CEDAR GIRLS' SECONDARY SCHOOL
Preliminary Examination 2018
Secondary Four

ADDITIONAL MATHEMATICS — 4047/01

Paper 1

17 August 2018

2 hours

Additional Materials: Answer Paper

READ THESE INSTRUCTIONS FIRST

Write your Centre number, index number and name on all the work you hand in.
Write in dark blue or black pen on both sides of the paper.
You may use a HB pencil for any diagrams or graphs.
Do not use staples, paper clips, highlighters, glue or correction fluid.

Answer **all** the questions.
Write your answers on the separate Answer Paper provided.
Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.
The use of an approved scientific calculator is expected, where appropriate.
You are reminded of the need for clear presentation in your answers.

At the end of the examination, fasten all your work securely together.
The number of marks is given in brackets [ ] at the end of each question or part question.
The total number of marks for this paper is 80.

This document consists of **6** printed pages and **1** cover page.

[Turn over

Mathematical Formulae

1. ALGEBRA

Quadratic Equation

For the equation $ax^2 + bx + c = 0$,

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Binomial expansion

$$(a+b)^n = a^n + \binom{n}{1} a^{n-1} b + \binom{n}{2} a^{n-2} b^2 + \ldots + \binom{n}{r} a^{n-r} b^r + \ldots + b^n,$$

where n is a positive integer and $\binom{n}{r} = \dfrac{n!}{r!(n-r)!} = \dfrac{n(n-1)\ \ldots\ (n-r+1)}{r!}$

2. TRIGONOMETRY

Identities

$$\sin^2 A + \cos^2 A = 1$$

$$\sec^2 A = 1 + \tan^2 A$$

$$cosec^2 A = 1 + \cot^2 A$$

$$\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$$

$$\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$$

$$\tan(A \pm B) = \frac{\tan A \pm \tan B}{1 \mp \tan A \tan B}$$

$$\sin 2A = 2 \sin A \cos A$$

$$\cos 2A = \cos^2 A - \sin^2 A = 2\cos^2 A - 1 = 1 - 2\sin^2 A$$

$$\tan 2A = \frac{2 \tan A}{1 - \tan^2 A}$$

Formulae for ΔABC

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$a^2 = b^2 + c^2 - 2bc \cos A$$

$$\Delta = \frac{1}{2} ab \sin C$$

Answer **all** the questions.

1 A cone has curved surface area $\pi\left(17-\sqrt{3}\right)$ cm^2 and slant height $\left(7-3\sqrt{3}\right)$ cm.
Without using a calculator, find the diameter of the base of the cone, in cm, in the form of $a+b\sqrt{3}$, where a and b are integers. [4]

2 The roots of the quadratic equation $5x^2-3x+1=0$ are $\dfrac{1}{\alpha}$ and $\dfrac{1}{\beta}$.

Find a quadratic equation with roots α^3 and β^3. [6]

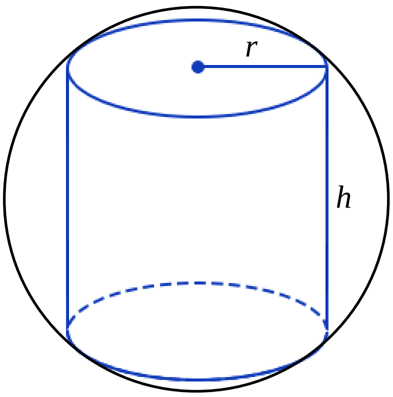
3 **(i)** Show that $2x^2+1$ is a factor of $2x^3-4x^2+x-2$. [2]

(ii) Express $\dfrac{11x-5x^2-11}{2x^3-4x^2+x-2}$ in partial fractions. [5]

4 **(i)** Sketch the graph of $y=\dfrac{4}{\sqrt{x}}$ for $x>0$. [2]

(ii) Find the coordinates of the point(s) of intersection of $y=\dfrac{4}{\sqrt{x}}$ and $y^2=81x$. [4]

5 The diagram shows a cylinder of height h cm and base radius r cm inscribed in a sphere of radius 35 cm.



(i) Show that the height of the cylinder, h cm, is given by $h=2\sqrt{1225-r^2}$. [2]

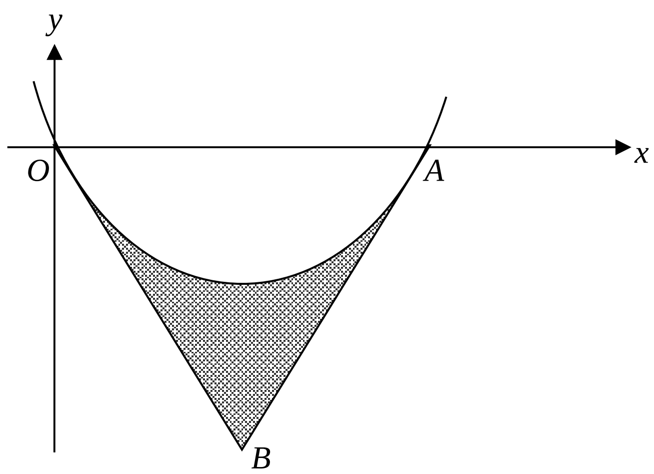
(ii) Given that r can vary, find the maximum volume of the cylinder. [4]

6 **(i)** Show that $\dfrac{2-\sec^2 x}{2\tan x+\sec^2 x}=\dfrac{\cos x-\sin x}{\cos x+\sin x}$. [3]

(ii) Hence find, for $0 \le x \le 2\pi$, the values of x for which $\dfrac{6-3\sec^2 x}{2\tan x+\sec^2 x}=\dfrac{3}{2}$. [3]

7 A curve is such that $\dfrac{\mathrm{d}^2 y}{\mathrm{d}x^2}=\dfrac{2}{e^{2x-3}}$ and the point $P(1.5, 2)$ lies on the curve. The gradient of the normal to the curve at P is 10. Find the equation of the curve. [6]

8 The diagram shows the graph of $y=x^{\frac{3}{2}}-4x$ which passes through the origin O and cuts the x-axis at the point $A(16, 0)$. Tangents to the curve at O and A meet at the point B.



