 3\sqrt{3}$	
	$x(2-\sqrt{3}) = 4 - 3\sqrt{3}$	
	$x = \frac{4-3\sqrt{3}}{2-\sqrt{3}} \times \frac{2+\sqrt{3}}{2+\sqrt{3}}$	M1 M1
	$= \frac{8-6\sqrt{3}+4\sqrt{3}-9}{4-3}$	
	$= -1-2\sqrt{3}$	A1
2	$\frac{x^4+x^3-5x^2+13x-6}{x^2-3x+2} = x^2-2x+3$	B3
3	$\left(2+\frac{x}{p}\right)^n = 2^n + \frac{(n)(2^{n-1})x}{p} + \frac{n(n-1)(2^{n-2})x^2}{2p^2} + \ldots$	
	$2^n + \frac{(n)(2^{n-1})x}{p} + \frac{n(n-1)(2^{n-2})x^2}{2p^2} = 256 + 256x + qx^2$	
	$2^n = 256 = 2^8$	A1
	$n = 8$	
	$\frac{8(128)}{p} = 256$	
	$p = 4$	A1
	$\frac{(8)(7)(64)}{2(4^2)} = q$	
	$q = 112$	A1
4i	$f(x) = 8x + x^2 - 2x^3$	
	$f'(x) = 8 + 2x - 6x^2$	M1
	$\because$ Decrea sin g	M1
	$\therefore 8 + 2x - 6x^2 < 0$	
	$4 + x - 3x^2 < 0$	
	$3x^2 - x - 4 > 0$	
	$(3x-4)(x+1) > 0$	M1
	Sketch	
	$x < -1, x > \frac{4}{3}$	A1
4ii	Drawing of Number Line	B1

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5i	$L.H.S = (\tan\theta + \sec\theta)^2$	
	$= \left(\frac{\sin\theta}{\cos\theta} + \frac{1}{\cos\theta}\right)^2$	M1
	$= \left(\frac{\sin\theta + 1}{\cos\theta}\right)^2$	
	$= \frac{(\sin\theta + 1)^2}{\cos^2\theta}$	
	$= \frac{(\sin\theta + 1)^2}{1 - \sin^2\theta}$	M1
	$= \frac{(\sin\theta + 1)^2}{(1 - \sin\theta)(1 + \sin\theta)}$	
	$= \frac{1 + \sin\theta}{1 - \sin\theta}$	A1
5ii	$(\tan\theta + \sec\theta)^2 = 2$	
	$\frac{1 + \sin\theta}{1 - \sin\theta} = 2$	M1
	$1 + \sin\theta = 2 - 2\sin\theta$	
	$3\sin\theta = 1$	
	$\sin\theta = \frac{1}{3}$	M1
	$B.A, \alpha = 19.471$	
	$\theta = 19.471, 180 - 19.471$	
	$= 19.471, 160.528$	
	$\approx 19.5°, 160.5°$	A1
6	$f''(x) = 12x - 4$	
	$f'(x) = \int 12x - 4dx$	
	$= 6x^2 - 4x + c$	M1
	$f'(2) = 11$	
	$6(2)^2 - 4(2) + c = 11$	
	$24 - 8 + c = 11$	
	$c = -5$	M1
	$f'(x) = 6x^2 - 4x - 5$	
	$f(x) = 2x^3 - 2x^2 - 5x + k$	M1
	$f(-1) = -7$	
	$2(-1)^3 - 2(-1)^2 - 5(-1) + k = -7$	M1
	$-2 - 2 + 5 + k = -7$	
	$k = -8$	
	$f(x) = 2x^3 - 2x^2 - 5x - 8$	A1

7i	$3x^2 + 5x - 4 = 0$	
	$\alpha + \beta = -\frac{5}{3}$	M1
	$\alpha\beta = -\frac{4}{3}$	M1
	$\alpha^2 + \beta^2 = (\alpha + \beta)^2 - 2\alpha\beta = \frac{25}{9} + \frac{8}{3} = \frac{25}{9} + \frac{24}{9} = \frac{49}{9}$	A1
7ii	$\alpha^2 - \frac{1}{\beta} + \beta^2 - \frac{1}{\alpha}$	
	$= \frac{49}{9} - \frac{\alpha + \beta}{\alpha\beta}$	
	$= \frac{49}{9} + \frac{5}{3} \div \left(-\frac{4}{3}\right) = \frac{49}{9} - \frac{5}{4} = \frac{151}{36}$	M1
	$\left(\alpha^2 - \frac{1}{\beta}\right)\left(\beta^2 - \frac{1}{\alpha}\right)$	
	$= \alpha^2\beta^2 - \alpha - \beta + \frac{1}{\alpha\beta}$	
	$= \left(-\frac{4}{3}\right)^2 + \frac{5}{3} - \frac{3}{4} = \frac{97}{36}$	M1
	New equation	
	$36x^2 - 151x + 97 = 0$	A1
8i	$f(x) = 2x^3 + ax^2 + bx + 6 = 0$	
	$f(-2) = -16 + 4a - 2b + 6 = 0$	M1
	$4a - 2b = 10$	
	$2a - b = 5$	
	$f(-\frac{1}{2}) = -\frac{1}{4} + \frac{a}{4} - \frac{b}{2} + 6 = 7.5$	M1
	$-1 + a - 2b + 24 = 30$	
	$a - 2b = 7$	
	$a = 2b + 7$	
	$4b + 14 - b = 5$	
	$3b = -9$	
	$a = 1, b = -3$	A2
8ii	$f(x) = 2x^3 + x^2 - 3x + 6$	
	$= (x + 2)(2x^2 - 3x + 3)$	M1
	$x = -2$	M1
	$2x^2 - 3x + 3 = 0$	
	$b^2 - 4ac = 9 - 4(2)(3) = -15 < 0$	
	$\therefore$ x = -2 is the only real root.	A1

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	$f(x) = 2x^3 - 2x^2 - 5x - 6$	A1

	$\therefore$ x = -2 is the only real root.	A1

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9i	$y = 9 - \frac{1}{x^2}$ $y = 0$ $9 - \frac{1}{x^2} = 0$ $\frac{1}{x^2} = 9$ $x = \frac{1}{3}, -\frac{1}{3}(rej)$ $A\left(\frac{1}{3}, 0\right)$	B1
9ii	$y = 9 - \frac{1}{x^2}$ $\frac{dy}{dx} = -\frac{(-2)}{x^3} = \frac{2}{x^3}$	B1
9iii	When $x = 0.5$ $\frac{dy}{dx} = \frac{2}{(0.5)^3} = 16$ $m_1 m_2 = -1$ $(16)m_2 = -1$ $m_2 = -\frac{1}{16}$ Subst (0.5, 5) , $m = -\frac{1}{16}$ in y = mx + c $5 = -\frac{1}{16}(0.5) + c$ $c = 5\frac{1}{32}$ $y = -\frac{1}{16}x + 5\frac{1}{32}$ $y = -\frac{1}{16}x + \frac{161}{32}$ $32y = -2x + 161$ When $y = 0$ $x = 80.5$ $B(80.5, 0)$	M1 M1 A1 A1

9iv	$Area = \int_{\frac{1}{3}}^{\frac{1}{2}} 9 - \frac{1}{x^2} dx + \frac{1}{2} \times 80 \times 5$ $= \left[9x + \frac{1}{x}\right]_{\frac{1}{3}}^{\frac{1}{2}} + 200$ $= (9 \times \frac{1}{2} + 2) - (9 \times \frac{1}{3} + 3) + 200$ $= 6.5 - 6 + 200$ $= 0.5 + 200$ $= 200.5 units^2$	M2 M1 A1
10i	$\frac{4}{3}\pi r^3 + \pi r^2 x = 150$ $x = \frac{150 - \frac{4}{3}\pi r^3}{\pi r^2}$ $= \frac{150}{\pi r^2} - \frac{4}{3}r$	M1 A1
10ii	$A = 4\pi r^2 + 2\pi r x$ $A = 4\pi r^2 + 2\pi r\left(\frac{150}{\pi r^2} - \frac{4}{3}r\right)$ $= 4\pi r^2 + \frac{300}{r} - \frac{8}{3}\pi r^2$ $= \frac{4}{3}\pi r^2 + \frac{300}{r}$	M1 M1 A1
10iii	$A = \frac{4}{3}\pi r^2 + \frac{300}{r}$ $\frac{dA}{dr} = \frac{8}{3}\pi r - \frac{300}{r^2}$ $\frac{8}{3}\pi r - \frac{300}{r^2} = 0$ $\frac{8}{3}\pi r = \frac{300}{r^2}$ $r^3 = \frac{900}{8\pi} = 35.809$ $r = 3.2961$ $A = \frac{4}{3}\pi(3.2961)^2 + \frac{300}{(3.2961)} = 227.54 \approx 228 mm^2$ $\frac{d^2A}{dr^2} = \frac{8}{3}\pi + \frac{600}{r^3} = 25.13 > 0$ $\therefore$ min $Area$	M1 M1 M1 A1 B1

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11ai	$A = \frac{1-(-5)}{2} = 3$ $120 \times 2 = \frac{360}{B}$ $B = 1.5$ $C = \frac{1+(-5)}{2} = -2$	B1 B1 B1
11aii	$60 - m = n - 60$ $m + n = 120$	B1
11bi	$2\cos 2x = 2\sin x - 1$ $2(1-2\sin^2 x) = 2\sin x - 1$ $2 - 4\sin^2 x = 2\sin x - 1$ $4\sin^2 x + 2\sin x - 3 = 0$ $\sin x = \frac{-2 \pm \sqrt{4-4(4)(-3)}}{8}$ $= 0.651387, -1.151387\ (rej)$ $\sin x = 0.651387$ $B.A, \alpha = \sin^{-1}(0.641387)$ $= 0.7094121$ $x = \alpha, \pi - \alpha$ $= 0.7094121, 2.43218$ $\approx 0.709, 2.43$ Ans: $x \approx 0.709, 2.43$	M1 M1 A1 (rej) M1 (Basic Angle) A1
11bii	$\sin(y+4) = 3\cos y$ $\sin y \cos 4 + \sin 4 \cos y = 3\cos y$ $\sin y \cos 4 = \cos y(3 - \sin 4)$ $\tan y = \frac{3 - \sin 4}{\cos 4} = -5.7373$ $B.A, \alpha = 1.398531$ $y = \pi - 1.398531\ .2\pi - 1.398531\ \approx 1.74, 4.88$	M1 M1 M1 A1

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Geylang Methodist School (Secondary)
Preliminary Examination 2016

ADDITIONAL MATHEMATICS **4044 / 01**
Paper 1

4 Normal (Academic)

Additional Materials provided: Writing Paper
Cover Sheet

1 hour 45 minutes

Setter: Ms Nainee Ismail

Thursday, 11 August 2016

READ THESE INSTRUCTIONS FIRST

Write your name, class and index number on all the work you hand in.
Write in dark blue or black pen on both sides of the paper.
You may use a soft pencil for any diagrams or graphs.
Do not use staples, paper clips, highlighters, glue or correction fluid.

Answer **all** the questions.
Write your answers on the writing papers provided.
Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.
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You are reminded of the need for clear presentation in your answers.

At the end of the examination, fasten all your work securely together.
The number of marks is given in brackets [ ] at the end of each question or part question.
The total score for this paper is **70**.

This document consists of **5** printed pages including the cover page and **1** blank page.

[Turn over

Mathematical Formulae

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For the equation $ax^2+bx+c=0$,

$$x=\frac{-b\pm\sqrt{b^2-4ac}}{2a}$$

Binomial expansion

$$(a+b)^n=a^n+\binom{n}{1}a^{n-1}b+\binom{n}{2}a^{n-2}b^2+\ldots+\binom{n}{r}a^{n-r}b^r+\ldots+b^n,$$

where n is a positive integer and $\binom{n}{r}=\frac{n!}{r!(n-r)!}=\frac{n(n-1)\ldots(n-r+1)}{r!}$

2. TRIGONOMETRY

Identities

$$\sin^2 A+\cos^2 A=1$$
$$\sec^2 A=1+\tan^2 A$$
$$\operatorname{cosec}^2 A=1+\cot^2 A$$
$$\sin(A\pm B)=\sin A\cos B\pm\cos A\sin B$$
$$\cos(A\pm B)=\cos A\cos B\mp\sin A\sin B$$
$$\tan(A\pm B)=\frac{\tan A\pm\tan B}{1\mp\tan A\tan B}$$
$$\sin 2A=2\sin A\cos A$$
$$\cos 2A=\cos^2 A-\sin^2 A=2\cos^2 A-1=1-2\sin^2 A$$
$$\tan 2A=\frac{2\tan A}{1-\tan^2 A}$$

Formulae for ΔABC

$$\frac{a}{\sin A}=\frac{b}{\sin B}=\frac{c}{\sin C}$$
$$a^2=b^2+c^2-2bc\cos A$$
$$\Delta=\frac{1}{2}ab\sin C$$

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Answer *all* the questions.

1 Find the set of values of p for which the equation $-3x^2+px-12=0$ has real roots. [3]

2 Solve the simultaneous equations

$$3^y \times 9^{-x} = \frac{1}{3},$$

$$x+2y-3=0.$$

[4]

3 The coordinates of the points A and B are $(-1,2)$ and $(3,10)$.
Find the equation of the perpendicular bisector of AB. [4]

4 **(i)** Factorise x^3-8. [2]

(ii) Explain why the curve $y=x^3-8$ only has one $x-$intercept at $x=2$. [2]

5 The volume, V cm^3, of a spherical ball is given by $V=\frac{4}{3}\pi r^3$, where r cm is the radius of the ball. The ball is deflated such that its volume decreases at a rate of 294 cm^{3}s^{-1}. Find the value of r when the radius is decreasing at a rate of $\frac{3}{2\pi}$ cms^{-1}. [4]

6 **(i)** Write down the first four terms in the expansion, in ascending powers of x, of $(1+ax)^8$, where a is a positive constant. [2]

(ii) Given that the coefficients of x and x^3 in the expansion of $(1+ax)^8$ are equal, find the value of the constant a, leaving your answer in surd form. [3]

7 Given that $y=(2x+1)^3x^{-2}$, find $\frac{dy}{dx}$, giving your answer in the form $\frac{(px+q)(2x+1)^2}{x^3}$ where p and q are integers. [5]

8 **(i)** Given that $y=(2x^2-1)^4$, show that $\frac{dy}{dx}=16x(2x^2-1)^3$. [2]

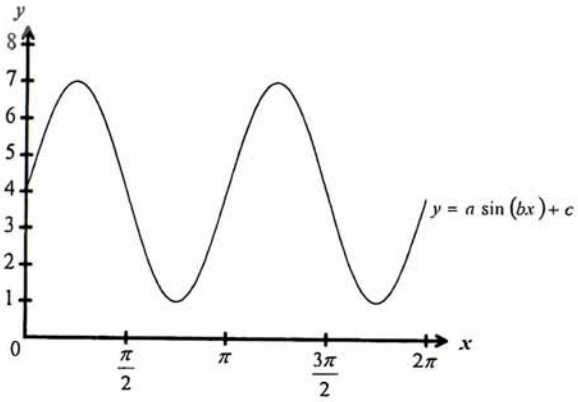
(ii) Hence, find $\int_0^2 x(2x^2-1)^3 \, dx$. [3]

9 The function $f(x)=ax^3-3x^2+b$, where x is a real number and that $f(2)=3$, has a stationary value at $x=1$.

(i) Find the value of the constants a and b. [4]

(ii) Explain, with justification, why the stationary point at $x=1$ is a minimum. [3]

10 **(a)** The diagram shows part of the graph $y=a\sin(bx)+c$.



State

(i) the period and amplitude of y, [2]

(ii) the value of a, b and c. [3]

(b) On a separate diagram, sketch the graph of $y=1-\cos 2x$ for $0° \le x \le 180°$. [2]

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$\frac{(px+q)(2x+1)^2}{x^3}$ where p and q are integers. [5]

11 The function $f(x)$ is defined as $f(x)=x^3-kx^2+3x+6$, for all real values of x.

(i) Find the range of values of x for which $f(x)$ is an increasing function. [3]

(ii) For the case $k=2$, find the equation of the normal to the curve $y=f(x)$ where $x=1$. [4]

12 (i) Sketch the parabola $y^2=2x$. [2]

The line $y=x$ intersects the parabola at the points A and B.

(ii) Find the midpoint of AB. [4]

(iii) Show that the midpoint of AB lies on the line $2x-y=1$. [1]

13 (i) Show that the equation

$$24\tan A-2\tan^2 A=3\sec^2 A-8$$

may be written in the form

$$5\tan^2 A-24\tan A-5=0.$$ [2]

(ii) Find, in degrees, the two principal values of A for which

$$24\tan A-2\tan^2 A=3\sec^2 A-8.$$ [4]

(iii) By using a suitable identity, explain why the values of A found in **(ii)** satisfy the equation $\tan 2A=-\frac{5}{12}$. [2]

- End Of Paper -

(b) On a separate diagram, sketch the graph of $y=1-\cos 2x$ for $0° \le x \le 180°$. [2]

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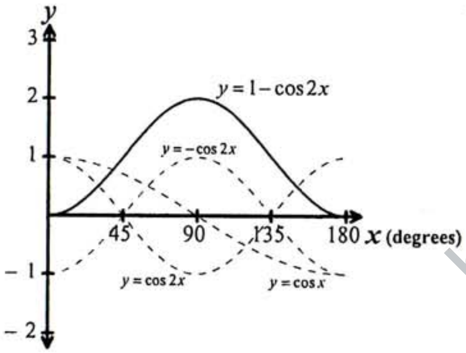
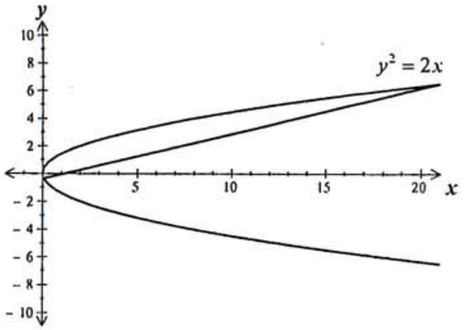
GMS(S) 2016 PRELIM 4N(A) A.Maths Paper 1 (4044 / 01) Answers

1	$p \le -12$ or $p \ge 12$	
2	$(x=1\ ,\ y=1)$	
3	$y=-\frac{1}{2}x+\frac{13}{2}$	
4	(i) $(x-2)(x^2+2x+4)$	(ii) $y=\underbrace{(x-2)}_{\substack{\text{cuts } x\text{-axis}\\ \text{at } x=2}}\underbrace{(x^2+2x+4)}_{\substack{\text{does not cut } x\text{-axis}\\ (b^2-4ac<0)}}$
5	$r=7$ cm	
6	(i) $1+8ax+28a^2x^2+56a^3x^3+\dots$	(ii) $a=\frac{1}{\sqrt{7}}$ or $\frac{\sqrt{7}}{7}$
7	$\frac{dy}{dx}=\frac{(2x-2)(2x+1)^2}{x^3}$ $(p=2\ ,q=-2)$	
8	(i) Use Chain Rule	(ii) 150
9	(i) $a=2\ ,\ b=-1$	(ii) $f''(1)=6\quad(>0)$ stationary point at $x=1$ is a minimum.
10	(a)(i) Period $=\pi$ radians Amplitude = 3 units	(a)(ii) $a=3\ ,\ b=2\ ,\ c=4$
	(b) 	
11	(i) $-3<k<3$	(ii) $y=-\frac{1}{2}x+\frac{17}{2}$

GMS(S) 2016 PRELIM 4N(A) A.Maths Paper 1 (4044 / 01) Answers

12	(i) 	
	(ii) $MP_{AB}(1\ ,1)$	(iii) Substitute $(1\ ,1)$ in $2x-y=1$ Show: $LHS = RHS$
13	(i) Use: $\sec^2 A=1+\tan^2 A$ and substitute into equation	(ii) $A=-11.3°\ ,\ 78.7°$
	(iii) Use: $\tan 2A=\frac{2\tan A}{1-\tan^2 A}$ and substitute into equation	

Mathematical Formulae

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Geylang Methodist School (Secondary)
Preliminary Examination 2016

ANSWER SCHEME

71

ADDITIONAL MATHEMATICS
Paper 1

4047/01

4 Normal (Academic)

Additional Materials provided: Writing Paper
Cover Sheet

1 hour 45 minutes

Setter: Ms Nainee Ismail

Thursday, 11 August 2016

READ THESE INSTRUCTIONS FIRST

Write your name, class and index number on all the work you hand in.
Write in dark blue or black pen on both sides of the paper.
You may use a soft pencil for any diagrams or graphs.
Do not use staples, paper clips, highlighters, glue or correction fluid.

Answer **all** the questions.
Write your answers on the writing papers provided.
Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.
The use of a scientific calculator is expected, where appropriate.
You are reminded of the need for clear presentation in your answers.

At the end of the examination, fasten all your work securely together.
The number of marks is given in brackets [ ] at the end of each question or part question.
The total score for this paper is 70.

This document consists of 17 printed pages including the cover page and 1 blank page

[Turn over

Mathematical Formulae

1. ALGEBRA

Quadratic Equation

For the equation $ax^2+bx+c=0$,

$$x=\frac{-b\pm\sqrt{b^2-4ac}}{2a}$$

Binomial expansion

$$(a+b)^n=a^n+\binom{n}{1}a^{n-1}b+\binom{n}{2}a^{n-2}b^2+\ldots+\binom{n}{r}a^{n-r}b^r+\ldots+b^n,$$

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Answer *all* the questions.

