Name ______________________________( ) Class:______

CHIJ KATONG CONVENT
PRELIMINARY EXAMINATION 2018
SECONDARY 4 EXPRESS / 5 NORMAL (ACADEMIC)

ADDITIONAL MATHEMATICS 4047/01
PAPER 1

Duration: 2 hours

Classes: 403, 405, 406, 502

READ THESE INSTRUCTIONS FIRST

Write your name, class and registration number on all the work you hand in.
Write in dark blue or black pen on both sides of the paper.
You may use an HB pencil for any diagrams or graphs.
Do not use staples, paper clips, glue or correction fluid/tape.

Answer **all** the questions.
Write your answers on the separate Answer Paper provided.
Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.
The use of an approved scientific calculator is expected, where appropriate.
You are reminded of the need for clear presentation in your answers.
Omission of essential working will result in loss of marks.

There are two sections in this paper.
At the end of the examination, fasten sections *A* and *B* separately.

The number of marks is given in brackets [ ] at the end of each question or part question.
The total number of marks for this paper is 80.

Mathematical Formulae

1. ALGEBRA

Quadratic Equation

For the equation $ax^2 + bx + c = 0$,

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Binomial expansion

$$(a+b)^n = a^n + \binom{n}{1} a^{n-1} b + \binom{n}{2} a^{n-2} b^2 + \ldots\ldots + \binom{n}{r} a^{n-r} b^r + \ldots\ldots + b^n ,$$

where n is a positive integer and $\binom{n}{r} = \frac{n!}{r!(n-r)!} = \frac{n(n-1)\ldots\ldots(n-r+1)}{r!}$

2. TRIGONOMETRY

Identities

$$\sin^2 A + \cos^2 A = 1$$

$$\sec^2 A = 1 + \tan^2 A$$

$$\operatorname{cosec}^2 A = 1 + \cot^2 A$$

$$\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$$

$$\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$$

$$\tan(A \pm B) = \frac{\tan A \pm \tan B}{1 \mp \tan A \tan B}$$

$$\sin 2A = 2 \sin A \cos A$$

$$\cos 2A = \cos^2 A - \sin^2 A = 2\cos^2 A - 1 = 1 - 2\sin^2 A$$

$$\tan 2A = \frac{2\tan A}{1 - \tan^2 A}$$

Formulae for ΔABC

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$a^2 = b^2 + c^2 - 2bc \cos A$$

$$\Delta = \frac{1}{2} bc \sin A$$

Answer **all** questions.

Section A

1 A metal cube with sides $2x$ mm is heated. The sides are expanding at a rate of 0.05 mm/s. Calculate the rate of change of the total surface area of the cube when $x = 0.57$ mm. [3]

2 Without using a calculator, find the integer value of a and of b for which the solution of the equation $2x\sqrt{5} = x\sqrt{2} + \sqrt{18}$ is $\dfrac{\sqrt{a}+b}{3}$. [4]

3 The equation of a curve is $y = \dfrac{3x^2}{\sqrt{4x-h}}$.
Given that the x-coordinate of the stationary point is 1, find the value of h. [4]

4 The roots of the quadratic equation $8x^2 - 49x + c = 0$ are $\dfrac{2\alpha}{\beta}$ and $\dfrac{2\beta}{\alpha}$.

(i) Show that $c = 32$. [1]

(ii) Given that $\alpha\beta = 4$, find two distinct quadratic equations whose roots are α and β. [4]

5 Given that $y = \dfrac{2 - 3\sec^2 2x}{\tan^2 2x + 1}$,

(i) express y in the form $\cos 4x + k$, [2]

(ii) sketch the graph of $|y|$ for $-\dfrac{\pi}{2} \le x \le \pi$ and state the value of n when $|y| = n$ has four solutions. [3]

6 The polynomial $\text{f}(x) = px^3 + 3x^2 + qx - 6$ is divisible by $x^2 + x - 6$.

(i) Find the value of p and of q. [4]

(ii) Find the remainder in terms of x when f(x) is divided by $x^2 - 1$. [2]

7 Given the equation $\frac{2}{\sin^2\theta} = 5 - \cot\theta$ where $0° < \theta < 360°$, find

(i) the values of θ, [4]

(ii) the exact values of cos θ. [2]

8 **(i)** Express $\frac{2x-1}{x^2(x+1)}$ in partial fractions. [4]

(ii) Hence, determine $\int \frac{2x-1}{x^2(x+1)}\,dx$. [2]

Section B

Begin this section on a new sheet of writing paper.

9 Given the curve $y = (m+1)x^2 - 8x + 3m$ has a minimum value, find the range of values of m

(i) for which the line $y = m - 4mx$ meets the curve, [5]

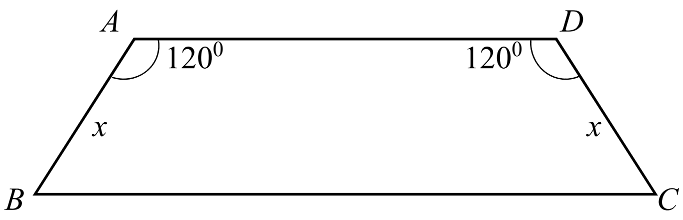
(ii) for which the y-intercept of the curve is greater than $-\frac{5}{2}$. [2]

10 **(i)** Solve the equation $3\log_{27}\left[\log_{1000}(x^2+9) - \log_{1000} x\right] = -1$. [3]

(ii) **(a)** On the same axes, sketch the graphs of $y = \log_{\frac{1}{2}} x - 1$ and $y = \log_2 x + 1$. [2]

(b) Explain why the two graphs are symmetrical about the x-axis. [2]

11



A piece of wire of length 80 cm is bent into the shape of a trapezium *ABCD*.
$AB = CD = x$ cm and angle $BAD =$ angle $ADC = 120°$.

(i) Show that the area of the trapezium *ABCD* is given by $\frac{\sqrt{3}}{2}x(40-x)$ cm^2. [4]

(ii) Given that x can vary, find the value of x for which the area has a stationary value. [2]

(iii) Determine whether this stationary value is a maximum or a minimum. [2]

12 A particle moves in a straight line so that its velocity, v m/s, is given by $v = 2 - \frac{18}{(t+2)^2}$ where t is the time in seconds, after leaving a fixed point O.
Its displacement from O is 9 m when it is at instantaneous rest.

Find

(i) the value of t when it is at instantaneous rest, [2]

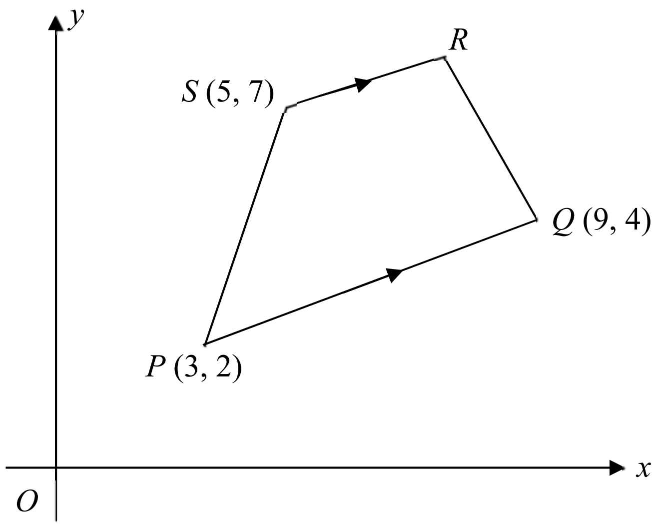
(ii) the distance travelled during the first 4 seconds. [4]

At $t = 7$, the particle starts with a new velocity, V m/s, given by $V = -h(t^2 - 7t) + k$.