(i) Show that B is the point $\left(5\dfrac{1}{3}, -21\dfrac{1}{3}\right)$. [3]

(ii) Find the area of the shaded region bounded by the curve and the lines OB and AB. [4]

9 A tram, moving along a straight road, passes station O with a velocity of 975 m/min. Its acceleration, a m/min^2, t mins after passing through station O, is given by $a = 2t - 80$.
The tram comes to instantaneous rest, first at station A and later at station B.
Find

(i) the acceleration of the tram at station A and at station B, [3]

(ii) the greatest speed of the tram as it travels from station A to station B, [2]

(iii) the distance between station A to station B. [2]

10 **(i)** By considering the general term in the binomial expansion of $\left(x^4 - \dfrac{1}{kx^2}\right)^6$, where k is a positive constant, explain why there are only even powers of x in this expansion. [2]

(ii) Given that the term independent of x in this binomial expansion is $\dfrac{5}{27}$, find the value of k. [2]

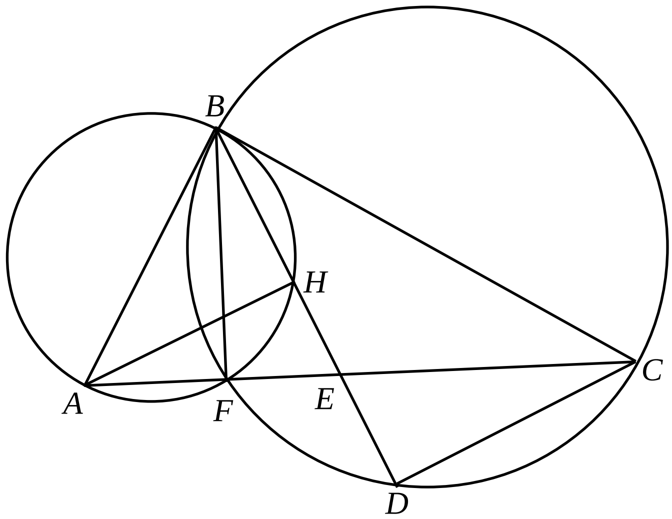
(iii) Using the value of k found in part **(ii)**, hence obtain the coefficient of x^{18} in $\left(2 - 3x^6\right)\left(x^4 - \dfrac{1}{kx^2}\right)^6$. [4]

11 M and N are two points on the circumference of a circle, where M is the point (6, 8) and N is the point (10, 16). The centre of the circle lies on the line $y = 2x + 1$.

(i) Find the equation of the circle in the form $x^2 + y^2 + ax + by + c = 0$, where a, b and c are constants. [6]

(ii) Explain whether the point (9, 10) lie inside the circle. Justify your answer with mathematical calculations. [2]

12



In the diagram, two circles intersect at B and F. BC is the diameter of the larger circle and is the tangent to the smaller circle at B.
Point A lies on the smaller circle such that $AFEC$ is a straight line.
Point D lies on the larger circle such that $BHED$ is a straight line.
Prove that

(i) CD is parallel to AH, [3]

(ii) AB is a diameter of the smaller circle, [2]

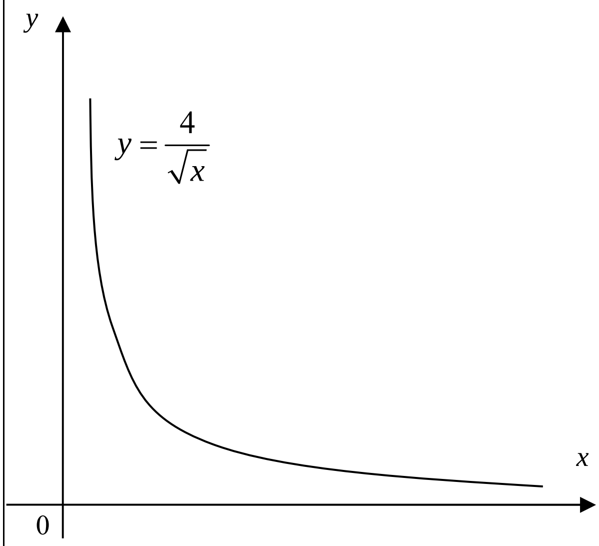
(iii) triangles ABC and BFC are similar, [2]

(iv) $AC^2 - AB^2 = CF \times AC$. [2]

End of Paper

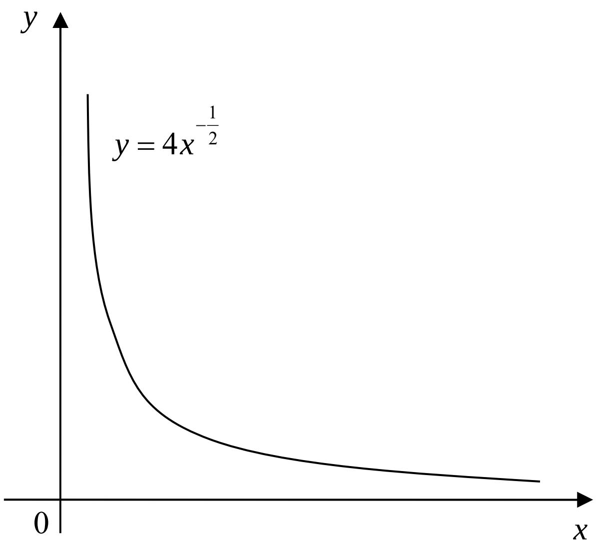
CEDAR GIRLS' SECONDARY SCHOOL
SECONDARY 4 ADDITIONAL MATHEMATICS
Answer Key for 2018 Preliminary Examination

PAPER 4047/1

1	$\left(10+4\sqrt{3}\right)$ cm
2	$x^2+18x+125=0$
3i	$2x^3-4x^2+x-2=(2x^2+1)(x-2)$ It is divisible by $2x^2+1$ with no remainder.
3ii	$\dfrac{-5x^2+11x-11}{2x^3-4x^2+x-2}=-\dfrac{1}{x-2}+\dfrac{5-3x}{2x^2+1}$
4i	
4ii	$\left(\dfrac{4}{9},6\right)$
5i	Using Pythagoras' Theorem: $\left(\dfrac{h}{2}\right)^2+r^2=35^2$
5ii	104 000 cm^3 (3 s.f.)
6ii	$x=0.322$ or $x=3.46$ (3 s.f.)
7	$y=\dfrac{1}{2}e^{3-2x}+\dfrac{9}{10}x+\dfrac{3}{20}$
8ii	68.3 units^2 (3 s.f.)
9i	Acceleration at A = – 50 m/min^2 Acceleration at B = 50 m/min^2
9ii	Greatest speed = 625 m/min
9iii	20.8 km (3 s.f.)
10i	General term $=\binom{6}{r}(x)^{24-6r}\left(-\dfrac{1}{k}\right)^r$ Since $6r$ is an even number, $24-6r$ will be even.
10ii	$k=3$
10iii	Coefficient of $x^{18}=2(-2)+(-3)\binom{5}{3}=-9$
11i	$x^2+y^2-12x-26y+180=0$
11ii	Length of point to centre of circle = 4.24 < 5. Yes, the point lies inside the circle as its length from the centre of the circle is less than the radius.
12i	$\angle AHD=\angle HDC$ (alternate angles)
12ii	AB is a diameter of the smaller circle ($\angle$ in semicircle).
12iii	Triangle ABC is similar to triangle BFC as all corresponding angles are equal.
12iv	$\dfrac{BC}{FC}=\dfrac{AC}{CB}$ (ratio of similar triangles) $BC^2=CF\times AC$ $BC^2=AC^2-AB^2$ (Pythagoras' Theorem) $\therefore AC^2-AB^2=CF\times AC$ (shown)

