BUKIT PANJANG GOVERNMENT HIGH SCHOOL

Preliminary Examination 2018

SECONDARY 4 EXPRESS/ 5 NORMAL

ADDITIONAL MATHEMATICS

4047/1

Paper 1

Date: 3 August, 2018
Duration: 2 h
Time: **1030 – 1230**

Additional Materials: *Answer Paper*

READ THESE INSTRUCTIONS FIRST

Write your class, register number and name on all the work you hand in.
Write in dark blue or black pen on both sides of the paper.
You may use a soft pencil for any diagrams or graphs.
Do not use staples, paper clips, highlighters, glue or correction fluid.

Answer **all** the questions.
Write your answers on the separate Answer Paper provided.
Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.
The use of an approved scientific calculator is expected, where appropriate.
You are reminded of the need for clear presentation in your answers.

At the end of the examination, fasten all your work securely together.
The number of marks is given in brackets [ ] at the end of each question or part question.

The total number of marks for this paper is 80.

Setter: Mr Choo Kong Lum **[Turn over]**

1. ALGEBRA

Quadratic Equation

For the equation $ax^2 + bx + c = 0$,

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Binomial expansion

$$(a+b)^n = a^n + \binom{n}{1}a^{n-1}b + \binom{n}{2}a^{n-2}b^2 + \cdots + \binom{n}{r}a^{n-r}b^r + \cdots + b^n,$$

where n is a positive integer and $\binom{n}{r} = \frac{n!}{r!(n-r)!} = \frac{n(n-1)\ldots(n-r+1)}{r!}$

2. TRIGONOMETRY

Identities

$$\sin^2 A + \cos^2 A = 1$$

$$\sec^2 A = 1 + \tan^2 A$$

$$\operatorname{cosec}^2 A = 1 + \cot^2 A$$

$$\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$$

$$\cos(A \pm B) = cosAcosB \mp \sin AsinB$$

$$\tan(A \pm B) = \frac{tanA \pm B}{1 \mp tanAtanB}$$

$$\sin 2A = 2 \sin A \cos A$$

$$\cos 2A = \cos^2 A - \sin^2 A = 2\cos^2 A - 1 = 1 - 2\sin^2 A$$

$$\tan 2A = \frac{2\tan A}{1 - \tan^2 A}$$

Formulae for ΔABC

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$a^2 = b^2 + c^2 - 2bc \cos A$$

$$\Delta = \tfrac{1}{2} ab \sin C$$

Answer* ALL *the questions.

1(a) Given that $(\sqrt{3}+1)x = \sqrt{3}-1$, find the value of $x+\frac{1}{x}$ without using a calculator. [4]

1(b) Given that $2\sqrt{2}-3 = \frac{\sqrt{h-k\sqrt{2}}}{1+\sqrt{2}}$, find the values of *h* and *k*. [3]

2(a) Show that for all real values of *p* and of *q*, $y = -(1+p^2)x^2 + 2pqx - (2q^2+1)$ is always negative for all real values of *x*. [4]

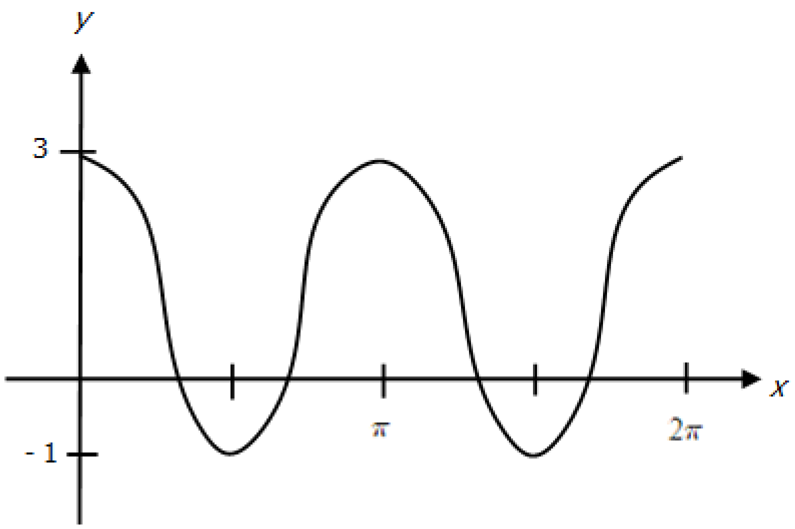
2(b) Find the range of values of *m* for which $\frac{-4}{m^2+3m+2} < 0$ [2]