(iii) Find the value of k. [1]

(iv) Given that the deceleration is 0.9 m/s^2 when $t = 8$, find the value of h. [2]

13 Solutions to this question by accurate drawing will not be accepted.



In the diagram, *PQ* is parallel to *SR* and the coordinates of *P*, *Q* and *S* are (3, 2), (9, 4) and (5, 7) respectively.

The gradient of the line *OR* is 1.

Find

(i) the coordinates of *R*, [4]

(ii) the area of the quadrilateral *PQRS*, [2]

(iii) the coordinates of the point *H* on the line $y = 1$ which is equidistant from *P* and *Q*. [4]

End of Paper

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ADDITIONAL MATHEMATICS **4047/01**
PAPER 1
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Solutions for students

1 A metal cube with sides $2x$ mm is heated. The sides are expanding at a rate of 0.05 mm/s.

Calculate the rate of change of the total surface area of the cube when $x = 0.57$ mm. [3]

Solution

Let $l = 2x$

Area $A = 6l^2$

$$\frac{dA}{dt} = \frac{dA}{dl} \times \frac{dl}{dt}$$

$$= 12l \times 0.05$$

$$= 12(2(0.57)) \times 0.05$$

$$= 0.684$$

chain rule

Answer: 0.684 mm^2/s.

OR

Most students applied this method but used 0.05 wrongly for $\frac{dx}{dt}$.

Some students used wrong formula for SA.

Area $A = 6(2x)^2 = 24x^2$

$$\frac{dA}{dt} = \frac{dA}{dx} \times \frac{dx}{dt}$$

$$= 48x \times 0.025$$

$$= 48(0.57) \times 0.025$$

$$= 0.684$$

chain rule

Answer: 0.684 mm^2/s.

2 Without using a calculator, find the integer value of *a* and of *b* for which the solution of the equation $2x\sqrt{5} = x\sqrt{2} + \sqrt{18}$ is $\dfrac{\sqrt{a}+b}{3}$. [4]

Solution

$$x\left(2\sqrt{5}-\sqrt{2}\right)=\sqrt{18}$$

$$x=\frac{\sqrt{18}}{2\sqrt{5}-\sqrt{2}}\times\frac{2\sqrt{5}+\sqrt{2}}{2\sqrt{5}+\sqrt{2}}$$

conjugate surds

$$=\frac{2\sqrt{90}+6}{18}$$

$$=\frac{6\sqrt{10}+6}{18}$$

$$=\frac{\sqrt{10}+1}{3}$$

$a = 10$, $b = 1$

OR

A handful used this method but did not reject one answer/ did not know why one of the answers is not acceptable.

$$\left(2x\sqrt{5}\right)^2=\left(x\sqrt{2}+\sqrt{18}\right)^2$$

$$20x^2=2x^2+2\sqrt{36}x+18$$

$$18x^2-12x-18=0$$

$$3x^2-2x-3=0$$

$$x=\frac{2+\sqrt{4-4(2)(-3)}}{2(3)}$$

$$=\frac{1+\sqrt{10}}{3}\text{ or }\frac{1-\sqrt{10}}{3}\text{(reject)}$$

$a=10, b=1$

3 The equation of a curve is $y=\dfrac{3x^2}{\sqrt{4x-h}}$.

Given that the *x*-coordinate of the stationary point is 1, find the value of *h*. [4]

Solution

$$\frac{dy}{dx}=\frac{\sqrt{4x-h}\,(6x)-3x^2\left(\frac{1}{2}\right)(4x-h)^{-\frac{1}{2}}(4)}{4x-h}$$

quotient OR product rule

$$=\frac{(4x-h)^{-\frac{1}{2}}\left[(6x)(4x-h)-6x^2\right]}{4x-h}$$

$$=\frac{18x^2-6hx}{(4x-h)^{\frac{3}{2}}}$$

At stationary point, $\dfrac{dy}{dx}=0$.

When $x = 1$, $\dfrac{18(1)^2-6h(1)}{(4(1)-h)^{\frac{3}{2}}}=0$

$h = 3$

4 The roots of the quadratic equation $8x^2 - 49x + c = 0$ are $\dfrac{2\alpha}{\beta}$ and $\dfrac{2\beta}{\alpha}$.

(i) Show that $c = 32$. [1]

(ii) Given that $\alpha\beta = 4$, find two distinct quadratic equations whose roots are α and β. [4]

<u>Solution</u>

(i)

$$\left(\frac{2\alpha}{\beta}\right)\left(\frac{2\beta}{\alpha}\right) = \frac{c}{8}$$

$$4 = \frac{c}{8}$$

$$c = 32$$

(ii)

$$\frac{2\alpha}{\beta} + \frac{2\beta}{\alpha} = \frac{49}{8}$$ SOR

$$\frac{2\alpha^2 + 2\beta^2}{\alpha\beta} = \frac{49}{8}$$

$$\frac{2\alpha^2 + 2\beta^2}{4} = \frac{49}{8}$$

$$\alpha^2 + \beta^2 = \frac{49}{4}$$

$$(\alpha+\beta)^2 - 2\alpha\beta = \frac{49}{4}$$ apply perfect square

$$(\alpha+\beta)^2 - 8 = \frac{49}{4}$$

$$(\alpha+\beta)^2 = \frac{81}{4}$$

$$\alpha + \beta = \pm\frac{9}{2}$$

Eqns are $2x^2 - 9x + 8 = 0$, $\quad 2x + 9x + 8 = 0$.

both eqns, accept fractional coefficients

5 Given that $y = \dfrac{2 - 3\sec^2 2x}{\tan^2 2x + 1}$,

(i) express y in the form $\cos 4x + k$, [2]

(ii) sketch the graph of $|y|$ for $-\frac{\pi}{2} \le x \le \pi$ and state the value of n when $|y| = n$ has four solutions. [3]

Solution

(i) $\frac{2-3\sec^2 2x}{\tan^2 2x+1} = \frac{2-3\sec^2 2x}{\sec^2 2x}$

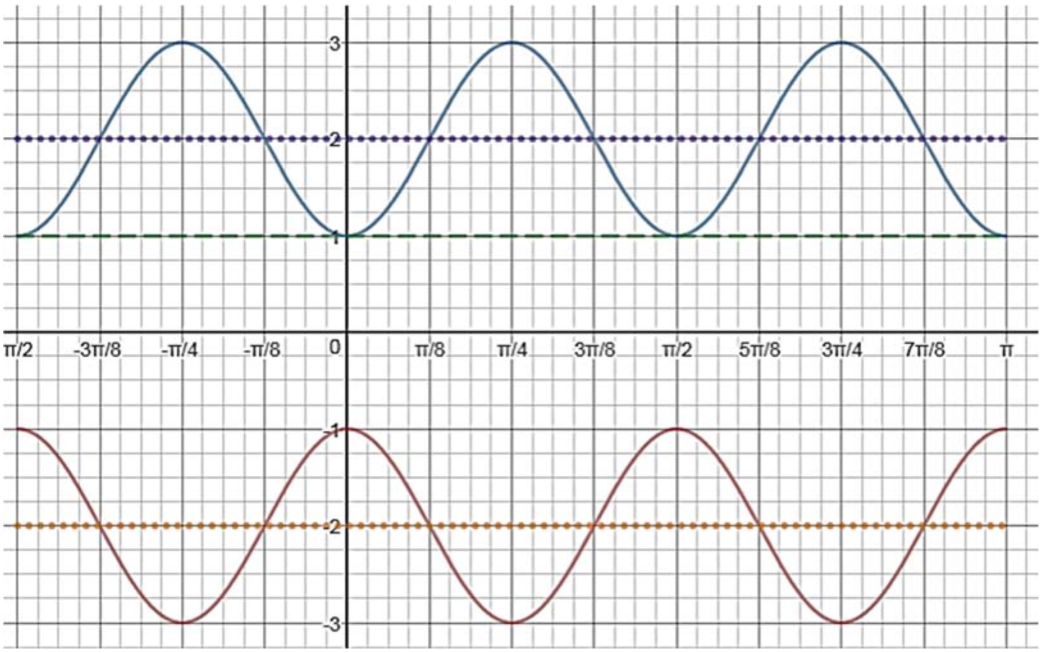
$= 2\cos^2 2x - 3$

$= 2\cos^2 2x - 1 - 2$

$= \cos 4x - 2$

(iii) graph

$n = 1$



6 The polynomial $f(x) = px^3 + 3x^2 + qx - 6$ is divisible by $x^2 + x - 6$.

(i) Find the value of p and of q. [4]