2018 Preliminary Examination 2
Additional Mathematics 4047 Paper 1
Solutions

Qn	Working
1	$\pi rl = \pi\left(17-\sqrt{3}\right)$ $r = \frac{\left(17-\sqrt{3}\right)}{7-3\sqrt{3}}$ $r = \frac{\left(17-\sqrt{3}\right)}{7-3\sqrt{3}} \times \frac{7+3\sqrt{3}}{7+3\sqrt{3}}$ $r = \frac{110+44\sqrt{3}}{22}$ $r = 5+2\sqrt{3}$ Diameter = $=10+4\sqrt{3}$ cm
2	$\frac{1}{\alpha}+\frac{1}{\beta} = -\frac{-3}{5}$ $=\frac{3}{5}$ $\frac{1}{\alpha\beta} = \frac{1}{5}$ $\alpha\beta = 5$ $\frac{1}{\alpha}+\frac{1}{\beta} = \frac{\alpha+\beta}{\alpha\beta}$ $\frac{\alpha+\beta}{5} = \frac{3}{5}$ $\alpha+\beta = 3$ $\alpha^3+\beta^3 = (\alpha+\beta)(\alpha^2-\alpha\beta+\beta^2)$ $=3[(\alpha+\beta)^2-3\alpha\beta]$ $=3[(3)^2-3(5)]$ $=-18$ $\alpha^3\beta^3 = (\alpha\beta)^3$ $=125$ Equation: $x^2+18x+125=0$

Qn	Working
3i	$2x^3-4x^2+x-2=(2x^2+1)(x-2)$ It is divisible by $2x^2+1$ with no remainder.
3ii	$\dfrac{-5x^2+11x-11}{2x^3-4x^2+x-2}=\dfrac{A}{x-2}+\dfrac{Bx+C}{2x^2+1}$ $-5x^2+11x-11=A(2x^2+1)+(Bx+C)(x-2)$ When $x = 2$, $A=-1$ Comparing x^2: $-5=2A+B$ $-5=-2+B$ $B=-3$ Comparing constant: $-11=A-2C$ $-11=-1-2C$ $C=5$ $\dfrac{-5x^2+11x-11}{2x^3-4x^2+x-2}=-\dfrac{1}{x-2}+\dfrac{5-3x}{2x^2+1}$
4	

4ii

$\left(\frac{4}{\sqrt{x}}\right)^2 = 81x$

$\frac{16}{x} = 81x$

$81x^2 = 16$

$x = \pm\frac{4}{9}$

$x = \frac{4}{9}$

$y = 6$

Point of intersection = $\left(\frac{4}{9}, 6\right)$

5i

$\left(\frac{h}{2}\right)^2 + r^2 = 35^2$ (Pythagoras' Theorem)

$\frac{h^2}{4} = 1225 - r^2$

$h^2 = 4(1225 - r^2)$

$h = 2\sqrt{1225 - r^2}$

(shown)

5ii

$V = \pi r^2(2\sqrt{1225 - r^2})$

$V = 2\pi r^2(1225 - r^2)^{\frac{1}{2}}$

$\frac{dV}{dr} = 2\pi r^2(\frac{1}{2}(-2r)(1225 - r^2)^{-\frac{1}{2}}) + (1225 - r^2)^{\frac{1}{2}}(4\pi r)$

$= -2\pi r^3(1225 - r^2)^{-\frac{1}{2}} + 4\pi r(1225 - r^2)^{\frac{1}{2}}$

$-2\pi r^3(1225 - r^2)^{-\frac{1}{2}} + 4\pi r(1225 - r^2)^{\frac{1}{2}} = 0$

$r^3 = 2r(1225 - r^2)$

$3r^3 = 2450r$

$r = 28.577$ (reject $r = 0$ and -ve r)

Using First Derivative Test,

x	28.577 (-)	28.577	28.577 (+)
Sign of $\frac{dV}{dr}$	+ve	0	-ve
slope	/	—	\

V is maximum at $r = 28.577$

Maximum volume:

$V = \pi(28.577)^2(2\sqrt{1225-(28.577)^2})$

$= 103\,688$

$= 104\,000$

$= 104\,000$ cm^3 (3 s.f.)

Qn	Working
6i	LHS: $\dfrac{2-\sec^2 x}{2\tan x+\sec^2 x} = \dfrac{2-(\tan^2 x+1)}{2\tan x+(\tan^2 x+1)}$ $= \dfrac{1-\tan^2 x}{2\tan x+\tan^2 x+1}$ $= \dfrac{(1-\tan x)(1+\tan x)}{(\tan x+1)^2}$ $= \dfrac{1-\tan x}{1+\tan x}$ $= \dfrac{1-\frac{\sin x}{\cos x}}{1+\frac{\sin x}{\cos x}}$ $= \dfrac{\cos x-\sin x}{\cos x}\times\dfrac{\cos x}{\cos x+\sin x}$ $= \dfrac{\cos x-\sin x}{\cos x+\sin x}$ (shown)
6ii	$3\times\dfrac{\cos x-\sin x}{\cos x+\sin x} = \dfrac{3}{2}$ $\dfrac{\cos x-\sin x}{\cos x+\sin x} = \dfrac{1}{2}$ $2\cos x-2\sin x = \cos x+\sin x$ $\cos x = 3\sin x$ $\tan x = \dfrac{1}{3}$ $x = 0.322$ or $x = 3.46$ (3 s.f.)

Qn	Working
7	$\frac{\mathrm{d}^2y}{\mathrm{d}x^2}=2e^{3-2x}$ $\frac{\mathrm{d}y}{\mathrm{d}x}=2[-\frac{1}{2}e^{3-2x}]+c$ $\frac{\mathrm{d}y}{\mathrm{d}x}=-e^{3-2x}+c$ Gradient at tangent at $P=-\frac{1}{10}$ $-e^{3-2x}+c=-\frac{1}{10}$ when $x=1.5$ $c=\frac{9}{10}$ $\frac{\mathrm{d}y}{\mathrm{d}x}=-e^{3-2x}+\frac{9}{10}$ $y=\frac{1}{2}e^{3-2x}+\frac{9}{10}x+c$ $2=\frac{1}{2}e^{3-2(1.5)}+\frac{9}{10}(1.5)+c$ $c=\frac{3}{20}$ Eqn: $y=\frac{1}{2}e^{3-2x}+\frac{9}{10}x+\frac{3}{20}$

Qn	Working
8i	$\frac{dy}{dx}=\frac{3}{2}x^{\frac{1}{2}}-4$ At O, $x=0$, $\frac{dy}{dx}=-4$ Equation OB: $y=-4x$ …(1) At A, $x=16$, $\frac{dy}{dx}=2$ $y=2x+c$ $0=2(16)+c$ $c=-32$ Equation AB: $y=2x-32$ $2x-32=-4x$ $x=5\frac{1}{3}$ Sub into (1), $y=-21\frac{1}{3}$ $B=\left(5\frac{1}{3},-21\frac{1}{3}\right)$ (shown)
ii	Area of curve = $\left\lvert\int_0^{16} x^{\frac{3}{2}}-4x\,dx\right\rvert=\left\lvert\left[\frac{2}{5}x^{\frac{5}{2}}-2x^2\right]_0^{16}\right\rvert$ =102.4 units^2 Area of triangle OAB = $\frac{1}{2}\times16\times21\frac{1}{3}$ $=170\frac{2}{3}$ units^2 Area of shaded region $=170\frac{2}{3}-102.4$ = 68.3 units^2 (3 s.f.)