3(a) (i) For the function $y = \sin x$, where $-1 \le y \le 1$, state the principal values of *x*, in radians. [1]

(ii) For the function $y = \cos x$, where $-1 \le y \le 1$, state the principal values of *x*, in radians. [1]

(iii) For the function $y = \tan x$, state the principal values of *x*, in radians. [1]

3(b) The diagram shows part of the graph for the function $y = a\cos bx + c$.



(i) Find the values of *a*, *b* and *c*. [3]

(ii) Copy the diagram and draw the line $y = \frac{x}{\pi} - 1$ on the same diagram. Hence state the number of solutions when $a\cos bx + c = \frac{x}{\pi} - 1$. [2]

4. (i) Sketch the graph of $y = x^{\frac{2}{3}}$ for $x \geq 0$. [1]

(ii) Find the equation of the line that must be inserted in the graph above in order to solve the equation $3x^{\frac{2}{3}} + 9x = 6$. [2]

5. Express $\frac{4x^5+2x^4+3x^3-x^2-x+1}{x^3+x}$ in partial fractions. [6]

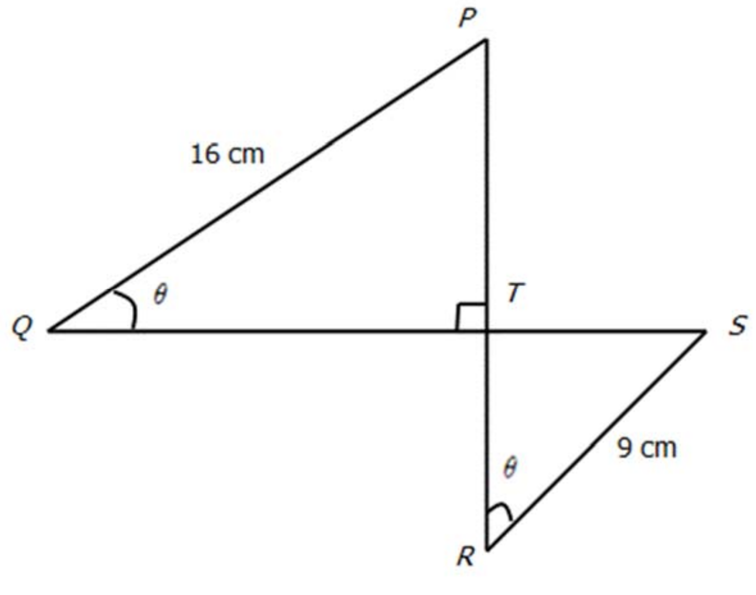
6. (i) Sketch the graphs of $y = |x-2| + 1$ and $y = x^2 + 3$ on the same diagram. For each graph, indicate the coordinates of the minimum point on the diagram. [4]

(ii) Find the coordinates of the point of intersection. [4]

7(a) Given that $y = \ln\sqrt{\frac{3x+1}{3x-1}}$, find an expression for $\frac{dy}{dx}$ and simplify your answer as a single fraction. [3]

7(b) Given that $y = 2e^{x^2+3}$, find the coordinates of the stationary point, leaving your answer in exact form. Determine the nature of the stationary point. [5]

8. The diagram shows two lines *PR* and *QS* which are perpendicular to each other. *RS* = 9 cm, *PQ* = 16 cm and $\angle PQT = \angle SRT = \theta$ radians.



