NAME: ______________________ _____ (   )   **CLASS: 4** (   )

ANGLICAN HIGH SCHOOL
SECONDARY FOUR
PRELIMINARY EXAMINATIONS 2018
ADDITIONAL MATHEMATICS PAPER 1 [4047/01]

S4

11 September 2018 Tuesday **2 hours**

Additional Materials: 6 Writing Papers

READ THESE INSTRUCTIONS FIRST
Write your name, index number and class on all the work you hand in.
Write in dark blue or black pen on both sides of the writing paper provided.
You may use a soft pencil for any diagrams or graphs.
Do not use staples, paper clips, highlighters and glue or correction fluid.

Answer **all** the questions.
Write your answers on the separate Writing Paper provided.
Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.
The use of an approved scientific calculator is expected, where appropriate.
You are reminded of the need for clear presentation in your answers.

At the end of the examination, fasten all your work securely together and attach the question paper on top of the scripts.
The number of marks is given in brackets [ ] at the end of each question or part question.
The total number of marks for this paper is 80.

For Examiners' Use

Question	Marks	Question	Marks	Question	Marks
1		7		13	
2		8		Table of Penalties	
3		9			
4		10		Units	
5		11		Presentation	
6		12		Accuracy	
Parent's Name & Signature: Date:		Total:		80	

This paper consists of **6** printed pages.

[Turn over

Mathematical Formulae

1. ALGEBRA

Quadratic Equation

For the equation $ax^2 + bx + c = 0$,

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Binomial expansion

$$(a+b)^n = a^n + \binom{n}{1}a^{n-1}b + \binom{n}{2}a^{n-2}b^2 + \ldots + \binom{n}{r}a^{n-r}b^r + \ldots + b^n,$$

where n is a positive integer and $\binom{n}{r} = \frac{n!}{r!(n-r)!} = \frac{n(n-1)\ldots(n-r+1)}{r!}$

2. TRIGONOMETRY

Identities

$$\sin^2 A + \cos^2 A = 1$$

$$\sec^2 A = 1 + \tan^2 A$$

$$\operatorname{cosec}^2 A = 1 + \cot^2 A$$

$$\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$$

$$\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$$

$$\tan(A \pm B) = \frac{\tan A \pm \tan B}{1 \mp \tan A \tan B}$$

$$\sin 2A = 2\sin A \cos A$$

$$\cos 2A = \cos^2 A - \sin^2 A = 2\cos^2 A - 1 = 1 - 2\sin^2 A$$

$$\tan 2A = \frac{2\tan A}{1 - \tan^2 A}$$

Formulae for ΔABC

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$a^2 = b^2 + c^2 - 2bc\cos A$$

$$\text{Area of } \Delta = \frac{1}{2}ab\sin C$$

Answer ALL questions

1. The product of the two positive numbers, x and y, where $x > y$, is 24. The difference between their squares is 14. Form two equations, and hence, find the exact values of the two numbers. [5]

2. Show that $\left(2+\sqrt{7}\right)^2 - \dfrac{18}{3-\sqrt{7}} = c + d\sqrt{7}$ where c and d are integers. [4]

3. **(a)** **(i)** Sketch the two curves $y = 0.5\sqrt[3]{x}$ and $y = \dfrac{8}{x}$ on the same axes for $x > 0$. [3]

(ii) Find the coordinates of the intersection point. [2]

(b) Solve the equation $2 = \left|e^{-x} - 3\right|$. [3]

4 **(i)** Given that the line $y = 2$ intersects the graph of $y = \log_{\frac{1}{5}} x$ at the point P, find the coordinates of P. [2]

(ii) Sketch the graph of $y = \log_{\frac{1}{5}} x$. [2]

(iii) State the range of values of x for which $y < 0$. [1]

5 **(i)** Sketch the graph of $y^2 = 169x$. [2]

(ii) Express $4x^2 - 181x = -9$ in the form $\left(px+q\right)^2 = 169x$, where p and q are constants. [2]

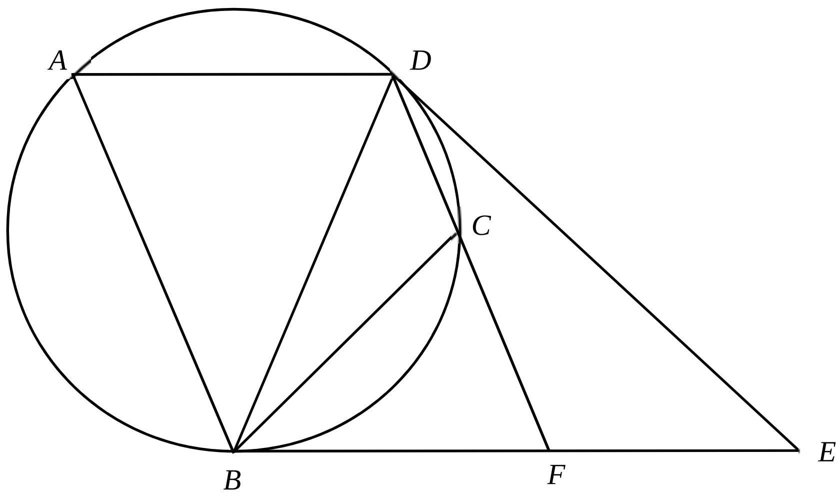
(iii) A suitable straight line can be drawn on the graph in **(i)** to solve the equation $4x^2 - 181x = -9$.

(a) State the equation of this straight line. [1]

(b) On the same axes, sketch the straight line and state the number of solutions of the equation $4x^2 - 181x = -9$. [2]

[Turn over

6.



The diagram shows a circle passing through points *A*, *B*, *C* and *D*. The tangents from *E* meets the circle at *B* and *D*. Given that $AD = BF$ and triangle *ABD* is isosceles, where $AB = BD$. Prove that

(i) *ABFD* is a parallelogram. [3]

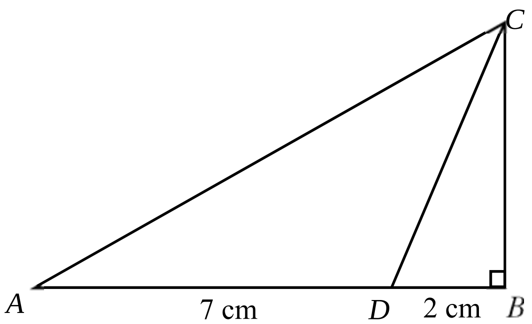
(ii) triangle *BCD* is similar to triangle *DFE*. [3]

(iii) $BD \times EF = CD \times DE$. [1]

7. (a) Sketch the graph of $y = 2\cos\left(\frac{x}{2}\right) - 1$, for the interval $0 \le x \le 2\pi$. [2]

(b) In the diagram, triangle *ABC* is a right-angle triangle, where $\angle ABC = 90°$.

D is a point of *AB* such that *AD* is 7 cm and *BD* is 2 cm.

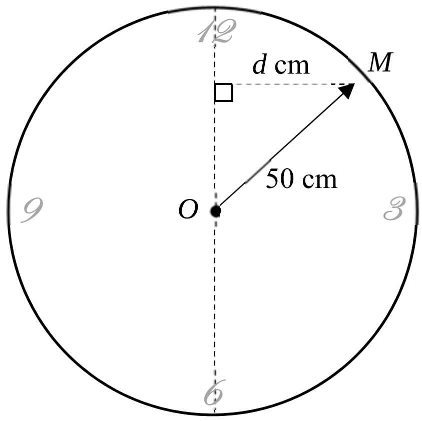


Given that $\cos \angle ADC = -\frac{1}{3}$,

(i) Find the exact length of *BC*. [1]

(ii) Find the value of $\tan \angle ACD$ in the form $a\sqrt{b}$, where *a* and *b* are integers. [2]

8.



The minute hand of a clock is 50 cm, measured from the centre of the clock, O, to the tip of the minute hand, M. The displacement, d cm, of M from the vertical line through O is given by $d = a\sin bt$, where t is the time in minutes past the hour.