1 Find the set of values of p for which the equation $-3x^2+px-12=0$ has real roots. [3]

1 $-3x^2+px-12=0$

$[\,a=-3\,;\,b=p\,;\,c=-12\,]$

[real roots → $b^2-4ac\geq 0$]

$$b^2-4ac\geq 0$$

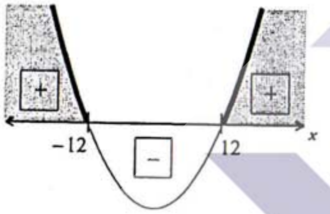
$$(p)^2-4(-3)(-12)\geq 0 \qquad \boxed{\text{M1}}$$

$$p^2-144\geq 0$$

$$p^2-12^2\geq 0$$

$$(p+12)(p-12)\geq 0 \qquad \boxed{\text{M1}}$$

[Leading to critical points : $p=-12$, $p=12$]



$(p+12)(p-12)\geq 0$

From graph:

$\Rightarrow p\leq -12$ or $p\geq 12$ $\boxed{\text{A1}}$

---

2 Solve the simultaneous equations

$$3^y\times 9^{-x}=\frac{1}{3},$$

$$x+2y-3=0.$$

[4]

2 $3^y\times 9^{-x}=\dfrac{1}{3}$ $\rightarrow(1)$

$x+2y-3=0$ $\rightarrow(2)$

(1): $3^y\times 9^{-x}=\dfrac{1}{3}$

$$(3)^y\times(3^2)^{-x}=(3)^{-1}$$

$$3^y\times 3^{-2x}=3^{-1}$$

$$3^{y+(-2x)}=3^{-1}$$

$$y-2x=-1$$

$\Rightarrow$ $y=2x-1$ $\rightarrow(3)$ $\boxed{\text{M1}}$

Substitute (3) into (2):

(2): $x+2y-3=0$

$$x+2(2x-1)-3=0 \qquad \boxed{\text{M1}}$$

$$x+4x-2-3=0$$

$$5x-5=0$$

$$5x=5 \qquad [+(5)\text{ on both sides}]$$

$\Rightarrow$ $x=1$ $\boxed{\text{A1}}$

Substitute $x=1$ into (3):

(3): $y=2x-1$

$$y=2(1)-1$$

$$y=2-1$$

$\Rightarrow$ $y=1$ $\boxed{\text{A1}}$

$\therefore$ $(x=1\,,\,y=1)$

---

3 The coordinates of the points A and B are $(-1,2)$ and $(3,10)$.

---

4 (i) Factorise x^3-8. [2]

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3 *The coordinates of the points A and B are $(-1, 2)$ and $(3, 10)$.*

Find the equation of the perpendicular bisector of AB. [4]

3 Given: $A(-1, 2)$, $B(3, 10)$

Mid-point of AB: $MP_{AB} = \left(\frac{x_1 + x_1}{2}, \frac{y_1 + y_2}{2}\right) = \left(\frac{-1+3}{2}, \frac{2+10}{2}\right)$

$MP_{AB} = \left(\frac{2}{2}, \frac{12}{2}\right) = (1, 6)$ **[M1]**

Gradient of AB: $m_{AB} = \frac{y_2 - y_1}{x_2 - x_1} = \frac{10-2}{3-(-1)} = \frac{8}{4} = 2$

$m_{AB} = \frac{8}{4} = 2$ **[M1]**

Gradient of $\perp$ bisector of AB: $(m_{\perp AB})(m_{AB}) = -1$

$m_{\perp AB} = \frac{-1}{m_{AB}} = \frac{-1}{2} = -\frac{1}{2}$ **[M1]**

Equation of $\perp$ bisector of AB: $(x_1, y_1) = (1, 6)$; $m_{\perp AB} = -\frac{1}{2}$

$y - y_1 = m_{\perp AB}(x - x_1)$

$y - 6 = -\frac{1}{2}(x - 1)$

$y = -\frac{1}{2}x + \frac{1}{2} + 6$

$\Rightarrow \quad y = -\frac{1}{2}x + \frac{13}{2}$ **[A1]**

4 (i) Factorise $x^3 - 8$. [2]

(ii) Explain why the curve $y = x^3 - 8$ only has one x – intercept at $x = 2$. [2]

4 (i) Difference of cubes:

$x^3 - y^3 = (x - y)(x^2 + xy + y^2)$

$x^3 - 8 = x^3 - 2^3$ **[M1]**

$x^3 - 2^3 = (x-2)(x^2 + x(2) + 2^2)$

$\therefore\ x^3 - 2^3 = (x-2)(x^2 + 2x + 4)$ **[A1]**

(ii) $y = x^3 - 8$

$y = (x-2)(x^2 + 2x + 4)$

x – intercept: $y = 0$

$y = x^3 - 8 \rightarrow 0 = x^3 - 8$

$y = (x-2)(x^2 + 2x + 4) \rightarrow 0 = (x-2)(x^2 + 2x + 4)$

$\Rightarrow x - 2 = 0 \qquad \Rightarrow x^2 + 2x + 4 = 0$

$\Rightarrow x = 2 \qquad \Rightarrow$ No solution $(b^2 - 4ac < 0)$

$(x^2 + 2x + 4) \Rightarrow a = 1$, $b = 2$, $c = 4$

Discriminant $= b^2 - 4ac$

$= 2^2 - 4(1)(4)$

$= 4 - 16$

Discriminant $= -12\ (< 0)$ **[M1]**

$$y = \underbrace{(x-2)}_{\text{cuts } x\text{-axis at } x=2}\ \underbrace{(x^2 + 2x + 4)}_{\text{does not cut } x\text{-axis } (b^2 - 4ac < 0)}$$ **[A1]**

Hence, $y = x^3 - 8$ has only one x – intercept at $x = 2$.

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5 The volume, V cm³, of a spherical ball is given by $V=\frac{4}{3}\pi r^3$, where r cm is the radius of the ball. The ball is deflated such that its volume decreases at a rate of 294 cm³s⁻¹. Find the value of r when the radius is decreasing at a rate of $\frac{3}{2\pi}$ cms⁻¹. [4]

5 Given: $V=\frac{4}{3}\pi r^3$;

$\frac{dV}{dt}=-294$ cm³s⁻¹ ; $\frac{dr}{dt}=-\frac{3}{2\pi}$ cms⁻¹ **[M1]**

(For values of $\frac{dV}{dt}$, $\frac{dr}{dt}$ stated as positive → increasing)

$\frac{dV}{dt}=294$ cm³s⁻¹ ; $\frac{dr}{dt}=\frac{3}{2\pi}$ cms⁻¹ – **[M1]** (Decreasing → negative)

$$V=\frac{4}{3}\pi r^3$$

$$\frac{dV}{dr}=\frac{4}{3}(3)\pi r^{3-1} \quad \boxed{\text{M1}}$$

$$\frac{dV}{dr}=4\pi r^2$$

$$\frac{dV}{dt}=\frac{dV}{dr}\times\frac{dr}{dt}$$

$$-294=4\pi r^2\times\left(-\frac{3}{2\pi}\right) \quad \boxed{\text{M1}}$$

$$-294=2r^2\times(-3)$$

$$-294=-6r^2$$

$$\frac{-294}{-6}=r^2$$

$$r^2=49$$

$$r=\pm\sqrt{49}$$

$$r=\pm 7$$

$$\therefore r=7\text{ cm} \quad \boxed{\text{A1}}$$

(Reject $r=-7$ cm, radius is positive)

6 (i) Write down the first four terms in the expansion, in ascending powers of x, of $(1+ax)^8$, where a is a positive constant. [2]

(ii) Given that the coefficients of x and x^3 in the expansion of $(1+ax)^8$ are equal, find the value of the constant a, leaving your answer in surd form. [3]

6 (i)

$$(1+ax)^8 =\binom{8}{0}1^{8-0}(ax)^0+\binom{8}{1}1^{8-1}(ax)^1+\binom{8}{2}1^{8-2}(ax)^2+\binom{8}{3}1^{8-3}(ax)^3+\ldots$$

$$=(1)1^8(1)+(8)1^7(ax)+(28)1^6(a^2x^2)+(56)1^5(a^3x^3)+\ldots \quad \boxed{\text{M1}}$$

$$\therefore (1+ax)^8 =1+8ax+28a^2x^2+56a^3x^3+\ldots \quad \boxed{\text{A1}}$$

(ii) Coefficients of x and x^3 are equal:

Term containing x $=8ax$

Term containing x^3 $=56a^3x^3$

$$\Rightarrow 8a=56a^3 \quad \boxed{\text{M1}}$$

$$0=56a^3-8a$$

$$8a(7a^2-1)=0$$

$$\rightarrow 8a=0 \qquad \rightarrow (7a^2-1)=0 \quad \boxed{\text{M1}}$$

$$a=0 \qquad 7a^2=1$$

$$a^2=\frac{1}{7}$$

$$a=\pm\sqrt{\frac{1}{7}}=\pm\frac{\sqrt{1}}{\sqrt{7}}=\pm\frac{1}{\sqrt{7}}$$

$$\therefore a=\frac{1}{\sqrt{7}} \quad (a \text{ is a positive constant})$$

$$OR \quad a=\frac{1}{\sqrt{7}}\times\frac{\sqrt{7}}{\sqrt{7}}$$

$$\therefore a=\frac{\sqrt{7}}{7} \quad \boxed{\text{A1}}$$

– **[A1]** (For answers in decimal)

(Reject $a=-\frac{1}{\sqrt{7}}$ or 0 or $-\frac{\sqrt{7}}{7}$ → a is a positive constant.)

$(2x^2-1)^4$, show that $\frac{dy}{dx}=16x(2x^2-1)^3$. [2]

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7 Given that $y=(2x+1)^3x^{-2}$, find $\frac{dy}{dx}$, giving your answer in the form $\frac{(px+q)(2x+1)^2}{x^3}$ where p and q are integers. [5]

7 Given: $y = (2x+1)^3 x^{-2}$ Let: $u=(2x+1)^3$

$$\frac{du}{dx}=3(2x+1)^{3-1}(2)$$

$$\frac{du}{dx}=6(2x+1)^2$$

$$v=x^{-2}$$

$$\frac{dv}{dx}=-2x^{-2-1}$$

$$\frac{dv}{dx}=-2x^{-3}$$

$$\frac{dv}{dx}=\frac{-2}{x^3}$$

[M1]

$$\frac{dy}{dx} = u\,dv + v\,du$$

$$\frac{dy}{dx} = (2x+1)^3\left(\frac{-2}{x^3}\right)+\left(x^{-2}\right)6(2x+1)^2 \quad \boxed{[M1]}$$

$$\frac{dy}{dx} = \frac{-2(2x+1)^3}{x^3}+\frac{6(2x+1)^2}{x^2}$$

[Extract common factor]

$$\frac{dy}{dx} = \frac{2(2x+1)^2}{x^2}\left[\frac{-(2x+1)}{x}+\frac{3}{1}\right] \quad \boxed{[M1]}$$

$$\frac{dy}{dx} = \frac{2(2x+1)^2}{x^2}\left[\frac{-(2x+1)}{x}+\frac{3}{1}\left(\frac{1}{1}\right)\right]$$

$$\frac{dy}{dx} = \frac{2(2x+1)^2}{x^2}\left[\frac{-(2x+1)}{x}+\frac{3}{1}\left(\frac{x}{x}\right)\right]$$

$$\frac{dy}{dx} = \frac{2(2x+1)^2}{x^2}\left[\frac{-2x-1}{x}+\frac{3x}{x}\right]$$

$$\frac{dy}{dx} = \frac{2(2x+1)^2}{x^2}\left[\frac{-2x-1+3x}{x}\right] \quad \boxed{[M1]}$$

$$\frac{dy}{dx} = \frac{2(2x+1)^2}{x^2}\left[\frac{x-1}{x}\right]$$

$$\frac{dy}{dx} = \frac{2(2x+1)^2(x-1)}{x^2(x)}$$

$$\frac{dy}{dx} = \frac{(2x-2)(2x+1)^2}{x^3} \quad \text{(Shown)} \quad \boxed{[A1]}$$

$$\frac{dy}{dx} = \frac{(px+q)(2x+1)^2}{x^3} \quad (p=2\,,\ q=-2)$$

8 (i) Given that $y=(2x^2-1)^4$, show that $\frac{dy}{dx}=16x(2x^2-1)^3$. [2]

(ii) Hence, find $\int_0^2 x(2x^2-1)^3\,dx$. [3]

8 (i) $y=(2x^2-1)^4$

$$\frac{dy}{dx}=4(2x^2-1)^{4-1}(4x) \quad \boxed{[M1]}$$

$$\therefore \frac{dy}{dx}=16x(2x^2-1)^3 \quad \text{(Shown)} \quad \boxed{[A1]}$$

(ii) From (i),

$$\int 16x(2x^2-1)^3\,dx = (2x^2-1)^4+c \quad \boxed{[M1]}$$

$$16\int x(2x^2-1)^3\,dx = (2x^2-1)^4+c$$

$$\int x(2x^2-1)^3\,dx = \left[\frac{1}{16}(2x^2-1)^4\right]+c \quad [\div(16) \text{ on both sides}]$$

$$\int_0^2 x(2x^2-1)^3\,dx = \frac{1}{16}\left[(2x^2-1)^4\right]_0^2$$

$$\int_0^2 x(2x^2-1)^3\,dx = \frac{1}{16}\left[(2(2)^2-1)^4 - (2(0)^2-1)^4\right] \quad \boxed{[M1]}$$

$$= \frac{1}{16}\left[(7)^4 - (-1)^4\right]$$

$$= \frac{1}{16}\left[2401 - 1\right]$$

$$= \frac{1}{16}\left[2400\right]$$

$$\therefore \int_0^2 x(2x^2-1)^3\,dx = 150 \quad \boxed{[A1]}$$

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9 The function $f(x) = ax^3 - 3x^2 + b$, where x is a real number and that $f(2) = 3$, has a stationary value at $x = 1$.

(i) Find the value of the constants a and b. [4]

(ii) Explain, with justification, why the stationary point at $x = 1$ is a minimum. [3]

9 (i) Equation (1): $f(x) = ax^3 - 3x^2 + b$; $f(2) = 3$

$f(2) = a(2)^3 - 3(2)^2 + b$ [M1]

$3 = 8a - 12 + b$

$3 + 12 = 8a + b$

$8a + b = 15 \quad \rightarrow (1)$

Equation (2): $f(x) = ax^3 - 3x^2 + b$ has a stationary value at $x = 1 \Rightarrow f'(1) = 0$

$f'(x) = 3ax^2 - 6x$ [M1]

$f'(1) = 3a(1)^2 - 6(1)$

$0 = 3a - 6$

$6 = 3a$

$\therefore a = 2$ [A1]

Substitute $a = 2$ into (1):

(1): $8a + b = 15$

$8(2) + b = 15$

$16 + b = 15$

$b = 15 - 16$

$\therefore b = -1$ [A1]

(ii) $a = 2, b = -1 \Rightarrow f(x) = 2x^3 - 3x^2 - 1$

$f'(x) = 6x^2 - 6x$ [M1]

$f''(x) = 12x - 6$ [M1]

At $x = 1 \Rightarrow f''(1) = 12(1) - 6$

$f''(1) = 12 - 6$

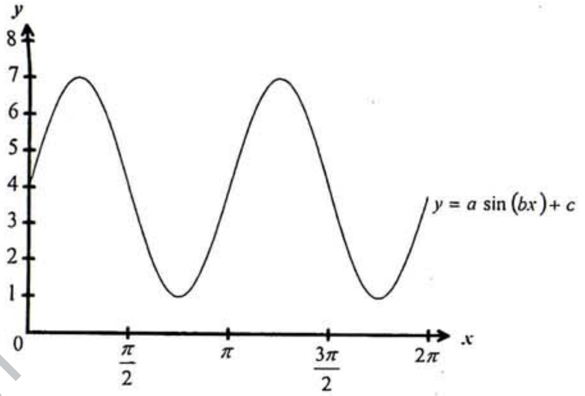
$f''(1) = 6 \quad (> 0)$ [A1]

Since $f''(1) = 6 \quad (> 0)$,

$\Rightarrow$ Second Derivative is positive at $x = 1$,

$\Rightarrow$ Hence stationary point at $x = 1$ is a minimum.

10 (a) The diagram shows part of the graph $y = a\sin(bx) + c$.



State

(i) the period and amplitude of y, [2]

(ii) the value of a, b and c. [3]

(b) On a separate diagram, sketch the graph of $y = 1 - \cos 2x$ for $0° \le x \le 180°$. [2]

10 (a) (i) Period $= \pi$ radians [A1]

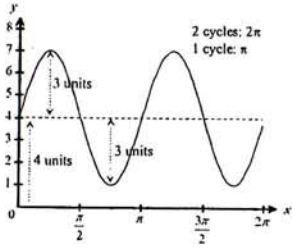
Amplitude = 3 units [A1]

(ii) $a = 3$ [A1]

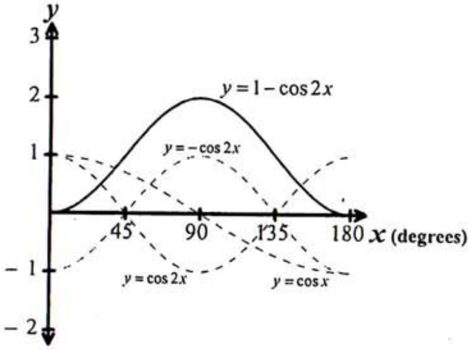
$b = 2$ [A1]

$c = 4$ [A1]

$y = 3\sin 2x + 4$



(b) $y = 1 - \cos 2x$ for $0° \le x \le 180°$.



- Shape
- Period/Range
- Max/Min point
- x, y – intercepts
- Label graph/axes

[A2]

– [A1] (For each missing point)

[2]

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$\Rightarrow$ Hence stationary point at $x=1$ is a minimum.

---

11 The function $f(x)$ is defined as $f(x)=x^3-kx^2+3x+6$, for all real values of x.

(i) Find the range of values of k for which $f(x)$ is an increasing function. [3]

(ii) For the case $k=2$, find the equation of the normal to the curve $y=f(x)$ where $x=1$. [4]

11 (i) $f(x)=x^3-kx^2+3x+6$

$f'(x)=3x^2-2kx+3$

$f(x)$ is increasing:

$\Rightarrow \quad f'(x)>0$

$\Rightarrow \quad 3x^2-2kx+3>0 \quad [\,a=3\,;\,b=-2k\,;\,c=3\,]$ **[M1]**

$[\,a>0\;;\;3x^2-2kx+3>0\,]$

$\rightarrow \quad a>0$, minimum curve

$\rightarrow \quad 3x^2-2kx+3>0$, always positive (above x – axis)

$\therefore \quad b^2-4ac<0$, does not cut x – axis (no real roots)

$\Rightarrow \quad b^2-4ac<0$

$(-2k)^2-4(3)(3)<0 \quad [\,a=3\,;\,b=-2k\,;\,c=3\,]$ **[M1]**

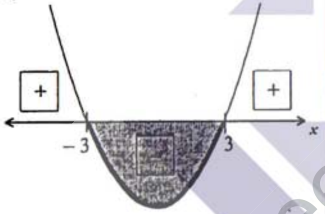
$4k^2-36<0 \quad [\div(4)$ on both sides$]$

$k^2-9<0$

$k^2-3^2<0$

$(k+3)(k-3)<0$

[Leading to critical points : $k=-3$, $k=3$]



$(k+3)(k-3)<0$

From graph:

$\therefore \quad -3<k<3$ **[A1]**

(ii) $y=f(x)=x^3-kx^2+3x+6$

Given: $k=2$, $x=1$

$y=f(1)=(1)^3-(2)(1)^2+3(1)+6$

$y=1-2+3+6$

$y=8 \quad (1\,,8)$ **[M1]**

Gradient of tangent at $x=1$: $\quad m_{tan}=f'(1)=3(1)^2-2(2)(1)+3=3-4+3$

$[\,f'(x)=3x^2-2kx+3\,] \quad m_{tan}=6-4=2$ **[M1]**

Gradient of normal at $x=1$: $\quad (m_{tan})(m_{nor})=-1$ **[M1]**

$$m_{nor}=\frac{-1}{m_{tan}}=\frac{-1}{2}=-\frac{1}{2}$$

Equation of normal to the curve at $(1\,,8)$: $\quad y-y_1=m_{nor}(x-x_1)$

$(x_1,y_1)=(1\,,8) \;;\; m_{nor}=-\dfrac{1}{2} \qquad y-8=-\dfrac{1}{2}(x-1)$

$$y=-\frac{1}{2}x+\frac{1}{2}+8$$

$$\Rightarrow \quad y=-\frac{1}{2}x+\frac{17}{2}$$ **[A1]**

[Turn over]

$-2\downarrow$

– [A1] (For each missing point)

---

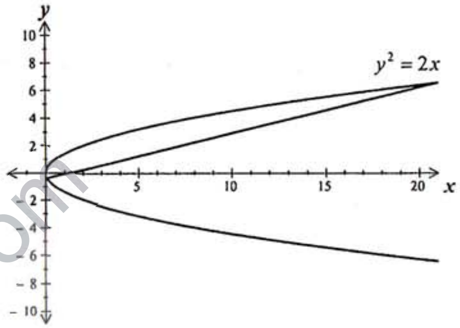
12 (i) Sketch the parabola $y^2=2x$. [2]

The line $y=x$ intersects the parabola at the points A and B.