(i) Show that $QS = 16cos\theta + 9sin\theta$. [1]

(ii) Express *QS* in the form of $Rsin(\theta + \alpha)$. [3]

(iii) Find the value of θ for which *QS* = 12 cm. [3]

(iv) Show that the area of the quadrilateral *PQRS* is $\frac{288+337sin2\theta}{4}$ cm^2 [4]

9. (i) Differentiate $(x-5)\sqrt{2x-1}$ with respect to x and simplify your answer as a single fraction. [2]

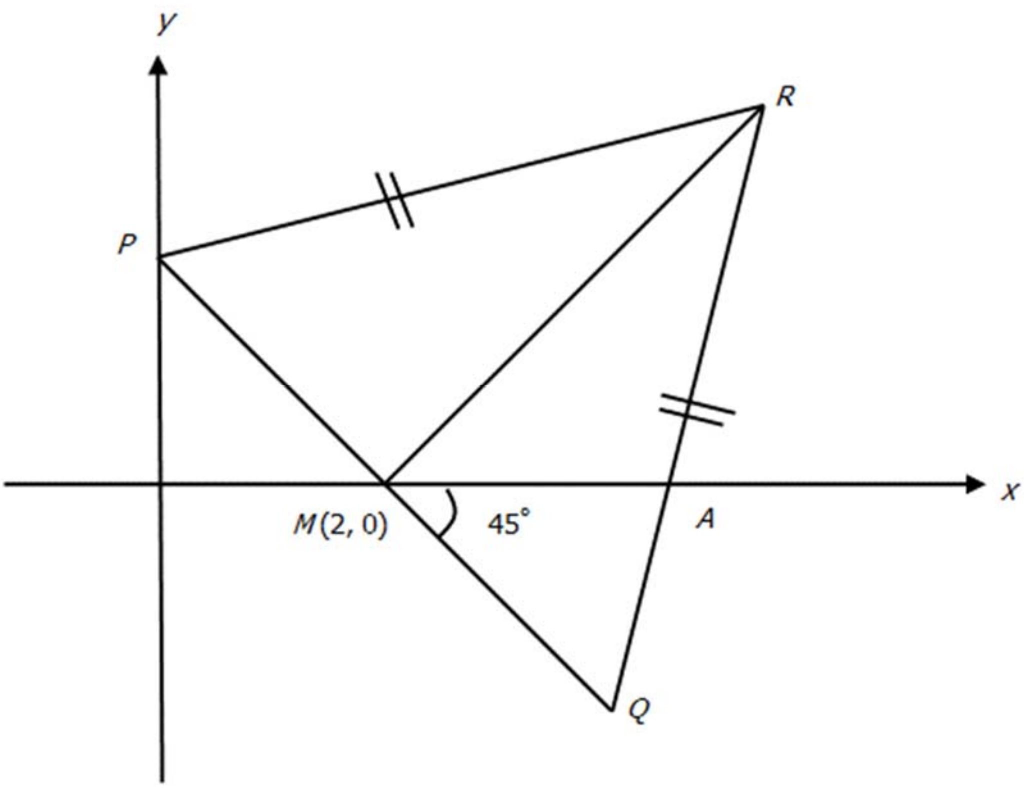
(ii) Hence evaluate $\int_1^2 \frac{3x-9}{\sqrt{2x-1}}\,dx$, leaving your answer in exact form. [4]

10. (i) Given that $\frac{dy}{dx}=\frac{5}{1+\cos 2x}$. Find the equation of the curve if the curve passes through the y – axis at $y=1$. [4]

(ii) Find the equation of the normal to the curve at $x=\frac{\pi}{4}$. [3]

11. **Solutions to this question by accurate drawing will not be accepted.**

The following diagram shows an isosceles triangle PQR, where $PR = QR$. It is given that M (2, 0) is the midpoint of PQ. The line QR intersects the x - axis at point A such that $\angle AMQ = 45°$.