(ii) Find the remainder in terms of x when $f(x)$ is divided by x^2-1. [2]

Solution

(i) $x^2+x-6=(x-2)(x+3)$

By the factor thm, $f(2)=0$

$p(2)^3+3(2)^2+q(2)-6=0$ factor thm

$8p+2q+6=0$

$4p+q=-3$ ....(1)

$f(-3)=0$

$p(-3)^3+3(-3)^2+q(-3)-6=0$ factor thm

$-27p-3q+21=0$

$9p+q=7$ ....(2)

Solve (1) and (2); $p=2,\ q=-11$

OR

$px^3+3x^2+qx-6=(x-2)(x+3)(px+1)$

(ii) Using $x^2=1$,

$f(x)=2x^3+3x^2-11x-6$

$=2x^2(x)+3x^2-11x-6$

$=2x+3-11x-6$

$=-9x-3$

OR long division (ecf)

Many used this method.

$$\begin{array}{r} 2x+3 \\ x^2-1\overline{)\,2x^3+3x^2-11x-6} \\ \underline{2x^3 \qquad\quad -2x} \\ 3x^2-9x-6 \\ \underline{3x^2 \qquad\ -3} \\ -9x-3 \end{array}$$

7 Given the equation $\dfrac{2}{\sin^2\theta}=5-\cot\theta$ where $0^0<\theta<360^0$, find

(i) the values of θ. [4]

(ii) the exact values of $\cos\theta$. [2]

Solution

(i) $2\operatorname{cosec}^2\theta=5-\cot\theta$

$2(1+\cot^2\theta)-5+\cot\theta=0$ identity

$2\cot^2\theta+\cot\theta-3=0$

$(2\cot\theta+3)(\cot\theta-1)=0$ factorisation

$\cot\theta=-\dfrac{3}{2}$ or $\cot\theta=1$

$\tan\theta=-\dfrac{2}{3}$ or $\tan\theta=1$

Basic angle = 33.69^0 , 45^0

$\theta=146.3^0, 326.3^0, 45^o, 225^0$

OR

$5\sin^2\theta-\sin\theta\cos\theta-2=0$

which is common to many but at the same time spells the end of qn 7.

$5\sin^2\theta-\sin\theta\cos\theta-2(\cos^2\theta+\sin^2\theta)=0$

$3\sin^2\theta-\sin\theta\cos\theta-2\cos^2\theta=0$

$(3\sin\theta+2\cos\theta)(\sin\theta-\cos\theta)=0$

$\tan\theta=-\dfrac{2}{3}$ or $\tan\theta=1$

(ii) $\tan\theta=-\dfrac{2}{3}$ (quadrants 2, 4) or $\tan\theta=1$ (quadrants 1, 3)

$\cos\theta = \pm\frac{3}{\sqrt{13}}$, $\cos\theta = \pm\frac{1}{\sqrt{2}}$

8 **(i)** Express $\frac{2x-1}{x^2(x+1)}$ in partial fractions. [4]

(ii) Hence, determine $\int \frac{2x-1}{x^2(x+1)}\,\mathrm{d}x$. [2]

Solution

(i) $\frac{2x-1}{x^2(x+1)} = \frac{A}{x} + \frac{B}{x^2} + \frac{C}{x+1}$ correct factors

$2x-1 = Ax(x+1) + B(x+1) + Cx^2$

Let $x = -1$, $-3 = C(-1)^2$ ==> $C = -3$ or comparing coeff.

Let $x = 0$, $B = -1$

Let $x = 1$, $1 = 2A - (2) - 3(1)^2$ ==> $A = 3$

Hence, $\frac{2x-1}{x^2(x+1)} = \frac{3}{x} - \frac{1}{x^2} - \frac{3}{x+1}$

(ii) $\int \frac{2x-1}{x^2(x+1)}\,\mathrm{d}x = \int\left(\frac{3}{x} - \frac{1}{x^2} - \frac{3}{x+1}\right)\mathrm{d}x$

9 Given the curve $y = (m+1)x^2 - 8x + 3m$ has a minimum value, find the range of values of m

(i) for which the line $y = m - 4mx$ meets the curve, [5]

(ii) for which y – intercept of the curve is greater than $-\frac{5}{2}$. [2]

Solution

(i) $(m+1)x^2 - 8x + 3m = m - 4mx$

$(m+1)x^2 + 4mx - 8x + 2m = 0$ quadratic eqn

$b^2 - 4ac \geq 0$

$(4m-8)^2 - 4(m+1)(2m) \geq 0$ discriminant, inequality

$(4(m-2))^2 - 8m(m+1) \geq 0$

$2(m^2 - 4m + 4) - m^2 - m \geq 0$ expansion, simplify

$m^2 - 9m + 8 \geq 0$

$(m-1)(m-8) \geq 0$ factorisation

$m \leq 1$ or $m \geq 8$

Since it is a minimum graph, $m + 1 > 0$, ie $m > -1$

So $-1 < m \leq 1$ or $m \geq 8$

(ii) At y – intercept, $x = 0$,

$$(m+1)x^2 - 8x + 3m > -\frac{5}{2}$$

$$m > -\frac{5}{6}$$

10 **(i)** Solve the equation $3\log_{27}\left[\log_{1000}(x^2+9) - \log_{1000} x\right] = -1$. [3]

Solution

$$\log_{1000}\frac{x^2+9}{x} = 27^{-\frac{1}{3}}$$

$$\log_{1000}\frac{x^2+9}{x} = \frac{1}{3}$$ index form

$$\frac{x^2+9}{x} = 1000^{\frac{1}{3}}$$

$$x^2 + 9 = 10x$$ index form

$$x^2 - 10x + 9 = 0$$

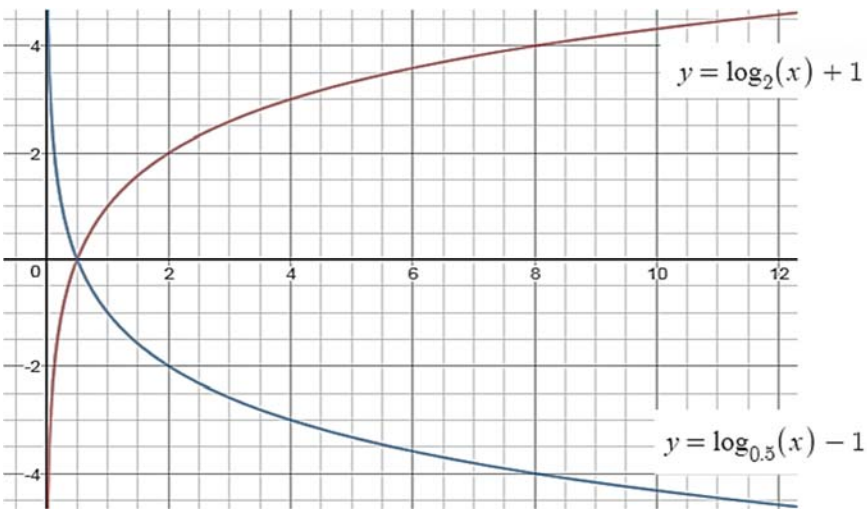
$$(x-1)(x-9) = 0$$

$$x = 1 \text{ or } 9$$

(ii) **(a)** On the same axes, sketch the graphs of $y = \log_{\frac{1}{2}} x - 1$ and $y = \log_2 x + 1$. [2]