(i) Find the exact value of a and of b. [3]

(ii) Find the duration, in each hour, where $|d| > 25$. [3]

9. **(i)** Prove that $\dfrac{\cos^4\theta - \sin^4\theta}{\sin^2\theta\cos^2\theta} = 4\cot 2\theta \operatorname{cosec}2\theta$. [3]

(ii) Hence, or otherwise, solve

$$\frac{\cos^4\theta - \sin^4\theta}{\sin^2\theta\cos^2\theta} = 4\operatorname{cosec}2\theta \text{ for } 0^\circ \le \theta \le 180^\circ.$$ [3]

10. A curve is such that $\dfrac{d^2y}{dx^2} = 6x - 2$ and $P(2, -8)$ is a point on the curve. The gradient of the normal at P is $-\dfrac{1}{2}$. Find the equation of the curve. [7]

11. Find and simplify $\dfrac{dy}{dx}$ for the following:

(i) $y = \ln\cos x$

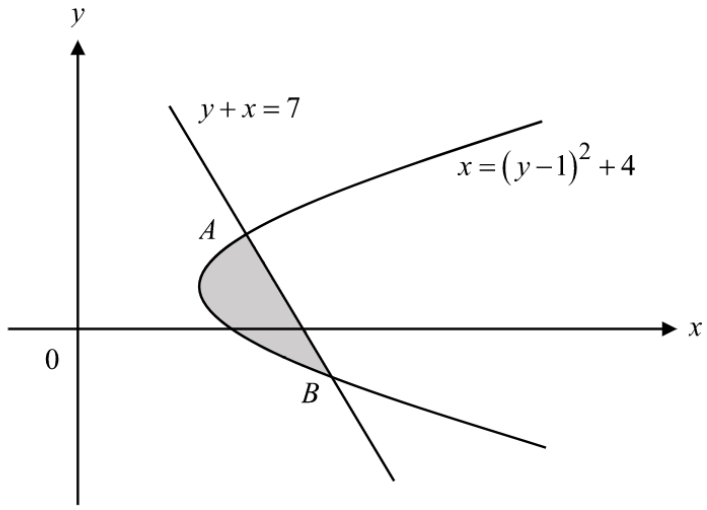
(ii) $y = e^{x^2} \times e^x$ [4]

[Turn over

12. In the diagram, the curve $x = (y - 1)^2 + 4$ and the line $y + x = 7$ intersect at A and B.

(i) Find the coordinates of A and of B. [3]

(ii) Calculate the area of the shaded region. [4]



13. A particle moves in a straight line so that its acceleration, a ms^{-2}, is given by $a = 2t - 13$, where t is the time in seconds after passing a fixed point O. The particle first comes to instantaneous rest at $t = 5$ s . Find,

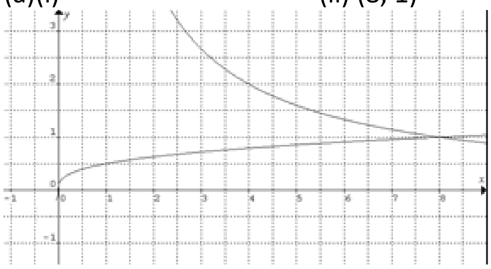
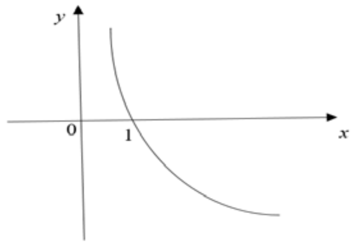
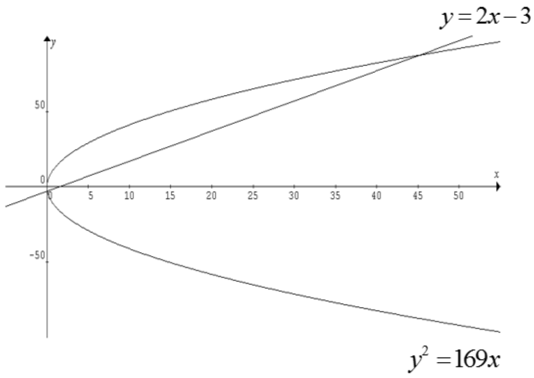
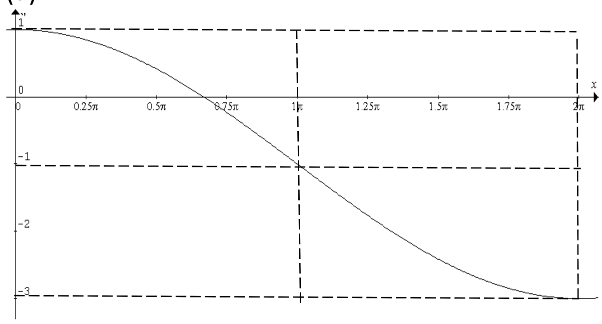
(i) the velocity when the particle passes through O. [2]

(ii) the total distance travelled by the particle when it next comes to rest. [5]

(iii) the minimum velocity of the particle. [2]

*** End of Paper ***

Answer key

1	$x = 4\sqrt{2} \quad y = 3\sqrt{2}$	2	$-16-5\sqrt{7}$
3	(a)(i) (ii) (8, 1)  (b) $x \approx -1.61$ or 0	4	(i) (0.04, 2) (iii) $x > 1$ (ii) 
5	(i)  (ii) $(2x-3)^2 = 169x$ (iii) (a) $y = 2x-3$ (b) 2 solutions	7	(a)  (b) (i) $4\sqrt{2}$ cm (ii) $\dfrac{14\sqrt{2}}{25}$
8	(i) $a = 50 \quad b = \dfrac{\pi}{30}$ (ii) 40 mins	9	(ii) $\theta = 22.5°$ or $112.5°$
10	$y = x^3 - 2x^2 - 6x$.	11	(i) $-\tan x$ (ii) $(2x+1)e^{x^2+x}$
12	(i) $A(5, 2)$, $B(8, -1)$ (ii) $4\dfrac{1}{2}$ units^2	13	(i) 40 ms^{-1} (ii) $83\dfrac{2}{3}m$ (iii) $-2\dfrac{1}{4}ms^{-1}$

2018 S4 AM Prel P1

1. The product of the two positive numbers, x and y, where $x > y$, is 24. The difference between their squares is 14. Form two equations, and hence, find the exact values of the two numbers. [5]

	Solutions	Marks
1	$xy = 24$	M1
	$y = \frac{24}{x}$ ... (1)	
	$x^2 - y^2 = 14$ ... (2)	M1
	Sub. (1) into (2):	
	$x^2 - \left(\frac{24}{x}\right)^2 = 14$	
	$x^2 - \frac{576}{x^2} = 14$	
	$x^4 - 14x^2 - 576 = 0$	M1
	$(x^2 - 32)(x^2 + 18) = 0$	
	$x^2 = 32$ or $x^2 = -18$ (rejected)	
	$x = \sqrt{32}$ $\left(-\sqrt{32}\text{ is rejected}\right)$	A1
	$= 4\sqrt{2}$	
	$\therefore\ y = \frac{24}{4\sqrt{2}} \times \frac{\sqrt{2}}{\sqrt{2}}$	A1
	$= 3\sqrt{2}$	

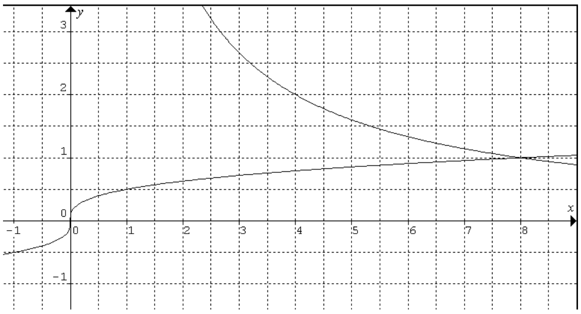
2. Show that $\left(2+\sqrt{7}\right)^2 - \frac{18}{3-\sqrt{7}} = c + d\sqrt{7}$ where c and d are integers. [4]

2	$\left(2+\sqrt{7}\right)^2 - \frac{18}{3-\sqrt{7}}$	
	$= 4 + 4\sqrt{7} + 7 - \frac{18(3+\sqrt{7})}{(3-\sqrt{7})(3+\sqrt{7})}$	M1, M1
	$= 11 + 4\sqrt{7} - \frac{54 + 18\sqrt{7}}{9-7}$	M1
	$= 11 + 4\sqrt{7} - \frac{1}{2}(54 + 18\sqrt{7})$	A1
	$= -16 - 5\sqrt{7}$	

3. (a) (i) Sketch the two curves $y = 0.5\sqrt[3]{x}$ and $y = \dfrac{8}{x}$ on the same axes for $x > 0$. [3]