Qn	Working
9i	$a = 2t - 80$ $v = t^2 - 80t + c$ $t = 0$, $v = 975$ $975 = (0)^2 - 80(0) + c$ $c = 975$ $v = t^2 - 80t + 975$ When $v = 0$, $t^2 - 80t + 975 = 0$ $(t - 15)(t - 65) = 0$ $t = 15$, $t = 65$ Acceleration at a = 2(15) – 80 = – 50 m/min^2 Acceleration at a = 2(65) – 80 = 50 m/min^2
9ii	When $a = 0$, $t = \frac{15 + 65}{2}$ $t = 40$ $v = (40)^2 - 80(40) + 975$ $v = -625$ m/min Greatest speed = 625 m/min
9iii	Distance $AB = \left\| \int_{15}^{65} t^2 - 80t + 975 \, dt \right\|$ $= \left\| \left[\frac{t^3}{3} - 40t^2 + 975t \right]_{15}^{65} \right\|$ $= 20833\frac{1}{3}$ m = 20 800 m (3 s.f.) = 20.8 km

Qn	Working
10(i)	General Term $=\binom{6}{r}\left(x^4\right)^{6-r}\left(-\frac{1}{k}x^{-2}\right)^r$ $=\binom{6}{r}\left(x\right)^{24-6r}\left(-\frac{1}{k}\right)^r$ Since $6r$ is an even number, $24-6r$ will be even.
(ii)	For independent term, $24-6r=0 \Rightarrow r=4$ $\binom{6}{4}\left(-\frac{1}{k}\right)^4=\frac{5}{27}$ $\frac{15}{k^4}=\frac{5}{27}$ $k=+\sqrt[4]{\frac{27\times15}{5}}=3$ (as $k>0$)
(iii)	$\left(2-3x^6\right)(\ \ldots+\text{Term in } x^{18}+\text{Term in } x^{12}+\ldots)$ For term in x^{18}, $24-6r=18 \Rightarrow r=1$ Therefore, term in $x^{18}=\binom{6}{1}\left(-\frac{1}{3}\right)x^{18}=-2x^{18}$ For term in x^{12}, $24-6r=12 \Rightarrow r=2$ Therefore, term in $x^{12}=\binom{6}{2}\left(-\frac{1}{3}\right)^2x^{12}=\frac{5}{3}x^{12}$ Coefficient of $x^{18}=2\left(-2\right)+(-3)\left(\frac{5}{3}\right)=-9$

Qn	Working
11i	Let MN be a chord of circle. Midpoint of $MN = \left(\frac{10+6}{2}, \frac{16+8}{2}\right)$ = (8, 12) Gradient of $MN = \frac{16-8}{10-6}$ = 2 Gradient of perpendicular bisector $= -\frac{1}{2}$ Equation of perpendicular bisector of MN: $y-12=-\frac{1}{2}(x-8)$ $y=-\frac{1}{2}x+16$ $-\frac{1}{2}x+16=2x+1$ $x=6$ $y=13$ Centre of circle = (6, 13) Radius = 13 – 8 = 5 units Equation of circle: $(x-6)^2+(y-13)^2=5^2$ $x^2+y^2-12x-26y+180=0$
11ii	Length of point to centre of circle $=\sqrt{(9-6)^2+(10-13)^2}$ $=\sqrt{18}$ $=4.24$ units < 5 (radius) Yes, the point lies inside the circle as its length from the centre of the circle is less than the radius.

Qn	Working
12i	$\angle BDC = 90°$ ($\angle$ in semicircle) $\angle BFC = 90°$ ($\angle$ in same segment) or ($\angle$ in semicircle) $\angle BFA = 180° - 90°$ (adj $\angle s$ on straight line) $= 90°$ $\angle BHA = \angle BFA = 90°$ ($\angle$ in same segment) $\angle AHD = 180° - 90°$ (adj $\angle s$ on straight line) $= 90°$ $\angle AHD = \angle BDC = \angle HDC$ (alternate angles) $\therefore CD // AH$
12ii	$\angle BHA = \angle BFA = 90°$ ($\angle$ in same segment) AB is a diameter of the smaller circle ($\angle$ in semicircle).
12iii	Since AB and BC are tangents to the smaller and bigger circle respectively, $\angle ABC = 90°$ (tan $\perp$ rad) $\angle ABC = \angle BFC$ $\angle BCA = \angle FCB$ (common $\angle$) Triangle ABC is similar to triangle BFC as all corresponding angles are equal.
12iv	$\frac{BC}{FC} = \frac{AC}{CB}$ (ratio of similar triangles) $BC^2 = CF \times AC$ $BC^2 = AC^2 - AB^2$ (Pythagoras' Theorem) $\therefore AC^2 - AB^2 = CF \times AC$ (shown)

CEDAR GIRLS' SECONDARY SCHOOL
Preliminary Examination
Secondary Four

ADDITIONAL MATHEMATICS

4047/02
20 August 2018

2 hours 30 minutes

Additional Materials: Answer Paper
Graph paper (1 sheet)

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Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.
The use of a scientific calculator is expected, where appropriate.
You are reminded of the need for clear presentation in your answers.

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The number of marks is given in brackets [ ] at the end of each question or part question.
The total number of marks for this paper is 100.

Mathematical Formulae

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Quadratic Equation

For the equation $ax^2 + bx + c = 0$,

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Binomial expansion

$$(a+b)^n = a^n + \binom{n}{1} a^{n-1} b + \binom{n}{2} a^{n-2} b^2 + \ldots + \binom{n}{r} a^{n-r} b^r + \ldots + b^n,$$

where n is a positive integer and $\binom{n}{r} = \frac{n!}{r!(n-r)!} = \frac{n(n-1)\ldots(n-r+1)}{r!}$

2. TRIGONOMETRY

Identities

$$\sin^2 A + \cos^2 A = 1$$

$$\sec^2 A = 1 + \tan^2 A$$

$$cosec^2 A = 1 + \cot^2 A$$

$$\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$$

$$\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$$

$$\tan(A \pm B) = \frac{\tan A \pm \tan B}{1 \mp \tan A \tan B}$$

$$\sin 2A = 2 \sin A \cos A$$

$$\cos 2A = \cos^2 A - \sin^2 A = 2\cos^2 A - 1 = 1 - 2\sin^2 A$$

$$\tan 2A = \frac{2 \tan A}{1 - \tan^2 A}$$

$$\sin A + \sin B = 2 \sin \tfrac{1}{2}(A+B) \cos \tfrac{1}{2}(A-B)$$

$$\sin A - \sin B = 2 \cos \tfrac{1}{2}(A+B) \sin \tfrac{1}{2}(A-B)$$

$$\cos A + \cos B = 2 \cos \tfrac{1}{2}(A+B) \cos \tfrac{1}{2}(A-B)$$

$$\cos A - \cos B = -2 \sin \tfrac{1}{2}(A+B) \sin \tfrac{1}{2}(A-B)$$

Formulae for ΔABC

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$a^2 = b^2 + c^2 - 2bc \cos A$$

$$\Delta = \frac{1}{2} ab \sin C$$

Answer **all** the questions.