(ii) Find the midpoint of AB. [4]

(iii) Show that the midpoint of AB lies on the line $2x-y=1$. [1]

12 (i)



- Shape of parabola
- x, y – intercepts
- Label graph/axes

[A2]

– **[A1]** (For each missing point)

Scale is not necessary

(ii) Points of intersection – Solve equations simultaneously:

$y^2=2x \quad \rightarrow(1)$

$y=x \quad \rightarrow(2)$

Substitute (2) into (1):

(1): $\quad y^2=2x$

$(x)^2=2x$ **[M1]**

$x^2-2x=0$

$x(x-2)=0$

$\Rightarrow \quad x=0 \qquad \Rightarrow \quad x-2=0$

$\Rightarrow \quad x=2$

Substitute $x=0$, $x=2$ into (2):

(2): $\; y=x \;\; (x=0) \qquad (2)$: $\; y=x \;\; (x=2)$

$y=0 \;\; (0\,,0) \qquad y=2 \;\; (2\,,2)$

$\Rightarrow \quad A(0\,,0)$ **[M1]** $\qquad \Rightarrow \quad B(2\,,2)$ **[M1]**

$$MP_{AB}=\left(\frac{x_1+x_2}{2},\frac{y_1+y_2}{2}\right)=\left(\frac{0+2}{2},\frac{0+2}{2}\right)=\left(\frac{2}{2},\frac{2}{2}\right)=(1\,,1)$$ **[A1]**

(iii) $(1\,,1) \;\Rightarrow\; x=1\,,\; y=1 \;\Rightarrow\;$ Substitute in $2x-y=1$

$LHS=2x-y=2(1)-(1)=1$

$RHS=1$

Since $LHS=RHS$, $(1\,,1)$ satisfies $2x-y=1$ [$(1\,,1)$ lies on $2x-y=1$] (Shown) **[B1]**

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13 (i) Show that the equation

$$24\tan A - 2\tan^2 A = 3\sec^2 A - 8$$

may be written in the form

$$5\tan^2 A - 24\tan A - 5 = 0.$$ [2]

(ii) Hence, find, in degrees, the two principal values of A for which

$$24\tan A - 2\tan^2 A = 3\sec^2 A - 8.$$ [4]

(iii) By using a suitable identity, explain why the values of A found in (ii) satisfy the equation $\tan 2A = -\frac{5}{12}$. [2]

13 (i) $24\tan A - 2\tan^2 A = 3\sec^2 A - 8$ [From: $\sec^2 A = 1 + \tan^2 A$]

$24\tan A - 2\tan^2 A = 3\left(1 + \tan^2 A\right) - 8$ [M1]

$24\tan A - 2\tan^2 A = 3 + 3\tan^2 A - 8$

$0 = 3 + 3\tan^2 A - 8 + 2\tan^2 A - 24\tan A$

$0 = 5\tan^2 A - 24\tan A - 5$

$5\tan^2 A - 24\tan A - 5 = 0$ (Shown) [A1]

(ii) $24\tan A - 2\tan^2 A = 3\sec^2 A - 8$ (Principal values)

[From (i)]:

$5\tan^2 A - 24\tan A - 5 = 0$ [M1] (Principal value of tangent: $-90° < \tan^{-1} A < -90°$)

$(\tan A - 5)(5\tan A + 1) = 0$ [M1]

$\tan A$	-5	$-25\tan A$
$5\tan A$	$+1$	$+\tan A$
$5\tan^2 A$	-5	$-24\tan A$

$\Rightarrow$ $\tan A - 5 = 0$

$\tan A = 5$

$A = \tan^{-1}(5)$

$\therefore$ $A = 78.69007° \approx 78.7°$ [A1]

$\Rightarrow$ $5\tan A + 1 = 0$

$5\tan A = -1$

$\tan A = \frac{-1}{5}$

$A = \tan^{-1}\left(-\frac{1}{5}\right)$

$\therefore$ $A = -11.30993° \approx -11.3°$ [A1]

$\therefore$ $A = -11.3°$, $78.7°$

(iii) $\tan 2A = -\frac{5}{12}$ [From: $\tan 2A = \frac{2\tan A}{1 - \tan^2 A}$]

$\frac{2\tan A}{1 - \tan^2 A} = \frac{5}{-12}$ [M1]

$(2\tan A)(-12) = (5)\left(1 - \tan^2 A\right)$

$(-24\tan A) = 5 - 5\tan^2 A$

$5\tan^2 A - 24\tan A - 5 = 0$ [A1]

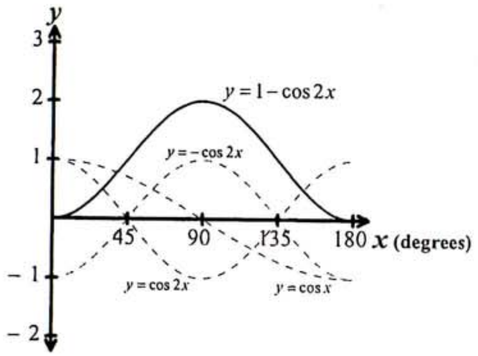
$24\tan A - 2\tan^2 A = 3\sec^2 A - 8$ [From (i)]

$\tan 2A = -\frac{5}{12}$ is equivalent to

$5\tan^2 A - 24\tan A - 5 = 0$; $24\tan A - 2\tan^2 A = 3\sec^2 A - 8$ (Shown)

- End Of Paper -

GMS(S) 2016 PRELIM 4N(A) A.Maths Paper 1 (4044/01) Answers

1	$p \le -12$ or $p \ge 12$	
2	$(x = 1\ ,\ y = 1)$	
3	$y = -\frac{1}{2}x + \frac{13}{2}$	
4	(i) $(x-2)\left(x^2+2x+4\right)$	(ii) $y = \underbrace{(x-2)}_{\substack{\text{cuts } x\text{-axis} \\ \text{at } x=2}} \underbrace{\left(x^2+2x+4\right)}_{\substack{\text{does not cut } x\text{-axis} \\ (b^2-4ac<0)}}$
5	$r = 7\,\text{cm}$	
6	(i) $1 + 8ax + 28a^2x^2 + 56a^3x^3 + \ldots$	(ii) $a = \frac{1}{\sqrt{7}}$ or $\frac{\sqrt{7}}{7}$
7	$\frac{\mathrm{d}y}{\mathrm{d}x} = \frac{(2x-2)(2x+1)^2}{x^3}$ $(p = 2\ , q = -2)$	
8	(i) Use Chain Rule	(ii) 150
9	(i) $a = 2\ ,\ b = -1$	(ii) $f''(1) = 6\ \ (>0)$ stationary point at $x = 1$ is a minimum.
10	(a)(i) Period $= \pi$ radians Amplitude = 3 units	(a)(ii) $a = 3\ ,\ b = 2\ ,\ c = 4$
	(b) 	
11	(i) $-3 < k < 3$	(ii) $y = -\frac{1}{2}x + \frac{17}{2}$

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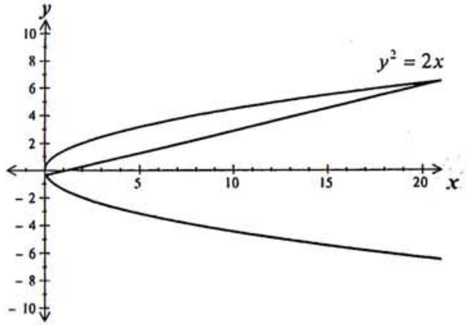
12
$5\tan^2 A-24\tan A-5=0$; $24\tan A-2\tan^2 A=3\sec^2 A-8$ (Shown)

GMS(S) 2016 PRELIM 4N(A) A Maths Paper 1 [redacted] Answers

12	(i) 	
	(ii) $MP_{AB}(1,1)$	(iii) Substitute $(1,1)$ in $2x-y=1$ Show: $LHS = RHS$
13	(i) Use: $\sec^2 A=1+\tan^2 A$ and substitute into equation	(ii) $A=-11.3°$, $78.7°$
	(iii) Use: $\tan 2A=\dfrac{2\tan A}{1-\tan^2 A}$ and substitute into equation	

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- End Of Paper -

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Geylang Methodist School (Secondary)
Preliminary Examination 2016

ADDITIONAL MATHEMATICS
Paper 2

4044 / 02

4 Normal (Academic)

1 hour 45 minutes

Additional Materials provided: Writing Paper
Cover Sheet

Setter: Ms Nainee Ismail

Monday, 15 August 2016

READ THESE INSTRUCTIONS FIRST

Write your name, class and index number on all the work you hand in.
Write in dark blue or black pen on both sides of the paper.
You may use a soft pencil for any diagrams or graphs.
Do not use staples, paper clips, highlighters, glue or correction fluid.

Answer **all** the questions.
Write your answers on the writing papers provided.
Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.
The use of a scientific calculator is expected, where appropriate.
You are reminded of the need for clear presentation in your answers.

At the end of the examination, fasten all your work securely together.
The number of marks is given in brackets [ ] at the end of each question or part question.
The total score for this paper is 70.

This document consists of 5 printed pages including the cover page and 1 blank page.

[Turn over

Mathematical Formulae

1. ALGEBRA

Quadratic Equation

For the equation $ax^2+bx+c=0$,

$$x=\frac{-b\pm\sqrt{b^2-4ac}}{2a}$$

Binomial expansion

$$(a+b)^n=a^n+\binom{n}{1}a^{n-1}b+\binom{n}{2}a^{n-2}b^2+\ldots+\binom{n}{r}a^{n-r}b^r+\ldots+b^n,$$

where n is a positive integer and $\binom{n}{r}=\frac{n!}{r!(n-r)!}=\frac{n(n-1)\ldots(n-r+1)}{r!}$

2. TRIGONOMETRY

Identities

$$\sin^2 A+\cos^2 A=1$$
$$\sec^2 A=1+\tan^2 A$$
$$\operatorname{cosec}^2 A=1+\cot^2 A$$
$$\sin(A\pm B)=\sin A\cos B\pm\cos A\sin B$$
$$\cos(A\pm B)=\cos A\cos B\mp\sin A\sin B$$
$$\tan(A\pm B)=\frac{\tan A\pm\tan B}{1\mp\tan A\tan B}$$
$$\sin 2A=2\sin A\cos A$$
$$\cos 2A=\cos^2 A-\sin^2 A=2\cos^2 A-1=1-2\sin^2 A$$
$$\tan 2A=\frac{2\tan A}{1-\tan^2 A}$$

Formulae for ΔABC

$$\frac{a}{\sin A}=\frac{b}{\sin B}=\frac{c}{\sin C}$$
$$a^2=b^2+c^2-2bc\cos A$$
$$\Delta=\frac{1}{2}ab\sin C$$

Answer *all* the questions.

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6 In triangle ABC, the length, in cm, of AB is $1+4\sqrt{2}$, of BC is $2\sqrt{2}-2$

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Answer all the questions.

1 Find the remainder when $x^4 - 2x^3 + 6x^2 - 4x + 9$ is divided by $x^2 - 3x + 1$. [3]

2 (i) Find the set of values of c for which the quadratic expression $-x^2 + 4x + c$ is always negative. [3]

(ii) Explain why the quadratic expression $-x^2 + 4x - 8$ is always negative. [1]

3 (i) Express $\sqrt{3}\cos\theta + \sin\theta$ in the form $R\cos(\theta - \alpha)$, where $R > 0$ and $0 < \alpha < \frac{\pi}{2}$ radians. [3]

(ii) Hence, or otherwise, state in radians, the principal value of $\sqrt{3}\cos\theta + \sin\theta = 0$. [2]

4 The roots of a quadratic equation are α and β, where $\alpha + \beta = -\frac{1}{2}$ and $\alpha\beta = 1$.

(i) Find this quadratic equation in the form $2x^2 + bx + c = 0$, where b and c are integers. [2]

The roots of the quadratic equation $x^2 + px + q = 0$ are $\frac{\alpha}{\beta}$ and $\frac{\beta}{\alpha}$.

(ii) Find the value of each of the constants p and q. [4]

5 (i) Prove that $\frac{\cos x}{1 - \sin x} + \frac{1 - \sin x}{\cos x} = 2\sec x$. [3]

(ii) Hence, state the exact value of $\tan(90° - x)$ such that x is an acute angle which satisfies

$$\frac{\cos x}{1 - \sin x} + \frac{1 - \sin x}{\cos x} = \frac{9}{4}.$$ [3]

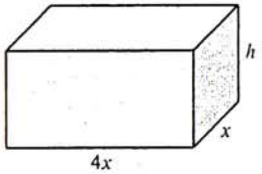
[Turn over

6 In triangle ABC, the length, in cm, of AB is $1 + 4\sqrt{2}$, of BC is $2\sqrt{2} - 2$ and of AC is $3\sqrt{5}$. Without using a calculator,

(i) explain why triangle ABC is right-angled and state which angle is $90°$, [4]

(ii) show that $\tan B\hat{C}A = \frac{a + b\sqrt{2}}{2}$, where a and b are integers to be found. [4]

7 The diagram shows a closed cuboid box. A contractor wants to use metal sheet of negligible thickness to make a cuboid box that holds exactly 500 cm^3 of sand. The cost is minimum if the box has the smallest possible surface area. The dimensions of the box is such that the sides of its base are of length $4x$ cm, width x cm and height h cm. The total surface area is A cm^2.



(i) Express h in terms of x and show that $A = 8x^2 + \frac{1250}{x}$. [3]

(ii) Given that x can vary, find the value of the surface area for which the cost is least. [4]

(iii) Calculate the minimum cost, to the nearest cent, of producing 1 box if a metal sheet costs 25 cents per 1000 cm^2. [1]

8 A circle C_1 has centre $(3, -4)$ and radius 5.

(i) State the equation of C_1. [2]

The equation of circle C_2 is $x^2 + y^2 - 10x = 0$.

(ii) Find the coordinates of the centre of C_2 and the radius of C_2. [4]

(iii) Justify with suitable calculations, if the centre of the circle C_1 lies within the circle C_2. [2]

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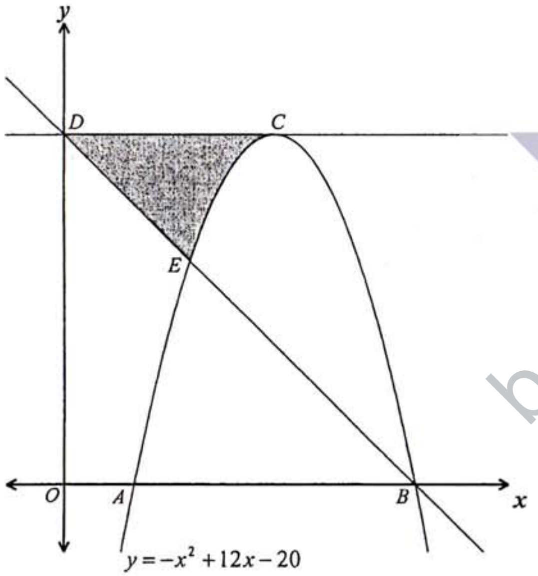
9 A polynomial $f(x)$ is such that $f''(x)=12x+10$, $f'(1)=12$ and $f(-1)=4$.

(i) Using integration, show that $f(x)=2x^3+5x^2-4x-3$. **[5]**

(ii) Show that $x-1$ is a factor of $f(x)$, and solve the equation $f(x)=0$. **[5]**

10 The diagram shows part of the curve $y=-x^2+12x-20$.
The curve crosses the x – axis at the points A and B.
The tangent to the curve at the point C is parallel to the x – axis.
The straight line BD passes through the curve at points B and E.
The tangent to the curve at the point C and the straight line BD meets at point D.



(i) Find the coordinates of A, B, C, D and E. **[7]**

(ii) Find the area of the shaded region CDE. **[5]**

- End Of Paper -

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(ii) Find the area of the shaded region CDE. [5]

GMS(S) 2016 PRELIM 4N(A) A Maths Paper 2 [redacted] Answers

1	$19x+1$		
2	(i) $c<-4$	(ii) $-x^2+4x-8$ $(c=-8 \rightarrow c<-4)$ $-x^2+4x-8$ is always negative	
3	(i) $\left(\sqrt{3}\cos\theta+\sin\theta\right)=2\cos\left(\theta-\frac{\pi}{6}\right)$	(ii) Principal value: $\frac{2\pi}{3}$	
4	(i) $2x^2+x+2=0 \Rightarrow b=1, c=2$	(ii) $p=\frac{7}{4}$; $q=1$	
5	(ii) $\frac{8}{\sqrt{17}}$ *OR* $\frac{8}{17}\sqrt{17}$		
6	(i) $\Rightarrow (AB)^2+(BC)^2=(AC)^2$ $\Rightarrow$ Obeys Pythagoras' Theorem $\Rightarrow \Delta ABC$ is a right – angled triangle with $AC \rightarrow$ hypotenuse $\therefore \angle ABC$ is right – angled, 90° (angle opposite the hypotenuse side)		
	(ii) $\frac{9+5\sqrt{2}}{2}$ $[\,a=9\ ,\ b=5\,]$		
7	(i) $h=\frac{125}{x^2}$, $A=8x^2+\frac{1250}{x}$	(ii) 439 cm^2 (3sf)	(iii) 11 cents
8	(i) $(x-3)^2+(y+4)^2=25$ *OR* $x^2+y^2-6x+8y=0$	(ii) $C(5, 0)$, 5 units	(iii) Centre of Circle C_1 lies within Circle C_2
9	(ii) Show $\text{f}(1)=0 \Rightarrow (x-1)$ is a factor of $\text{f}(x)=2x^3+5x^2-4x-3$ $\text{f}(x)=0 \rightarrow x=-3\ ,-\frac{1}{2}\ , 1$		
10	(i) $A(2,0)$, $B(10,0)$, $C(6,16)$, $D(0,16)$, $E(3.6, 10.24)$		
	(ii) 14.976 square units		

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Geylang Methodist School (Secondary)
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ANSWER SCHEME

ADDITIONAL MATHEMATICS 4044 / 02
Paper 2
4 Normal (Academic)

Additional Materials provided: Writing Paper
Cover Sheet
1 hour 45 minutes

Setter: Ms Nainee Ismail
Monday, 15 August 2016

READ THESE INSTRUCTIONS FIRST

Write your name, class and index number on all the work you hand in.
Write in dark blue or black pen on both sides of the paper.
You may use a soft pencil for any diagrams or graphs.
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Answer all the questions.
Write your answers on the writing papers provided.
Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.
The use of a scientific calculator is expected, where appropriate.
You are reminded of the need for clear presentation in your answers.

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The number of marks is given in brackets [ ] at the end of each question or part question.
The total score for this paper is 70.

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[Turn over

Mathematical Formulae

1. ALGEBRA

Quadratic Equation

For the equation $ax^2+bx+c=0$,

$$x=\frac{-b\pm\sqrt{b^2-4ac}}{2a}$$

Binomial expansion

$$(a+b)^n=a^n+\binom{n}{1}a^{n-1}b+\binom{n}{2}a^{n-2}b^2+\ldots+\binom{n}{r}a^{n-r}b^r+\ldots+b^n,$$

where n is a positive integer and $\binom{n}{r}=\frac{n!}{r!(n-r)!}=\frac{n(n-1)\ldots(n-r+1)}{r!}$

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2. TRIGONOMETRY

Identities

$$\sin^2 A+\cos^2 A=1$$
$$\sec^2 A=1+\tan^2 A$$
$$\operatorname{cosec}^2 A=1+\cot^2 A$$
$$\sin(A\pm B)=\sin A\cos B\pm\cos A\sin B$$
$$\cos(A\pm B)=\cos A\cos B\mp\sin A\sin B$$
$$\tan(A\pm B)=\frac{\tan A\pm\tan B}{1\mp\tan A\tan B}$$
$$\sin 2A=2\sin A\cos A$$
$$\cos 2A=\cos^2 A-\sin^2 A=2\cos^2 A-1=1-2\sin^2 A$$
$$\tan 2A=\frac{2\tan A}{1-\tan^2 A}$$

Formulae for ΔABC

$$\frac{a}{\sin A}=\frac{b}{\sin B}=\frac{c}{\sin C}$$
$$a^2=b^2+c^2-2bc\cos A$$
$$\Delta=\frac{1}{2}ab\sin C$$

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*Answer **all** the questions.*

1 Find the remainder when $x^4 - 2x^3 + 6x^2 - 4x + 9$ is divided by $x^2 - 3x + 1$. [3]

1

$$\begin{array}{r|l} & x^2 + x + 8 \\ x^2 - 3x + 1 & \overline{x^4 - 2x^3 + 6x^2 - 4x + 9} \\ & \underline{-\left(x^4 - 3x^3 + x^2\right)} \downarrow \downarrow \\ & x^3 + 5x^2 - 4x \downarrow \\ & \underline{-\left(x^3 - 3x^2 + x\right)} \downarrow \\ & 8x^2 - 5x + 9 \\ & \underline{-\left(8x^2 - 24x + 8\right)} \\ & \underline{19x + 1}. \end{array}$$

[M1]

[M1]

Since $\dfrac{x^4 - 2x^3 + 6x^2 - 4x + 9}{x^2 - 3x + 1} = x^2 + x + 8 + \dfrac{19x + 1}{x^2 - 3x + 1}$

Remainder = $19x + 1$ [A1]

2 **(i)** Find the set of values of c for which the quadratic expression $-x^2 + 4x + c$ is always negative. [3]

(ii) Explain why the quadratic expression $-x^2 + 4x - 8$ is always negative. [1]

2 **(i)** Given: Quadratic expression $-x^2 + 4x + c$ is always negative

$\Rightarrow \quad -x^2 + 4x + c < 0 \qquad [\, a = -1 \,;\, b = 4 \,;\, c = c \,]$ [M1]

$[\, a < 0 \;;\; -x^2 + 4x + c < 0 \,]$

$\rightarrow \quad a < 0$, maximum curve

$\rightarrow \quad -x^2 + 4x + c < 0$, always negative (below x – axis)

$\therefore \quad b^2 - 4ac < 0$, does not cut x – axis (no real roots) [M1]

$\Rightarrow \quad b^2 - 4ac < 0$

$(4)^2 - 4(-1)(c) < 0 \qquad [\, a = -1 \,;\, b = 4 \,;\, c = c \,]$

$16 + 4c < 0$

$4c < -16 \qquad [\div (4)$ on both sides$]$

$\therefore \quad c < -4$ [A1]

(ii) For: Quadratic expression $-x^2 + 4x + c$ to be always negative $\Rightarrow c < -4$ [From **(i)**]

Quadratic expression $-x^2 + 4x - 8 \quad (c = -8 \rightarrow c < -4)$ [A1]

Hence, $-x^2 + 4x - 8$ is always negative.

[Accept working that states: $a < 0 \;\; (a = -1)$ and $b^2 - 4ac < 0 \;\; (a = -1, b = 4, c = -8)$]

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3 (i) Express $\sqrt{3}\cos\theta+\sin\theta$ in the form $R\cos(\theta-\alpha)$,

where $R>0$ and $0<\alpha<\frac{\pi}{2}$ radians. [3]

(ii) Hence, or otherwise, state in radians, the principal value of $\sqrt{3}\cos\theta+\sin\theta=0$. [2]

3 (i) The *R*-Formula:

$$(a\cos\theta+b\sin\theta)=R\cos(\theta-\alpha),\quad R=\sqrt{a^2+b^2}\ \text{ and }\ \tan\alpha=\frac{b}{a}$$

$$\left(\sqrt{3}\cos\theta+\sin\theta\right)\Rightarrow a=\sqrt{3},b=1$$

$$R=\sqrt{a^2+b^2}=\sqrt{\left(\sqrt{3}\right)^2+1^2}=\sqrt{3+1}=\sqrt{4}=2 \qquad \boxed{\text{M1}}$$

$$\tan\alpha=\frac{b}{a}=\frac{1}{\sqrt{3}}\Rightarrow\alpha=\tan^{-1}\left(\frac{1}{\sqrt{3}}\right)\Rightarrow\alpha=\frac{\pi}{6}\text{ radians} \qquad \boxed{\text{M1}}$$

$$\therefore\quad\left(\sqrt{3}\cos\theta+\sin\theta\right)=2\cos\left(\theta-\frac{\pi}{6}\right) \qquad \boxed{\text{A1}}$$

(ii)

$$\sqrt{3}\cos\theta+\sin\theta=0$$

$$2\cos\left(\theta-\frac{\pi}{6}\right)=0 \qquad \text{[From (i)]}$$

$$\cos\left(\theta-\frac{\pi}{6}\right)=0 \qquad (\text{Principal value of cosine: } 0\le\cos^{-1}x\le\pi)$$

$$\left(\theta-\frac{\pi}{6}\right)=\cos^{-1}0 \qquad \boxed{\text{M1}}$$

$$\theta-\frac{\pi}{6}=\frac{\pi}{2}$$

$$\theta=\frac{\pi}{2}+\frac{\pi}{6}$$

$$\theta=\frac{3\pi}{6}+\frac{\pi}{6}$$

$$\therefore\text{Principal value: }\ \theta=\frac{4\pi}{6}=\frac{2\pi}{3} \qquad \boxed{\text{A1}}$$

4 The roots of a quadratic equation are α and β, where $\alpha+\beta=-\frac{1}{2}$ and $\alpha\beta=1$.

(i) Find this quadratic equation in the form $2x^2+bx+c=0$, where b and c are integers. [2]

The roots of the quadratic equation $x^2+px+q=0$ are $\frac{\alpha}{\beta}$ and $\frac{\beta}{\alpha}$.