(ii) Find the coordinates of the intersection point. [2]

(b) Solve the equation $2 = \left|e^{-x} - 3\right|$. [3]

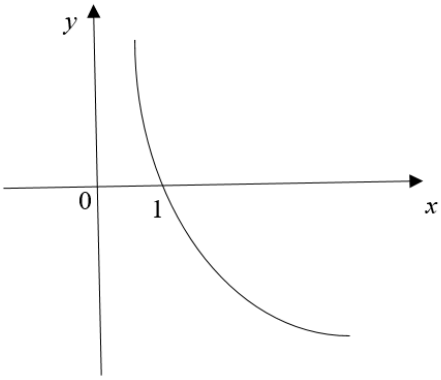
3(a) (i)		B1 B1 (for each curve) ONLY for $x > 0$ B1(label)
(ii)	$0.5\sqrt[3]{x} = \dfrac{8}{x}$ $x^{\frac{4}{3}} = 16$ $x = (2^4)^{\frac{3}{4}}$ $= 8$ $y = \dfrac{8}{8} = 1$ (8, 1)	M1 A1
(b)	$2 = \left\|e^{-x} - 3\right\|$ $e^{-x} - 3 = 2$ or $e^{-x} - 3 = -2$ $e^{-x} = 5$ $e^{-x} = 1$ $e^{x} = 5^{-1}$ $e^{x} = 1$ $x = -\ln 5$ $x = \ln 1$ $= -1.6094$ $x = 0$ ≈ -1.61	M1 A1, A1

4 (i) Given that the line $y = 2$ intersects the graph of $y = \log_{\frac{1}{5}} x$ at the point P, find the coordinates of P. [2]

(ii) Sketch the graph of $y = \log_{\frac{1}{5}} x$. [2]

(iii) State the range of values of x for which $y < 0$. [1]

[Solution]

(i)	$\log_{\frac{1}{5}} x = 2$ $x = 0.2^2$ $= 0.04$ Coordinates of P are (0.04, 2)	**M1** **A1**
(ii)		* Shape – 1 mark * x-intercept – 1 mark
(iii)	$y < 0 \Rightarrow \log_{\frac{1}{5}} x < 0$ $\Rightarrow x > 1$	**B1**

5 (i) Sketch the graph of $y^2 = 169x$. [2]

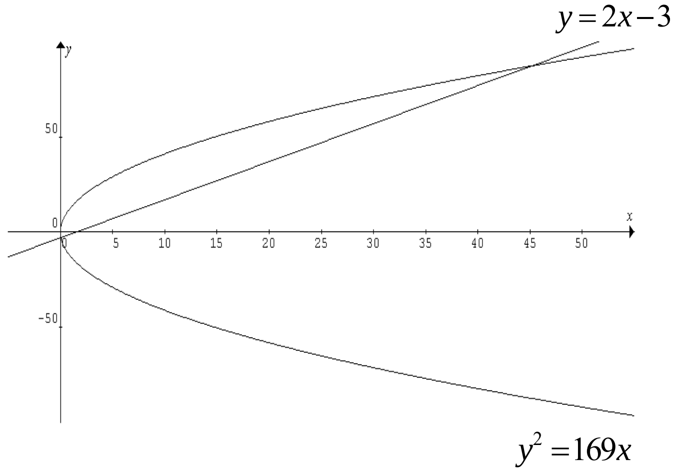
(ii) Express $4x^2 - 181x = -9$ in the form $(px+q)^2 = 169x$, where p and q are constants. [2]

(iii) A suitable straight line can be drawn on the graph in (i) to solve the equation $4x^2 - 181x = -9$.

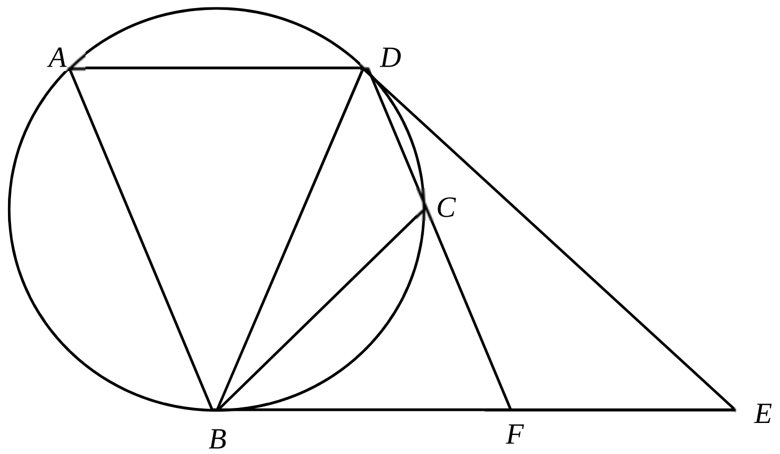
(a) State the equation of this straight line. [1]

(b) On the same axes, sketch the straight line and state the number of solutions of the equation $4x^2 - 181x = -9$. [2]

Solution

5(i)		* upper portion – 1 mark * lower portion – 1 mark * graph of y = 2x – 3
(ii)	$4x^2 - 181x = -9$ $4x^2 - 181x + 169x = -9 + 169x$ $4x^2 - 12x + 9 = 169x$ $(2x-3)^2 = 169x$	**M1** **A1**
(iii)	(a) $y = 2x - 3$	**B1**
	2 solutions	**A1**

6



The diagram shows a circle passing through points *A*, *B*, *C* and *D*. The tangents from *E* meets the circle at *B* and *D*. Given that $AD = BF$ and triangle *ABD* is isosceles, where $AB = BD$. Prove that

i) *ABFD* is a parallelogram. [3]
ii) triangle *BCD* is similar to triangle *DFE*. [3]
iii) $BD \times EF = CD \times DE$. [1]

Solution

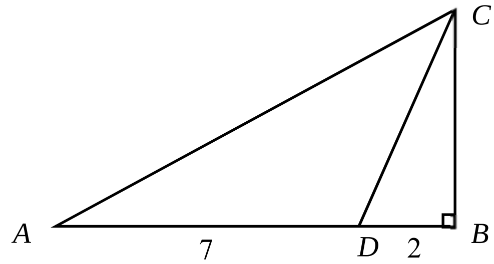
i)	$\angle DBF = \angle BAD$ (alt. seg. thm) $= \angle ADB$ (ΔABD is isosceles)	M1
	By alternate angles, $AD // BF$	M1
	Since $AD = BF$, *ABFD* is a parallelogram.	M1
ii)	$\angle EDF = \angle DBC$ (alt. seg. thm) $\angle DFE = 180° - \angle BFD$ (adj $\angle$ on a str. line) $= 180° - \angle BAD$ (opp. $\angle$ in parallelogram) $= 180° - (180° - \angle DCB)$ ($\angle$ in opp. seg) $= \angle DCB$	M1 M1 M1
iii)	By AA, ΔBCD is similar to ΔDFE. $\frac{BD}{DE} = \frac{CD}{EF}$ $BD \times EF = CD \times DE$	M1

.

7. (a) Sketch the graph of $y = 2\cos\left(\frac{x}{2}\right) - 1$, for the interval $0 \le x \le 2\pi$. [2]

(b) In the diagram, triangle *ABC* is a right-angle triangle, where $\angle ABC = 90°$.

D is a point of *AB* such that *AD* is 7 cm and *BD* is 2 cm.