(i) Show that the gradient of the line MR is 1. [1]

(ii) Find the equation of the line PQ. [2]

(iii) Find the coordinates of Q. [2]

(iv) Given that the area of ΔPQR is 20 units², find the coordinates of R. [5]

END OF PAPER

ANSWERS (SEC 4 EXP / 5 NA AM PAPER 1 – PRELIM 2018)

1(a)	4
1(b)	$h = 3,\ k = 2$
2(b)	$m < -2$ or $m > -1$
3(a)(i)	$-\frac{\pi}{2} \le x \le \frac{\pi}{2}$
3(a)(ii)	$0 \le x \le \pi$
3(a)(iii)	$-\frac{\pi}{2} < x < \frac{\pi}{2}$
3(b)(i)	$a = 2,\ \ b = 2,\ \ c = 1$
3(b)(ii)	4
4(ii)	$y = -3x + 2$
5.	$4x^2 + 2x - 1 + \frac{1}{x} - \frac{4x}{x^2+1}$
6(ii)	(2, 1) and (0, 3)
6(iii)	(0, 3) and (-1, 4)
7(a)	$\frac{-3}{(3x+1)(3x-1)}$ or $\frac{3}{(1+3x)(1-3x)}$
7(b)	$(0,\ 2e^3)$ minimum point
8(ii)	$\sqrt{337}\sin(\theta + 1.06)$ or $18.4\sin(\theta + 1.06)$
8(iii)	1.37 radians
9(i)	$\frac{3x-6}{\sqrt{2x-1}}$
9(ii)	$7 - 6\sqrt{3}$
10(i)	$y = \frac{5}{2}\tan x + 1$
10(ii)	$y = -\frac{1}{5}x + \frac{\pi}{20} + \frac{7}{2}$
11(ii)	$y = -x + 2$
11(iii)	(4, - 2)
11(iv)	(7, 5)

Name : ____( ) Class : __________

BUKIT PANJANG GOVERNMENT HIGH SCHOOL

Preliminary Examinations 2018

SECONDARY FOUR EXPRESS/FIVE NORMAL

ADDITIONAL MATHEMATICS

4047/02

Paper 2

Date: 13 August, 2018

Duration: 2 hours 30 min

Time: 07 45 – 10 15

READ THESE INSTRUCTIONS FIRST

Write your name, class and register number on all the work you hand in.
Write in dark blue or black pen on both sides of the paper.
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Setter: Mrs Chiu H W

Mathematical Formulae

1. ALGEBRA

Quadratic Equation

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Binomial Expansion

$$(a+b)^n = a^n + \binom{n}{1} a^{n-1} b + \binom{n}{2} a^{n-2} b^2 + \cdots + \binom{n}{r} a^{n-r} b^r + \cdots + b^n$$

where n is a positive integer and $\binom{n}{r} = \frac{n!}{r!(n-r)!} = \frac{n(n-1)\ldots(n-r+1)}{r!}$

2. TRIGONOMETRY

Identities

$$\sin^2 A + \cos^2 A = 1$$
$$\sec^2 A = 1 + \tan^2 A$$
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$$\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$$

$$\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$$

$$\tan(A \pm B) = \frac{\tan A \pm \tan B}{1 \mp \tan A \tan B}$$

$$\sin 2A = 2 \sin A \cos A$$
$$\cos 2A = \cos^2 A - \sin^2 A = 2\cos^2 A - 1 = 1 - 2\sin^2 A$$
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Formulae for ΔABC

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

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1 Expand $(1 + a\,x)^4(1 - 4x)^3$ in ascending powers of x up to and including the term containing x^2. [4]

Given that the first two terms in the above expansion are $p + qx^2$, where p and q are constants, find the value of p and of q. [3]

2 (i) Given that $u = 4^x$, express $4^x - 3(4^{1-x}) = 11$ as an equation in u. [2]