(ii) Find the value of each of the constants p and q. [4]

4 (i) Given: Sum of Roots (*SOR*): $\alpha+\beta=-\frac{1}{2}$

Product of Roots (*POR*): $\alpha\beta=1$

Quadratic Equation:

$$x^2-(SOR)x+(POR)=0$$

$$x^2-(\alpha+\beta)x+(\alpha\beta)=0$$

$$x^2-\left(-\frac{1}{2}\right)x+(1)=0 \qquad \boxed{\text{M1}}$$

$$x^2+\frac{1}{2}x+1=0 \qquad [\times(2)\ \text{throughout}]$$

$$2x^2+x+2=0 \qquad \boxed{\text{A1}}$$

$$2x^2+bx+c=0\ \Rightarrow\ b=1,c=2$$

(ii) Given: Roots of $x^2+px+q=0$ are $\frac{\alpha}{\beta}$ and $\frac{\beta}{\alpha}$.

Sum of Roots (*SOR*)

$$=\frac{\alpha}{\beta}+\frac{\beta}{\alpha}$$

$$=\frac{\alpha^2}{\alpha\beta}+\frac{\beta^2}{\alpha\beta}$$

$$=\frac{\alpha^2+\beta^2}{\alpha\beta}$$

$$=\frac{(\alpha+\beta)^2-2\alpha\beta}{\alpha\beta}$$

$$=\frac{\left(-\frac{1}{2}\right)^2-2(1)}{(1)}$$

$$=\frac{1}{4}-2$$

$$=\frac{1}{4}-\frac{8}{4}$$

$$=-\frac{7}{4} \qquad \boxed{\text{M1}}$$

Product of Roots (*POR*)

$$=\left(\frac{\alpha}{\beta}\right)+\left(\frac{\beta}{\alpha}\right)$$

$$=1 \qquad \boxed{\text{M1}}$$

Quadratic Equation:

$$x^2-(SOR)x+(POR)=0$$

$$x^2-\left(-\frac{7}{4}\right)x+(1)=0$$

$$x^2+\frac{7}{4}x+1=0$$

$$x^2+px+q=0$$

Constants: $p=\frac{7}{4}$ $\boxed{\text{A1}}$

$q=1$ $\boxed{\text{A1}}$

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5 (i) Prove that $\frac{\cos x}{1-\sin x}+\frac{1-\sin x}{\cos x}=2\sec x$. [3]

(ii) Hence, state the exact value of $\tan(90°-x)$ such that x is an acute angle which satisfies

$$\frac{\cos x}{1-\sin x}+\frac{1-\sin x}{\cos x}=\frac{9}{4}.$$ [3]

5 (i) $\frac{\cos x}{1-\sin x}+\frac{1-\sin x}{\cos x}=2\sec x$

$$LHS=\frac{\cos x}{1-\sin x}+\frac{1-\sin x}{\cos x}$$

$$=\frac{\cos x}{1-\sin x}\left(\frac{\cos x}{\cos x}\right)+\frac{1-\sin x}{\cos x}\left(\frac{1-\sin x}{1-\sin x}\right)$$

$$=\frac{\cos^2 x}{(\cos x)(1-\sin x)}+\frac{(1-\sin x)(1-\sin x)}{(\cos x)(1-\sin x)}$$

$$=\frac{\cos^2 x+(1-\sin x)(1-\sin x)}{(\cos x)(1-\sin x)}$$ [M1]

$$=\frac{\cos^2 x+(1-\sin x-\sin x+\sin^2 x)}{(\cos x)(1-\sin x)}$$

$$=\frac{\cos^2 x+\sin^2 x+1-2\sin x}{(\cos x)(1-\sin x)}$$ [From: $\sin^2 A+\cos^2 A=1$]

$$=\frac{1+1-2\sin x}{(\cos x)(1-\sin x)}=\frac{2-2\sin x}{(\cos x)(1-\sin x)}=\frac{2\quad(1-\sin x)}{(\cos x)(1-\sin x)}$$ [M1]

$$=\frac{2}{\cos x}$$

$=2\sec x$ [A1]

$=RHS$ (Shown)

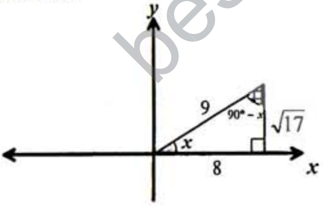
(ii) $\frac{\cos x}{1-\sin x}+\frac{1-\sin x}{\cos x}=\frac{9}{4}$, x is an acute angle → Quadrant 1

$2\sec x=\frac{9}{4}$ [From (i)]

$\frac{2}{\cos x}=\frac{9}{4}$

$8=9\cos x$

$\cos x=\frac{8}{9}$ [M1]



By Pythagoras' Theorem,

$9^2=8^2+opp^2$

$9^2-8^2=opp^2$

$opp^2=81-64$

$opp^2=17$

$opp=\sqrt{17}$ [M1]

From Quadrant 1,

$\therefore\quad \tan(90°-x)=\frac{8}{\sqrt{17}}$

OR $\tan(90°-x)=\frac{8}{\sqrt{17}}\times\frac{\sqrt{17}}{\sqrt{17}}$

$\therefore\quad \tan(90°-x)=\frac{8}{17}\sqrt{17}$ [A1]

[Turn over

6 In triangle ABC, the length, in cm, of AB is $1+4\sqrt{2}$, of BC is $2\sqrt{2}-2$ and of AC is $3\sqrt{5}$. Without using a calculator,

(i) explain why triangle ABC is right-angled and state which angle is 90°, [4]

(ii) show that $\tan B\hat{C}A=\frac{a+b\sqrt{2}}{2}$, where a and b are integers to be found. [4]

6 (i)

$(AB)^2=(1+4\sqrt{2})^2$

$=(1)^2+2(1)(4\sqrt{2})+(4\sqrt{2})^2$

$=1+8\sqrt{2}+(4)^2(\sqrt{2})^2$

$=1+8\sqrt{2}+(16)(2)$

$=1+8\sqrt{2}+32$

$(AB)^2=33+8\sqrt{2}$

$(BC)^2=(2\sqrt{2}-2)^2$

$=(2\sqrt{2})^2-2(2\sqrt{2})(2)+(-2)^2$

$=(2)^2(\sqrt{2})^2-8\sqrt{2}+4$

$=(4)(2)-8\sqrt{2}+4$

$=8-8\sqrt{2}+4$

$(BC)^2=12-8\sqrt{2}$

$(AC)^2=(3\sqrt{5})^2$

$=(3)^2(\sqrt{5})^2$

$=(9)(5)$

$(AC)^2=45$ [M1]

$(AB)^2=33+8\sqrt{2}$, $(BC)^2=12-8\sqrt{2}$

$(AB)^2+(BC)^2=(33+8\sqrt{2})+(12-8\sqrt{2})$

$=33+12+8\sqrt{2}-8\sqrt{2}$

$(AB)^2+(BC)^2=45$ [M1]

From above, $(AC)^2=45$ and $(AB)^2+(BC)^2=45$

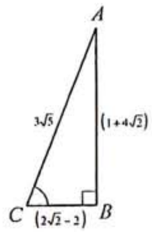
Hence, $(AB)^2+(BC)^2=(AC)^2$

⇒ Obeys Pythagoras' Theorem [A1]

⇒ ΔABC is a right – angled triangle with AC → hypotenuse

$\therefore \angle ABC$ is right – angled, 90° (angle opposite the hypotenuse side) [A1]

(ii)



$$\tan B\hat{C}A=\frac{AB}{BC}$$

$$=\frac{1+4\sqrt{2}}{2\sqrt{2}-2}$$ [M1]

$$=\frac{1+4\sqrt{2}}{2\sqrt{2}-2}\times\frac{2\sqrt{2}+2}{2\sqrt{2}+2}$$ (Rationalize)

$$=\frac{(1+4\sqrt{2})(2\sqrt{2}+2)}{(2\sqrt{2}-2)(2\sqrt{2}+2)}$$ [M1]

$$=\frac{(1)(2\sqrt{2})+(1)(2)+(4\sqrt{2})(2\sqrt{2})+(4\sqrt{2})(2)}{(2\sqrt{2})^2-(2)^2}$$

$$=\frac{2\sqrt{2}+2+(8)(2)+8\sqrt{2}}{(2)^2(\sqrt{2})^2-4}=\frac{2+16+10\sqrt{2}}{(4)(2)-4}=\frac{18+10\sqrt{2}}{8-4}$$

$$=\frac{18+10\sqrt{2}}{4}=\frac{2(9+5\sqrt{2})}{2\quad(2)}$$

$$\therefore \tan B\hat{C}A=\frac{9+5\sqrt{2}}{2}$$

$\tan B\hat{C}A=\frac{a+b\sqrt{2}}{2}$ [$a=9$] [A1] , [$b=5$] [A1]

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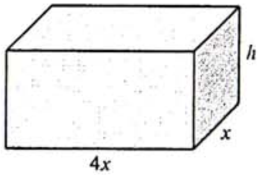
7 The diagram shows a closed cuboid box. A contractor wants to use metal sheet of negligible thickness to make a cuboid box that holds exactly 500 cm^3 of sand. The cost is minimum if the box has the smallest possible surface area. The dimensions of the box is such that the sides of its base are of length $4x$ cm, width x cm and height h cm. The total surface area is A cm^2.



(i) Express h in terms of x and show that $A = 8x^2 + \frac{1250}{x}$. [3]

(ii) Given that x can vary, find the value of the surface area for which the cost is least. [4]

(iii) Calculate the minimum cost, to the nearest cent, of producing 1 box if a metal sheet costs 25 cents per 1000 cm^2. [1]

7 (i) Given: $V = 500\,\text{cm}^3$

$V = L \times B \times H = 500\,\text{cm}^3$

$V = (4x)\times(x)\times(h) = 500\,\text{cm}^3$

$\Rightarrow 4x^2h = 500$

$h = \frac{500}{4x^2}$

$h = \frac{125}{x^2} \quad \to (1) \quad \boxed{\text{M1}}$

Total Surface Area (A)

$= (\text{Top} + \text{Bottom}) +$
$(\text{Front} + \text{Back}) +$
$(\text{Left} + \text{Right})$

$= 2\,(4x)(x) +$
$2\,(4x)(h) +$
$2\,(x)(h)$

$A = 8x^2 + 8xh + 2xh \quad \boxed{\text{M1}}$

$= 8x^2 + 10xh$

[Substitute (1)]

$= 8x^2 + 10x\left(\frac{125}{x^2}\right)$

$\therefore A = 8x^2 + \frac{1250}{x} \quad \boxed{\text{A1}}$

(Shown)

(ii) $A = 8x^2 + \frac{1250}{x} = 8x^2 + 1250x^{-1}$

$\frac{dA}{dx} = 16x - 1250x^{-2} \quad \boxed{\text{M1}}$

$\frac{dA}{dx} = 0$ [Stationary Value]

$\Rightarrow 16x - 1250x^{-2} = 0 \quad \boxed{\text{M1}}$

$16x = 1250x^{-2}$

$16x = \frac{1250}{x^2}$

$16x^3 = 1250$

$x^3 = \frac{1250}{16}$

$x^3 = \frac{625}{8}$

$x = \sqrt[3]{\frac{625}{8}}$

$x = 4.274939867\ldots\text{cm} \quad \boxed{\text{M1}}$

$A = 8x^2 + \frac{1250}{x}$

$A = 8(4.275\ldots)^2 + \frac{1250}{(4.275\ldots)}$

$\therefore A = 438.60266\ldots \approx 439\ \text{cm}^2\,(3\text{sf}) \quad \boxed{\text{A1}}$

(iii) 1000 cm^2 → 25 cents

1 cm^2 → 0.025 cents

438.6026607… cm^2 → 0.025 cents × 438.60266 07

= 10.96506652 cents

≈ 11 cents (nearest cent) $\boxed{\text{A1}}$

8 A circle C_1 has centre $(3, -4)$ and radius 5.

(i) State the equation of C_1. [2]

The equation of circle C_2 is $x^2 + y^2 - 10x = 0$.

(ii) Find the coordinates of the centre of C_2 and the radius of C_2. [4]

(iii) Justify with suitable calculations, if the centre of the circle C_1 lies within the circle C_2. [2]

8 (i) Centre C_1: $C_1(3, -4)$

Radius C_1: 5

Standard Equation of a circle:

$(x-a)^2 + (y-b)^2 = r^2$

$(x-3)^2 + (y-(-4))^2 = 5^2 \quad \boxed{\text{M1}}$

$(x-3)^2 + (y+4)^2 = 25$

General Equation of a circle:

$(x-3)^2 + (y+4)^2 = 25$

$x^2 - 6x + 9 + y^2 + 8y + 16 - 25 = 0$

$OR \quad x^2 + y^2 - 6x + 8y = 0 \quad \boxed{\text{A1}}$

(ii) **Method 1:**

Using the "***General Form***" equation

$x^2 + y^2 - 10x = 0$

$x^2 + y^2 - 10x + 0y + 0 = 0$

$(x^2 + y^2 + 2gx + 2fy + c = 0)$

$c = 0$

$2g = -10 \Rightarrow g = -5$

$2f = 0 \Rightarrow f = 0 \quad \boxed{\text{M1}}$

Centre of Circle C_2:

$C(-g, -f) = C_2(5, 0) \quad \boxed{\text{A1}}$

Radius of Circle C_2:

$\sqrt{g^2 + f^2 - c} \quad \boxed{\text{M1}}$

$= \sqrt{(-5)^2 + (0)^2 - (0)}$

$= \sqrt{25}$

$= 5$ units $\boxed{\text{A1}}$

Method 2:

Using the "***Standard Form***" equation

[By using "Completing the Square"]

$x^2 + y^2 - 10x = 0$

$(x^2 - 10x) + (y^2) = 0$

$\left(x^2 - 10x + \left(\frac{-10}{2}\right)^2\right) + (y-0)^2 = +\left(\frac{-10}{2}\right)^2 \quad \boxed{\text{M1}}$

$(x^2 - 10x + (-5)^2) + (y-0)^2 = (-5)^2$

$(x-5)^2 + (y-0)^2 = 5^2 \quad \boxed{\text{M1}}$

$(x-a)^2 + (y-b)^2 = r^2$

$\Rightarrow$ Centre of Circle C_2:

$C(a,b) \to C_2(5,0) \quad \boxed{\text{A1}}$

$\Rightarrow$ Radius of Circle C_2:

$r = 5$ units $\boxed{\text{A1}}$

(iii) Centre of Circle C_1: $C_1(3, -4)$

Centre of Circle C_2: $C_2(5, 0)$

Distance from $(3, -4)$ to $(5, 0)$:

$\sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2}$

$= \sqrt{(3-5)^2 + (-4-0)^2}$

$= \sqrt{(-2)^2 + (-4)^2}$

$= \sqrt{4+16}$

$= \sqrt{20}$

$= 4.472135955 \quad (< 5\text{units}) \quad \boxed{\text{M1}}$

$\Rightarrow$ Distance is less than Radius of C_2

Radius of Circle C_2: $r = 5$ units

$\therefore$ Centre of Circle C_1 lies within Circle C_2 $\boxed{\text{A1}}$

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≈ 11 cents (nearest cent) **[A1]**

9 A polynomial $f(x)$ is such that $f''(x)=12x+10$, $f'(1)=12$ and $f(-1)=4$.

(i) Using integration, show that $f(x)=2x^3+5x^2-4x-3$. [5]

(ii) Show that $x-1$ is a factor of $f(x)$, and solve the equation $f(x)=0$. [5]

9 (i) Given: $f''(x)=12x+10$, $f'(1)=12$

$$f'(x)=\int f''(x)\ dx$$

$$f'(x)=\int (12x+10)\ dx$$

$$=12\left[\frac{x^{1+1}}{1+1}\right]+10\left[\frac{x^{0+1}}{0+1}\right]+c$$

$$f'(x)=12\left[\frac{x^2}{2}\right]+10\left[\frac{x^1}{1}\right]+c$$

$f'(x)=6x^2+10x+c$ **[M1]**

$f'(1)=12$,

$f'(1)=6(1)^2+10(1)+c$

$12 \ =6+10+c$

$c \ =12-6-10$

$c \ =-4$

$\Rightarrow f'(x)=6x^2+10x-4$ **[M1]**

Since: $f'(x)=6x^2+10x-4$, $f(-1)=4$

$$f(x)\ =\int f'(x)\ dx$$

$f(x)\ =\int (6x^2+10x-4)\ dx$ **[M1]**

$$=6\left[\frac{x^{2+1}}{2+1}\right]+10\left[\frac{x^{1+1}}{1+1}\right]-4\left[\frac{x^{0+1}}{0+1}\right]+c$$

$$=6\left[\frac{x^3}{3}\right]+10\left[\frac{x^2}{2}\right]-4\left[\frac{x^1}{1}\right]+c$$

$f(x)\ =2x^3+5x^2-4x+c$ **[M1]**

$f(-1)=4$,

$f(-1)=2(-1)^3+5(-1)^2-4(-1)+c$

$4 \ =-2+5+4+c$

$c \ =4+2-5-4$

$c \ =-3$

$\therefore f(x)=2x^3+5x^2-4x-3$ (Shown) **[A1]**

(ii) Show: $(x-1)$ is a factor of $f(x)=2x^3+5x^2-4x-3$.

Try : $f(1)$ $=2(1)^3+5(1)^2-4(1)-3$

$=2+5-4-3$

$=0$

$f(1) \ =0 \ \Rightarrow (x-1)$ is a factor of $f(x)=2x^3+5x^2-4x-3$ **[B1]**

$f(x) \ =2x^3+5x^2-4x-3=0$

$\rightarrow \ 2x^3+5x^2-4x-3=0$

$\Rightarrow \ (x-1)(ax^2+bx+c)=0$

$(x-1)(2x^2+7x+3)=0$

$(x-1)(2x+1)(x+3)=0$

$\Rightarrow \ (x-1)=0 \quad \therefore x=1$ **[A1]**

$\Rightarrow \ (2x+1)=0 \quad \therefore x=-\dfrac{1}{2}$ **[A1]**

$\Rightarrow \ (x+3)=0 \quad \therefore x=-3$ **[A1]**

$\therefore f(x)=0 \rightarrow x=-3,\ -\dfrac{1}{2},\ 1$

$2x$	$+1$	$+x$
x	$+3$	$+6x$
$2x^2$	$+3$	$+7x$

$$\begin{array}{r}
2x^2+7x+3 \\
x-1\overline{)\,2x^3+5x^2-4x-3} \\
\underline{-(2x^3-2x^2)}\ \downarrow\ \ \downarrow \\
7x^2-4x\ \ \downarrow \\
\underline{-(7x^2-7x)}\ \downarrow \\
3x-3 \\
\underline{-(3x-3)} \\
\underline{0}
\end{array}$$

OR By Synthetic Division: **[M1]**

	x^3	x^2	x^1	x^0
	2	5	−4	−3
1		2	7	3
	2	7	3	0
	x^2	x^1	x^0	

$=4.472135955$ $(<5\text{units})$ **[M1]**

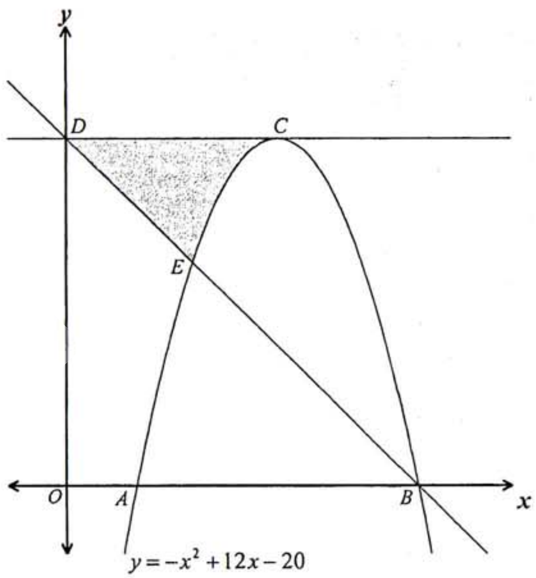
10 The diagram shows part of the curve $y=-x^2+12x-20$.

The curve crosses the $x-$axis at the points A and B.

The tangent to the curve at the point C is parallel to the $x-$axis.

The straight line BD passes through the curve at points B and E.

The tangent to the curve at the point C and the straight line BD meets at point D.