(b) Explain why the two graphs are symmetrical about the x-axis. [2]

Solution



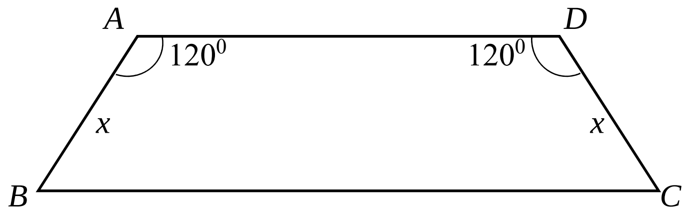
(ii) $-(\log_{\frac{1}{2}} x - 1) = -\dfrac{\log_2 x}{\log_2 \frac{1}{2}} + 1$ [M1]

$= -\dfrac{\log_2 x}{\log_2 2^{-1}} + 1$

$= \log_2 x + 1$

The functions are negative of each other. [A1]

11



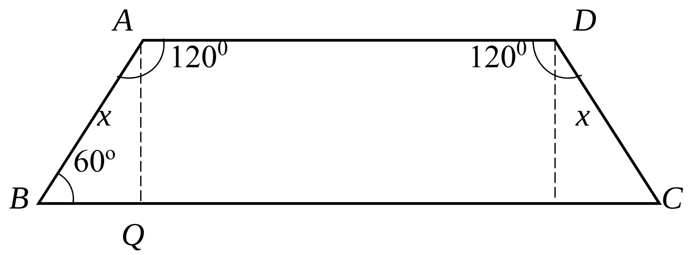
A piece of wire of length 80 cm is bent into the shape of a trapezium *ABCD*.
$AB = CD = x$ cm and angle BAD = angle $ADC = 120°$.

(i)

Show that the area of the trapezium *ABCD* is given by $\dfrac{\sqrt{3}}{2}x(40-x)$ cm^2. [4]

(ii) Given that x can vary, find the value of x for which the area has a stationary value. [2]

(iii) Determine whether this stationary value is a maximum or a minimum. [2]



Solution

$\angle ABC = 180 - 120 (\text{int.}\angle s, AD // BC)$

$= 60^0$

(i) $\cos\angle ABC = \dfrac{BQ}{x}$

$BQ = \dfrac{x}{2}$

Perimeter $= BC + 2x + AD$

$80 = \dfrac{x}{2} + AD + \dfrac{x}{2} + x + AD + x$

$AD = \dfrac{80-3x}{2}$

$\sin 60° = \dfrac{AQ}{x}$ ➔ $AQ = \dfrac{\sqrt{3}}{2}x$

$\text{Area} = \dfrac{1}{2}(AD+BC)\left(\dfrac{\sqrt{3}}{2}x\right)$

OR (Most used this method)

$AD + BC = 80 - 2x$

$\angle ABC = 180 - \angle BAD$ (int.angles, $AD // BC$)

$= 60°$

AQ = height of the trapezium

$\sin 60 = \dfrac{AQ}{x}$

$AQ = \dfrac{\sqrt{3}}{2}x$

$\text{Area} = \dfrac{1}{2}\left(\dfrac{\sqrt{3}}{2}x\right)(AD+BC)$

$= \dfrac{1}{2}\left(\dfrac{\sqrt{3}}{2}x\right)(80-2x)$

$= \dfrac{\sqrt{3}}{2}x(40-x)$ (shown)

$= \frac{1}{4}(80 - 3x + x)\sqrt{3}x$

$= \frac{\sqrt{3}}{4}x(80 - 2x)$

$= \frac{\sqrt{3}}{2}x(40 - x)$ (Shown)

(ii) $\frac{dA}{dx} = 0$ when the area has a stationary value

$20\sqrt{3} - \frac{\sqrt{3}}{2}(2x) = 0$ differentiation

$x = 20$

(iii) $\frac{d^2A}{dx^2} = -\sqrt{3} < 0$. second derivative or using first derivative

Area is a maximum

12 A particle moves in a straight line so that its velocity, v m/s, is given by $v = 2 - \frac{18}{(t+2)^2}$ where t is the time in seconds, after leaving a fixed point O.

Its displacement from O is 9 m when it is at instantaneous rest.

(i) the value of t when it is at instantaneous rest, [2]

(ii) the distance travelled during the first 4 seconds. [4]

At $t = 7$, the particle starts with a new velocity, V ms^{-1}, given by $V = -h(t^2 - 7t) + k$.

(iii) Find the value of k. [1]

(iv) Given that the deceleration is 1.9 m/s^2 when $t = 8$, find the value of h. [2]

Solution

(i) At turning pt, $v = 0$

$2 - \frac{18}{(t+2)^2} = 0$

$t = 1$ or -5 (NA)

(ii)

$s = \int \frac{dv}{dt} dt \quad = 2t + \frac{18}{t+2} + c$

When $t = 1$, $s = 9$

$2(1) + \frac{18}{1+2} + c = 9$

$c = 1$, so $\quad s = 2t + \frac{18}{t+2} + 1$

When $t = 0$, s = 10 m
When $t = 1$, $s = 9$ m
When $t = 4$, $s = 12$ m

Total distance travelled = 10 - 9 +12 – 9 = 4 m

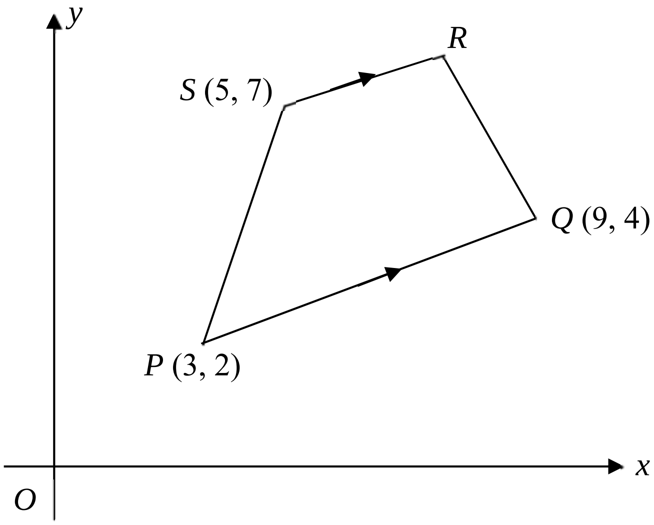
(iii) When $t = 7$, $v = 2 - \frac{18}{(7+2)^2} = \frac{16}{9}$

$$V = -h(t-7) + k = \frac{16}{9}, \text{ hence } k = \frac{16}{9}$$

(iv) $V = -h(t^2 - 7t) + k = -ht^2 + 7ht + k$

$$a = \frac{\mathrm{d}V}{\mathrm{d}t} = -2ht + 7h$$
$$-2h(8) + 7h = -0.9$$
$$-16h + 7h = -0.9$$
$$-9h = -0.9$$
$$h = 0.1$$

13 Solutions to this question by accurate drawing will not be accepted.



In the diagram, PQ is parallel to SR and the coordinates of P, Q and S are (3, 2), (9, 4) and (5, 7) respectively. The gradient of the line OR is 1.

Find

(i) the coordinates of R, [4]

(ii) the area of the quadrilateral $PQRS$, [2]

(iii) the coordinates of the point H on the line $y = 1$ which is equidistant from P and Q. [4]

Solution

(i) $m_{PQ} = \frac{1}{3}$

Since PQ // SR, $m_{SR} = \frac{1}{3}$

Eqn of SR, $(y-7)=\frac{1}{3}(x-5)$ $==>$ $y=\frac{x}{3}+\frac{16}{3}$

Sub. $R(a, a)$ into $y=\frac{x}{3}+\frac{16}{3}$, $a=8$ OR use eqn of OR as $y=x$

$\therefore R=(8,\ 8)$

(ii) Area of $PQRS=\frac{1}{2}\begin{vmatrix}3 & 9 & 8 & 5 & 3\\ 2 & 4 & 8 & 7 & 2\end{vmatrix}$ [M1]

$=\frac{1}{2}(39)=19.5\text{ units}^2$ [A1]

(iii) Since the point H lies on the line $y=1$ and is equidistant from P and Q, H must lie on the $\perp$ bisector of PQ.