1 **(a)** Given that $3\lg\left(x\sqrt[3]{y}\right) = 2 + 2\lg x - \lg y$, where x and y are positive numbers, express, in its simplest form, y in terms of x. [3]

(b) Given that $p = \log_8 q$, express, in terms of p,

(i) $\log_8\left(\dfrac{1}{q}\right)$, [2]

(ii) $\log_2 4q$. [2]

2 **(i)** Show that $\dfrac{\mathrm{d}}{\mathrm{d}x}(\sin x \cos x) = 2\cos^2 x - 1$. [2]

(ii) Hence, without using a calculator, find the value of each of the constants a and b for which

$$\int_0^{\frac{\pi}{4}} \cos^2 x \, \mathrm{d}x = a + b\pi. \quad [4]$$

3 The variables x and y are such that when values of $\dfrac{1}{y} + \dfrac{1}{x}$ are plotted against $\dfrac{1}{x}$, a straight line with gradient m is obtained. It is given that $y = \dfrac{1}{6}$ when $x = 1$ and that $y = \dfrac{1}{2}$ when $x = \dfrac{1}{2}$.

(i) Find the value of m. [4]

(ii) Find the value of x when $\dfrac{3}{y} + \dfrac{3}{x} = 3$. [2]

(iii) Express y in terms of x. [2]

4 The equation of a curve is $y = x^3 + px^2$, where p is a positive constant.

(i) Show that the origin is a stationary point on the curve and find the x-coordinate of the other stationary point in terms of p. [3]

(ii) Find the nature of each of the stationary points. [3]

Another curve has equation $y = x^3 + px^2 + px$.

(iii) Find the set of values of p for which this curve has no stationary points. [3]

5 A quadratic function f(x) is given by $\mathrm{f}(x) = k(x-2)^2 - (x-3)(x+2)$, where k is a constant and $k \neq 1$.

(i) Find the value of k such that the graph of $y = \mathrm{f}(x)$ touches the x-axis at one point. [3]

(ii) Find the range of values of k for which the function possesses a maximum point. [1]

(iii) Find the range of values of k for which the value of the function never exceeds 18. [3]

6 **(a)** A substance is decaying in such a way that its mass, m kg, at a time t years from now is given by the formula

$$m = 240e^{-0.04t}.$$

(i) Find the time taken for the substance to halve its mass. [2]

(ii) Find the value of t for which the mass is decreasing at a rate of 2.1 kg per year. [3]

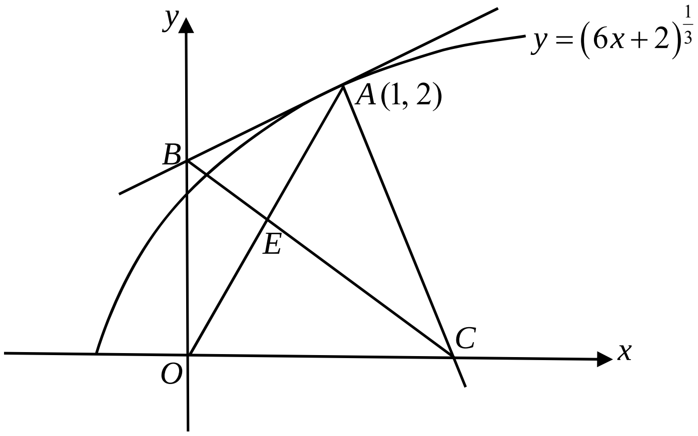
(b) The noise rating, N and its intensity, I are connected by the formula

$$N = 10\left(\lg\frac{I}{k}\right), \text{ where } k \text{ is a constant.}$$

A hot water pump has a noise rating of 50 decibels.
A dishwasher, however, has a noise rating of 62 decibels.

Find the value of $\dfrac{\text{Intensity of the noise from the dishwasher}}{\text{Intensity of the noise from the hot water pump}}$. [3]

7



The diagram shows the curve $y=(6x+2)^{\frac{1}{3}}$ and the point $A\ (1, 2)$ which lies on the curve. The tangent to the curve at A cuts the y-axis at B and the normal to the curve at A cuts the x-axis at C.

(i) Find the equation of the tangent AB and the equation of the normal AC. [4]

(ii) Find the length of BC. [2]

(iii) Find the coordinates of the point of intersection, E, of OA and BC. [4]

8 It is given that $y_1=\tan x$ and $y_2=2\cos 2x+1$.

(i) State the period, in radians, of y_1 and the amplitude of y_2. [2]

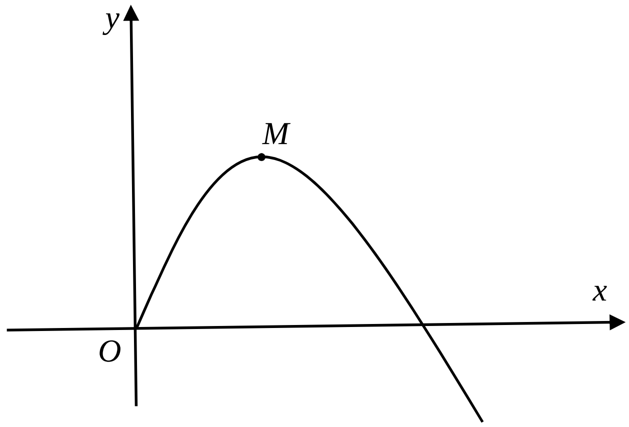
For the interval $0\le x\le 2\pi$,

(ii) sketch, on the same diagram, the graphs of y_1 and y_2, [3]

(iii) state the number of roots of the equation $|\tan x|-2\cos 2x=1$, [1]

(iv) find the range(s) of values of x for which y_1 and y_2 are both increasing as x increases. [2]

9 **(a)**



The diagram shows part of the curve,

$$y = \tan x \cos 2x,$$

and its maximum point M.

(i) Show that $\dfrac{dy}{dx} = 4\cos^2 x - \sec^2 x - 2$. [5]

(ii) Hence find the x-coordinate of M. [3]

(b) A particle moves along the line $y = \ln\sqrt{\dfrac{5x}{x-2}}$ in such a way that the x-coordinate is increasing at a constant rate of 0.4 units per second. Find the rate at which the y-coordinate of the particle is increasing at the instant when $x = 2.5$. [3]

10 **(a)** The function f is defined for all real values of x by $f(x) = e^{2x} - 3e^{-2x}$.

(i) Show that $f'(x) > 0$ for all values of x. [2]

(ii) Show that $f''(x) = hf(x)$, where h is an integer. [2]

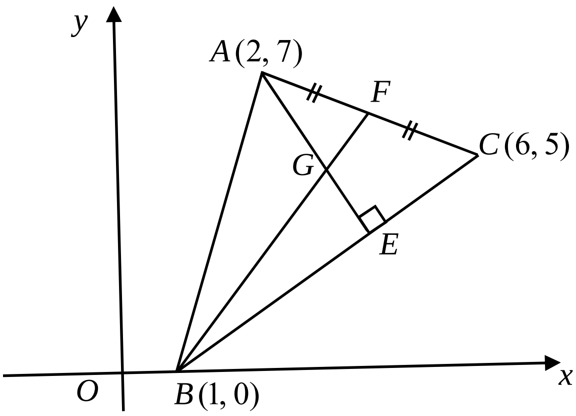
(iii) Find the value of x for which $f''(x) = 0$ in the form $x = p\ln q$, where p and q are rational numbers. [2]

(b) The function g is defined for all real values of x by $g(x) = e^{2x} + 3e^{-2x}$.