(i) Find the coordinates of A, B, C, D and E. [7]

(ii) Find the area of the shaded region CDE. [5]

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10 (i)

Coordinates of A and B:

$x-$ intercepts of $y=-x^2+12x-20$

$x-$ intercept: $y=0$

$0=-x^2+12x-20$

$x^2-12x+20=0$

x	-2	$-2x$
x	-10	$-10x$
x^2	$+20$	$-12x$

$(x-2)(x-10)=0$

$\Rightarrow (x-2)=0$

$\rightarrow x=2 \quad , y=0$

$\therefore A(2,0)$ $\boxed{\text{[A1]}}$

$\Rightarrow (x-10)=0$

$\rightarrow x=10 \quad , y=0$

$\therefore B(10,0)$ $\boxed{\text{[A1]}}$

Coordinates of C:

Maximum point of $y=-x^2+12x-20$

$y=-x^2+12x-20$

$\frac{dy}{dx}=-2x+12$ $\boxed{\text{[M1]}}$

Stationary value (Maximum point):

$\Rightarrow \frac{dy}{dx}=0$

$\rightarrow -2x+12=0$

$-2x=-12$

$\rightarrow \quad x=6$

Substitute into $y=-x^2+12x-20$

$\rightarrow y=-(6)^2+12(6)-20$

$=-36+72-20$

$\rightarrow y=16$

$\therefore C(6,16)$ $\boxed{\text{[A1]}}$

Alternative method:

- Find the average of $x=2,10$
- And substitute answer in $y=-x^2+12x-20$

Coordinates of D:

Equation of $CD \rightarrow y=16$

(CD is parallel to $x-$ axis)

D is a $y-$ intercept of CD : $x=0$

$\therefore D(0,16)$ $\boxed{\text{[A1]}}$

Coordinates of E:

E is the point of intersection of

(1): Line BD

(2): Curve $y=-x^2+12x-20$

(1): BD passes through points

$B(10,0)$, $D(0,16)$

$m_{BD}=\frac{y_2-y_1}{x_2-x_1}=\frac{16-0}{0-10}=\frac{16}{-10}=-1.6$

Equation of Line BD: $m=-1.6$, $c=16$

$y=mx+c$

$y=-1.6x+16$

$\Rightarrow y=-1.6x+16 \quad \rightarrow (1)$

$\Rightarrow y=-x^2+12x-20 \quad \rightarrow (2)$

Substitute (1) into (2):

(2): $(-1.6x+16)=-x^2+12x-20$ $\boxed{\text{[M1]}}$

$x^2-12x-1.6x+16+20=0$

$x^2-13.6x+36=0$

x	-3.6	$-3.6x$
x	-10	$-10x$
x^2	$+36$	$-13.6x$

$(x-3.6)(x-10)=0$

$\Rightarrow (x-3.6)=0 \qquad \Rightarrow (x-10)=0$

$\rightarrow x=3.6 \qquad \rightarrow x=10$

$\rightarrow B(10,0)$

Substitute $x=3.6$ in $y=-x^2+12x-20$

$\rightarrow y=-(3.6)^2+12(3.6)-20$

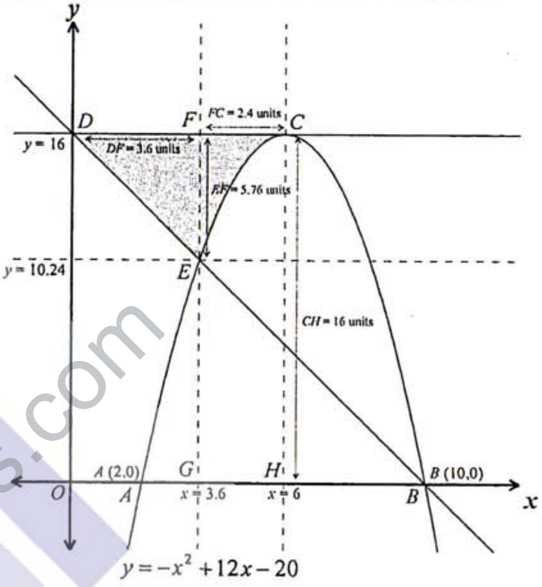
$=-12.96+43.2-20$

$\rightarrow y=10.24$

$\therefore E(3.6,10.24)$ $\boxed{\text{[A1]}}$

10 (ii)

$$\boxed{\text{Area of Shaded Region} = \underset{\text{Region 1}}{\text{Shaded }\Delta DEF} + \underset{\text{Region 2}}{\text{Shaded Area } CEF}}$$



Region 1: $\boxed{\text{Shaded } \Delta DEF}$

$DF=3.6-0=3.6$ units

$EF=16-10.24=5.76$ units

Area of ΔDEF

$=\frac{1}{2}\times DF\times EF$

$=\frac{1}{2}\times 3.6\times 5.76$

$=\frac{1}{2}\times 3.6\times 5.76$

$=10.368$ square units $\boxed{\text{[M1]}}$

Region 2: $\boxed{\text{Shaded Area } CEF}$

$CH=16-0=16$ units

$FC=6-3.6=2.4$ units

Shaded Area CEF

$=\text{Area of Rectangle } FCHG-\int_{3.6}^{6}(y)\,dx=(CH\times FC)-\int_{3.6}^{6}(-x^2+12x-20)\,dx$

$=(16\times 2.4)-\left[-\left(\frac{x^{2+1}}{2+1}\right)+12\left(\frac{x^{1+1}}{1+1}\right)-20\left(\frac{x^{0+1}}{0+1}\right)\right]_{3.6}^{6}$ $\boxed{\text{[M1]}}$

$=(38.4)-\left[-\left(\frac{x^3}{3}\right)+12\left(\frac{x^2}{2}\right)-20\left(\frac{x^1}{1}\right)\right]_{3.6}^{6}=(38.4)-\left[-\frac{1}{3}x^3+6x^2-20x\right]_{3.6}^{6}$

$\boxed{\text{[M1]}}$

$=(38.4)-\left[\left(-\frac{1}{3}(6)^3+6(6)^2-20(6)\right)-\left(-\frac{1}{3}(3.6)^3+6(3.6)^2-20(3.6)\right)\right]$

$=(38.4)-[(24)-(-9.792)]=(38.4)-[24+9.792]=38.4-33.792$

$=4.608$ square units $\boxed{\text{[M1]}}$

$\therefore \text{Area of Shaded Region}=\underset{\text{Region 1}}{\text{Shaded }\Delta DEF}+\underset{\text{Region 2}}{\text{Shaded Area } CEF}$

$=\underset{\text{Region 1}}{10.368}+\underset{\text{Region 2}}{4.608}$

$\therefore$ Area of Shaded Region $=14.976$ square units $\boxed{\text{[A1]}}$

- End Of Paper -

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$\therefore$ Area of Shaded Region = 14.976 square units [A1]

GMS(S) 2016 PRELIM 4N(A) A.Maths Paper 2 [redacted] Answers

1	$19x+1$		
2	(i) $c<-4$	(ii) $-x^2+4x-8$ $(c=-8 \to c<-4)$ $-x^2+4x-8$ is always negative	
3	(i) $\left(\sqrt{3}\cos\theta+\sin\theta\right)=2\cos\left(\theta-\frac{\pi}{6}\right)$	(ii) Principal value: $\frac{2\pi}{3}$	
4	(i) $2x^2+x+2=0 \Rightarrow b=1, c=2$	(ii) $p=\frac{7}{4}$; $q=1$	
5		(ii) $\frac{8}{\sqrt{17}}$ OR $\frac{8}{17}\sqrt{17}$	
6	(i) $\Rightarrow (AB)^2+(BC)^2=(AC)^2$ $\Rightarrow$ Obeys Pythagoras' Theorem $\Rightarrow \Delta ABC$ is a right – angled triangle with $AC \to$ hypotenuse $\therefore \angle ABC$ is right – angled, 90° (angle opposite the hypotenuse side)		
		(ii) $\frac{9+5\sqrt{2}}{2}$ $[\,a=9\ ,\ b=5\,]$	
7	(i) $h=\frac{125}{x^2}$, $A=8x^2+\frac{1250}{x}$	(ii) 439 cm^2 (3sf)	(iii) 11 cents
8	(i) $(x-3)^2+(y+4)^2=25$ OR $x^2+y^2-6x+8y=0$	(ii) $C(5,0)$, 5 units	(iii) Centre of Circle C_1 lies within Circle C_2
9		(ii) Show $f(1)=0 \Rightarrow (x-1)$ is a factor of $f(x)=2x^3+5x^2-4x-3$ $f(x)=0 \to x=-3, -\frac{1}{2}, 1$	
10	(i) $A(2,0)$, $B(10,0)$, $C(6,16)$, $D(0,16)$, $E(3.6,10.24)$		
		(ii) 14.976 square units	

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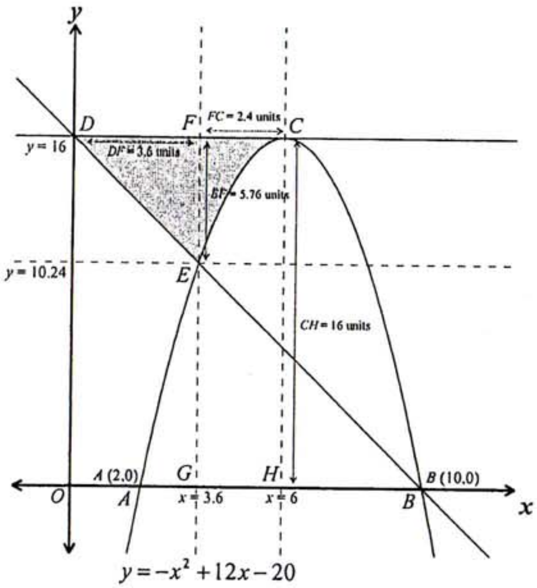

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HUA YI SECONDARY SCHOOL

4NA Preliminary Examination 2016 4NA

ADDITIONAL MATHEMATICS

4044/01

Paper 1

15 August 2016

1 h 45 min

Additional materials: Answer Paper

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Write your Name, Class and Index Number on all the work you hand in.
Write in dark blue or black pen.
You may use a soft pencil for any diagrams or graphs.
Do not use staples, paper clips, highlighters, glue or correction fluid.

Answer all questions.

Write your answers on separate Answer Paper provided.
Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.
The use of a scientific calculator is expected, where appropriate.
You are reminded of the need for clear presentation in your answers.

At the end of the examination, fasten all your work securely together.
The number of marks is given in brackets [ ] at the end of each question or part question.

The total number of marks for this paper is 70.

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70

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Mathematical Formulae

1. ALGEBRA

Quadratic Equation

For the equation $ax^2 + bx + c = 0$,

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Binomial Expansion

$$(a+b)^n = a^n + \binom{n}{1}a^{n-1}b + \binom{n}{2}a^{n-2}b^2 + \ldots + \binom{n}{r}a^{n-r}b^r + \ldots + b^n,$$

where n is a positive integer and $\binom{n}{r} = \frac{n!}{r!(n-r)!} = \frac{n(n-1)\ldots(n-r+1)}{r!}$

2. TRIGONOMETRY

Identities

$$\sin^2 A + \cos^2 A = 1$$
$$\sec^2 A = 1 + \tan^2 A$$
$$\cos ec^2 A = 1 + \cot^2 A$$
$$\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$$
$$\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$$
$$\tan(A \pm B) = \frac{\tan A \pm B}{1 \mp \tan A \tan B}$$
$$\sin 2A = 2\sin A \cos A$$
$$\cos 2A = 2\cos^2 A - 1 = 1 - 2\sin^2 A = \cos^2 A - \sin^2 A$$
$$\tan 2A = \frac{2\tan A}{1 - \tan^2 A}$$

Formulae for ΔABC

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$
$$a^2 = b^2 + c^2 - 2bc\cos A$$
$$\Delta = \frac{1}{2}bc\sin A$$

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1 In the expansion of $(3+ax)^5$, where $a>0$, the sum of the coefficients of x and x^2 is 270.

(i) Find the value of a. [4]

(ii) Using the value of a found in part (i), find the coefficient of x^2 in the expansion of $(1-5x)(3+ax)^5$. [2]

2 Given that $f(x) = 2x^3 - 11x^2 + 10x + 8$,

(i) show that $(x-2)$ is a factor of $f(x)$, [2]

(ii) solve the equation $f(x)=0$. [3]

3 Without using a calculator, express $\frac{1}{1+\sqrt{3}} - \frac{3}{1-\sqrt{3}}$ in the form $a+b\sqrt{3}$, where a and b and integers. [3]

4 The volume, V cm^3, of a spherical ice is given by $V = \frac{4}{3}\pi r^3$, where r cm is the radius of the ice.
When the ice is placed in a glass, the volume of the ice decreases at a constant rate of 2.94 cm^3/s. Find the value of r when the radius is decreasing at $\frac{1}{6\pi}$ cm/s. [4]

5 The function f is defined by $f(x) = ax^3 + 5x^2 + bx + 4$, where a and b are constants.
It is given that $f(2) = 48$ and that when $f(x)$ is divided by $x+1$, the remainder is 15.
Find the value of a and of b. [4]

6 (i) Find the range of values of p for which the equation $x^2 - px + 2p = 3$ has real roots. [3]

(ii) Find the range of values of k for which the curve $y = x^2 - 2x + 3k - 2$ is always positive. [3]

7 Given that $\sin(A+B) = 2\sin(A-B)$, show that $\tan A = 3\tan B$. [4]

8 It is given that $f(x) = 3\cos 2x + 1$ and $0 \le x \le 2\pi$.

(i) State the amplitude and period of f. [2]

(ii) Sketch the graph of $y = f(x)$. [3]

(iii) State the number of solutions for $3\cos 2x = 2$. [2]

9 Solve the inequality $4x(4-x) > 15$. [3]

10 The parabola $y^2 = \frac{1}{2}kx$, where k is a constant, passes through the point $P(6, 3)$.

(i) Find the value of k. [2]

(ii) Sketch the graph of this parabola. [1]

A line $x = 6$ passes through the curve at the points P and Q.

(iii) Find the coordinates of the point Q. [1]

(iv) Hence, find the area of triangle OPQ. [1]

11 By using a suitable substitution or otherwise, solve $2^{2x+3} - 33(2^x) + 4 = 0$. [5]

12 The equation of the curve is $y = 2x^2 - kx - 1$, where k is a constant.

Given that the tangent at $x = 2$ passes through the point (4, 3),

(i) show that the value of k is 5, [4]

(ii) find the equation of the normal at the point where the curve crosses the y-axis. [3]

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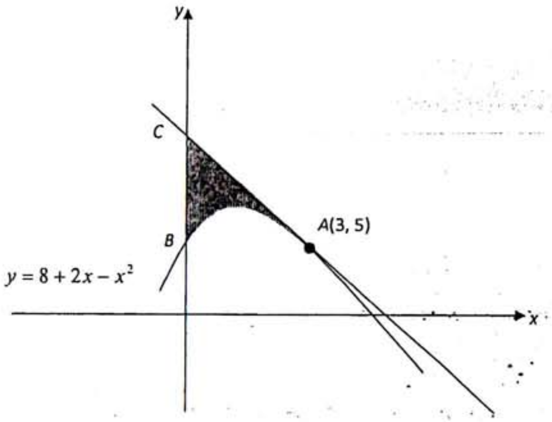
13 The diagram shows part of the curve $y = 8 + 2x - x^2$ which crosses the y-axis at B. The tangent to the curve at $A(3, 5)$ meets the y-axis at C.



(i) Find the coordinates of the points B. [1]

(ii) Find the coordinates of the points C. [3]

(iii) Find the area of the shaded region. [5]

14 Write down the principal value, in radians as a multiple of π, of

(i) $\sin^{-1}\left(\frac{\sqrt{3}}{2}\right)$ [1]

(ii) $\cos^{-1}\left(\frac{1}{\sqrt{2}}\right)$ [1]

END OF PAPER

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HUA YI SECONDARY SCHOOL
MATHEMATICS DEPARTMENT

2016 Sec 4NA Additional Mathematics Paper 1 Prelim Marking Scheme

Question	Solution	Mark Allocation
1 (i)	$\binom{5}{1}3^4(a\ )+\binom{5}{2}3^3(a^2)=270$ .... M1 $5(81)a\ +10(27)a^2-270=0$ .... M1 Divided by 135, $3a+2a^2-2=0$ $2a^2+3a-2=0$ $(2a-1)(a+2)=0$ Since $a>0$, $a=\frac{1}{2}$	--- M1 --- M1 --- M1 --- A1
(ii)	$(1-5x)(3+\frac{1}{2}x)^5 = (1-5x)(243+202.5x+67.5x^2+....\quad)$ coefficient of x^2 = $67.5-5(202.5)$ $=-945$	--- M1 --- A1
2 (i)	$f(2)=2(2)^3-11(2)^2\ +10(2)+8$ $=0$ [shown]	--- M1 --- A1
2 (ii)	f(x)=$(x-2)\ (2x^2-7x-4)$ f(x)= $(x-2)(2x+1)(x-4)$ x= $2,-\frac{1}{2},4$	--- M1(long division) --- M1 --- A1
3	$\frac{1}{1+\sqrt{3}}-\frac{3}{1-\sqrt{3}}$ $=\frac{1-\sqrt{3}-3(1+\sqrt{3})}{(1+\sqrt{3})(1-\sqrt{3})}$ $=\frac{1-\sqrt{3}-3-3\sqrt{3}}{1-3}$ $=\frac{-2-4\sqrt{3}}{-2}$ $=1+2\sqrt{3}$	--- M1 --- M1 --- A1
4	$V=\frac{4}{3}\pi r^3$ $\frac{dV}{dr}=4\pi r^2$ $-2.94=4\pi r^2\ \times-\frac{1}{6\pi}$ $2.94=\frac{2r^2}{3}$ $2r^2=8.82$	--- M1 --- M2(1 mark for –ve sign)

Question	Solution	Mark Allocation
	$r^2=4.41$ $r=\sqrt{4.41}$ $r=2.1$	--- A1
5	$f(2)=a(2)^3+5(2)^2+b(2)+4=48$ $8a+20+2b+4=48$ $4a+b=12$ .... Equation (1) $f(-1)=a(-1)^3+5(-1)^2+b(-1)+4=15$ $-a+5-b+4=15$ $a+b=-6$ .... Equation (2) (1) – (2) $3a=18$ $a=6$ $6+b=-6$ $b=-12$	--- M1 --- M1 --- A2
6 (i)	$x^2-px+2p=3$ $x^2-px+2p-3=0$ $b^2-4ac\geq 0$ $(-p)^2-4(1)(2p-3)\geq 0$ $p^2-8p+12\geq 0$ $(p-2)(p-6)\geq 0$ $p\leq 2$ or $p\geq 6$	--- M1 --- M1 --- A1
6 (ii)	$y=x^2-2x+3k-2$ $b^2-4ac<0$ $(-2)^2-4(1)(3k-2)<0$ $4-12k+8<0$ $12k>12$ $k>1$	--- M1 --- M1 --- A1
7	$\sin(A+B)=2\sin(A-B)$ $\sin A\cos B+\cos A\sin B=2\sin A\cos B-2\cos A\sin B$ $\sin A\cos B=3\cos A\sin B$ $\frac{\sin A}{\cos A}=\frac{3\sin B}{\cos A}$ $\tan A=3\tan B$ (shown)	--- M1 --- M1 --- M1 --- A1

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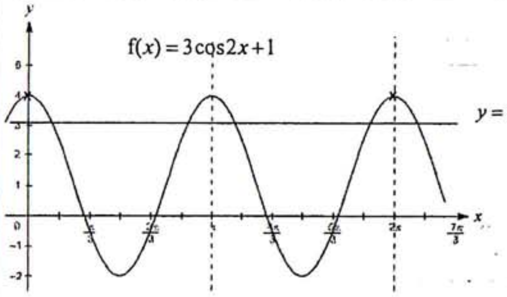
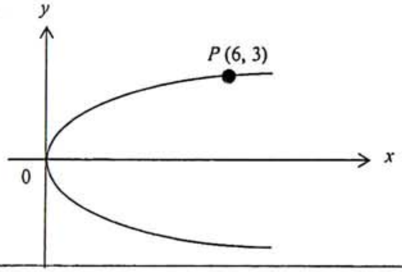
Question		Solution	Mark Allocation
8	(i)	Amplitude = 3 Period= π	--- B1 --- B1
8	(ii)(iii)	 4 solutions	ii) B1- correct no. of cycles B1- shape B1- max, min iii) B1- draw y=3 --- B1
9		$4x(4-x) > 15$ $16x - 4x^2 > 15$ $-4x^2 + 16x - 15 > 0$ $4x^2 - 16x + 15 < 0$ $(2x-5)(2x-3) < 0$ $1.5 < x < 2.5$	--- M1 ---M1 --- A1
10	(i)	$y^2 = \frac{1}{2}kx$ Sub $P(6,3)$ $3^2 = \frac{1}{2}k(6)$ $k = 3$	--- M1 --- A1
10	(ii)	$y^2 = \frac{3}{2}x$ 	--- B1

Question		Solution	Mark Allocation
10	(iii)	Coordinates of Q is (6, –3).	--- B1
10	(iv)	Area of $\Delta OPQ = \frac{1}{2} \times 6 \times 6$ $= 18 \text{ units}^2$	--- B1
11		$2^{2x+3} - 33(2^x) + 4 = 0$ $2^3(2^{2x}) - 33(2^x) + 4 = 0$ Let $y = 2^x$ $8y^2 - 33y + 4 = 0$ $(8y-1)(y-4) = 0$ $y = \frac{1}{8}$ or $y = 4$ $2^x = 2^{-3}$ or $2^x = 2^2$ $x = -3$ or $x = 2$	--- M1 --- M1 --- M1 --- A2
12	(i)	$y = 2x^2 - kx - 1$ $\frac{dy}{dx} = 4x - k$ $\left.\frac{dy}{dx}\right\|_{x=2} = 8 - k$ $x = 2, \quad y = 7 - 2k$ $\therefore \quad 8 - k = \frac{7-2k-3}{2-4}$ $-16 + 2k = 4 - 2k$ $4k = 20$ $k = 5$	--- M1 --- M1 --- M1 --- A1
12	(ii)	Crosses y-axis, $x = 0$, $y = -1$ $\left.\frac{dy}{dx}\right\|_{x=0} = -5, \qquad m_N = \frac{1}{5}$ $y - (-1) = \frac{1}{5}x$ $y = \frac{1}{5}x - 1$ $5y = x - 5$	--- M1 --- M1 --- A1
13	(i)	Sub $x = 0$, $B = (0,8)$	--- B1
13	(ii)	$\frac{dy}{dx} = 2 - 2x$ At $x = 3$, $\frac{dy}{dx} = -4$	--- M1

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Question	Solution	Mark Allocation
	$5=-4(3)+c$ $c=17$ $C=(0,17)$	--- M1 --- A1
13 (iii)	area of the shaded region = area of trapezium – area under curve $=\frac{1}{2}(17+5)(3)-\int_0^3 8+2x-x^2\ \mathrm{d}x$ $=33-\left[8x+x^2-\frac{x^3}{3}\right]_0^3$ $=33-[24+9-9]$ = 9 square units	--- M2 --- M1 --- M1 --- A1
14 (i)	$\frac{\pi}{3}$	--- B1
14 (ii)	$\frac{\pi}{4}$	--- B1

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HUA YI SECONDARY SCHOOL

4NA Preliminary Examination 2016 **4NA**

ADDITIONAL MATHEMATICS **4044/02**

Paper 2

18 August 2016

1 h 45 min

Additional materials: Answer Paper

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Write your answers on separate Answer Paper provided.
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The total number of marks for this paper is 70.

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Mathematical Formulae

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Binomial Expansion

$$(a+b)^n=a^n+\binom{n}{1}a^{n-1}b+\binom{n}{2}a^{n-2}b^2+\ldots+\binom{n}{r}a^{n-r}b^r+\ldots+b^n,$$

where n is a positive integer and $\binom{n}{r}=\frac{n!}{r!(n-r)!}=\frac{n(n-1)\ldots(n-r+1)}{r!}$

2. TRIGONOMETRY

Identities

$$\sin^2 A+\cos^2 A=1$$
$$\sec^2 A=1+\tan^2 A$$
$$\cos ec^2 A=1+\cot^2 A$$
$$\sin(A\pm B)=\sin A\cos B\pm\cos A\sin B$$
$$\cos(A\pm B)=\cos A\cos B\mp\sin A\sin B$$
$$\tan(A\pm B)=\frac{\tan A\pm B}{1\mp\tan A\tan B}$$
$$\sin 2A=2\sin A\cos A$$
$$\cos 2A=2\cos^2 A-1=1-2\sin^2 A=\cos^2 A-\sin^2 A$$
$$\tan 2A=\frac{2\tan A}{1-\tan^2 A}$$

Formulae for ΔABC

$$\frac{a}{\sin A}=\frac{b}{\sin B}=\frac{c}{\sin C}$$
$$a^2=b^2+c^2-2bc\cos A$$
$$\Delta=\frac{1}{2}bc\sin A$$

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1 Find the range of values of k for which $2x^2+kx+2>0$ for all real values of x. [3]

2 Integrate $(3x+4)^8$ with respect to x. [2]

3 The equation $x^2-2x+a=0$ has roots α and β. Another equation $x^2+bx+9=0$ has roots α^2 and β^2. Calculate the possible values of a and of b. [5]

4 (i) Given that $y=\dfrac{x^2}{(1+x^2)}$, find $\dfrac{dy}{dx}$. [2]

(ii) Hence, find $\int_3^7 \dfrac{5x}{(1+x^2)^2}dx$. [3]

5 A circle passes through the points $P(-2,0)$ and $Q(4,0)$ and has its centre lying on the line $y=x+4$.

(i) Show that the equation of the perpendicular bisector of PQ is $x=1$. [3]

(ii) Find the coordinate of the centre of the circle. [2]

(iii) Find the equation of the circle. [3]

(iv) Find the coordinates of a point R such that PR is the diameter of the circle. [3]

6 (i) Express $3\cos x-5\sin x$ in the form $R\cos(x-\alpha)$, where $R>0$ and $0<\alpha<\dfrac{\pi}{2}$ radians. [3]

(ii) The height, h centimetres, of a wave of water in a tank being used for a physics experiment is given by

$$h=3\cos t-5\sin t,$$

where t is the time in seconds after the start of the experiment. After how many seconds does the wave first reach a height of 3.5 centimetres? [4]

7 A curve has the equation $y=x^3-x^2-5x+4$.

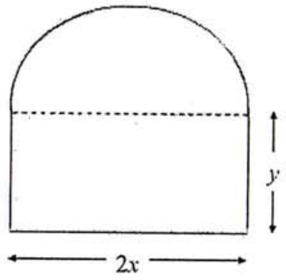
(i) Find the coordinates of the stationary points of the curve. [4]

(ii) Determine the nature of these stationary points. [4]

(iii) Find the set of values of x for which y is increasing. [2]

8 Solve the equation $8\cot x=3\sin x$ for $0°\le x\le 360°$ [5]

9 The diagram shows a window pane which is formed by a rectangle with sides $2x$ metres and y metres and a semicircle of radius x metres. The perimeter of the window is 12 metres.