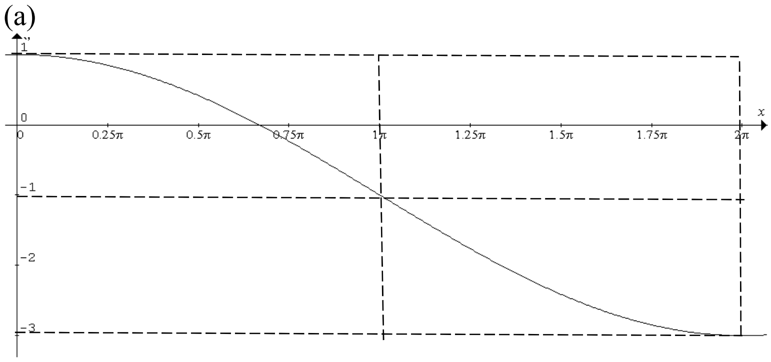


Given that $\cos \angle ADC = -\frac{1}{3}$,

(i) Find the exact length of *BC*. [1]

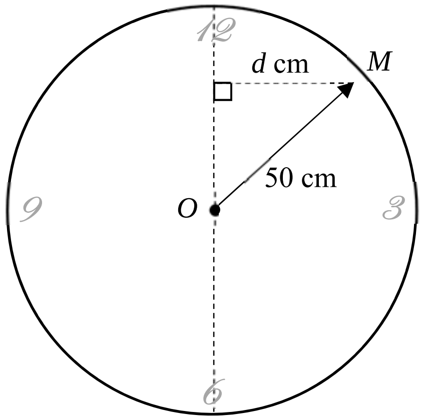
(ii) Find the value of $\tan \angle ACD$ in the form $a\sqrt{b}$, where *a* and *b* are integers. [2]

Solution

(a) 	G1: Correct shape. G1: Label all key points and axes clearly
(bi) $\cos \angle BDC = -\cos \angle ADC$ $\frac{BD}{CD} = -\left(-\frac{1}{3}\right)$ $\frac{2}{CD} = \frac{1}{3}$ $CD = 6$ cm $BC = \sqrt{6^2 - 2^2}$ $= 4\sqrt{2}$ cm	A1
(bii)	

$\tan\angle ACD = \tan(\angle ACB - \angle BCD)$ $= \dfrac{\tan\angle ACB - \tan\angle BCD}{1+(\tan\angle ACB)(\tan\angle BCD)}$ $= \dfrac{\frac{9}{4\sqrt{2}} - \frac{2}{4\sqrt{2}}}{1+\left(\frac{9}{4\sqrt{2}}\right)\left(\frac{2}{4\sqrt{2}}\right)}$ $= \dfrac{\frac{7}{4\sqrt{2}}}{\frac{25}{16}}$ $= \dfrac{14\sqrt{2}}{25}$	M1

8.



The minute hand of a clock is 50 cm, measured from the centre of the clock, *O*, to the tip of the minute hand, *M*. The displacement, *d* cm, of *M* from the vertical line through *O* is given by $d = a\sin bt$, where *t* is the time in minutes past the hour.

i) Find the exact value of *a* and of *b*. [3]

ii) Find the duration, in each hour, where $|d| > 25$. [3]

Solution

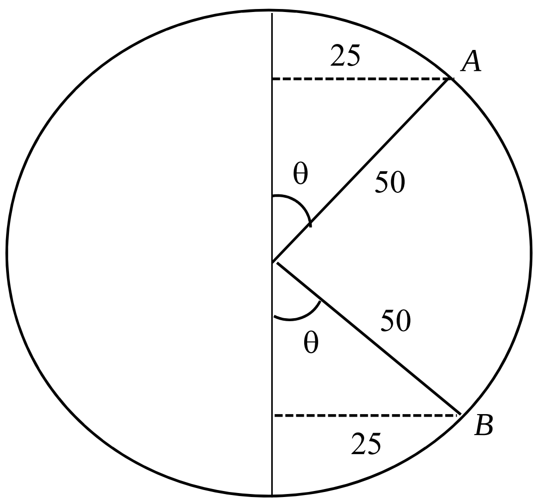
i)	$a = 50$ Period = 60 $\dfrac{2\pi}{b} = 60$ $b = \dfrac{\pi}{30}$	B1 -- for *a* M1 -- for period = 60 A1 – for *b*

<table>
<tr>
<td>ii)</td>
<td>$|d| > 25$
$| \quad |$ when $d = 25$
$\left|50\sin\left(\frac{\pi}{30}t\right)\right| = 25$
$50\sin\left(\frac{\pi}{30}t\right) = \pm 25$
$\sin\left(\frac{\pi}{30}t\right) = \pm\frac{1}{2}$
basic angle $= \sin^{-1}\left(\frac{1}{2}\right) = 30^\circ$
$\frac{\pi}{30}t = \frac{\pi}{6}, \frac{5\pi}{6}, \frac{7\pi}{6}, \frac{11\pi}{6}$
$t = 5, 25, 35, 55$

For $|d| > 25$
Duration $= (25 - 2) + (55 - 35)$
$= 40$ mins</td>
<td>M1

A1

A1</td>
</tr>
<tr>
<td></td>
<td colspan="2">Alternative Solution :



Observe that at the first instance when $d = 25$ at the point A,

$\cos\theta = \frac{25}{50} = \frac{1}{2} \Rightarrow \theta = \frac{\pi}{6}$.
This happened again at the point B.

Between points A and B, $|d| > 25$.

Time taken from A to $B = \frac{\pi - 2\theta}{\pi} \times 30 = \frac{\pi - \frac{\pi}{3}}{\pi} \times 30 = \frac{2}{3} \times 30 = 20$ minutes.
By symmetry, the time for $|d| > 25$ in the other half of the clock face would be 20 minutes as well.
Hence total time for $|d| > 25$ is 20 + 20 = 40 minutes.</td>
</tr>
</table>

9. i) Prove that $\dfrac{\cos^4\theta-\sin^4\theta}{\sin^2\theta\cos^2\theta}=4\cot 2\theta\cos ec2\theta$. [3]

ii) Hence, or otherwise, solve

$$\frac{\cos^4\theta-\sin^4\theta}{\sin^2\theta\cos^2\theta}=4\text{cosec}\,2\theta \text{ for } 0°\le\theta\le180°.$$ [3]

Solution

i)	$LHS=\dfrac{\cos^4\theta-\sin^4\theta}{\sin^2\theta\cos^2\theta}$ $=\dfrac{(\cos^2\theta-\sin^2\theta)(\cos^2\theta+\sin^2\theta)}{(\sin\theta\cos\theta)^2}$ $=\dfrac{\cos 2\theta}{\left(\frac{1}{2}\sin 2\theta\right)^2}$ $=\dfrac{\cos 2\theta}{\frac{1}{4}\sin^2 2\theta}$ $=4\left(\dfrac{\cos 2\theta}{\sin 2\theta}\right)\left(\dfrac{1}{\sin 2\theta}\right)$ $=4\cot 2\theta\cos ec2\theta \quad (RHS)$	M1: factorise M1: double angle formulae M1: getting expression
ii)	$\dfrac{\cos^4\theta-\sin^4\theta}{\sin^2\theta\cos^2\theta}=4\cos ec2\theta$ $4\cot 2\theta\cos ec2\theta=4\cos ec2\theta$ $4\cos ec2\theta(\cot 2\theta-1)=0$ $\cos ec\theta=0 \Rightarrow \dfrac{1}{\sin\theta}=0$ (no solution) OR $\cot 2\theta=1$ $\tan 2\theta=1$ basic angle $=\tan^{-1}1$ $=45°$ $0°\le\theta\le180° \Rightarrow 0°\le2\theta\le360°$ $2\theta=45°$ or $180°+45°$ $\theta=22.5°$ or $112.5°$	M1 M1 A1

10. A curve is such that $\dfrac{d^2y}{dx^2}=6x-2$ and $P(2,-8)$ is a point on the curve. The gradient of the normal at P is $-\dfrac{1}{2}$. Find the equation of the curve. [7]

Solution:

Given $\dfrac{d^2y}{dx^2}=6x-2$	

$\frac{dy}{dx} = \int (6x-2)dx$ $= 3x^2 - 2x + c$	M[1] – no mk if there is no 'c'
Gradient of normal at (2, – 8) = $-\frac{1}{2}$ Gradient of tangent at $P = -\frac{1}{-\frac{1}{2}}$	B[1] –grad. of tangent at P
$\frac{dy}{dx} = 2$ Sub $x = 2$, $3(2)^2 - 2(2) + c = 2$ $c = -6$ $\therefore \frac{dy}{dx} = 3x^2 - 2x - 6$	M[1] –substitution A[1] – for 1st derivative
$y = \int (3x^2 - 2x - 6)dx$ $= x^3 - x^2 - 6x + c_1$ Sub (2, – 8), $-8 = (2)^3 - (2)^2 - 6(2) + c_1$ $c_1 = 0$ Hence, the equation of the curve is $y = x^3 - 2x^2 - 6x$.	A[1] – no mk if there is no 'c_1' M[1] –substitution A[1] - eqn

11. Find and simplify $\frac{dy}{dx}$ for the following:

(i) $y = \ln \cos x$

(ii) $y = e^{x^2} \times e^x$ [4]