The curve $y = g(x)$ and the line $x = \frac{1}{4}\ln 3$ intersect at point Q.

Show that the y-coordinate of Q is $k\sqrt{3}$, where k is an integer. [2]

11 **Solutions to this question by accurate drawing will not be accepted.**



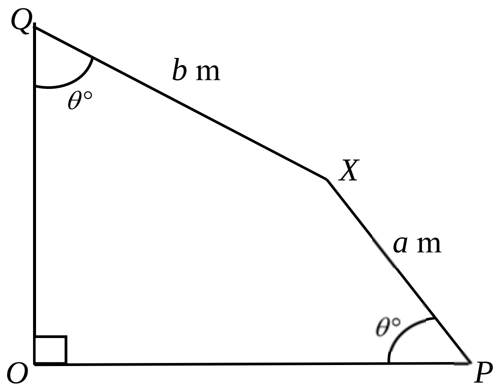
The diagram, which is not drawn to scale, shows a triangle ABC with vertices A (2, 7), B (1, 0) and C (6, 5) respectively. E and F are points on BC and AC respectively for which AE is perpendicular to BC and BF bisects AC.
G is the point of intersection of lines AE and BF.
Find

(i) the coordinates of G, [4]

(ii) the coordinates of the point D such that $ABCD$ is a parallelogram, [2]

(iii) the area of $ABCD$. [2]

12



The diagram above shows a quadrilateral in which $PX = a$ m and $QX = b$ m.
Angle OQX = Angle OPX = $\theta°$ and OQ is perpendicular to OP.

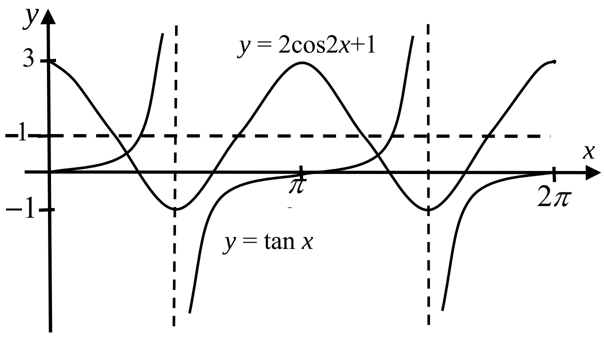
(i) Show that $OP = a\cos\theta + b\sin\theta$. [3]

(ii) It is given that the maximum length of OP is $\sqrt{5}$ m and the corresponding value of θ is $63.43°$.
By using $OP = R\cos(\theta - \alpha)$, where $R > 0$ and θ is acute, find the value of a and of b. [5]

(iii) Given that $OP = 2.15$ m, find the value of θ . [2]

End of Paper

CEDAR GIRLS' SECONDARY SCHOOL
SECONDARY 4 ADDITIONAL MATHEMATICS
Answer Key for Prelim Examination 2018

PAPER 4047/02			
1a	$y=\dfrac{10}{\sqrt{x}}$	**8(i)**	Period of $y_1=\pi$ radians
1bi	$-p$		Amplitude of $y_2=2$
bii	$2+3p$	**8(ii)**	
2ii	$a=\dfrac{1}{4}, b=\dfrac{1}{8}$		
3(i)	$m=-3$		
3(ii)	$x=\dfrac{1}{3}$		
3(iii)	$y=\dfrac{x}{10x-4}$		
4(i)	$x=-\dfrac{2p}{3}$		
4(ii)	$(0, 0)$ is a minimum point.	**8(iii)**	4
	maximum point at $x=-\dfrac{2p}{3}$	**8(iv)**	$\dfrac{\pi}{2}<x<\pi$, $\dfrac{3\pi}{2}<x<2\pi$
4(iii)	$\{p: 0<p<3\}$	**9a(ii)**	0.452 or $25.9°$
5(i)	$k=\dfrac{25}{16}$	**9b**	-0.32 units per second
5(ii)	$k<1$	**10a(iii)**	$x=\dfrac{1}{4}\ln 3$
5(iii)	$k\le\dfrac{47}{56}$	**10b**	$2\sqrt{3}$
6ai	17.3 years	**11(i)**	$G\left(3\frac{2}{3}, 5\frac{1}{3}\right)$
6aii	$t=38.0$	**11(ii)**	$(7, 12)$
6b	15.8	**11(iii)**	30 sq units
7(i)	Eqn of AB: $y=\frac{1}{2}x+\frac{3}{2}$	**12 (ii)**	$a=1.00$, $b=2.00$
	Eqn of AC: $y=-2x+4$	**12(iii)**	$\theta=79.4$ or 47.5
7(ii)	2.5 units		
7(iii)	Coordinates of $E=\left(\dfrac{6}{11}, 1\dfrac{1}{11}\right)$		

2018 Preliminary Examination 2
Additional Mathematics 4047/2
Solutions

Qn	Working	Marks	Total	Remarks
1a	$3\lg\left(x\sqrt[3]{y}\right)=2+2\lg x-\lg y$ $3\lg x+\lg y=2+2\lg x-\lg y$ $\lg x+2\lg y=2$ $\lg\left(xy^2\right)=2$ $xy^2=10^2=100$ $y=\sqrt{\dfrac{100}{x}}=\dfrac{10}{\sqrt{x}}=\dfrac{10\sqrt{x}}{x}$		[3]	
b(i)	$\log_8\dfrac{1}{q}=\log_8 1-\log_8 q$ $=0-p=-p$		[2]	
b(ii)	$\log_2 4q=\log_2 4+\log_2 q$ $=2+\dfrac{\log_8 q}{\log_8 2}$ $=2+3p$		[2]	
		Total	[7]	
2(i)	$\dfrac{\mathrm{d}}{\mathrm{d}x}(\sin x\cos x)$ $=\sin x(-\sin x)+\cos x(\cos x)$ $=\cos^2 x-\sin^2 x$ $=\cos^2 x-\left(1-\cos^2 x\right)$ $=2\cos^2 x-1$		[2]	
(ii)	$\int_0^{\frac{\pi}{4}}\left(2\cos^2 x-1\right)dx=\left[\sin x\cos x\right]_0^{\frac{\pi}{4}}$ $\int_0^{\frac{\pi}{4}}\left(2\cos^2 x\right)\mathrm{d}x-\int_0^{\frac{\pi}{4}}1\,\mathrm{d}x=\left[\sin x\cos x\right]_0^{\frac{\pi}{4}}$ $=\dfrac{\sqrt{2}}{2}\times\dfrac{\sqrt{2}}{2}=\dfrac{1}{2}$ $2\int_0^{\frac{\pi}{4}}\left(2\cos^2 x\right)\mathrm{d}x=\dfrac{1}{2}+\left[x\right]_0^{\frac{\pi}{4}}$ $\int_0^{\frac{\pi}{4}}\left(\cos^2 x\right)\mathrm{d}x=\dfrac{1}{4}+\dfrac{\pi}{8}\Rightarrow a=\dfrac{1}{4},\ b=\dfrac{1}{8}$		[4]	
		Total	[6]	