(i) Show that $y=\dfrac{1}{2}(12-2x-\pi x)$. [2]

(ii) Hence, show that the area, A square metres, of the window pane is given by

$$A=12x-2x^2-\frac{\pi x^2}{2}.$$ [2]

Given that x and y can vary, find

(iii) the value of x for which A is a maximum, [3]

10 Given that $\tan A=\dfrac{3}{4}$ and $\sin A$ is negative, without using a calculator, find the value of

(i) $\sin 2A$, [4]

(ii) $\tan(-A)$, [1]

(iii) $\sec A$. [1]

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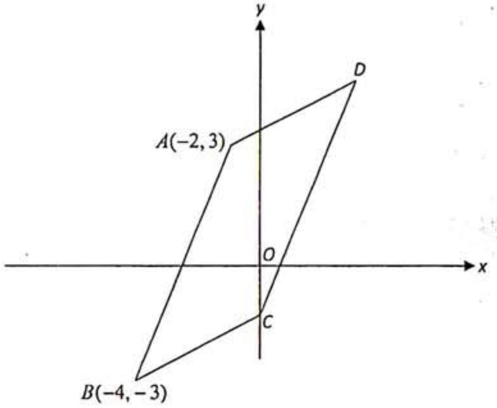
11 **Answers to this question by accurate drawing will not be accepted.**



In the figure above, the coordinates of *A* and *B* are $(-2, 3)$ and $(-4, -3)$ respectively.

Find

(i) the equation of the perpendicular bisector of *AB*, [4]

(ii) the area of triangle *ABC* if the perpendicular bisector of *AB* meets the *y*-axis at *C*, [3]

(iii) the coordinates of *D* if *ABCD* is a parallelogram. [2]

~ END OF PAPER ~

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HUA YI SECONDARY SCHOOL
MATHEMATICS DEPARTMENT

2016 Sec 4NA Additional Mathematics Paper 2 Prelims Marking Scheme

Question	Solution	Mark Allocation
1	$2x^2+kx+2>0$ $b^2-4ac<0$ $k^2-4(2)(2)<0$ $k^2-16<0$ $(k+4)(k-4)<0$ $-4<k<4$	--- M1 --- M1 --- A1
2	$\int(3x+4)^8\ \mathrm{dx}=\dfrac{(3x+4)^9}{9(3)}+c$ $=\dfrac{(3x+4)^9}{27}+c$	--- M1 --- A1
3	$x^2-2x+a=0$ $\alpha+\beta=2$ $\alpha\beta=a$ $x^2+bx+9=0$ $\alpha^2+\beta^2=-b$ $\alpha^2\beta^2=9$ $(\alpha\beta)^2=9$ $\alpha\beta=\pm3$ $\therefore\ a=\pm3$ $(\alpha+\beta)^2-2\alpha\beta=-b$ $(2)^2-2(\pm3)=-b$ $4\pm6=-b$ $b=-10,\ b=2$	--- B1 --- B1 ---A1 --- M1 ---A1
4 (i)	$y=\dfrac{x^2}{(1+x^2)}$ $\dfrac{dy}{dx}=\dfrac{(1+x^2)(2x)-x^2(2x)}{(1+x^2)^2}$ $=\dfrac{2x(1+x^2-x^2)}{(1+x^2)^2}$ $=\dfrac{2x}{(1+x^2)^2}$	--- M1 --- A1

Question	Solution	Mark Allocation
4 (ii)	$\int_3^7\dfrac{5x}{(1+x^2)^2}dx$ $=\dfrac{5}{2}\int_3^7\dfrac{2x}{(1+x^2)^2}dx$ $=\dfrac{5}{2}\left[\dfrac{x^2}{(1+x^2)}\right]_3^7$ $=\dfrac{5}{2}\left[\dfrac{7^2}{(1+7^2)}-\dfrac{3^2}{(1+3^2)}\right]$ $=\dfrac{1}{5}$	--- M1 --- M1 --- A1
5 (i)	Midpoint $PQ=\left(\dfrac{-2+4}{2},0\right)=(1,0)$ Gradient $PQ=0$ Equation of ⊥ bisector of PQ: $x=1[shown]$	--- M1 --- M1 --- A1
5 (ii)	Sub $x=1$ into $y=x+4$: $y=5$ Centre of circle: (1, 5)	M1 --- B1
5 (iii)	Radius $=\sqrt{(5-0)^2+(1-4)^2}$ $=\sqrt{34}$ units Equation of circle: $(x-1)^2+(y-5)^2=34$	--- M1 --- M1 --- A1
5 (iv)	Let $R=(x,y)$ $\left(\dfrac{-2+x}{2},\dfrac{0+y}{2}\right)=(1,5)$ $x=4, y=10$ $R=(4,10)$	--- M1 --- A1,A1
6 (i)	$3\cos\theta-5\sin\theta=R\cos(\theta+\alpha)$ $R^2=3^2+5^2$ $=34$ $R=\sqrt{34}$ $\tan\alpha=\dfrac{5}{3}$ $\alpha=1.030$	--- M1 --- M1

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Question	Solution	Mark Allocation
	$\therefore\ 3\cos\theta - 5\sin\theta = \sqrt{34}\cos(\theta + 1.03)$	--- A1
6 (ii)	$\therefore\ 3\cos\theta - 5\sin\theta = \sqrt{34}\cos(t + 1.03)$ When h =3.5 $3.5 = \sqrt{34}\cos(t + 1.03)$ $\cos(t + 1.03) = \frac{3.5}{\sqrt{34}}$ $t + 1.03 = 0.9269, 2\pi - 0.9626, \ldots$ $t = -0.1031, 5.356, ..$ $\therefore time = 5.37s(3S.f)$	--- M1 --- M1 --- M1 --- A1
7 (i)	$\frac{dy}{dx} = 3x^2 - 2x - 5$ when $\frac{dy}{dx} = 0$, $(3x-5)(x+1) = 0$ $x = \frac{5}{3}$ or $x = -1$ $y = -\frac{67}{27}$ or $y = 7$ The coordinates of the stationary points are $\left(\frac{5}{3}, -\frac{67}{27}\right)$ and $(-1, 7)$	--- M1 --- M1 ---M1 --- A1
7 (ii)	$\frac{d^2y}{dx^2} = 6x - 2$ when $x = \frac{5}{3}$, $\frac{d^2y}{dx^2} = 6\left(\frac{5}{3}\right) - 2 = 8 > 0$ $\therefore \left(\frac{5}{3}, -\frac{67}{27}\right)$ is a minimum point. when $x = -1$, $\frac{d^2y}{dx^2} = 6(-1) - 2 = -8 < 0$ $\therefore (-1, 7)$ is a maximum point.	--- M1 --- A1 --- M1 --- A1
7 (iii)	For y to be increasing, $\frac{dy}{dx} > 0$, $(3x-5)(x+1) > 0$ $x < -1$ or $x > \frac{5}{3}$	--- M1 --- A1

Question	Solution	Mark Allocation
8	$8\cot x = 3\sin x$ $\frac{8\cos}{\sin x} = 3\sin x$ $3\sin^2 x - 8\cos x = 0$ $3(1 - \cos^2 x) - 8\cos x = 0$ $3 - 3\cos^2 x - 8\cos x = 0$ $3\cos^2 x + 8\cos x - 3 = 0$ $(3\cos x - 1)(\cos x + 3) = 0$ $3\cos x - 1 = 0$ or $\cos x = -3$ (no solution) $\cos x = \frac{1}{3}$ $x = 70.5^0$ (1 d.p.) , 289.5^0 (1 d.p.)	--- M1 --- M1 --- M1 --- M1 --- A1
9 (i)	$2x + 2y + \frac{\pi(2x)}{2} = 12$ $2x + 2y + \pi x = 12$ $y = \frac{12 - 2x - \pi x}{2}$	--- M1 --- A1
9 (ii)	$A = 2xy + \frac{\pi x^2}{2}$ $= 2x\left(\frac{12 - 2x - \pi x}{2}\right) + \frac{\pi x^2}{2}$ $= 12x - 2x^2 - \pi x^2 + \frac{\pi x^2}{2}$ $= 12x - 2x^2 - \frac{\pi x^2}{2}$ (shown)	--- M1 --- A1
9 (iii)	$\frac{dA}{dx} = 12 - 4x - \pi x$ $\frac{dA}{dx} = 0$ $12 - 4x - \pi x = 0$ $x = \frac{12}{4 + \pi} \approx 1.68$ Check : $\frac{d^2A}{dx^2} = -4 - \pi < 0$ $\therefore x = \frac{12}{4 + \pi} \approx 1.68$, when A is a max.	--- M1 --- M1 --- A1

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Question	Solution	Mark Allocation
10 (i)	$\tan A = \frac{3}{4}$ and $\sin A$ is negative $\Rightarrow 180° < \theta < 270°$ $\tan\theta = \frac{3}{4} \Rightarrow \frac{\text{opp}}{\text{adj}}$ $\text{hyp} = \sqrt{3^2+4^2}$ $= 5$ $\sin 2\theta = 2\sin\theta\cos\theta$ $= 2\left(\frac{-3}{5}\right)\left(\frac{-4}{5}\right)$ $= \frac{24}{25}$	--- M1 --- M1 --- M1 --- A1
10 (ii)	$-\frac{3}{4}$	--- B1
10 (iii)	$-\frac{5}{4}$	--- B1
11 (i)	Mid-point of $AB = \left(\frac{-2+(-4)}{2}, \frac{3+(-3)}{2}\right)$ $= (-3, 0)$ Gradient of $AB = \frac{3-(-3)}{-2+4}$ $= \frac{6}{2}$ $= 3$ Gradient of $\perp$ bisector of $AB = -\frac{1}{3}$ Equation of perpendicular bisector of AB, $y - 0 = -\frac{1}{3}(x+3)$ $y = -\frac{1}{3}x - 1$	--- M1 --- M1 --- M1 --- A1
11 (ii)	At C, $x = 0$, $y = -1$ $\therefore C(0, -1)$	--- M1

Question	Solution	Mark Allocation
	Area of $\Delta ABC = \frac{1}{2}\begin{vmatrix} -2 & -4 & 0 & -2 \\ 3 & -3 & -1 & 3 \end{vmatrix}$ $= \frac{1}{2}\lvert(6+4+0)-(-12+0+2)\rvert$ $= 10$ sq units	--- M1 --- A1
11 (iii)	Let the coordinates of D be (a, b). Mid-point AC = Mid-point BD $\left(\frac{-2+0}{2}, \frac{3-1}{2}\right) = \left(\frac{-4+a}{2}, \frac{-3+b}{2}\right)$ $(-1, 1) = \left(\frac{-4+a}{2}, \frac{-3+b}{2}\right)$ $\frac{-4+a}{2} = -1$ $a = 2$ Or $\frac{-3+b}{2} = 1$ $b = 5$ Therefore coordinates of D is (2, 5)	--- M1 --- A1

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Name:

Name: Class Index no.

Class	Index no.

Jurong West Secondary School

Preliminary Examinations 2016

/70

ADDITIONAL MATHEMATICS **4044/01**
Secondary Four Normal Academic **22 August 2016**
Paper 1 **1100 – 1245**
1 hour 45 minutes

Additional Materials: Answer Paper

READ THESE INSTRUCTIONS FIRST

Write your name, class and index number on all the work you hand in.
Write in dark blue or black pen on both sides of the paper.
You may use an HB pencil for any diagrams or graphs.
Do not use staples, paper clips, glue or correction fluid.

Answer **all** questions.
Write your answers on the separate Answer Paper provided.
Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.
The use of an approved scientific calculator is expected, where appropriate.
You are reminded of the need for clear presentation in your answers.

At the end of the examination, fasten all your work securely together.
The number of marks is given in brackets [ ] at the end of each question or part question.
The total of the marks for this paper is 70.

After checking of answer script		
Checked	Signature	Date
Student		

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Mathematical Formulae

1. ALGEBRA

Quadratic Equation

For the equation $ax^2 + bx + c = 0$,

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}.$$

Binomial Expansion

$$(a+b)^n = a^n + \binom{n}{1}a^{n-1}b + \binom{n}{2}a^{n-2}b^2 + \ldots + \binom{n}{r}a^{n-r}b^r + \ldots + b^n,$$

where n is a positive integer and $\binom{n}{r} = \frac{n!}{r!(n-r)!} = \frac{n(n-1)\ldots(n-r+1)}{r!}$.

2. TRIGONOMETRY

Identities

$$\sin^2 A + \cos^2 A = 1$$

$$\sec^2 A = 1 + \tan^2 A$$

$$\operatorname{cosec}^2 A = 1 + \cot^2 A$$

$$\sin(A \pm B) = \sin A\cos B \pm \cos A\sin B$$

$$\cos(A \pm B) = \cos A\cos B \mp \sin A\sin B$$

$$\tan(A \pm B) = \frac{\tan A \pm \tan B}{1 \mp \tan A\tan B}$$

$$\sin 2A = 2\sin A\cos A$$

$$\cos 2A = \cos^2 A - \sin^2 A = 2\cos^2 A - 1 = 1 - 2\sin^2 A$$

$$\tan 2A = \frac{2\tan A}{1 - \tan^2 A}$$

Formulae forΔABC

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$a^2 = b^2 + c^2 - 2bc\cos A$$

$$\Delta = \frac{1}{2}ab\sin C$$

---

1 (i) Factorise $a^3 + b^3$. [1]

(ii) Hence, express $2^3 + 27$ as a product of two integers. [2]

2 Find $\int 2(4x - 3)^{-7}dx$. [3]

3 Given that $\operatorname{cosec} A = \frac{5}{3}$, where A is acute, without the use of calculator, find the values of the following.

(i) $\tan A$ [2]

(ii) $\sin 2A$ [2]

4 (i) Show that $27^{x+1} \times 3^{3x}$ may be written as 3^{6x+3}. [2]

(ii) Hence find the value of $\frac{27^{x+1} \times 3^{3x}}{5(3^3) \times 3^{6x}}$. [2]

5 The volume, V cm^3, of liquid in a container is given by $V = 4x^3$, where x cm is the depth of the liquid. Liquid is poured into the container at a constant rate of 24 cm^3/s. Determine the rate of increase of x when $x = 3$. [4]

6 (i) The curve $y^2 = -4x$ and the line $y = 3x + 1$ intersect at points A and B. Find the coordinates of A and B. [3]

(ii) Sketch, on the same diagram, the graphs of $y^2 = -4x$ and $y = 3x + 1$, marking the points A and B on your diagram. [3]

7 The points A, B and C have coordinates $(2, 10)$, $(15,8)$ and $(13,1)$ respectively. D is the point on the x-axis such that CD is parallel to AB.

(i) Find the equation of CD. [3]

(ii) Find the coordinates of D. [1]

(iii) Hence, find the area of quadrilateral $ABCD$. [2]

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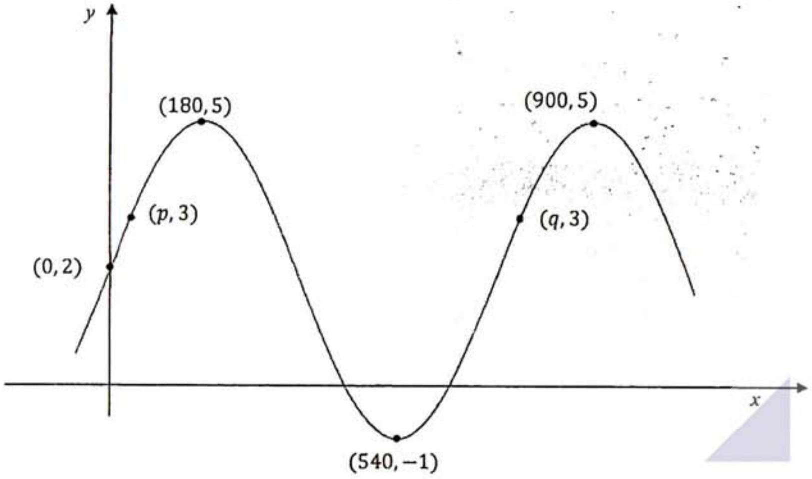
8 The sketch below shows part of the graph of $y = a\sin\frac{x}{b} + c$, where x is in degrees.



(i) Write down the values of the integers a, b and c. [3]

(ii) When $x = 60$, $y = 3.5$. Use the sketch to write down two further values of x between 0 and 900 for which $y = 3.5$. [2]

(iii) Find an equation connecting p and q. [1]

9 The equation of a curve is $y = \frac{x-5}{x+3}$.

(i) By using the quotient rule, find $\frac{dy}{dx}$. [2]

(ii) Find the equation of the normal to the curve at the point $(1, -1)$. [3]

(iii) Given that $y = f(x)$ for $x \geq 0$, show that f is an increasing function. [1]

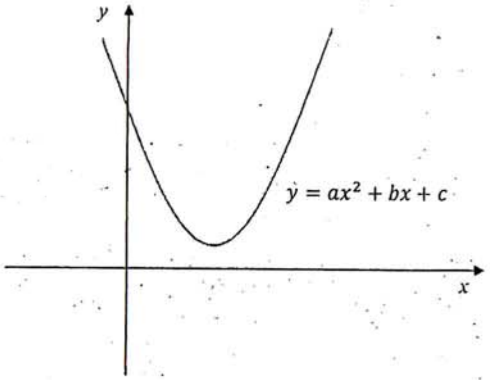
10 (i) Differentiate $(3x^2 - 5x)^4$ with respect to x. [2]

(ii) Hence evaluate $\int_{\sqrt{2}}^{3}(6x-5)(3x^2-5x)^3\,dx$. [4]

11 (a) Show that $\frac{\cos 2A}{1-\cos^2 A} = \operatorname{cosec}^2 A - 2$. [3]

(b) Hence, solve the equations $\frac{\cos 2A}{1-\cos^2 A} = 0$, for $0 \leq x \leq \pi$. [3]

12 The sketch below shows part of the graph of $y = ax^2 + bx + c$. For each of the following expressions state, with a reason, whether it is positive, zero or negative.



(i) c [2]

(ii) $b^2 - 4ac$ [2]

(iii) $\frac{dy}{dx}$ at y intercept [2]

(iv) $\frac{d^2y}{dx^2}$ [2]

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13 (i) Without using a calculator, express $\frac{5-\sqrt{3}}{2+\sqrt{3}}$ in the form $a + b\sqrt{3}$, where a and b are integers. [3]

(ii) Hence or otherwise, find the values of the integers p and q for which $\frac{5-\sqrt{3}}{2+\sqrt{3}} = (p - \sqrt{3})(2q - \sqrt{3})$. [5]

End of Paper

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Name: **Marked Scheme**

Class Index no.

Jurong West Secondary School

Preliminary Examinations 2016

70

ADDITIONAL MATHEMATICS **4044/01**

Secondary Four Normal Academic **22 August 2016**

Paper 1 **1100 – 1245**

1 hour 45 minutes

Additional Materials: Answer Paper

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After checking of answer script		
Checked by	**Signature**	**Date**
Student		

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where n is a positive integer and $\binom{n}{r} = \frac{n!}{r!(n-r)!} = \frac{n(n-1)\ldots(n-r+1)}{r!}$.

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$$\Delta = \frac{1}{2}ab\sin C$$

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1 (i) Factorise $a^3 + b^3$. [1]

$a^3 + b^3 = (a + b)(a^2 - ab + b^2)$ [B1]

(ii) Express $2^3 + 27$ as a product of two integers. [2]

$$\begin{aligned} 2^3 + 27 &= 2^3 + 3^3 \\ &= (2 + 3)(2^2 - 2(3) + 3^2) \text{ [M1]} \\ &= (5)(7) \text{ [A1]} \end{aligned}$$

2 Find $\int 2(4x - 3)^{-7}\mathrm{d}x$. [3]

$$\begin{aligned} \int 2(4x - 3)^{-7}\mathrm{d}x &= \frac{2(4x-3)^{-6}}{-6(4)} + c \text{ [M1 for denominator, M1 for numerator]} \\ &= -\frac{1}{12}(4x - 3)^{-6} + c \text{ [A1]} \end{aligned}$$

3 Given that $\operatorname{cosec} A = \frac{5}{3}$, where A is acute, find the value of the following.

(i) $\tan A$ [2]

$\sin A = \frac{3}{5}$ [M1 for reciprocal]
$\tan A = \frac{3}{4}$ [A1]

(ii) $\sin 2A$ [2]

$\sin 2A = 2 \sin A \cos A$
$= 2\left(\frac{3}{5}\right)\left(\frac{4}{5}\right)$ [M1 for $\cos A$]
$= \frac{24}{25}$ [A1]

4 (i) Show that $27^{x+1} \times 3^{3x}$ may be written as 3^{6x+3}. [2]

$$\begin{aligned} 27^{x+1} \times 3^{3x} &= 3^{3x+3}.3^{3x} \text{ [M1 for } 3^{3x+3}] \\ &= 3^{6x+3} \text{ (shown) [A1]} \end{aligned}$$

(ii) Hence or otherwise, find the value of $\frac{27^{x+1} \times 3^{3x}}{5(3^3) \times 3^{6x}}$. [2]

$$\begin{aligned} \frac{27^{x+1} \times 3^{3x}}{5(3^3) \times 3^{6x}} &= \frac{3^{6x+3}}{5 \times 3^{6x+3}} \text{ [M1 for } 5 \times 3^{6x+3}] \\ &= \frac{1}{5} \text{ [A1]} \end{aligned}$$

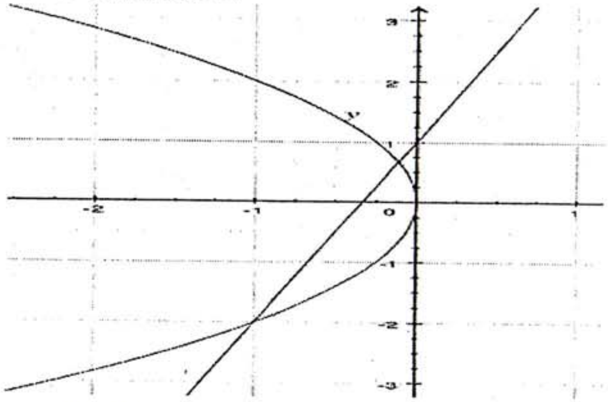
5 The volume, V cm^3, of liquid in a container is given by $V = 4x^3$, where x cm is the depth of the liquid. Liquid is poured into the container at a constant rate of 24 cm^3/s. Determine the rate of increase of x when $x = 3$. [4]

$\frac{dV}{dx} = 12x^2$ [M1 for differentiation]
$\frac{dV}{dt} = 24$
$\frac{dx}{dt} = \frac{dx}{dV} \times \frac{dV}{dt}$ [M1 for chain rule]
$= \frac{1}{12(3^2)} \times 24$ [M1 for substitution]
$= \frac{2}{9}$ cm/s [A1]

6 (i) Find the coordinates of the intersection points A and B for $y^2 = -4x$ and $y = 3x + 1$. [3]

$(3x + 1)^2 = -4x$ [M1 for substitution]
$9x^2 + 6x + 1 = -4x$
$9x^2 + 10x + 1 = 0$
$(9x + 1)(x + 1) = 0$
$x = -\frac{1}{9}$ or $x = -1$
$y = \frac{2}{3}$ $y = -2$
$A\left(-\frac{1}{9}, \frac{2}{3}\right), B(-1, -2)$ *or* $B\left(-\frac{1}{9}, \frac{2}{3}\right), A(-1, -2)$ [A1, A1]

(ii) Sketch, on the same diagram, the graphs of $y^2 = -4x$ and $y = 3x + 1$, marking the points A and B on your diagram. [3]



Correct negative parabola [B1]
Correct line [B1]
Correct intersections [B1]

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7 The points A, B and C have coordinates $(2, 10)$, $(15,8)$ and $(13,1)$ respectively. [3]
D is the point on the x-axis such that CD is parallel to AB.