Mid-point of $PQ=(6, 3)$

gradient of $\perp$ bisector $=-3$.

Equation, $(y-3)=-3(x-6)$

$y=-3x+21$

Since $y=1$,

$1=-3x+21,\quad x=6\frac{2}{3}$

$\therefore H(6\frac{2}{3},\ 1)$

OR

$PH=QH$

$\sqrt{(2-1)^2+(3-x)^2}=\sqrt{(4-1)^2+(9-x)^2}$ using length

$1+9-6x+x^2=9+81-18x+x^2$ expansion

$12x=80$

$x=\frac{20}{3}$

$H=(\frac{20}{3},1)$

Name ______________________________( ) Class:______

CHIJ KATONG CONVENT
PRELIMINARY EXAMINATION 2018
SECONDARY 4 EXPRESS/
5 NORMAL (ACADEMIC)

ADDITIONAL MATHEMATICS **4047/02**

PAPER 2 **Duration: 2 hours 30 minutes**

Classes: 403, 405, 406, 502

READ THESE INSTRUCTIONS FIRST

Write your name, class and registration number on all the work you hand in.
Write in dark blue or black pen on both sides of the paper.
You may use an HB pencil for any diagrams or graphs.
Do not use staples, paper clips, glue or correction fluid/tape.

Answer **all** the questions.
Write your answers on the separate Answer Paper provided.
Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.
The use of an approved scientific calculator is expected, where appropriate.
You are reminded of the need for clear presentation in your answers.
Omission of essential working will result in loss of marks.

At the end of the examination, fasten all your work securely together.
The number of marks is given in brackets [ ] at the end of each question or part question.

The total number of marks for this paper is 100.

Mathematical Formulae

1. ALGEBRA

Quadratic Equation

For the equation $ax^2 + bx + c = 0$,

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Binomial expansion

$$(a+b)^n = a^n + \binom{n}{1}a^{n-1}b + \binom{n}{2}a^{n-2}b^2 + \ldots\ldots + \binom{n}{r}a^{n-r}b^r + \ldots\ldots + b^n,$$

where n is a positive integer and $\binom{n}{r} = \frac{n!}{r!(n-r)!} = \frac{n(n-1)\ldots\ldots(n-r+1)}{r!}$

2. TRIGONOMETRY

Identities

$$\sin^2 A + \cos^2 A = 1$$

$$\sec^2 A = 1 + \tan^2 A$$

$$\operatorname{cosec}^2 A = 1 + \cot^2 A$$

$$\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$$

$$\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$$

$$\tan(A \pm B) = \frac{\tan A \pm \tan B}{1 \mp \tan A \tan B}$$

$$\sin 2A = 2 \sin A \cos A$$

$$\cos 2A = \cos^2 A - \sin^2 A = 2\cos^2 A - 1 = 1 - 2\sin^2 A$$

$$\tan 2A = \frac{2\tan A}{1 - \tan^2 A}$$

Formulae for ΔABC

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$a^2 = b^2 + c^2 - 2bc \cos A$$

$$\Delta = \frac{1}{2}bc \sin A$$

Name: ____________________________ (   ) Class: ______

1 A rectangular garden, with length x m and breadth y m, has an area of 270 m^2. It has a path of width 2.5 m all round it. Given that the outer perimeter of the path is 87 m, find the length and breadth of the garden. [5]

2 **(a)** Solve $2\left(9^{x-1}\right)-5\left(3^{x}\right)=27$. [4]

(b) Given that $f(x)=\ln(5x-2)^{3}$,

(i) State the range of x for $f(x)$ to be defined. [1]

(ii) Show that $5f'(x)+(5x-2)f''(x)=0$. [4]

3 **(a)** **(i)** Write down the first four terms in the expansion of $(1+x)^{50}$ and $(1-x)^{50}$.

Hence, write down the first two terms for $(1+x)^{50}-(1-x)^{50}$. [3]

(ii) Without the use of calculator, deduce if 1.01^{50} or $1^{50}+0.99^{50}$ is larger. [3]

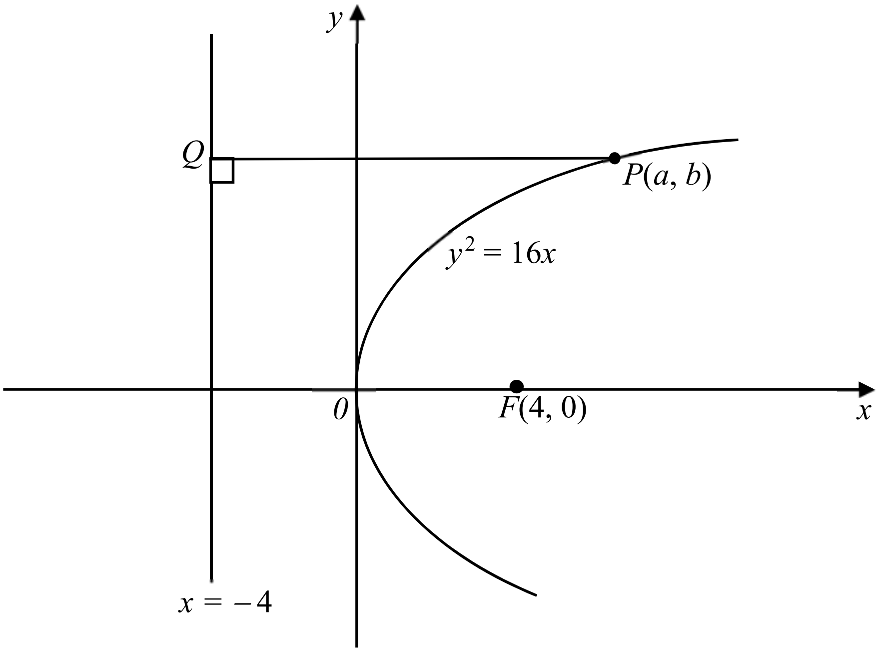
(b) The term independent of x in $x^{11}\left(2x+\dfrac{k}{x^{2}}\right)^{7}$ is 896.

Find the two possible values of k. [4]

4 **(i)** Prove that $\tan A+\cot A=\dfrac{2}{\sin 2A}$. [4]

(ii) Hence, or otherwise, solve $\tan A+\cot A=2.5$ for $0°<A<270°$. [4]

5 In the diagram, not drawn to scale, $P(a, b)$ is a point on the graph $y^2 = 16x$, and Q is a point on the line $x = -4$. PQ is the perpendicular distance from P to this line. $F(4, 0)$ is a point on the x-axis.



(i) Find the length PF in terms of a. [3]

(ii) Given that the tangent to the curve at P cuts the y-axis at G, find the coordinates of G in terms of a. [4]

(iii) Show that G is the the mid-point of QF. [2]

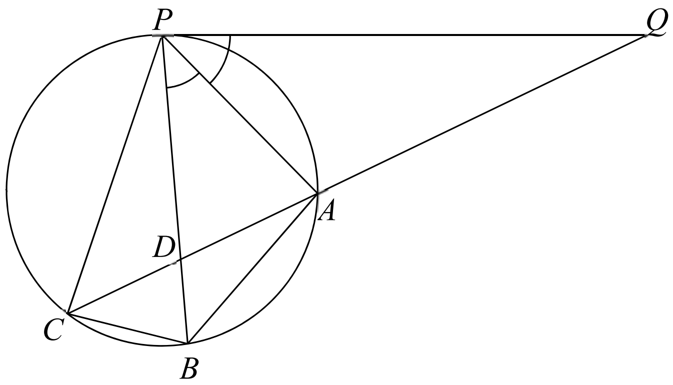
(iv) Find the equation of the normal at P in terms of a. [2]

6 **(a)** Evaluate $\int_0^{\frac{\pi}{6}} \sin\left(2x + \frac{\pi}{6}\right) dx$, leaving your answer in surd form. [3]

(b) **(i)** Find $\frac{d}{dx}\left[e^{2x}\left(\cos 3x + \frac{3}{2}\sin 3x\right)\right]$. [4]

(ii) Hence find $\int e^{2x} \cos 3x \; dx$. [2]

7 The diagram shows a point P on a circle and PQ is a tangent to the circle. Points A, B and C lie on the circle such that PA bisects angle QPB and QAC is a straight line. The lines QC and PB intersect at D.