(ii) Hence find the value(s) of x for which $4^x - 3(4^{1-x}) = 11$. [4]

(iii) Given that $p > 0$, determine the number of real roots in the equation $4^x - 3(4^{1-x}) = p$. **Show your working clearly.** [3]

3 (i) Show that $\frac{1}{\operatorname{cosec} x - 1} - \frac{1}{\operatorname{cosec} x + 1} = 2\tan^2 x$. [3]

(ii) Hence solve $\frac{1}{\operatorname{cosec} x - 1} - \frac{1}{\operatorname{cosec} x + 1} = 4 + \sec x$ for $0° < x < 360°$. [4]

4 A curve has the equation $y = \frac{2x-7}{x-1} - 20x$.

(i) Obtain an expression for $\frac{dy}{dx}$. [3]

(ii) Determine the values of x for which y is a decreasing function. [3]

The variables are such that, when $x = 2$, y is decreasing at the rate of 1.5 units per second.

(iii) Find the rate of change of x when $x = 2$. [2]

It is given further that the variable z is such that $z = \frac{2}{y}$.

(iv) Find the rate of change of z when $x = 2$. [3]

5 It is given that $f(x) = (kx + 1)(x^2 - 3x + k)$.

(a) (i) Find the value(s) of k if $3 - x$ is a factor of $f(x)$. [2]

(ii) For the values(s) of k found in (i), write down an expression for $f(x)$ with $(3 - x)$ as a factor. [2]

(b) Find the smallest integer value of k such that there is only one real solution for $(kx + 1)(x^2 - 3x + k) = 0$. [3]

6 The table below shows values of the variables x and y which are related by the equation $ay = x(1 - bx)$ where a and b are constants. One of the values of y is believed to be inaccurate.

x	2	3.5	4.5	6	7
y	5.0	9.1	14.0	21.0	26.3

(i) Plot $\frac{y}{x}$ against x and draw a straight line graph. [3]

(ii) Determine which value of y is inaccurate and estimate its correct value. [2]

(iii) Estimate the value of a and b. [4]

An alternative method for obtaining straight line graph for the equation $ay = x(1 - bx)$ is to plot x on the vertical axis and $\frac{y}{x}$ on the horizontal axis.

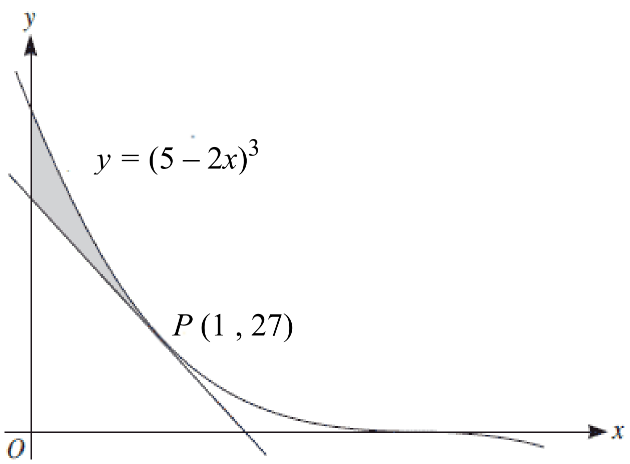
(iv) Without drawing a second graph, use your values of a and b to estimate the gradient and intercept on the vertical axis of the graph of x plotted against $\frac{y}{x}$. [3]

7 The roots of the quadratic equation $x^2 - 4x + 2 = 0$ are α and β.

(i) Find the exact value of $\alpha - \beta$ if $\alpha < \beta$. [4]

(ii) Form a quadratic equation with roots $\frac{\alpha-1}{\beta}$ and $\frac{\beta-1}{\alpha}$. [5]

8



The diagram shows the curve $y = (5 - 2x)^3$ and the tangent to the curve at the point $P(1, 27)$.