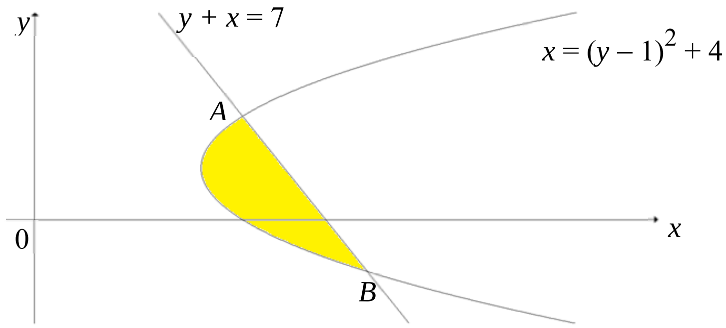
Solution:

(i)	$y = \ln \cos x$ $\frac{dy}{dx} = \frac{-\sin x}{\cos x}$ $= -\tan x$	M[1] A[1]
(ii)	$y = e^{x^2} \times e^x$ $y = e^{x^2+x}$ $\frac{dy}{dx} = (2x+1)e^{x^2+x}$	M[1] - Simplification A[1]
	OR $\frac{dy}{dx} = (2xe^{x^2})(e^x) + (e^{x^2})(e^x)$ $= (2x+1) \times e^{x^2} e^x$ $= (2x+1)e^{x^2+x}$	OR M[1] A[1] - Simplification

12. In the diagram, the curve $x = (y - 1)^2 + 4$ and the line $y + x = 7$ intersect at A and B.

(i) Find the coordinates of A and of B. [3]

(ii) Calculate the area of the shaded region. [4]



Solution:

(i) given $y + x = 7$ $y = -x + 7$ ……..① sub ① into $x = (y - 1)^2 + 4$ $x = (-x + 7 - 1)^2 + 4$ $= x^2 - 12x + 36 + 4$ $x^2 - 13x + 40 = 0$ $(x - 5)(x - 8) = 0$ $x = 5$ or $x = 8$ sub x into ①, $y = -5 + 7$ or $y = -8 + 7$ $= 2$ $= -1$ $\therefore A(5, 2)$, $B(8, -1)$	M[1] any QE [$x^2 - 13x + 40 = 0$ or $y^2 - y - 2 = 0$] A[1] for 1st set of ans [both x or both y] A[1] ans in coordinates form
Area of shaded region $= \frac{1}{2}(2-(-1))(5+8) - \int_{-1}^{2}\left((y-1)^2+4\right) dy$ $= \frac{39}{2} - \left[\frac{(y-1)^3}{3} + 4y\right]_{-1}^{2}$ $= \frac{39}{2} - \left[\left(\frac{(2-1)^3}{3} + 4(2)\right) - \left(\frac{(-1-1)^3}{3} + 4(-1)\right)\right]$ $= 19\frac{1}{2} - 15$ $= 4\frac{1}{2}$ units^2	M[2]—1mk for each integration M[1] Substitution A[1]

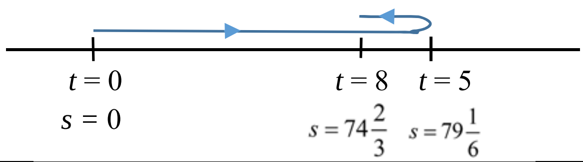
13. A particle moves in a straight line so that its acceleration, a ms^{-2}, is given by $a = 2t - 13$, where t is the time in seconds after passing a fixed point O. The particle first comes to instantaneous rest at $t = 5$ s. Find,

i) the velocity when the particle passes through O. [2]

ii) the total distance travelled by the particle when it next comes to rest. [5]

iii) the minimum velocity of the particle. [2]

Solution

i)	$a = 2t - 13$ $v = \int 2t - 13\ dt$ $= t^2 - 13t + c$ When $t = 5$, $v = 0$. $0 = 5^2 - 13(5) + c$ $c = 40$ Velocity when passes through $O = 40$ ms^{-1}	M1 A1
ii)	$t^2 - 13t + 40 = 0$ $(t-5)(t-8) = 0$ $t = 5$ *or* $t = 8$ $v = t^2 - 13t + 40$ $s = \int \left(t^2 - 13t + 40\right) dt$ $= \frac{t^3}{3} - \frac{13t^2}{2} + 40t + c$ When $t = 0, c = 0$, $s = \frac{t^3}{3} - \frac{13t^2}{2} + 40t$ When $t = 5$, $s = \frac{5^3}{3} - \frac{13(5)^2}{2} + 40(5) = 79\frac{1}{6}$ When $t = 8$, $s = \frac{8^3}{3} - \frac{13(8)^2}{2} + 40(8) = 74\frac{2}{3}$ 	M1 M1 A1 M1 A1

	Total distance $= 79\frac{1}{6} + \left(79\frac{1}{6} - 74\frac{2}{3}\right)$ $= 83\frac{2}{3}m$	
iii)	$a = 2t - 13$ $2t - 13 = 0$ $t = \frac{13}{2}$ $v = \left(\frac{13}{2}\right)^2 - 13\left(\frac{13}{2}\right) + 40$ $= -2\frac{1}{4}ms^{-1}$	M1 A1

NAME: ______________________________ (   )   **CLASS: 4 (   )**

ANGLICAN HIGH SCHOOL
SECONDARY FOUR
PRELIMINARY EXAMINATIONS 2018
ADDITIONAL MATHEMATICS PAPER 2 [4047/02]

S4

14 September 2018   **2 hours 30 minutes**

Additional Materials: 8 Writing Papers and 1 Graph Paper

READ THESE INSTRUCTIONS FIRST
Write your name, index number and class on all the work you hand in.
Write in dark blue or black pen on both sides of the writing paper provided.
You may use a soft pencil for any diagrams or graphs.
Do not use staples, paper clips, highlighters and glue or correction fluid.

Answer **all** the questions.
Write your answers on the separate writing paper provided.
Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.
The use of an approved scientific calculator is expected, where appropriate.
You are reminded of the need for clear presentation in your answers.

At the end of the examination, fasten all your work securely together and attach the question paper on top of the scripts.
The number of marks is given in brackets [ ] at the end of each question or part question.
The total number of marks for this paper is 100.

For Examiners' Use

Question	**Marks**	**Question**	**Marks**	**Table of Penalties**	
1		**7**			
2		**8**		**Units**	
3		**9**		**Presentation**	
4		**10**		**Accuracy**	
5		**11**		**Total:**	
6					
Parent's Name & Signature: **Date:**				**100**	

This paper consists of **6** printed pages.

[Turn over

Mathematical Formulae

1. ALGEBRA

Quadratic Equation

For the equation $ax^2 + bx + c = 0$,

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Binomial expansion

$$(a+b)^n = a^n + \binom{n}{1}a^{n-1}b + \binom{n}{2}a^{n-2}b^2 + \ldots + \binom{n}{r}a^{n-r}b^r + \ldots + b^n,$$

where n is a positive integer and $\binom{n}{r} = \frac{n!}{r!(n-r)!} = \frac{n(n-1)\ldots(n-r+1)}{r!}$

2. TRIGONOMETRY

Identities

$$\sin^2 A + \cos^2 A = 1$$

$$\sec^2 A = 1 + \tan^2 A$$

$$\operatorname{cosec}^2 A = 1 + \cot^2 A$$

$$\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$$

$$\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$$

$$\tan(A \pm B) = \frac{\tan A \pm \tan B}{1 \mp \tan A \tan B}$$

$$\sin 2A = 2 \sin A \cos A$$

$$\cos 2A = \cos^2 A - \sin^2 A = 2\cos^2 A - 1 = 1 - 2\sin^2 A$$

$$\tan 2A = \frac{2 \tan A}{1 - \tan^2 A}$$

Formulae for ΔABC

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$a^2 = b^2 + c^2 - 2bc \cos A$$

$$\text{Area of } \Delta = \frac{1}{2}ab \sin C$$

Answer **all** questions.