(i) Prove that $AP = AB$. [4]

(ii) Prove that CD bisects angle PCB. [4]

(iii) Prove that triangles CDP and CBA are similar. [2]

8 The table below shows experimental values of two variables x and y obtained from an experiment.

x	1	2	3	4	5	6
y	5.1	17.5	37.5	60.5	98	137

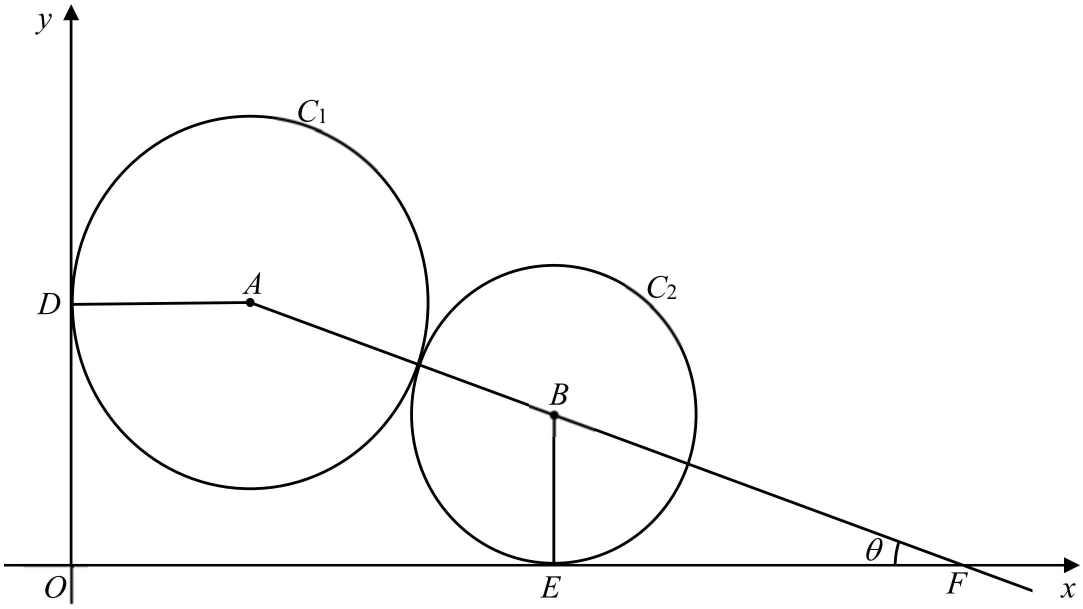
It is also given that x and y are related by the equation $y = ax + bx^2$, where a and b are constants.

(i) Plot $\frac{y}{x}$ against x and draw a straight line graph. Use 2 cm to represent 1 unit on the horizontal axis and 4 cm to represent 10 units on the vertical axis. [4]

(ii) Use the graph to estimate the value of a and of b. [2]

(iii) By drawing a suitable straight line, estimate the value of x for which $(b+5)x = 38 - a$. [4]

9 The figure below shows two circles, C_1 and C_2, touching each other in the first quadrant of the Cartesian plane. C_1 has radius 5 and touches the y-axis at D. C_2 has radius 4 and touches the x-axis at E. The line AB joining the centre of C_1 and C_2, meets the x-axis at F. Angle BFO is θ.



(i) Find expressions for OD and OE in terms of θ and show that

$$DE^2 = 122 + 90\cos\theta + 72\sin\theta .$$ [3]

(ii) Hence express DE^2 in the form $122 + R\cos(\theta - \alpha)$, where R > 0 and α is acute. [3]

(iii) Calculate the greatest possible length of DE and state the corresponding value of θ. [3]

10 The population of a town is estimated to increase by k % per year. The population at the end of 2017 was 20000. The population, y, after x years can be modelled by $y = A(1.11)^x$.

(i) Deduce the value of A and of k with the information provided. [2]

(ii) Sketch the graph of y. [1]

(iii) Find the value of x when $y = 9600$.
Explain the meaning of this value of x. [3]

(iv) Calculate the population of the town at the end of 2027. [2]

11 Given that $y = 2x^3 + 3x^2 + 11x + 5$,

(i) show that

(a) y is an increasing function for all values of x, [2]

(b) y has only one real root at $x = -\frac{1}{2}$. [3]

(ii) sketch the graph of y, [2]

(iii) hence, calculate the area bounded by $y = 2x^3 + 3x^2 + 11x + 5$, the x-axis and the lines $x = -1$ and $x = 1$. [4]

End of paper

1	$xy = 270$ $y = \frac{270}{x}$ .... (1) $2(x+5+y+5) = 87$ $x + y = \frac{67}{2}$ .... (2) Substitute (1) into (2), $x + \frac{270}{x} = \frac{67}{2}$ $2x^2 - 67x + 540 = 0$ $(2x-27)(x-20) = 0$ $2x - 27 = 0$ or $x - 20 = 0$ $x = 13.5$ or $x = 20$ When x = 13.5, y = 20 When x = 20, y = 13.5 Since x is the length, then x = 20 m and y = 13.5 m.
2a	$2\left(3^{2x} \bullet \frac{1}{9}\right) - 5\left(3^x\right) = 27$ Let 3^x, $\frac{2}{9}y^2 - 5y - 27 = 0$ $2y^2 - 45y - 243 = 0$ $(2y+9)(y-27) = 0$ $y = -\frac{9}{2}$ or $y = 27$ $3^x = -\frac{9}{2}$ (rejected) or $3^x = 3^3$ $\therefore x = 3$
2bi	$5x - 2 > 0$ $x > \frac{2}{5}$
	$f'(x) = \frac{3(5x-2)^2 \bullet 5}{(5x-2)^3}$ $= \frac{15}{5x-2}$

	OR $f'(x)=\frac{3\bullet 5}{(5x-2)}$ $=\frac{15}{5x-2}$ $f''(x)=-\frac{15}{(5x-2)^2}\bullet 5$ $=-\frac{75}{(5x-2)^2}$ $\therefore 5f'(x)+(5x-2)f''(x)$ $=\frac{75}{5x-2}-\frac{75}{5x-2}$ $=0$ (shown)
3ai	$(1+x)^{50}=1^{50}+50x+{}^{50}C_2x^2+{}^{50}C_3x^3+...+x^{50}$ $=1+50x+1225x^2+19600x^3+...+x^{50}$ $(1-x)^{50}=1-50x+1225x^2-19600x^3+...-x^{50}$ $(1+x)^{50}-(1-x)^{50}=100x+39200x^3$
ii	Let $x = 0.01$, $1.01^{50}-0.99^{50}=100(0.01)+39200(0.01)^3$ $=1+0.0392$ $1.01^{50}=1+0.0392+0.99^{50}$ $>1+0.99^{50}$ Hence, 1.01^{50} is larger.
3b	$T_{r+1}={}^7C_r(2x)^{7-r}\left(\frac{k}{x^2}\right)^r$ $={}^7C_r2^{7-r}k^rx^{7-3r}$ For $7-3r=-11$ $r=6$