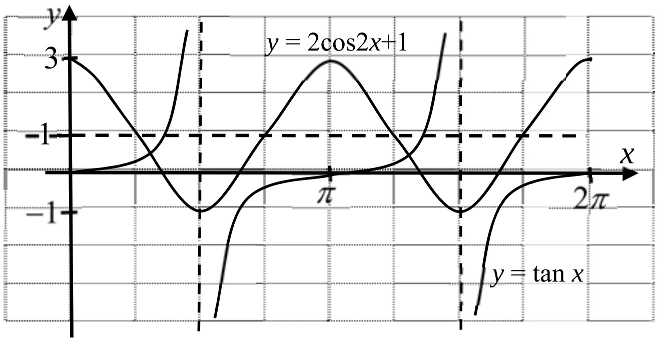
Qn	Working	Marks	Total	Remarks
3(i)	The linear equation is $\frac{1}{y}+\frac{1}{x}=m\left(\frac{1}{x}\right)+c$ Subst $y=\frac{1}{6}$ and $x=1$, $6+1=m+c \Rightarrow m+c=7$ Subst $y=\frac{1}{2}$ and $x=\frac{1}{2}$ $2+2=2m+c \Rightarrow 2m+c=4$ $m=-3$ and $c=10$		**[4]**	
(ii)	Since $\frac{3}{y}+\frac{3}{x}=3 \Rightarrow \frac{1}{y}+\frac{1}{x}=1$, $1=\frac{-3}{x}+10 \Rightarrow x=\frac{1}{3}$		**[2]**	
(iii)	$\frac{1}{y}+\frac{1}{x}=-3\left(\frac{1}{x}\right)+10$ $\frac{x+y}{xy}=\frac{-3+10x}{x}$ $y=\frac{x}{10x-4}$		**[2]**	
		Total	**[8]**	

Qn	Working	Marks	Total	Remarks
4(i)	$y = x^3 + px^2$ $\frac{\mathrm{d}y}{\mathrm{d}x} = 3x^2 + 2px = x(3x+2p)$ For stationary point, $\frac{\mathrm{d}y}{\mathrm{d}x} = 0$ $\therefore x = 0$ or $x = -\frac{2p}{3}$ When $x = 0$, $y = 0$. Therefore, $(0, 0)$ is a stationary point. The other x-coordinate of stationary point is $x = -\frac{2p}{3}$		**[3]**	
(ii)	$\frac{\mathrm{d}^2y}{\mathrm{d}x^2} = 6x + 2p$ When $x = 0$, $\frac{\mathrm{d}^2y}{\mathrm{d}x^2} = 2p > 0$ as $p > 0$ Therefore, $(0, 0)$ is a minimum point. When $x = -\frac{2p}{3}$, $\frac{\mathrm{d}^2y}{\mathrm{d}x^2} = 6\left(-\frac{2p}{3}\right) + 2p = -2p < 0$ as $p > 0$ Therefore, there is a maximum point at $x = -\frac{2p}{3}$		**[3]**	
(iii)	$y = x^3 + px^2 + px$ $\frac{\mathrm{d}y}{\mathrm{d}x} = 3x^2 + 2px + p$ Since $\frac{\mathrm{d}y}{\mathrm{d}x} \neq 0$, $b^2 - 4ac < 0$ $(2p)^2 - 4(3)(p) < 0$ $4p^2 - 12p < 0$ $4p(p-3) < 0$ The set is $\{p : 0 < p < 3\}$		**[3]**	
		Total	**[9]**	

Qn	Working	Marks	Total	Remarks
5(i)	$f(x)=k(x-2)^2-(x-3)(x+2)$ $=k(x^2-4x+4)-(x^2-x-6)$ $=kx^2-4kx+4k-x^2+x+6$ $=(k-1)x^2+(1-4k)x+4k+6$ Since it touches the x-axis at one point, $b^2-4ac=0$ $(1-4k)^2-4(k-1)(4k+6)=0$ $25-16k=0$ $k=\frac{25}{16}$		[3]	
(ii)	$k<1$		[1]	
(iii)	$(k-1)x^2+(1-4k)x+4k+6\leq 18$ $(k-1)x^2+(1-4k)x+4k-12\leq 0$ $b^2-4ac\leq 0$ and $k<1$ $(1-4k)^2-4(k-1)(4k-12)\leq 0$ and $k<1$ $56k-47\leq 0$ and $k<1$ $k\leq\frac{47}{56}$ and $k<1$ The solution is $k\leq\frac{47}{56}$		[3]	
		Total	[7]	

Qn	Working	Marks	Total	Remarks
6a(i)	When $t = 0,\ m = 240$ When $240e^{-0.04t} = 120$ $e^{-0.04t} = 0.5$ $t = \dfrac{\ln 0.5}{-0.04}$ $t = 17.3$ No. of years = 17.3		[2]	
a(ii)	$\dfrac{\mathrm{d}m}{\mathrm{d}t} = 240(-0.04)e^{-0.04t} = -9.6e^{-0.04t}$ $-9.6e^{-0.04t} = -2.1$ $t = \dfrac{\ln\left(\dfrac{2.1}{9.6}\right)}{-0.04} = 38.0$		[3]	
b	$10\lg\left(\dfrac{I_P}{k}\right) = 50 \Rightarrow \left(\dfrac{I_P}{k}\right) = 10^5$ where I_P = intensity of pump $\lg\dfrac{I_D}{k} = \dfrac{62}{10} = 6.2 \Rightarrow \left(\dfrac{I_D}{k}\right) = 10^{6.2}$ where I_D = intensity of dishwasher $\dfrac{I_D}{I_P} = \dfrac{10^{6.2}k}{10^5 k} = 15.8$		[3]	
		Total	[8]	

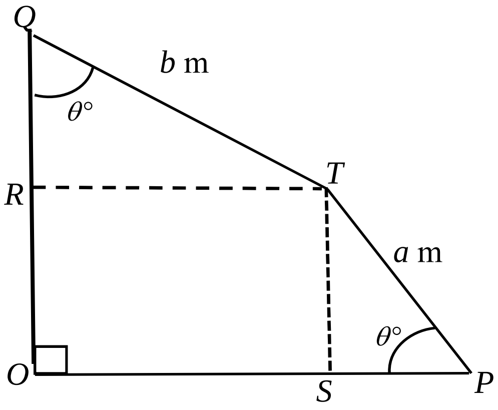
Qn	Working	Marks	Total	Remarks
7(i)	$y=(6x+2)^{\frac{1}{3}}$			
	$\frac{dy}{dx}=\frac{1}{3}(6x+2)^{-\frac{2}{3}}\bullet 6=\frac{2}{(6x+2)^{\frac{2}{3}}}$			Use of chain rule
	When $x=1$, $\frac{dy}{dx}=\frac{2}{(6(1)+2)^{\frac{2}{3}}}=\frac{1}{2}$			Correct substitution
	Eqn of AB: $y-2=\frac{1}{2}(x-1)\Rightarrow y=\frac{1}{2}x+\frac{3}{2}$			
	Eqn of AC: $y-2=-2(x-1)\Rightarrow y=-2x+4$		**[4]**	
7ii	When $x=0$, $y=1.5$			
	Coordinates of B = $(0, 1.5)$			
	When $y=0$, $-2x+4=0\Rightarrow x=2$			
	Coordinates of C = $(2, 0)$			
	$BC=\sqrt{1.5^2+2^2}=2.5$ units		**[2]**	
7iii	Gradient of OA = $\frac{2-0}{1-0}=2$			
	Therefore, eqn of OA: $y=2x$			
	Gradient of BC = $\frac{1.5}{-2}=-\frac{3}{4}$			
	Therefore, eqn of BC: $y=-\frac{3}{4}x+\frac{3}{2}$			
	At E,			
	$2x=-\frac{3}{4}x+\frac{3}{2}$			
	$\frac{11x}{4}=\frac{3}{2}\Rightarrow x=\frac{6}{11}$			
	$y=2\left(\frac{6}{11}\right)=\frac{12}{11}=1\frac{1}{11}$		**[4]**	
	Coordinates of E = $\left(\frac{6}{11}, 1\frac{1}{11}\right)$			
		Total	**[10]**	