1 **(a)** Given that the curve $y = x^2 + (3k-1)x + (2k+10)$ has a minimum value greater than 0, calculate the range of values of k. [4]

(b) Find the range of values of x for which $(x+4)(x-1)-6 \geq 0$. [2]

(c) The equation $2x^2 - x + 18 = 0$ has roots α and β. Find the quadratic equation whose roots are $\left(\frac{\alpha}{\beta}\right)^{\frac{1}{2}}$ and $\left(\frac{\beta}{\alpha}\right)^{\frac{1}{2}}$. [4]

2 **(a)** Simplify $\dfrac{25^p \times 10^{1+p}}{2^{p-1} \times 5^{2+3p}}$. [3]

(b) Given that n is a positive integer, show that $8^n + 8^{n+2} + 8^{n+4}$ is always divisible by 24. [2]

(c) Solve $2 - 2^a = 2^{a+3} - 4^{a+1}$. [4]

3 **(a)** Express $\dfrac{2x^3 - 3x - 1}{(x+3)(x-1)}$ as partial fractions. [5]

(b) The polynomial $P(x) = 2x^3 - hx^2 - 48x - 20$ leaves a remainder of 11 when divided by $x + 1$.

(i) Show that $h = 15$. [2]

(ii) Factorise $P(x)$ completely. [3]

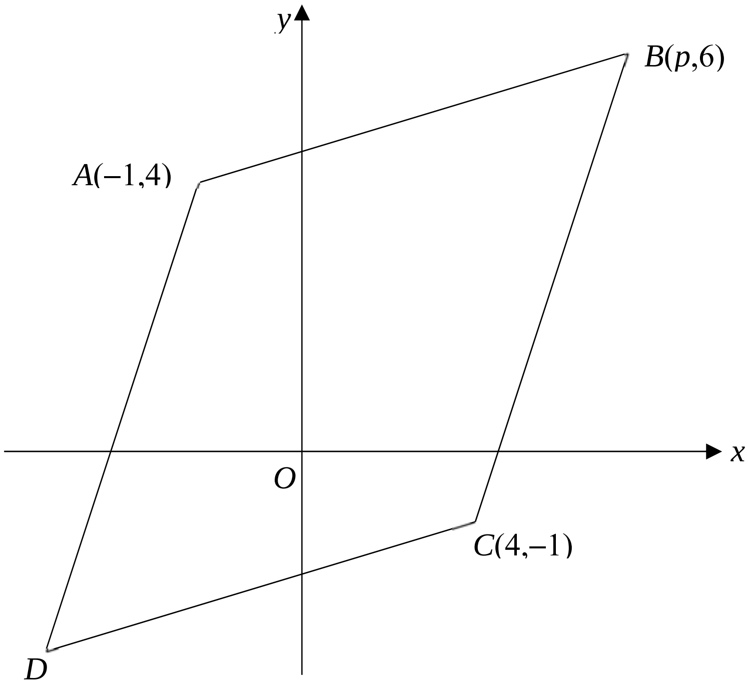
4 **(a)** **(i)** Write down, and simplify, the first 3 terms in the expansion of $(2-x)^8$ in ascending powers of x. [1]

(ii) Hence, determine the coefficient of y^2 in the expansion of $256(1-y)^8$. [3]

(b) **(i)** Write down the general term in the expansion of $\left(3x - \dfrac{1}{2x^2}\right)^{11}$. [1]

(ii) Hence, explain why the term in x^3 does not exist. [2]

5 Solutions to this question by accurate drawing will not be accepted.



The diagram shows a parallelogram with vertices $A(-1, 4)$, $B(p, 6)$, $C(4, -1)$ and D.

(i) Given that AC is perpendicular to BD, show that $p = 6$. [4]

(ii) Find the coordinates of D. [2]

(iii) Find the area of the parallelogram $ABCD$. [2]

6 A container in the shape of a pyramid has a volume of V cm^3, given by

$$V = \frac{1}{3}x(ax^2 + b),$$

where x is the height of the container in cm, and $(ax^2 + b)$ is the area of the rectangular base, of which a and b are unknown constants.
Corresponding values of x and V are shown in the table below.

x (cm)	5	10	15	20
V (cm^3)	150	600	1650	3600

(i) Using suitable variables, draw on graph paper, a straight line graph. [4]

(ii) Use your graph to estimate the value of a and of b. [4]

(iii) Explain how another straight line drawn on your graph can lead to an estimate of the value of x when the base area of the pyramid is three times the square of its height. Draw this line and find an estimate for the value of x. [3]

7 A circle has a diameter AB. The point A has coordinates (1, – 6) and the equation of the tangent to the circle at B is $3x + 4y = k$.

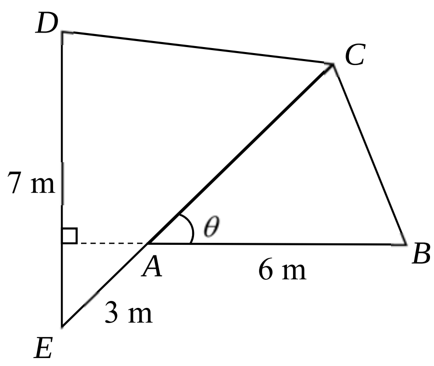
(i) Show that the equation of the normal to the circle at the point A is $4x - 3y = 22$. [3]

Given also that the line $x = -1$ touches the circle at the point (–1, –2).

(ii) Find the coordinates of the centre and the radius of the circle. [4]

(iii) Find the value of k. [3]

8 The diagram shows a lawn made up of two triangles, ABC and CDE. Triangle ABC is an isosceles triangle where $AB = AC = 6$ m. $DE = 7$ m, $AE = 3$ m, and BA produced is perpendicular to DE. Angle BAC is θ and the area of the lawn is S m^2.



(i) Show that $S = 18\sin\theta + 31.5\cos\theta$. [3]

(ii) Hence, express S as a single trigonometric term. [4]

(iii) Given that θ can vary, find the maximum area of the lawn and the corresponding value of θ . [2]

9 A curve has the equation $y = (1 - x)\sqrt{1 + 2x}$.

(i) Find $\dfrac{dy}{dx}$ in its simplest form. [3]

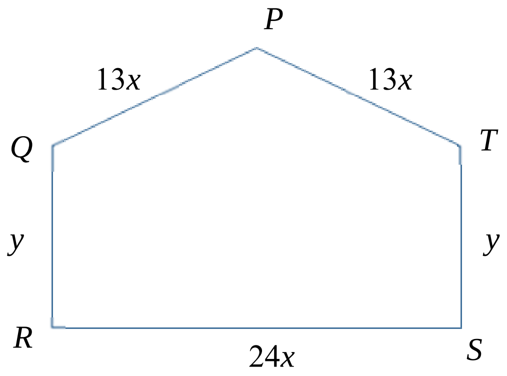
Hence,

(ii) determine the interval where y is increasing, [3]

(iii) find the rate of change of x when $x = 4$, given that y is decreasing at a constant rate of 2 units per second, [2]

(iv) evaluate $\int_1^4 \dfrac{x}{\sqrt{1 + 2x}}\,dx$. [2]

10 A piece of wire of length 180 cm is bent into the shape *PQRST* shown in the diagram.



Show that the area, A cm^2, enclosed by the wire is given by

$$A = 2160 - 540x^2.$$

Find the value of x and of y for which A is a maximum. [8]

11 **(a)** Find the following indefinite integrals.

(i) $\int \frac{e^{2x}}{2} dx$

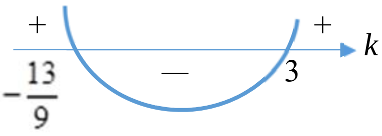
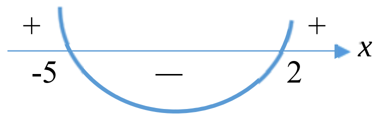
(ii) $\int \left(\frac{4}{x} + \frac{1}{x^2} \right) dx$ [3]

(b) Evaluate $\int_{-\frac{\pi}{6}}^{\frac{\pi}{6}} \frac{1}{2\operatorname{cosec}^2 x} dx$, leaving your answer in terms of π. [5]

END OF PAPER.