	OR $x^{11}T_{r+1} = {}^7C_r\left(2x\right)^{7-r}\left(\frac{k}{x^2}\right)^r x^{11}$ $= {}^7C_r 2^{7-r} k^r x^{18-3r}$ For $18 - 3r = 0$ $r = 6$ Term independent of $x = 896$ $x^{11}\left(2x+\frac{k}{x^2}\right)^7 = 896$ ${}^7C_6 2^{7-6} k^6 = 896$ $k^6 = 64$ $k = \pm 2$ Alternative method: $x^{11}\left(2x+\frac{k}{x^2}\right)^7$ $= x^{11}\left(\begin{array}{l} 2^7x^7 + 7(2x)^6\left(\frac{k}{x^2}\right) + {}^7C_2(2x)^5\left(\frac{k}{x^2}\right)^2 \\ + {}^7C_3(2x)^4\left(\frac{k}{x^2}\right)^3 + {}^7C_4(2x)^3\left(\frac{k}{x^2}\right)^4 \\ + {}^7C_5(2x)^5\left(\frac{k}{x^2}\right)^2 + {}^7C_6(2x)^6\left(\frac{k}{x^2}\right) \\ + \left(\frac{k}{x^2}\right)^7 \end{array}\right)$ $= x^{11}\left(\begin{array}{l} 2^7x^7 + 7\left(2^6\right)kx^4 + {}^7C_2 2^5 k^2 x + {}^7C_3 2^4 k^3 x^{-2} \\ + {}^7C_4 2^3 k^4 x^{-5} + {}^7C_5 2^2 k^5 x^{-5} + {}^7C_6 2k^6 x^{-11} + k^7 x^{-15} \end{array}\right)$ Term independent term of $x = 896$ $896 = x^{11}\left({}^7C_6 2k^6 x^{-11}\right)$ $896 = 14k^6$ $k^6 = 64$ $k = \pm 2$

4i	$\text{LHS} = \tan A + \cot A$ $= \dfrac{\sin A}{\cos A} + \dfrac{\cos A}{\sin A}$ $= \dfrac{\sin^2 A + \cos^2 A}{\sin A \cos A}$ $= \dfrac{1}{\frac{1}{2}(2\sin A\cos A)}$ $= \dfrac{2}{\sin 2A}$ $= \text{RHS} \quad \text{(shown)}$ OR $\text{LHS} = \tan A + \cot A$ $= \tan A + \dfrac{1}{\tan A}$ $= \dfrac{\tan^2 A + 1}{\tan A}$ $= \dfrac{\sec^2 A}{\tan A}$ $= \dfrac{1}{\cos^2 A} \bullet \dfrac{\cos}{\sin A}$ $= \dfrac{1}{\sin A \cos A}$ $= \dfrac{2}{2\sin A\cos A}$ $= \dfrac{2}{\sin 2A}$ $= \text{RHS} \quad \text{(shown)}$
4ii	$\dfrac{2}{\sin 2A} = \dfrac{5}{2}$ $\sin 2A = \dfrac{4}{5}$ $\alpha = 53.13°$ $2A = 53.13°, 126.87°, 413.13°, 486.67°$ $A = 26.6°, 63.4°, 206.6°, 243.4°$

5i	$y^2 = 16x$ At P, $b^2 = 16a$ $\text{PF} = \sqrt{(a-4)^2 + b^2}$ $= \sqrt{a^2 - 8a + 16 + 16a}$ $= \sqrt{(a+4)^2}$ $= a + 4$
ii	$y^2 = 16x$ $y = 4\sqrt{x}$ $\frac{\mathrm{d}y}{\mathrm{d}x} = \frac{2}{\sqrt{x}}$ At P, $\frac{\mathrm{d}y}{\mathrm{d}x} = \frac{2}{\sqrt{a}}$ Equation of tangent at P, $y - b = \frac{2}{\sqrt{a}}(x - a)$ $y = \frac{2x}{\sqrt{a}} - 2\sqrt{a} + 4\sqrt{a}$ $= \frac{2x}{\sqrt{a}} + 2\sqrt{a}$ When $x = 0$, $y = 2\sqrt{a}$ $\therefore$ G (0, $2\sqrt{a}$)
iii	Mid-point of QF $= \left(\frac{-4+4}{2}, \quad \frac{b+0}{2}\right)$ $= \left(0, \quad \frac{4\sqrt{a}}{2}\right)$ $= \left(0, \quad 2\sqrt{a}\right)$ Hence, G lies in the centre of QF. OR find lengths of QG and GP.

iv	Gradient of normal at P = $-\frac{\sqrt{a}}{2}$ Equation of normal at P: $y-b=-\frac{\sqrt{a}}{2}(x-a)$ $y=-\frac{\sqrt{a}}{2}x+\frac{a\sqrt{a}}{2}+4a$
6a	$\int_0^{\frac{\pi}{6}}\sin\left(2x+\frac{\pi}{6}\right)dx$ $=\left[-\frac{\cos\left(2x+\frac{\pi}{6}\right)}{2}\right]_0^{\frac{\pi}{6}}$ $=-\frac{\cos\frac{\pi}{2}}{2}-\left(-\frac{\cos\frac{\pi}{6}}{2}\right)$ $=0+\frac{\sqrt{3}}{4}$ $=\frac{\sqrt{3}}{4}$
6bi	$\frac{d}{dx}\left[e^{2x}\left(\cos 3x+\frac{3}{2}\sin 3x\right)\right]$ $=2e^{2x}\left(\cos 3x+\frac{3}{2}\sin 3x\right)+e^{2x}\left(-3\sin 3x+\frac{9}{2}\cos 3x\right)$ $=e^{2x}\left(2\cos 3x+3\sin 3x-3\sin 3x+\frac{9}{2}\cos 3x\right)$ $=\frac{13}{2}e^{2x}\cos 3x$
6bii	$\int e^{2x}\cos 3x\ dx=\frac{2}{13}\int\frac{13}{2}e^{2x}\cos 3x\ dx$ $=\frac{2}{13}e^{2x}\left(\cos 3x+\frac{3}{2}\sin 3x\right)+C$
7i	$\measuredangle ABP=\measuredangle APQ$ (alt. segment theorem) Since PA bisects ∡QPB, $\measuredangle APQ=\measuredangle APB$ $\therefore \measuredangle ABP=\measuredangle APB$ (base ∡s of isosceles triangle APB) Hence, AP = AB.

7ii

$\measuredangle ACB = \measuredangle APB$ (∡s in the same segemnt)

$\measuredangle ACP = \measuredangle ABP$ (∡s in the same segemnt)

$= \measuredangle APB$ (shown)

∡ACB=∡ACP

Hence, CD bisects $\measuredangle PCB$.

7iii

$\measuredangle ACB = \measuredangle ACP$ (from ii)