(i) Find the equation of the tangent to the curve at P. [4]

(ii) Find the area of the shaded region. [5]

9 A particle moves in a straight line so that t seconds after leaving a fixed point O, its velocity, v m s^{-1}, is given by $v = 2\left(3 - e^{-t/2}\right)$.

(i) Find the initial velocity of the particle. [1]

(ii) Find the acceleration of the particle when $v = 5$. [3]

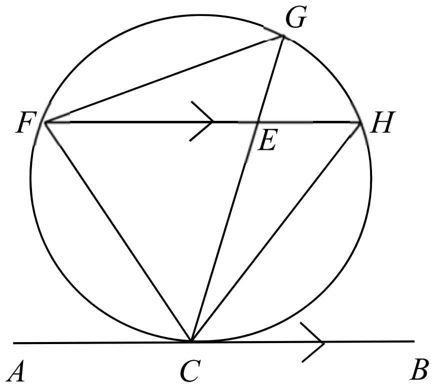
(iii) Calculate the displacement of the particle from O when $t = 10$. [3]

(iv) Does the particle reverses its direction of motion? Justify your answer with working clearly shown. [2]

10 The diagram shows a point C on a circle and line ACB is a tangent to the circle. Points F, G and H lie on the circle such that FH is parallel to AB. The lines GC and FH intersect at E.

(i) Prove that triangles ECF and FCG are similar.
Hence show that $(EC)(CG) = (CF)^2$. [4]

(ii) By using similar triangles, show that $(FE)(EH) = CF^2 - EC^2$. [5]



11 The equation of a circle, C_1, with centre A, is given by $x^2 + y^2 + 4x + 6y - 12 = 0$.

(i) Find the coordinates of A and the radius of C_1. [2]

Given that the circle passes through a point P $(-5, -7)$ and a point Q such that PQ is the diameter of the circle

(ii) write down the coordinates of Q. [2]

The tangent to the circle at point Q intersects the x-axis at point R.
A second circle, C_2, centre B, is drawn passing through A, Q and R.

(iii) Find the coordinates of R. [3]

(iv) Determine the coordinates of the centre, B and the radius of C_2. [4]

– END –

BPGH Preliminary Examination 2019 (Sec 4E/5N)
Additional Mathematics Paper 2 (Answers)

1 $(1 + ax)^4(1 - 4x)^3 = 1 + (4a - 12)x + (48 - 48a + 6a^2)\,x^2$
$p = 1 \quad a = 3 \quad q = -42$

2 (i) $u - \frac{12}{u} = 11$ (ii) $x = 1.79$, $4^x = -1$ (no real solution)

(iii) $u - \frac{12}{u} = p$
$u^2 - pu - 12 = 0$
$u = \dfrac{p + \sqrt{p^2 + 48}}{2}$ or $\dfrac{p - \sqrt{p^2 + 48}}{2}$
$4^x = \dfrac{p + \sqrt{p^2 + 48}}{2}$ or $4^x = \dfrac{p - \sqrt{p^2 + 48}}{2}$
Since $\frac{p+\sqrt{p^2+48}}{2} > 0$, $4^x > 0$ and there is real solution for x.
Since $\frac{p-\sqrt{p^2+48}}{2} < 0$, $4^x < 0$ and there is NO real solution for x.

Number of real solutions = 1

3 (i) L.H.S $= \frac{1}{cosec\,x-1} - \frac{1}{cosec x+1}$ (ii) $x = 30°, 131.8°, 228.2°, 300°$

$= \frac{cosec\,x + 1 - (cosec\,x - 1)}{cosec^2 x - 1}$

$= \frac{2}{cosec^2 x - 1}$

$= \frac{2}{cot^2 x - 1}$
$= 2\,tan^2 x$

4 (i) $\frac{dy}{dx} = \frac{5}{(x-1)^2} - 20$ (ii) $x < \frac{1}{2}$ or $x > \frac{3}{2}$