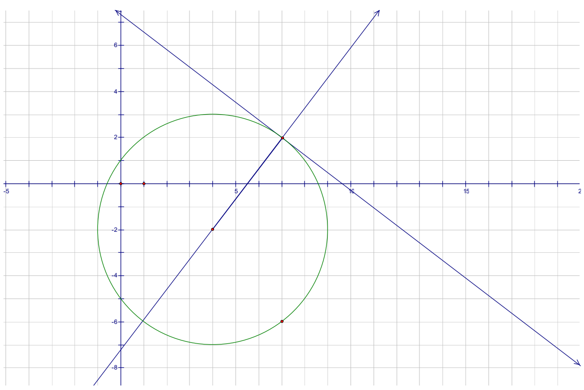
ANSWER KEY

1	(a) $-\frac{13}{9} < k < 3$ (b) $x \le -5$ or $x \ge 2$ (c) $x^2 - \frac{1}{6}x + 1 = 0$ or $6x^2 - x + 6 = 0$	2	(a) $\frac{4}{5}$ (b) $24 \times 1387 \times 8^{n-1}$ Since $n \ge 1$, $8^{n-1} \ge 1$, hence $8^n + 8^{n+2} + 8^{n+4}$ is divisible by 24. (c) $a = -2$ or $a = 1$
3	(a) $2x - 4 + \frac{23}{2(x+3)} - \frac{1}{2(x-1)}$ (b)(ii) $(x+2)(2x+1)(x-10)$		
4	(a)(i) $256 - 1024x + 1792x^2 + \ldots$ (ii) coefficient of y^2 = 7168 (b)(i) $T_{r+1} = \binom{11}{r}(3x)^{11-r}\left(-\frac{1}{2x^2}\right)^r$ (ii) $r = \frac{8}{3}$. As r is a not a whole number, the term in x^3 does not exist.	5	(ii) $(-3, -3)$ (iii) 45 units^2
		7	(ii) centre is $(4, -2)$ radius = 5 units (iii) $k = 29$
		8	(ii) $36.3\sin(\theta + 60.3°)$ (iii) Max $S \approx 36.3$ m^2 $\theta \approx 29.7°$
9	(i) $\frac{dy}{dx} = -\frac{3x}{\sqrt{1+2x}}$ (ii) y in increasing when $-0.5 < x < 0$. (iii) $\frac{dx}{dt} = \frac{1}{2}$ units/sec (iv) 3	10	$x = 2$ cm and $y = 40$ cm when A is a maximum.
11	(a) (i) $\int \frac{e^{2x}}{2} dx = \frac{e^{2x}}{4} + c$ (ii) $\int \left(\frac{4}{x} + \frac{1}{x^2}\right) dx = 4\ln x - \frac{1}{x} + c$	11	(b) $\frac{\pi}{12} - \frac{\sqrt{3}}{8}$ or $\frac{2\pi - 3\sqrt{3}}{24}$

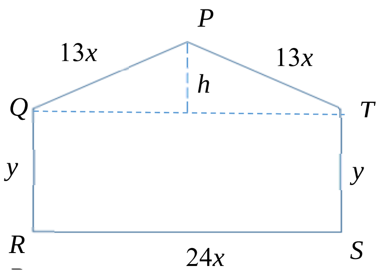
	Solutions
1(a)	$x^2+(3k-1)x+(2k+10)>0$ $b^2-4ac<0$ $(3k-1)^2-4(1)(2k+10)<0$ $9k^2-6k+1-8k-40<0$ $9k^2-14k-39<0$ $(9k+13)(k-3)<0$  $-\frac{13}{9}<k<3$
(b)	$(x+4)(x-1)-6\geq 0$ $x^2+3x-4-6\geq 0$ $x^2+3x-10\geq 0$ $(x+5)(x-2)\geq 0$  $x\leq -5$ or $x\geq 2$
(c)	$2x^2-x+18=0$ $\alpha+\beta=\frac{1}{2}$ $\alpha\beta=9$ $\left(\frac{\alpha}{\beta}\right)^{\frac{1}{2}}+\left(\frac{\beta}{\alpha}\right)^{\frac{1}{2}}=\frac{\alpha+\beta}{\alpha^{\frac{1}{2}}\beta^{\frac{1}{2}}}$ $=\frac{(\alpha+\beta)}{(\alpha\beta)^{\frac{1}{2}}}$ $=\frac{\frac{1}{2}}{9^{\frac{1}{2}}}$ $=\frac{1}{6}$ $\left(\frac{\alpha}{\beta}\right)^{\frac{1}{2}}\times\left(\frac{\beta}{\alpha}\right)^{\frac{1}{2}}=\left(\frac{\alpha\beta}{\beta\alpha}\right)^{\frac{1}{2}}$ $=1$ Required equation is $x^2-\frac{1}{6}x+1=0$ or $6x^2-x+6=0$

2(a)	$\frac{25^p \times 10^{1+p}}{2^{p-1}\times 5^{2+3p}} = \frac{5^{2p}\times(2\times 5)^{1+p}}{2^{p-1}\times 5^{2+3p}}$ $= \frac{5^{2p}\times 2^{1+p}\times 5^{1+p}}{2^{p-1}\times 5^{2+3p}}$ $= 2^{1+p-(p-1)}\times 5^{2p+1+p-(2+3p)}$ $= 2^2 \times 5^{-1}$ $= \frac{4}{5}$
(b)	$8^n + 8^{n+2} + 8^{n+4} = 8^n + 8^n \times 8^2 + 8^n \times 8^4$ $= 8^n(1+64+4096)$ $= 8^n(4161)$ $= 8^1 \times 8^{n-1} \times 3 \times 1387$ $= 24 \times 1387 \times 8^{n-1}$ Since $n \ge 1, 8^{n-1} \ge 1$ and $24\times 1387 \times 8^{n-1}$ is divisible by 24. o.e.
(c)	$2-2^a = 2^{a+3} - 4^{a+1}$ $2-2^a = 2^3(2^a) - 2^{2(a+1)}$ $2-2^a = 8(2^a) - 2^2(2^{2a})$ $2-2^a = 8(2^a) - 4(2^a)^2$ Let u be 2^a. $2-u = 8u-4u^2$ $4u^2-9u+2=0$ $(4u-1)(u-2)=0$ $u=\frac{1}{4}$ or $u=2$ $2^a = 2^{-2}$ or $2^a = 2$ $a=-2$ or $a=1$
3(a)	$\frac{2x^3-3x-1}{(x+3)(x-1)} = \frac{2x^3-3x-1}{x^2+2x-3}$ Long division of $2x^3-0x^2-3x-1$ by x^2+2x-3: quotient $2x-4$ $-(2x^3+4x^2-6x)$ $-4x^2+3x-1$ $-(-4x^2-8x+12)$ $11x-13$ $\frac{2x^3-3x-1}{(x+3)(x-1)} = 2x-4+\frac{11x-13}{(x+3)(x-1)}$

	$\frac{11x-13}{(x+3)(x-1)}=\frac{P}{x+3}+\frac{Q}{x-1}$ $=\frac{P(x-1)+Q(x+3)}{(x+3)(x-1)}$ $\Rightarrow\ 11x-13=Px-P+Qx+3Q$ $=(P+Q)x+(-P+3Q)$ $P+Q=11\quad ...(1)$ $-P+3Q=-13\quad ...(2)$ (1)+(2): $4Q=-2\ \Rightarrow Q=-\frac{1}{2}$ $\therefore\ P-\frac{1}{2}=11\ \Rightarrow\ P=\frac{23}{2}$ $\frac{2x^3-3x-1}{(x+3)(x-1)}=2x-4+\frac{23}{2(x+3)}-\frac{1}{2(x-1)}$
(b) (i)	$P(x)=2x^3-hx^2-48x-20$ $P(-1)=11$ $2(-1)^3-h(-1)^2-48(-1)-20=11$ $-2-h+48-20=11$ $h=15$ (shown)
(ii)	$P(x)=2x^3-15x^2-48x-20$ By trial and error, $x+2$ is a factor. $2x^3-15x^2-48x-20$ $=(x+2)(ax^2+bx+c)$ $=ax^3+bx^2+cx+2ax^2+2bx+2c$ $=ax^3+(b+2a)x^2+(c+2b)x+2c$ By comparing coefficients of x^3: $a=2$ x^2: $b+2(2)=-15$ $b=-19$ constant: $2c=-20$ $c=-10$ $\therefore P(x)=(x+2)(2x^2-19x-10)$ $=(x+2)(2x+1)(x-10)$
4(a) (i)	$(2-x)^8=2^8+\binom{8}{1}2^7(-x)+\binom{8}{2}2^6(-x)^2+\dots$ $=256-1024x+1792x^2+\dots$
(ii)	$256(1-y)^8=2^8(1-y)^8$ $=[2(1-y)]^8$ $=(2-2y)^8$ Taking $x=2y$, $(2-2y)^8=256-1024(2y)+1792(2y)^2+\dots$ Hence, coefficient of $y^2=1792\times 2^2=7168$.