(i) Find the equation of CD.

$\frac{10-8}{2-15} = -\frac{2}{13}$ [M1 for gradient AB]
$1 = \left(-\frac{2}{13}\right)(13) + c$ [M1 for attempting to find c]
$c = 3$
$y = -\frac{2}{13}x + 3$ [A1]

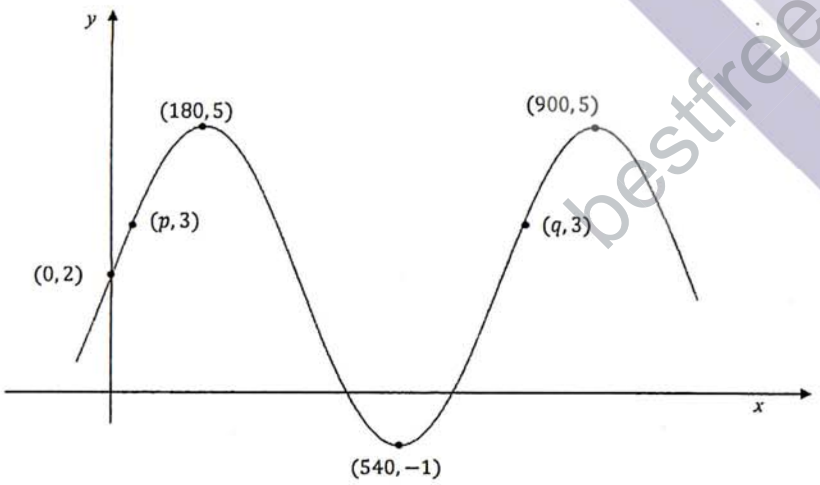
(ii) Find the coordinates of D. [1]

$D\left(19\frac{1}{2}, 0\right)$ [B1]

(iii) Hence, find the area of quadrilateral $ABCD$. [2]

$$\text{Area} = \frac{1}{2}\begin{vmatrix} 2 & 13 & 19.5 & 15 & 2 \\ 10 & 1 & 0 & 8 & 10 \end{vmatrix} \text{ [M1 for attempting to use area formula]}$$
$$= \frac{1}{2}(308 - 165.5)$$
$$= \frac{285}{4} = 71.25 \text{ units}^2 \text{ [A1]}$$

8 The sketch below shows part of the graph of $y = a\sin\frac{x}{b} + c$, where x is in degrees.



(i) Write down the values of the integers a, b and c. [3]

$a = 3, b = 2, c = 2$ [B1, B1, B1]

(ii) When $x = 60$, $y = 3.5$. Use the sketch to write down two further values of x between 0 and 900 for which $y = 3.5$. [2]

300° and 780° [B1, B1]

(iii) Find an equation connecting p and q. [1]

$q = p + 720°$ [B1]

9 The equation of a curve is $y = \frac{x-5}{x+3}$.

(i) By using the quotient rule, find $\frac{dy}{dx}$. [2]

$\frac{dy}{dx} = \frac{(x+3)-(x-5)}{(x+3)^2}$ [M1 for attempting to use quotient rule]
$= \frac{8}{(x+3)^2}$ [A1]

(ii) Find the equation of the normal to the curve at the point $(1, -1)$. [3]

$\frac{dy}{dx} = \frac{8}{(1+3)^2} = \frac{1}{2}$ [M1 for gradient of tangent]
m normal $= -2$
$-1 = -2(1) + c$ [M1 attempt to find c]
$c = 1$
$y = -2x + 1$ [A1]

(iii) Given that $y = f(x)$ for $x \geq 0$, show that f is an increasing function. [1]

Since $(x+3)^2 > 0$, $\frac{8}{(x+3)^2} > 0$, $f'(x) > 0$ [B1]
f is increasing

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10 (i) Differentiate $(3x^2 - 5x)^4$ with respect to x. [2]

$\frac{d(3x^2-5x)^4}{dx} = 4(3x^2 - 5x)^3(6x - 5)$ [B1, B1]

(ii) Hence evaluate $\int_{\sqrt{2}}^{3}(6x - 5)(3x^2 - 5x)^3\,\mathrm{d}x$. [4]

$\frac{1}{4}\int_{\sqrt{2}}^{3} 4(6x - 5)(3x^2 - 5x)^3\,\mathrm{d}x$

[M1 for making the expression $4(6x - 5)(3x^2 - 5x)^3$]

$= \frac{1}{4}[(3x^2 - 5x)^4]_{\sqrt{2}}^{3}$ [A1]

$= \frac{1}{4}\left\{[3(3)^2 - 5(3)]^4 - \left[3\left(\sqrt{2}\right)^2 - 5\left(\sqrt{2}\right)\right]^4\right\}$ [M1 for substituting the values]

$= 5183.7 = 5180$ units^2 [A1]

11 (a) Show that $\frac{\cos 2A}{1-\cos^2 A} = \operatorname{cosec}^2 A - 2$. [3]

$\frac{\cos 2A}{1-\cos^2 A} = \frac{1-2\sin^2 A}{\sin^2 A}$ [M1 for denominator, M1 for numerator]

$= \operatorname{cosec}^2 A - 2$ (shown) [A1]

(b) Hence, solve the equation $\frac{\cos A}{1-\cos^2 A} = 0$, for $0 \le x \le \pi$. [3]

$\operatorname{cosec}^2 A - 2 = 0$ [M1 for using part (a)]

$\operatorname{cosec}^2 A = 2$

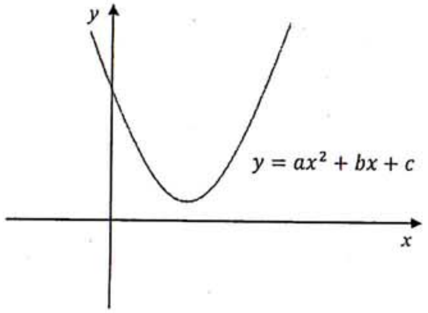
$\sin^2 A = \frac{1}{2}$ [M1 for converting reciprocal]

$\sin A = \pm\frac{1}{\sqrt{2}}$

$\alpha = \frac{\pi}{4}$

$A = \frac{\pi}{4}, \frac{3\pi}{4}$ [A1]

12 The sketch below shows part of the graph of $y = ax^2 + bx + c$. For each of the following expressions state, with a reason, whether it is positive, zero or negative.



(i) c [2]

Positive, [B1]
y-intercept is positive [B1]

(ii) $b^2 - 4ac$ [2]

Negative, [B1]
No real roots [B1]

(iii) $\frac{\mathrm{d}y}{\mathrm{d}x}$ at y intercept [2]

Negative [B1]
Decreasing [B1]

(iv) $\frac{\mathrm{d}^2y}{\mathrm{d}x^2}$ [2]

Positive [B1]
Only have minimum point [B1]

13 (i) Without using a calculator, express $\frac{5-\sqrt{3}}{2+\sqrt{3}}$ in the form $a + b\sqrt{3}$, where a and b are integers. [3]

$\frac{5-\sqrt{3}}{2+\sqrt{3}} \times \frac{2-\sqrt{3}}{2-\sqrt{3}}$ [M1 for rationalizing]

$= \frac{10-2\sqrt{3}-5\sqrt{3}+3}{4-3}$ [M1 for expansion]

$= 13 - 7\sqrt{3}$ [A1]

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(ii) Hence or otherwise, find the values of the integers p and q for which [5]
$\frac{5-\sqrt{3}}{2+\sqrt{3}} = (p-\sqrt{3})(2q-\sqrt{3})$.

$(p-\sqrt{3})(2q-\sqrt{3}) = 2pq - p\sqrt{3} - 2q\sqrt{3} + 3$
$= (2pq+3) - (p+2q)\sqrt{3}$
$= 13 - 7\sqrt{3}$
$2pq + 3 = 13$ [M1 for attempting to compare coefficient]
$pq = 5$
$p = \frac{5}{q} \ldots (1)$
$p + 2q = 7 \ldots (2)$ [M1 for attempting to compare coefficient of $\sqrt{3}$]
Sub (1) into (2),
$2q + \frac{5}{q} = 7$ [M1 for substitution]
$2q^2 - 7q + 5 = 0$
$(q-1)(2q-5) = 0$
$q = 1$ $q = \frac{5}{2}$ [A1]
$p = 5$ $p = 2$ [A1]

End of Paper

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Name:

Class	Index no.

Jurong West Secondary School

Preliminary Examinations 2016

70

ADDITIONAL MATHEMATICS 4044/02
Secondary Four Normal Academic **25 August 2016**
Paper 2 **0800 – 0945**
1 hour 45 minutes

Additional Materials: Answer Paper

READ THESE INSTRUCTIONS FIRST

Write your name, class and index number on all the work you hand in.
Write in dark blue or black pen on both sides of the paper.
You may use an HB pencil for any diagrams or graphs.
Do not use staples, paper clips, glue or correction fluid.

Answer **all** questions.
Write your answers on the separate Answer Paper provided.
Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.
The use of an approved scientific calculator is expected, where appropriate.
You are reminded of the need for clear presentation in your answers.

At the end of the examination, fasten all your work securely together.
The number of marks is given in brackets [ ] at the end of each question or part question.
The total of the marks for this paper is 70.

After checking of answer script		
Checked	Signature	Date
Student		

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Mathematical Formulae

1. ALGEBRA

Quadratic Equation

For the equation $ax^2 + bx + c = 0$,

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}.$$

Binomial Expansion

$$(a+b)^n = a^n + \binom{n}{1}a^{n-1}b + \binom{n}{2}a^{n-2}b^2 + \ldots + \binom{n}{r}a^{n-r}b^r + \ldots + b^n,$$

where n is a positive integer and $\binom{n}{r} = \frac{n!}{r!(n-r)!} = \frac{n(n-1)\ldots(n-r+1)}{r!}$.

2. TRIGONOMETRY

Identities

$$\sin^2 A + \cos^2 A = 1$$

$$\sec^2 A = 1 + \tan^2 A$$

$$\operatorname{cosec}^2 A = 1 + \cot^2 A$$

$$\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$$

$$\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$$

$$\tan(A \pm B) = \frac{\tan A \pm \tan B}{1 \mp \tan A \tan B}$$

$$\sin 2A = 2\sin A \cos A$$

$$\cos 2A = \cos^2 A - \sin^2 A = 2\cos^2 A - 1 = 1 - 2\sin^2 A$$

$$\tan 2A = \frac{2\tan A}{1 - \tan^2 A}$$

Formulae for ΔABC

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$a^2 = b^2 + c^2 - 2bc\cos A$$

$$\Delta = \frac{1}{2}ab\sin C$$

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1 (i) Write down the principal value, in radians as a multiple of π, of $\cos^{-1}\left(\frac{1}{2}\right)$. [1]

(ii) Given that the principal value of $\sin^{-1} a = x$, find the principal value of $\sin^{-1}(-a)$ in terms of x. [1]

2 (i) Given that $y = 3x^2 - 14x + 15$, determine the range of values of x for which $y > 20$. [3]

(ii) Represent this set of values on a number line. [1]

3 (i) Expand $(2 - x)^8$ in ascending powers of x, up to and including the term in x^3. [3]

(ii) Hence find the coefficient of x^2 in the expansion $(3 - 4x)(2 - x)^8$. [3]

4 A polynomial $f(x)$ is such that $f''(x) = 24x - 4$, $f'(1) = 23$ and $f(-1) = -10$.
Find $f(x)$. [5]

5 Given that $y = x^3(5x - 2)^7$, find $\frac{dy}{dx}$, giving your answer in the form $(px + q)(5x - 2)^6x^2$, where p and q are integers. [5]

6 The roots of a quadratic equation $x^2 - 5x = 3$ are α and β.

(i) Evaluate $\alpha^2 + \beta^2$. [4]

(ii) Find a quadratic equation with roots $\frac{1}{\alpha^2}$ and $\frac{1}{\beta^2}$. [4]

7 (i) Find the coordinates of the stationary points on the graph of $y = x^3 + x^2 - 21x + 7$. [5]

(ii) Determine the nature of each of these stationary points. [2]

8 (a) (i) Show that the equation

$$4\sec^2 x - 9\tan x = 2\tan^2 x + 9$$

may be written in the form

$$2\tan^2 x - 9\tan x - 5 = 0.$$ [1]

(ii) Hence solve the equation

$$4\sec^2 x - 9\tan x = 2\tan^2 x + 9, \text{ for } 0° \le x \le 180°.$$ [3]

(b) (i) Express $3\sin\theta + \sqrt{3}\cos\theta$ in the form of $R\sin(\theta + \alpha)$, where $R > 0$ and $0 < \alpha < \frac{\pi}{2}$ radians. [4]

(ii) In an experiment the height, h m, of a wave of a water tank is given by $h = 3\sin t + \sqrt{3}\cos t$, where t is the time in seconds after the start of the experiment.
Find the time when the height of the wave first takes the maximum value. [3]

9 The equation of a circle is $x^2 + y^2 - 4x + 6y - 12 = 0$.

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The point $A(-1, 1)$ lies on the circle.

(ii) Find the coordinates of the point B such that AB is a diameter of the circle. [2]

(iii) Given that the coordinates of D is $(1, 5)$, find the equation of AD. [2]

(iv) Hence show that the line AD cuts the circle at two distinct points. [3]

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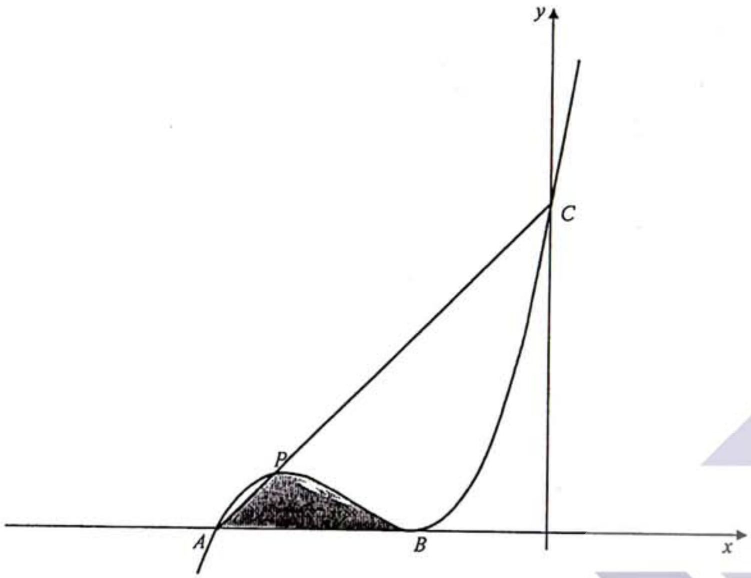
10 The diagram shows part of the graph of $y = \mathrm{f}(x)$, where $\mathrm{f}(x) = x^3 + 13x^2 + kx + 63$. The curve crosses the x- and y- axes at the points A and C respectively and meet the x-axis at point B.



Given that $x + 7$ is a factor of $\mathrm{f}(x)$,

(i) show that $k = 51$, [1]

(ii) solve $\mathrm{f}(x) = 0$ and hence state the x-coordinate of the point B. [4]

Given that the equation of line AC is $y = 9x + 63$,

(iii) show that the coordinates of P is $(-6, 9)$, [3]

(iv) find the area of shaded region APB. [4]

End of Paper

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Class	Index no.

Jurong West Secondary School

Preliminary Examinations 2016

70

ADDITIONAL MATHEMATICS 4044/02
Secondary Four Normal Academic 25 August 2016
Paper 2 0800 – 0945
1 hour 45 minutes

Additional Materials: Answer Paper

READ THESE INSTRUCTIONS FIRST

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The total of the marks for this paper is 70.

After checking of answer script		
Checked	Signature	Date
Student		

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Mathematical Formulae

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where n is a positive integer and $\binom{n}{r} = \frac{n!}{r!(n-r)!} = \frac{n(n-1)\ldots(n-r+1)}{r!}$.

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$$\Delta = \frac{1}{2}ab\sin C$$

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1 (i) Write down the principal value, in radians as a multiple of π, of $\cos^{-1}\left(\frac{1}{2}\right)$. [1]

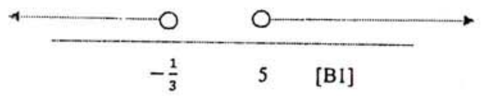
$\boxed{\frac{\pi}{3}\text{ [B1]}}$

(ii) Given that the principal value of $\sin^{-1} a = x$, find the principal value of $\sin^{-1}(-a)$ in terms of x. [1]

$\boxed{-x\text{ [B1]}}$

2 Given that $y = 3x^2 - 14x + 15$, determine the range of values of x for which $y > 20$. [3]

$3x^2 - 14x + 15 > 20$ [M1]
$3x^2 - 14x - 5 > 0$
$(3x + 1)(x - 5) > 0$ [M1 for factorization]
$x < -\frac{1}{3}$ or $x > 5$ [A1]



3 (i) Expand $(2 - x)^8$ in ascending powers of x, up to and including the term in x^3. [3]

$(2-x)^8 = 2^8 + \binom{8}{1}2^7(-x) + \binom{8}{2}2^6(-x)^2 + \binom{8}{3}2^5(-x)^3$
[M1 for binomial expansion]
$= 256 - 1024x + 1792x^2 - 1792x^3 + \cdots$ [A1 for 2 to 3 correct simplification, A2 for all correct]

(ii) Hence find the coefficient of x^2 in the expansion $(3 - 4x)(2 - x)^8$. [3]

Coefficient of $x^2 = 3(1792) + (-4)(-1024)$ [M1, M1 for correct coefficient]
$= 9472$ [A1]

4 A polynomial $f(x)$ is such that $f''(x) = 24x - 4$, $f'(1) = 23$ and $f(-1) = -10$. Find $f(x)$. [5]

$f'(x) = \int (24x - 4)dx$
$= \frac{24x^2}{2} - 4x + c$ [M1 for integration]
$f'(1) = 12(1^2) - 4(1) + c = 23$ [M1 for attempt to find c]
$c = 15$
$f'(x) = 12x^2 - 4x + 15$
$f(x) = \int (12x^2 - 4x + 15)dx$
$= \frac{12x^3}{3} - \frac{4x^2}{2} + 15x + c$ [M1]
$f(-1) = 4(-1)^3 - 2(-1)^2 + 15(-1) + c = -10$ [M1 for attempt to find c]
$c = 11$
$f(x) = 4x^3 - 2x^2 + 15x + 11$ [A1]

5 Given that $y = x^3(5x - 2)^7$, find $\frac{dy}{dx}$, giving your answer in the form $(px + q)(5x - 2)^6x^2$, where p and q are integers. [5]

$\frac{dy}{dx} = (5x - 2)^7(3)x^2 + 7(5)(5x - 2)^6x^3$ [M1 for attempt to use product rule]
$= 3x^2(5x - 2)^7 + 35x^3(5x - 2)^6$ [A1, A1]
$= x^2(5x - 2)^6[(15x - 6 + 35x)]$ [M1 for factorisation]
$= (50x - 6)(5x - 2)^6x^2$ [A1]

6 The roots of a quadratic equation $x^2 - 5x = 3$ are α and β.

(i) Evaluate $\alpha^2 + \beta^2$. [4]

$\alpha + \beta = 5$ [B1]
$\alpha\beta = -3$ [B1]
$\left.\begin{aligned}\alpha^2 + \beta^2 &= (\alpha + \beta)^2 - 2\alpha\beta \\ &= 5^2 - 2(-3)\end{aligned}\right\}$ [M1 for identity]
$= 31$ [A1]

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(ii) Find a quadratic equation with roots $\frac{1}{\alpha^2}$ and $\frac{1}{\beta^2}$. [3]

New sum $= \frac{1}{\alpha^2} + \frac{1}{\beta^2}$

$= \frac{\alpha^2+\beta^2}{(\alpha\beta)^2}$ [M1 for common denominator]

$= \frac{31}{(-3)^2}$

$= \frac{31}{9}$ [A1]

New Product $= \left(\frac{1}{\alpha^2}\right)\left(\frac{1}{\beta^2}\right) = \frac{1}{(\alpha\beta)^2} = \frac{1}{(-3)^2} = \frac{1}{9}$ [B1]

New equation: $x^2 - \frac{31}{9}x + \frac{1}{9} = 0$ [B1]

7 **(i)** Find the coordinates of the stationary points on the graph of $y = x^3 + x^2 - 21x + 7$. [5]

$\frac{dy}{dx} = 3x^2 + 2x - 21$ [M1 for differentiation]

$\frac{dy}{dx} = 0$ at stationary points

$3x^2 + 2x - 21 = 0$ [M1 for $\frac{dy}{dx} = 0$]

$(3x-7)(x+3) = 0$ [M1 for factorisation]

$x = -3$ or $x = \frac{7}{3} = 2\frac{1}{3}$

$y = 52 \qquad y = -\frac{644}{27} = -23\frac{23}{27}$

$(-3, 52), \left(2\frac{1}{3}, -23\frac{23}{27}\right)$ [A1, A1]

(ii) Determine the nature of each of these stationary points. [2]

$\frac{d^2y}{dx^2} = 6x + 2$

When $x = -3$,
$\frac{d^2y}{dx^2} < 0 \Rightarrow$ Maximum [B1]

When $x = \frac{7}{3}$,
$\frac{d^2y}{dx^2} > 0 \Rightarrow$ Minimum [B1]

8 **(a)** **(i)** Show that the equation [1]

$$4\sec^2 x - 9\tan x = 2\tan^2 x + 9$$

may be written in the form

$$2\tan^2 x - 9\tan x - 5 = 0.$$

$4(1+\tan^2 x) - 9\tan x = 2\tan^2 x + 9$

$4 + 4\tan^2 x - 9\tan x = 2\tan^2 x + 9$

$2\tan^2 x - 9\tan x - 5 = 0$ (shown) [B1]

(ii) Hence solve the equation
$4\sec^2 x - 9\tan x = 2\tan^2 x + 9$, for $0° \le x \le 180°$. [3]

$2\tan^2 x - 9\tan x - 5 = 0$

$(2\tan x + 1)(\tan x - 5) = 0$ [M1 for factorization]

$\tan x = -\frac{1}{2} \qquad \tan x = 5$

$\alpha = 26.5651° \qquad \alpha = 78.6901°$

$x = 153.4° \qquad x = 78.7°$ [A1, A1]

(b) **(i)** Express $3\sin\theta + \sqrt{3}\cos\theta$ in the form of $R\sin(\theta+\alpha)$, where $R > 0$ and $0 < \alpha < \frac{\pi}{2}$ radians. [4]

$3\sin\theta + \sqrt{3}\cos\theta = R\sin\theta\cos\alpha + R\cos\theta\sin\alpha$

$R\cos\alpha = 3$

$R\sin\alpha = \sqrt{3}$ [M1 for comparing coefficient]

$\alpha = \frac{\pi}{6}$ or 0.524 rad [A1]

$R = 2\sqrt{3}$ or 3.46 [B1]

$\sqrt{3}\sin\theta + 3\cos\theta = 2\sqrt{3}\sin(\theta + \frac{\pi}{6})$ [A1]

(ii) In an experiment, the height, h m of a wave of a water tank is given by [3]
$h = 3\sin t + \sqrt{3}\cos t$,
where t is the time in seconds after the start of the experiment.
Find the time when the height of the wave first takes the maximum value.