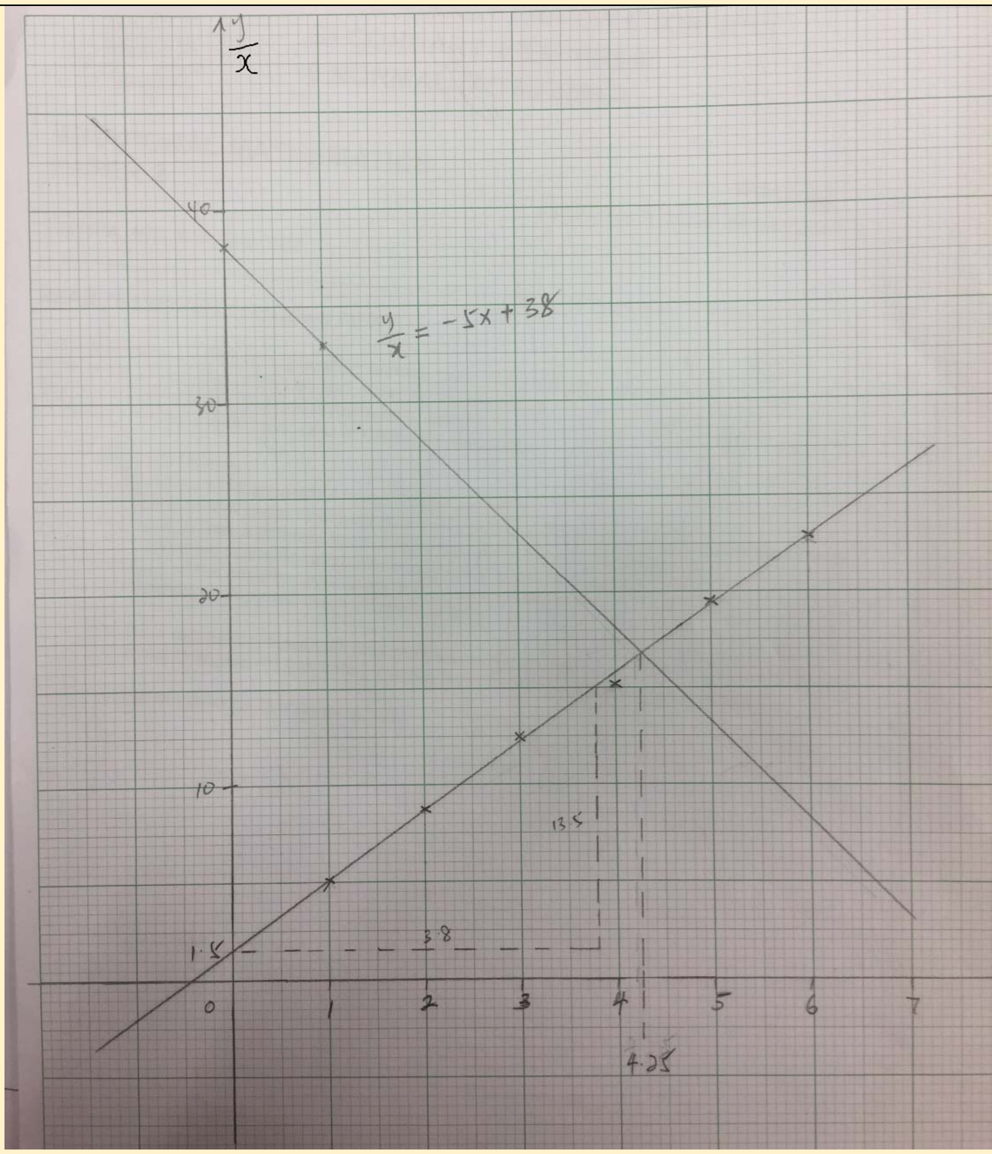
$\measuredangle CPD = \measuredangle CAB$ (∡s in the same segemnt)

Hence, ΔCDX and ΔCBA are similar.

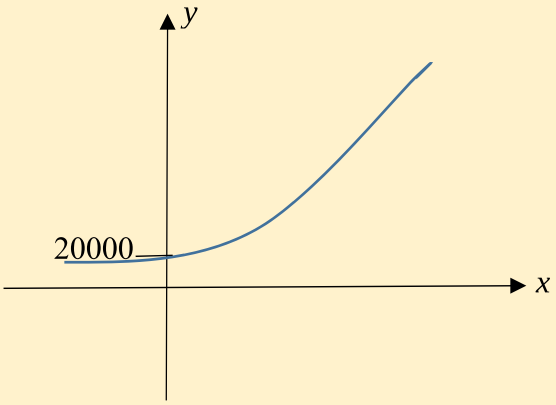
8i

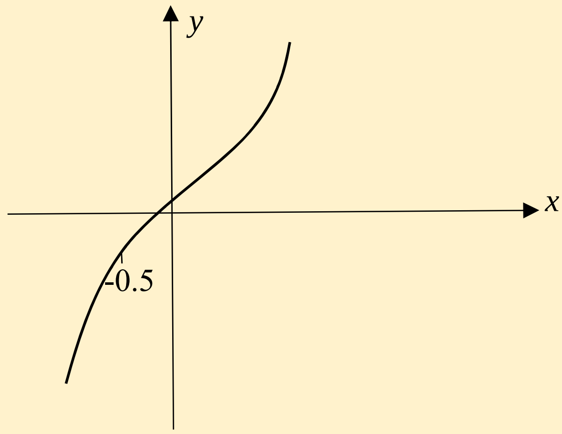
$\frac{y}{x} = bx + a$

x	1	2	3	4	5	6
y/x	5.1	8.75	12.5	15.13	19.6	22.83



ii	$a = \frac{y}{x} - \text{intercept}$ $= 1.5$ $b = gradient$ $= \frac{13.5}{3.8}$ $= 3.55$
iii	$(b+5)x = 38 - a$ $bx + 5x = 38 - a$ $bx + a = 38 - 5x$ Draw $\frac{y}{x} = 38 - 5x$, at point of intersection, x = 4.25
9i	$OE = 5 + 9\cos\theta$ $OD = 4 + 9\sin\theta$ $DE^2 = OE^2 + OD^2$ $= (5 + 9\cos\theta)^2 + (4 + 9\sin\theta)^2$ $= 25 + 90\cos\theta + 81\cos^2\theta$ $+16 + 72\sin\theta + 81\sin^2\theta$ $= 41 + 81 + 90\cos\theta + 72\sin\theta$ $= 122 + 90\cos\theta + 72\sin\theta$
ii	Let $90\cos\theta + 72\sin\theta = R\cos(\theta - \alpha)$. $R = \sqrt{90^2 + 72^2}$ $= \sqrt{13284}$ $= 115$ (3 s.f.) $\theta = \tan^{-1}\frac{72}{90}$ $= 38.65°$ $DE^2 = 122 + 115\cos(\theta - 38.7°)$ OR $122 + \sqrt{13284}\cos(\theta - 38.7°)$

iii	DE is greatest when $\cos(\theta - 38.7°) = 1$ $DE = \sqrt{122 + 115}$ $= 15.4$ units (3 s.f.) Corresponding θ is $38.7°$.
10i	A = 20000, k = 11
ii	
iii	When y = 9600, $9600 = 20000(1.11)^x$ $x = \lg\frac{9600}{20000} \div \lg 1.11$ $= -7.03$ (3 s.f.) The population of the town was 9600 approximately 7 years ago.
iv	When x = 10, $y = 20000(1.11)^{10}$ $= 56788$ The population of the town would be 56788 (or 56800) at the end of 2027.
11i	$y = 2x^3 + 3x^2 + 11x + 5$ $\frac{dy}{dx} = 6x^2 + 6x + 11$ $= 6\left(x + \frac{1}{2}\right)^2 + \frac{19}{2}$ $\frac{dy}{dx} > 0$ as $\left(x + \frac{1}{2}\right)^2 \geq 0$ for all values of x, hence y is an increasing function for all values of x.

ii	Using long division, $y=(2x+1)(x^2+x+5)$ But for x^2+x+5 , discriminant = -19 < 0, hence x^2+x+5 has no real roots. Therefore, y has only one real root at $x=-\frac{1}{2}$.
iii	
iv	Area required $=\int_{-1}^{1} y \text{ dx}$ $=\left\lvert\int_{-1}^{-0.5} 2x^3+3x^2+11x+5 \text{ dx}\right\rvert$ $+\int_{-0.5}^{1} 2x^3+3x^2+11x+5 \text{ dx}$ $=\left\lvert\frac{x^4}{2}+x^3+\frac{11}{2}+5x\right\rvert_{-1}^{-0.5}+\left[\frac{x^4}{2}+x^3+\frac{11}{2}+5x\right]_{-0.5}^{1}$ $=\left\lvert-\frac{39}{32}\right\rvert+\left[12-\left(-\frac{39}{32}\right)\right]$ $=14\frac{7}{16}$ *or* $=14.4$ sq. units (3 s.f.)