(b) (i)	$T_{r+1} = \binom{11}{r}(3x)^{11-r}\left(-\frac{1}{2x^2}\right)^r$
(ii)	For term in x^3, $11 - r - 2r = 3$ $3r = 8 \Rightarrow r = \frac{8}{3}$ As r is a not a whole number, the term in x^3 does not exist. o.e.
5 (i)	Grad $AC = \frac{4-(-1)}{-1-4} = -1$ Mid-point of $AC = \left(\frac{-1+4}{2}, \frac{4-1}{2}\right) = \left(\frac{3}{2}, \frac{3}{2}\right)$ As BD and AC share the same mid-point (property of parallelogram), gradient of $BD = \frac{6-\frac{3}{2}}{p-\frac{3}{2}} = \frac{9}{2p-3}$ $$\left(\frac{9}{2p-3}\right)(-1) = -1$$ $2p - 3 = 9 \Rightarrow 2p = 12 \Rightarrow p = 6$ (shown)
(ii)	Let D be (a, b). $$\left(\frac{a+6}{2}, \frac{b+6}{2}\right) = \left(\frac{3}{2}, \frac{3}{2}\right)$$ Comparing coordinates, $\frac{a+6}{2} = \frac{3}{2}$ $a + 6 = 3 \Rightarrow a = -3$. Similarly, $b = -3$. Therefore, coordinates of D are $(-3, -3)$.
(iii)	Area of the parallelogram $ABCD$ $= \frac{1}{2}\begin{vmatrix} -3 & 4 & 6 & -1 & -3 \\ -3 & -1 & 6 & 4 & -3 \end{vmatrix}$ $= \frac{1}{2}\{[(-3)(-1) + 4(6) + 6(4) + (-1)(-3)]$ $- [4(-3) + 6(-1) + (-1)(6) + (-3)(4)]\}$ $= 45$ units^2
7 (i)	 The normal to the circle at point A will pass through the centre of the circle, and point B also, and is perpendicular to the tangent to the circle at B. $3x + 4y = k \Rightarrow y = -\frac{3}{4}x + \frac{k}{4}$ Grad of tangent at $B = -\frac{3}{4}$ Grad of normal at $A = \frac{4}{3}$ Equation of normal at A: $$y - (-6) = \frac{4}{3}(x - 1)$$ $$y = \frac{4}{3}x - \frac{22}{3}$$ $\Rightarrow 4x - 3y = 22$. (shown)
(ii)	Since the line $x = -1$ touches the circle at the point $(-1, -2)$, so the equation of the normal at $(-1, -2)$ is $y = -2$. Solving the equations $4x - 3y = 22$ and

	$y=-2$, $4x-3(-2)=22 \Rightarrow 4x=16 \Rightarrow x=4$. Thus the centre is (4, –2). Radius $=\sqrt{(4-1)^2+[(-2)-(-6)]^2}$ $=\sqrt{9+16}$ $= 5$ units
(iii)	Let the coordinates of B be (p, q). $\left(\frac{p+1}{2}, \frac{q-6}{2}\right)=(4,-2)$ $p = 2(4)-1= 7$ and $q = 2(-2) + 6 = 2$ Therefore, B is (7, 2). Sub. (7, 2) into $3x + 4y = k$, $k = 3(7) + 4(2) = 29$
8 (i)	Area $\Delta ABC=\frac{1}{2}(6)^2\sin\theta$ $=18\sin\theta$ Area $\Delta CDE=\frac{1}{2}(7\times 9)\sin(90°-\theta)$ $=31.5\cos\theta$ $S=18\sin\theta+31.5\cos\theta$ (shown)
(ii)	$S=18\sin\theta+31.5\cos\theta$ $R=\sqrt{18^2+31.5^2}$ $=36.28016$ $\tan\alpha=\frac{31.5}{18}$ $\alpha=\tan^{-1}\left(\frac{31.5}{18}\right)$ $=60.2551187°$ $S=36.28016\sin(\theta+60.2551187°)$ $\approx 36.3\sin(\theta+60.3°)$
(iii)	$S=36.28016\sin(\theta+60.2551187°)$ Max $S \approx 36.3$ m^2 $\sin(\theta+60.2551187°)=1$ $0°<\theta<90°$ $60.2551187°<\theta+60.2551187°<150.2551187°$ $\theta+60.2551187°=90°$ $\theta=29.7448813°$ $\approx 29.7°$

9 (i)	$y=(1-x)\sqrt{1+2x}$ $\frac{dy}{dx}=(1-x)\left(\frac{1}{2}\right)(1+2x)^{-\frac{1}{2}}(2)+(1+2x)^{\frac{1}{2}}(-1)$ $=(1+2x)^{-\frac{1}{2}}(1-x-1-2x)$ $=-\frac{3x}{\sqrt{1+2x}}$
(ii)	For $\frac{dy}{dx}>0$, $-\frac{3x}{\sqrt{1+2x}}>0$ $\Rightarrow$ $1+2x>0$ and $-3x>0$ $x>-0.5$ $x<0$ $\therefore$ y in increasing when $-0.5<x<0$.
(iii)	$\frac{dy}{dt}=\frac{dy}{dx}\times\frac{dx}{dt}$ When $x=4$, $\frac{dy}{dt}=-2$, $-2=-\frac{3(4)}{\sqrt{1+2(4)}}\times\frac{dx}{dt}$ $\frac{dx}{dt}=\frac{1}{2}$ units/sec
(iv)	$\int_1^4\frac{x}{\sqrt{1+2x}}dx$ $=\left[-\frac{1}{3}(1-x)\sqrt{1+2x}\right]_1^4$ $=\left(-\frac{1}{3}(1-(4))\sqrt{1+2(4)}\right)$ $-\left(-\frac{1}{3}(1-1)\sqrt{1+2(1)}\right)$ $=3$
10	$13x+13x+y+24x+y=180$ $50x+2y=180$ $y=90-25x$ 

	Let h cm be the perpendicular distance from P to QT. $h^2 = (13x)^2 - (\frac{24x}{2})^2$ $= 25x^2$ $h = 5x$ Area $= y(24x) + \frac{1}{2}(24x)(5x)$ $A = (90 - 25x)(24x) + 60x^2$ $= 2160x - 600x^2 + 60x^2$ $= 2160x - 540x^2$ (shown) $\frac{dA}{dx} = 2160 - 1080x$ When $\frac{dA}{dx} = 0$, $2160 - 1080x = 0$ $x = 2160 \div 1080$ $= 2$ Sub $x = 2$, into $y = 90 - 25x$ $y = 90 - 25(2)$ $= 40$ $\frac{d^2A}{dx^2} = -1080$, $\therefore$ A is a maximum. $x = 2$ cm and $y = 40$ cm when A is a maximum.
11(a)(i)	$\int \frac{e^{2x}}{2} dx = \frac{e^{2x}}{4} + c$
(ii)	$\int \left(\frac{4}{x} + \frac{1}{x^2} \right) dx = 4 \ln x - \frac{1}{x} + c$

(b)	$\int_{-\frac{\pi}{6}}^{\frac{\pi}{6}} \frac{1}{2\operatorname{cosec}^2 x} dx$ $= \int_{-\frac{\pi}{6}}^{\frac{\pi}{6}} \frac{\sin^2 x}{2} dx$ $= \int_{-\frac{\pi}{6}}^{\frac{\pi}{6}} \frac{1}{2} \times \frac{1}{2}(1-\cos 2x)\ dx$ $= \int_{-\frac{\pi}{6}}^{\frac{\pi}{6}} \frac{1}{4}(1-\cos 2x)\ dx$ $= \left[\frac{1}{4}x - \frac{1}{8}\sin 2x\right]_{-\frac{\pi}{6}}^{\frac{\pi}{6}}$ $= \left(\frac{1}{4}\left(\frac{\pi}{6}\right) - \frac{1}{8}\sin 2\left(\frac{\pi}{6}\right)\right) -$ $\left(\frac{1}{4}\left(-\frac{\pi}{6}\right) - \frac{1}{8}\sin 2\left(-\frac{\pi}{6}\right)\right)$ $= \frac{\pi}{24} - \frac{\sqrt{3}}{16} + \frac{\pi}{24} - \frac{\sqrt{3}}{16}$ $= \frac{\pi}{12} - \frac{\sqrt{3}}{8}$ or $\frac{2\pi - 3\sqrt{3}}{24}$