(iii) $\frac{dx}{dt} = 0.1\ units/s$

(iv) $\frac{dz}{dy} = -\frac{2}{y^2}$
When $x = 2$, $y = -43$
$\frac{dz}{dt} = \frac{dz}{dy} \times \frac{dy}{dt} = 1.62 \times 10^{-3}\ units/s$

5 (a) (i) $k = 0$, $k = -\frac{1}{3}$
(ii) When $k = 0$, $f(x) = -x(3 - x)$

When $k = -\frac{1}{3}$, $f(x) = \frac{1}{3}(3 - x)\left(x^2 - 3x - \frac{1}{3}\right)$

(b) $x^2 - 3x + k = 0$
No real solution when $b^2 - 4ac < 0$, $k > 2\frac{1}{4}$. Smallest integer value of k is 3.

6 (ii) Inaccurate value of $y = 9.1$
Correct value of $\frac{y}{x} = 2.9$. When $x = 3.5$, correct value of $y = 2.9 \times 3.5 = 10.15$

(iii) Equation is $\frac{y}{x} = \frac{1}{a} - \frac{b}{a}x$
From graph, $\frac{1}{a} = 2$, $a = \frac{1}{2}$
$-\frac{b}{a} = 0.25$, $b = -0.125$

(iv) Equation is $x = \frac{1}{b} - \frac{a}{b}\left(\frac{y}{x}\right)$
Gradient = $-\frac{a}{b} = 4$
Intercept on vertical axis = $\frac{1}{b} = -8$

7 (i) $\propto - \beta = -\sqrt{8}$ (given $\alpha < \beta$)

(ii) $\frac{\propto -1}{\beta} + \frac{\beta - 1}{\propto} = 4$, $\left(\frac{\propto -1}{\beta}\right)\left(\frac{\beta - 1}{\propto}\right) = -\frac{1}{2}$
Equation is $x^2 - 4x - \frac{1}{2} = 0$ or $2x^2 - 8x - 1 = 0$

8 (i) $\frac{dy}{dx} = -6(5 - 2x)^2$, equation of tangent is $y = -54x + 81$
(ii) Shaded area = $\int_0^1 (5-2x)^3 dx - \int_0^1 (-54x+81)\, dx = 68 - 54 = 14$ units^2

9 (i) $v = 4\ m\ s^{-1}$ (ii) $a = \frac{1}{2}\ m\ s^{-2}$ (iii) $s = 6t + 4e^{-t/2} - 4 = 56.0$ m (when $t = 10$ s)
(iv) When $v = 0$, $t = -2.20\ s$. Since time cannot have a negative value, the particle did not reverse its direction of motion.

10 (i) ∠ACF = ∠FGC (alternate segment theorem/tangent-chord theorem)
∠ACF = ∠EFC (alternate angles)
∴ ∠FGC = ∠EFC

∠EFC = ∠FCG (common angle)
ΔECF and ΔFCG are similar triangles (AA similarity test)
$\frac{\text{EC}}{\text{FC}} = \frac{\text{CF}}{\text{CG}}$
(EC)(CG) = $(\text{CF})^2$

(ii) ∠GEF = ∠HEC (vertically opposite angles)
∠FGE = ∠CHE (angles in same segment)
ΔFGE and ΔCHE are similar triangles (AA similarity test)
$\frac{\text{FE}}{\text{EC}} = \frac{\text{EG}}{\text{EH}}$
(FE)(EH) = (EG)(EC)
= (CG – EC)(EC)
= (CG)(EC) – $(\text{EC})^2$
= $\text{CF}^2 - \text{EC}^2$ [(EC)(CG) = $(\text{CF})^2$ in (i)]

11 (i) Centre, A = (−2 , −3) , radius = 5 units (ii) Q(1 , 1)
(iii) R$\left(\frac{7}{3}, 0\right)$ (iv) B$\left(\frac{1}{6}, -\frac{3}{2}\right)$, radius = 2.64 units