Qn	Working	Marks	Total	Remarks
8i	Period of $y_1 = \pi$ radians Amplitude of $y_2 = 2$		**[2]**	
ii				
iv	$\frac{\pi}{2} < x < \pi$, $\frac{3\pi}{2} < x < 2\pi$		**[2]**	

Qn	Working	Marks	Total	Remarks
9a(i)	$y=\tan x\cos 2x$ $\frac{dy}{dx}=\tan x\left(-2\sin 2x\right)+\cos 2x\left(\sec^2 x\right)$ $=\frac{\sin x}{\cos x}\left(-2\times 2\sin x\cos x\right)+\left(2\cos^2 x-1\right)\left(\frac{1}{\cos^2 x}\right)$ $=-4\sin^2 x+2-\sec^2 x$ $=-4(1-\cos^2 x)+2-\sec^2 x$ $=4\cos^2 x-\sec^2 x-2$		**[5]**	
(ii)	When $\frac{dy}{dx}=0$, $4\cos^2 x-\sec^2 x-2=0$ $4\cos^4 x-2\cos^2 x-1=0$ $\cos^2 x=\frac{2\pm\sqrt{4-4(4)(-1)}}{8}$ $=0.80902$ $\cos x=0.89945$ $x=0.452$ or $25.9°$ The x-coordinate of M is 0.452.		**[3]**	
b	$y=\frac{1}{2}\left[\ln 5x-\ln(x-2)\right]$ $\frac{dy}{dx}=\frac{1}{2}\left(\frac{5}{5x}\right)-\frac{1}{2}\left(\frac{1}{x-2}\right)$ $=\frac{1}{2x}-\frac{1}{2(x-2)}$ $\frac{dy}{dt}=\frac{dy}{dx}\times\frac{dx}{dt}$ When $x=2.5$, $\frac{dy}{dt}=\left(\frac{1}{5}-\frac{1}{2(0.5)}\right)\times 0.4=-\frac{8}{25}=-0.32$ The rate is -0.32 units per second.		**[3]**	
		Total	**[11]**	

Qn	Working	Marks	Total	Remarks
10(a)(i)	$\mathrm{f}(x) = e^{2x} - 3e^{-2x}$ $\mathrm{f}'(x) = 2e^{2x} + 6e^{-2x}$ Since $e^{2x} > 0$ and $e^{-2x} > 0$, $\mathrm{f}'(x) > 0$		[2]	
(ii)	$\mathrm{f}''(x) = 4e^{2x} - 12e^{-2x} = 4(e^{2x} - 3e^{-2x})$ Therefore $\mathrm{f}''(x) = 4\mathrm{f}(x)$		[2]	
(iii)	$e^{2x} - 3e^{-2x} = 0$ $e^{2x} = \frac{3}{e^{2x}}$ $e^{4x} = 3$ $4x \ln e = \ln 3$ $x = \frac{1}{4}\ln 3$		[2]	
(b)	$g(x) = e^{2x} + 3e^{-2x}$, When $x = \frac{1}{4}\ln 3$, $g(x) = e^{2(\frac{1}{4}\ln 3)} + ke^{-2(\frac{1}{4}\ln 3)} = e^{\frac{1}{2}\ln 3} + ke^{-\frac{1}{2}\ln 3}$ $= \sqrt{3} + \frac{3}{\sqrt{3}} = 2\sqrt{3}$ Therefore the y-coordinate is $2\sqrt{3}$.		[2]	
		Total	[8]	

Qn	Working	Marks	Total	Remarks
11i	Mid-point of AC, $F=\left(\frac{2+6}{2},\ \frac{7+5}{2}\right)=(4,6)$ Gradient of $BF=\frac{6-0}{4-1}=2$ Eqn of BF: $y-0=2(x-1)\Rightarrow y=2x-2$ Gradient of $BC=\frac{5-0}{6-1}=1$ Gradient of $AE=-1$ Eqn of AE: $y-7=-1(x-2)\Rightarrow y=-x+9$ $-x+9=2x-2$ $x=3\frac{2}{3}$ $\therefore y=-3\frac{2}{3}+9=5\frac{1}{3}$ $G\left(3\frac{2}{3},5\frac{1}{3}\right)$		[4]	
(ii)	Let (x, y) be coordinates of D. $\left(\frac{1+x}{2},\ \frac{0+y}{2}\right)=(4,6)$ $\Rightarrow x=7,\ y=12$ Coordinates of $D=(7,12)$		[2]	
(iii)	Area of $ABCD=\frac{1}{2}\begin{vmatrix}2 & 1 & 6 & 7 & 2\\ 7 & 0 & 5 & 12 & 7\end{vmatrix}=30$ sq units		[2]	
		Total	**[8]**	

Qn	Working	Marks	Total	Remarks
12				
(i)	$\cos\theta = \dfrac{SP}{a} \Rightarrow SP = a\cos\theta$ $\sin\theta = \dfrac{OS}{b} \Rightarrow OS = b\sin\theta$ $OP = SP + OS$ $OP = a\cos\theta + b\sin\theta.$		[3]	
(ii)	$\sqrt{R} = \sqrt{a^2+b^2} \Rightarrow a^2+b^2 = 5$ Max. value of OP occurs at $\theta = 63.43°$. $\cos(\theta-\alpha) = 1 \Rightarrow \theta-\alpha = 0 \Rightarrow \alpha = \theta = 63.43$ $\tan\alpha = \dfrac{b}{a} \Rightarrow \dfrac{b}{a} = \tan 63.43 = 1.9996 \Rightarrow b = 1.9996a$ Subst $b = 1.9996a$ in $a^2+b^2 = 5$ $a^2 + (1.9996a)^2 = 5 \Rightarrow a = 1.00$ $\therefore b = 2.00$		[5]	
(iii)	$\cos\theta + 2\sin\theta = 2.15$ $\sqrt{5}\cos(\theta - 63.43) = 2.15$ $(\theta - 63.43) = \cos^{-1}\left(\dfrac{2.15}{\sqrt{5}}\right)$ $\theta = 79.4$ or 47.5		[2]	
		Total	**[10]**	