$\sin\left(t + \frac{\pi}{6}\right) = 1$ [M1]

$t + \frac{\pi}{6} = \frac{\pi}{2}$ [M1]

$t = \frac{\pi}{3}$

$= 1.05\ s$ [A1]

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9 The equation of a circle is $x^2 + y^2 - 4x + 6y - 12 = 0$.

(i) Find the coordinates of the centre of the circle and determine the length of the radius. [3]

$2g = -4$
$2f = 6$
$g = -2$
$f = 3$ [M1 for any method]
Centre $(2, -3)$ [A1]
Radius $\sqrt{(-2)^2 + 3^2 - (-12)} = 5$ units[A1]

The point $A(-1, 1)$ lies on the circle.

(ii) Find the coordinates of the point B such that AB is a diameter of the circle. [3]

$\frac{x_B+(-1)}{2} = 2$ [M1 for realizing that the centre is the midpoint of AB]
$\frac{y_B+1}{2} = -3$ [M1 for attempting to apply midpoint concept]
$B(5, -7)$ [A1]

(iii) Given that the coordinates of D is $(1, 5)$, find the equation of AD. [2]

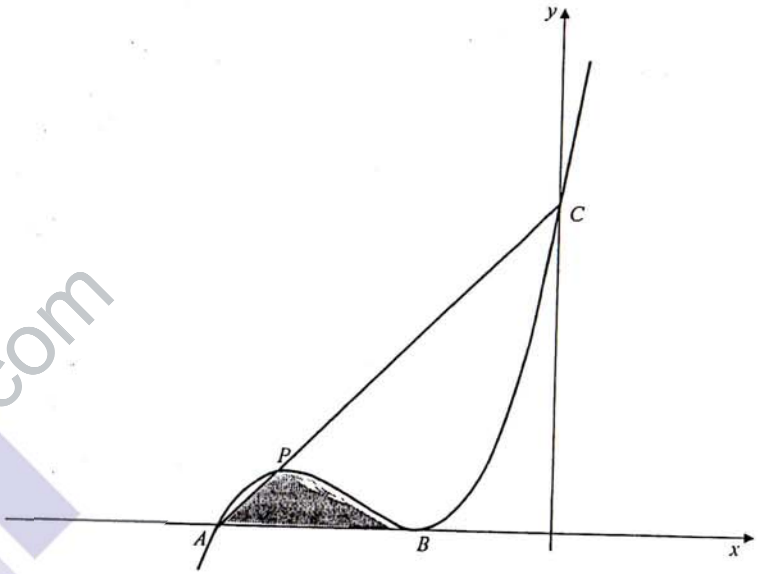
Gradient $AD = \frac{1-5}{-1-1} = 2$
$1 = 2(-1) + c$ [M1 for attempt to find c]
$c = 3$
Equation AD: $y = 2x + 3$ [A1]

(iv) Hence show that the line AD cuts the circle at two distinct points. [3]

$x^2 + (2x+3)^2 - 4x + 6(2x+3) - 12 = 0$ [M1 for substitution]
$x^2 + 4x^2 + 12x + 9 - 4x + 12x + 18 - 12 = 0$

$5x^2 + 20x + 15 = 0$
$x^2 + 4x + 3 = 0$
$4^2 - 4(1)(3) > 0$ [M1 for any method]
2 intersection points [A1]

10 The diagram shows part of the graph of $y = f(x)$, where $f(x) = x^3 + 13x^2 + kx + 63$. The curve crosses the x- and y- axes at the points A and C respectively and meet the x-axis at point B.



Given that $x + 7$ is a factor of $f(x)$,

(i) show that $k = 51$. [1]

$f(-7) = (-7)^3 + 13(-7)^2 + k(-7) + 63 = 0$
$k = 51$ (shown) [B1]

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(ii) solve $f(x) = 0$ and hence state the x-coordinates of the point B. [4]

$$\begin{array}{rl} & \underline{x^2 + 6x + 9\qquad} \\ x+7) & x^3 + 13x^2 + 51x + 63 \\ & \underline{x^3 + \ 7x^2} \\ & \qquad 6x^2 + 51x \\ & \qquad \underline{6x^2 + 42x} \\ & \qquad\qquad 9x + 63 \\ & \qquad\qquad \underline{9x + 63} \quad \text{[M1 for long division]} \end{array}$$

$f(x) = (x+7)(x^2+6x+9)$ [A1 for quotient]

$= (x+7)(x+3)^2$ [M1]

$x = -3$ [A1]

Given that the equation of line AC is $y = 9x + 63$,

(iii) find the coordinates of P. [3]

$9x + 63 = x^3 + 13x^2 + 51x + 63$ [M1 for attempting to solve simultaneous Equation]

$x^3 + 13x^2 + 42x = 0$

$x(x^2 + 13x + 42) = 0$

$x(x+7)(x+6) = 0$ [M1 for factorization]

When $x = -6, y = 9$

$P(-6, 9)$ [A1]

(iv) find the area of shaded region APB. [4]

Area of $APB = \frac{1}{2}(1)(9) + \int_{-6}^{-3}(x^3 + 13x^2 + 51x + 63)\mathrm{d}x$ [M1 for area of triangle; M1 for correct definite integral]

$= \frac{9}{2} + \left[\frac{x^4}{4} + \frac{13x^3}{3} + \frac{51x^2}{2} + 63x\right]_{-6}^{-3}$ [M1 for integration]

$= \frac{9}{2} + \left\{\left[\frac{(-3)^4}{4} + \frac{13(-3)^3}{3} + \frac{51(-3)^2}{2} + 63(-3)\right] - \left[\frac{(-6)^4}{4} + \frac{13(-6)^3}{3} + \frac{51(-6)^2}{2} + 63(-6)\right]\right\}$

$= \frac{9}{2} + \frac{63}{4}$

$= \frac{81}{4} = 20.25$ units^2 [A1]

End of Paper

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YUAN CHING SECONDARY SCHOOL
Secondary Four Normal (Academic) Course
Preliminary Examination 2016

CANDIDATE NAME			
CLASS		INDEX NUMBER	

ADDITIONAL MATHEMATICS

4044/01

Paper 1

11 Aug 2016

1 hour 45 minutes

Additional Materials : Answer Paper

READ THESE INSTRUCTIONS FIRST

Write your name, class and index number on all the work you hand in.
Write in dark blue or black pen.
You may use a pencil for any diagrams or graphs.
Do not use staple, paper clips, highlighters, glue or correction fluid.

Answer **all** questions.
Write your answers on the separate foolscap papers provided.
Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.
The use of an approved scientific calculator is expected, where appropriate.
You are reminded of the need for clear presentation in your answers.

At the end of the examination, fasten all your work securely together.
The number of marks is given in brackets [ ] at the end of each question or part question.
The total number of marks for this paper is 70.

	MARKS
Total	/ 70

This paper consists of 4 printed pages.

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Mathematical Formulae

1. ALGEBRA

Quadratic Equation

For the equation $ax^2+bx+c=0$,

$$x=\frac{-b\pm\sqrt{b^2-4ac}}{2a}$$

Binomial Expansion

$$(a+b)^n=a^n+\binom{n}{1}a^{n-1}b+\binom{n}{2}a^{n-2}b^2+\ldots+\binom{n}{r}a^{n-r}b^r+\ldots+b^n,$$

where n is a positive integer and $\binom{n}{r}=\frac{n!}{r!(n-r)!}=\frac{n(n-1)\ldots(n-r+1)}{r!}$

2. TRIGONOMETRY

Identities

$$\sin^2 A+\cos^2 A=1$$
$$\sec^2 A=1+\tan^2 A$$
$$\operatorname{cosec}^2 A=1+\cot^2 A$$
$$\sin(A\pm B)=\sin A\cos B\pm\cos A\sin B$$
$$\cos(A\pm B)=\cos A\cos B\mp\sin A\sin B$$
$$\tan(A\pm B)=\frac{\tan A\pm\tan B}{1\mp\tan A\tan B}$$
$$\sin 2A=2\sin A\cos A$$
$$\cos 2A=\cos^2 A-\sin^2 A=2\cos^2 A-1=1-2\sin^2 A$$
$$\tan 2A=\frac{2\tan A}{1-\tan^2 A}$$

Formulae for ΔABC

$$\frac{a}{\sin A}=\frac{b}{\sin B}=\frac{c}{\sin C}$$
$$a^2=b^2+c^2-2bc\cos A$$
$$\Delta=\frac{1}{2}ab\sin C$$

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Answer all the questions.

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Answer all the questions.

1 Write down the exact principal value, in radians, of

(i) $\sin^{-1}\left(-\frac{\sqrt{3}}{2}\right)$, [1]

(ii) $\cos^{-1}\left(\frac{\sqrt{2}}{2}\right)$. [1]

2 (i) Expand $(1-3x)^8$ in ascending powers of x, up to the term in x^3. [2]

(ii) Hence find the coefficient of x^2 in the expansion of $\left(x+2+\frac{3}{x}\right)(1-3x)^8$. [2]

3 The volume of a sphere V cm^3 is increasing at a constant rate of 30 cm^3/s. Calculate the rate of increase of its radius r cm when $r = 0.4$. [3]

4 (i) Show that $(x-a)$ is a factor of $x^3 - a^3$. [1]

(ii) Hence factorise completely $3x^3 - 24$. [2]

5 (a) Solve the following pair of simultaneous equations

$$\frac{1}{9} = 3^x \times 27^{2y} \quad \text{and} \quad 5^x \times 25^{2y} = 1.$$ [5]

(b) By means of a suitable substitution, solve the equation $16^x + 4 = 5(4^x)$. [4]

6 Find the range of values of x for the following inequalities.

(a) $2(x+1)^2 < 8$, [2]

(b) $x(1-x) \le x-1$. [2]

7 (a) Without the use of a calculator, express $\frac{\sqrt{3}-1}{\sqrt{3}+2}$ in the simplest form $a+b\sqrt{3}$ where a and b are integers. [3]

(b) Solve the equation $x^2\sqrt{3} - 5x + \sqrt{12} = 0$, expressing your answers in the form of $\frac{c}{\sqrt{3}}$ where c is an integer. [2]

8 (a) Solve $\cos 2\theta - \cos\theta = 0$ for $0° \le \theta \le 180°$. [4]

(b) Find x in terms of π when $\sqrt{3}\cos\left(x-\frac{\pi}{2}\right) = \cos x$ for $0 \le x \le \pi$. [4]

9 The curve $y = a + b\sin cx$ where a, b and c are real numbers has a maximum value of 3, minimum value of -5 and a period of 60°.

(i) Find the value of a, of b and of c. [3]

(ii) Hence sketch the graph for $0° \le x \le 120°$. [3]

10 The equation of a curve is $y = x^4(x+1)$.

(i) Find the x-coordinates of the stationary points of the curve. [3]

(ii) Determine the nature of each of these stationary points. [2]

11 (i) Sketch the graph of the curve $y^2 = 4-x$. [2]

(ii) State the equation of the tangent to the curve $y^2 = 4-x$ at $x = 4$. [1]

(iii) The line $y = -x+2$ cuts the curve $y^2 = 4-x$ at points P and Q. Find the equation of the perpendicular bisector of P and Q. [5]

12 Find (a) $\int 3(1+2x)^6\,dx$, [2]

(b) $\int\left(\frac{1}{x}+x\right)^2 dx$. [2]

13 Given that $15-6x = A(x^2+2)+(Bx+C)(2x-1)$ for all values of x, find the value of A, of B and of C. [3]

14 (a) Given that $(A+B)$ is acute and $\sin(A+B) = \frac{3}{7}$ and $\sin A\cos B = \frac{1}{7}$, without using a calculator, find the exact value of

(i) $\cos A\sin B$, [2]

(ii) $\frac{\tan A}{\tan B}$, [1]

(iii) $\cos(A+B)$. [1]

(b) Given that $\cot\theta = \frac{5}{12}$ where θ is acute, find the exact value of cosec θ. [2]

----- **END OF PAPER** -----

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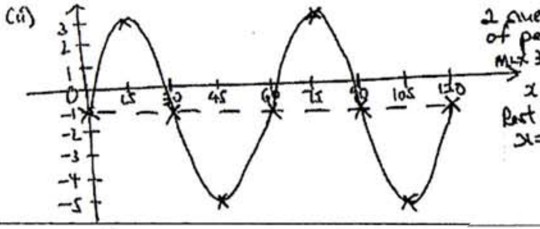
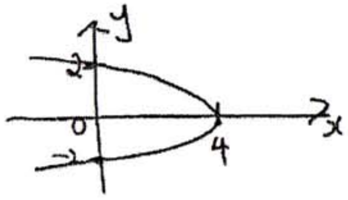
Answer Key 2016.4N.AM.Prelim P1

Qn	Answers3
1	(i) $-\frac{\pi}{3}$ (ii) $\frac{\pi}{4}$
2	(i) $1 - 24x + 252x^2 - 1512x^3 + \ldots$ (ii) -4056
3	14.9 cm/s
4	(i) $f(a) = 0$ (ii) $3(x-2)(x^2+2x+4)$
5	(a) $x = 4, y = -1$ (b) $x = 1, 0$
6	(a) $-3 < x < 1$ (b) $x \leq -1$ or $x \geq 1$
7	(a) $-5 + 3\sqrt{3}$ (b) $\frac{3}{\sqrt{3}}$ *or* $\frac{2}{\sqrt{3}}$
8	(a) 0°, 120° (b) $\frac{\pi}{6}$
9	(i) a = −1, b = 4, c = 6 (ii)  2 sine graph of period 60° [1] max 3, min −5 [1] Best line at x = −1 [1]
10	(i) $x = 0, -\frac{4}{5}$ (ii) min *pt when* $x = 0$, max *pt when* $x = -\frac{4}{5}$
11	(i)  correct shape [1] x- & y intercept properly labelled [1] (ii) $x = 4$ (iii) $y = x - 1$
12	(a) $\frac{3(1+2x)^7}{14}$ (b) $-\frac{1}{x} + 2x + \frac{x^3}{3} + c$
13	$A = 5\frac{1}{3}, B = -4\frac{1}{3}, C = -2\frac{2}{3}$
14	(a) (i) $\frac{2}{7}$ (ii) $\frac{1}{2}$ (iii) $\frac{2\sqrt{10}}{7}$ (b) $\frac{13}{12}$

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CANDIDATE NAME	
CLASS	
INDEX NUMBER	

ADDITIONAL MATHEMATICS

4044/02

Paper 2

15 Aug 2016

1 hour 45 minutes

Additional Materials : Answer Paper

READ THESE INSTRUCTIONS FIRST

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Write in dark blue or black pen.
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The total number of marks for this paper is 70.

	MARKS
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$$\sin 2A = 2 \sin A \cos A$$
$$\cos 2A = \cos^2 A - \sin^2 A = 2\cos^2 A - 1 = 1 - 2\sin^2 A$$
$$\tan 2A = \frac{2 \tan A}{1 - \tan^2 A}$$

Formulae for ΔABC

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$
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Answer **all** the questions.

1 (a) Find the range of values of k for which $kx^2+4x-k+5$ is always positive. [3]

(b) Find the values of m for which the line $y=2+mx$ is a tangent to the curve $y=x^2-4x+3$. [4]

2 The roots of a quadratic equation are α and β, where $\alpha+\beta=\frac{2}{3}$ and $\alpha\beta=-3$.

(i) Find this quadratic equation in the form of $ax^2+bx+c=0$ where a, b and c are integers. [2]

(ii) The roots of another quadratic equation $x^2+px+q=0$ are $\alpha-\beta$ and $\beta-\alpha$. Find the value of each of the constants p and q. [4]

3 Express $5\cos\theta-12\sin\theta$ in the form of $R\cos(\theta+\alpha)$ where $R>0$ and $0°<\alpha<90°$. [3]
Hence

(i) find the maximum value of $5\cos\theta-12\sin\theta$ and the corresponding value of θ at which this occurs, [2]

(ii) solve for the values of θ when $5\cos\theta-12\sin\theta=1$, for $0°\le\theta\le 360°$. [3]

4 (a) (i) Prove the identity $\sec A=\frac{\cos A}{1+\cos 2A}+\frac{\sin A}{\sin 2A}$. [3]

(ii) **Hence** find, in terms of π, all the angles between 0 and 2π which satisfy the equation $\frac{\cos A}{1+\cos 2A}+\frac{\sin A}{\sin 2A}=\sqrt{2}$. [2]

(b) Solve the equation $3\sec^2 x-\tan^2 x+5\tan x=6$ for $0°\le x\le 180°$. [4]

5 (a) Find the equation of the line that is parallel to $x+4y=1$ and which bisects the line AB where the coordinates of A and B are $(-1, 11)$ and $(7, -3)$ respectively. [3]

(b) A curve has the equation $y=x^4-6x+k$, where k is a constant.

(i) Find the gradient of the tangent to the curve at $x=1$. [2]

(ii) Show that the tangent to the curve at $x=1$ is perpendicular to the line $4y-2x=5$. [2]

(iii) Given that the tangent to the curve at $x=1$ cuts the y-axis at $y=-2$, show that $k=1$. [2]

(iv) Hence, find the equation of the normal to the curve at $x=1$. [2]

6 Given that $y=\frac{1-x}{x+1}$, $x\neq -1$, find $\frac{dy}{dx}$ and determine if y is an increasing or decreasing function. [4]

7 The equation of a circle is $x^2+y^2-4x+8y=5$.

(i) Find the centre and the radius of the circle. [2]

(ii) Show that the origin is inside the circle. [1]

(iii) Show that $P(2, 1)$ lies on the circle. [1]

(iv) State the equation of the tangent to the circle at $P(2, 1)$. [1]

8 The expression $f(x)=x^3+2x^2-x+7$ can be written as $(x^2-1)Q(x)+a$.

(a) Find

(i) $Q(x)$, [2]

(ii) the value of a. [1]

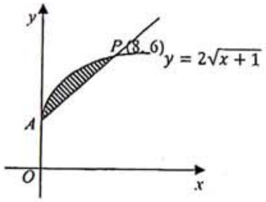
(b) Find the remainder when $f(x)$ is divided by $x-1$. [1]

9 The diagram shows a part of a curve $y=2\sqrt{x+1}$ which intersects the y-axis at A. A straight line passing though A meets the curve again at $P(8, 6)$. Find

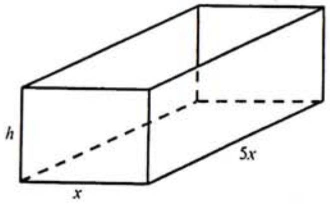
(i) the coordinates of A, [1]

(ii) the equation of line AP, [2]

(iii) the area of the shaded region. [4]



10 The diagram shows an open trough for water of rectangular base x metres by $5x$ metres and height h metres. The internal surface area of the water trough is 36 m^2. The thickness of the base and sides can be ignored.



(i) Show that the height, h m, of the water trough is $h=\frac{36-5x^2}{12x}$. [2]

(ii) Show that the volume, V m^3, of the water trough is $V=\frac{180x-25x^3}{12}$. [2]

(iii) Given that x and h can vary, find the value of x for which V is a maximum. [5]

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Answer Key 2016.4N.AM.Prelim P2

Qn	Answers
1	(a) $1 < k < 4$ (b) $m = -2$ or -6
2	(i) $3x^2 - 2x - 9 = 0$ (ii) $p = 0, q = -12\frac{4}{9}$
3	$5\cos\theta - 12\sin\theta = 13\cos(\theta + 67.4°)$ (i) Max = 13 when θ =292.6° (ii) $\theta = 18.2°$ or 207.0°
4	(aii) $A = \frac{\pi}{4}$ *or* $\frac{7\pi}{4}$ (b) $\theta = 26.6°$, 108.4°
5	(a) $4y = -x + 19$ b(i) -2 (ii) *since* $\frac{1}{2} \times (-2) = -1$ (iii) Use gradient of $(1, -5 + k)$ and $(0, -2) = -2$ (iv) $2y = x - 9$
6	(b) $\frac{dy}{dx} = \frac{-2}{(x+1)^2} < 0$; *therefore* y is an increasing function.
7	(i) Centre $(2, -4)$ radius = 5 (ii) distance of centre to origin = $\sqrt{20} < 5$. (iii) Show that when $x = 2, y = 1$. (iv) $y = 1$
8	(a) $Q(x) = x + 2$ (ii) $a = 9$ (b) 9
9	(i) $A(0, 2)$ (ii) $2y = x + 4$ (iii) $2\frac{2}{3}$
10	(i) $5x^2 + 12xh = 36$ (ii) $V = 5x^2h$ (iii) $x = 1.55$, Need to show V is max.

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