Name:	Register No.:	Class:

CRESCENT GIRLS' SCHOOL
SECONDARY FOUR
PRELIMINARY EXAMINATION

ADDITIONAL MATHEMATICS

4047/01

Paper 1

16 August 2018

2 hours

Additional Materials:	Answer Paper Mark Sheet

READ THESE INSTRUCTIONS FIRST

Write your name, register number and class in the spaces provided at the top of this page and on all the work you hand in.
Write in dark blue or black pen.
You may use a soft pencil for any diagrams or graphs.
Do not use paper clips, highlighter, glue or correction fluid.

Answer **all** the questions.
Write your answers on the separate Answer Paper provided.
Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.
The use of an electronic calculator is expected, where appropriate.
You are reminded of the need for clear presentation in your answers.

At the end of the examination, fasten all your work and mark sheet securely together.
The number of marks is given in brackets [ ] at the end of each question or part question.
The total number of marks for this paper is 80.

Mathematical Formulae

1. ALGEBRA

Quadratic Equation

For the equation $ax^2 + bx + c = 0$,

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Binomial expansion

$$(a+b)^n = a^n + \binom{n}{1}a^{n-1}b + \binom{n}{2}a^{n-2}b^2 + \ldots + \binom{n}{r}a^{n-r}b^r + \ldots + b^n,$$

where n is a positive integer and $\binom{n}{r} = \frac{n!}{r!(n-r)!} = \frac{n(n-1)\ldots(n-r+1)}{r!}$

2. TRIGONOMETRY

Identities

$$\sin^2 A + \cos^2 A = 1$$

$$\sec^2 A = 1 + \tan^2 A$$

$$\operatorname{cosec}^2 A = 1 + \cot^2 A$$

$$\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$$

$$\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$$

$$\tan(A \pm B) = \frac{\tan A \pm \tan B}{1 \mp \tan A \tan B}$$

$$\sin 2A = 2 \sin A \cos A$$

$$\cos 2A = \cos^2 A - \sin^2 A = 2\cos^2 A - 1 = 1 - 2\sin^2 A$$

$$\tan 2A = \frac{2\tan A}{1 - \tan^2 A}$$

Formulae for ΔABC

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$a^2 = b^2 + c^2 - 2bc\cos A$$

$$\Delta = \frac{1}{2}ab\sin C$$

1 The straight line $y-1=2m$ does not intersect the curve $y=x+\frac{m^2}{x}$.
Find the largest integer value of m. [5]

2 The line $2y+x=5$ intersects the curve $y^2=6-xy$ at the points P and Q.
Determine, with explanation, if the point $(1, 2)$ lies on the line joining the midpoint of PQ and $(3, 1)$. [5]

3 **(i)** Sketch on the same graph $y=|3\cos 2x|$ and $y+\frac{8}{3\pi}x=2$ for $0\le x\le \pi$. [3]

(ii) Hence, showing your working clearly, deduce the number of solutions in $|\cos 2x|-\frac{2}{3}+\frac{8x}{3\pi}=0$ in the interval $0\le x\le\pi$. [2]

4 **(i)** Find the value of a and of b if the curve $f(x)=ax+\frac{b}{x}$ where $x\ne 0$ has a stationary point at $(-2, -8)$. [4]

(ii) By considering the sign of $f'(x)$, determine the nature of the stationary point. [2]

5 It is given that $\int f'(x)\,dx=\frac{x}{2}-\frac{\sin kx}{8}+c$ where c is a constant of integration, and that $\int_0^{\frac{\pi}{8}} f'(x)\,dx=\frac{\pi}{16}-\frac{1}{8}$.

(i) Show that $k=4$. [2]

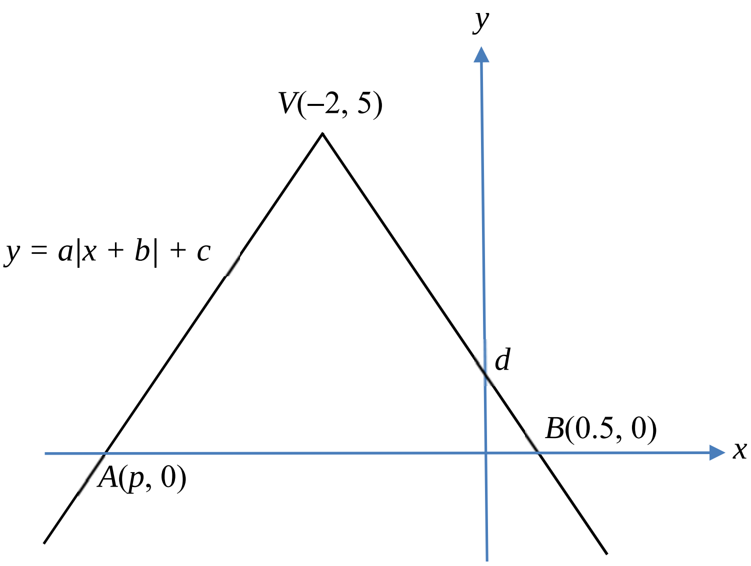
(ii) Hence find $f'(x)$, expressing your answer in $\sin^2 px$, where p is a constant. [2]

(iii) Find the equation of the curve $y=f(x)$ given that the point $\left(\frac{\pi}{4}, 0\right)$ lies on the curve. [2]

6 **(a)** The length of each side of a square of area $\left(49+20\sqrt{6}\right)$ m^2 can be expressed in the form $\left(\sqrt{c}+\sqrt{d}\right)$ m where c and d are integers and $c<d$.
Find the value of c and of d. [3]

(b) A parallelogram with base equals to $\left(4-\sqrt{12}\right)$ m has an area of $\left(22-\sqrt{48}\right)$ m^2.
Find, without using a calculator, the height of the parallelogram in the form $\left(p+q\sqrt{3}\right)$ m. [3]

7 The diagram shows part of the graph $y=a\,|\,x+b\,|+c$. The graph cuts the x-axis at A (p, 0) and at B (0.5, 0). The graph has a vertex point at V (−2, 5) and y-intercept, d.



(i) Explain why $p=-4.5$. [1]

(ii) Determine the value of each of a, b and c. [4]

(iii) State the set of values of k for which the line $y=kx+d$ intersects the graph at two distinct points. [2]

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(ii) Hence find $\int \frac{x^2 \ln x}{2}\,dx$. [4]

9 **(a)** If $32^y \times 5^{4y} = 2^{4y+4} \times 5^{3y-1}$, determine the value of 10^y. [3]

(b) **(i)** Sketch on the same axes, the graphs of $y = x^{-2}$ and $y = \sqrt{3x}$. [2]

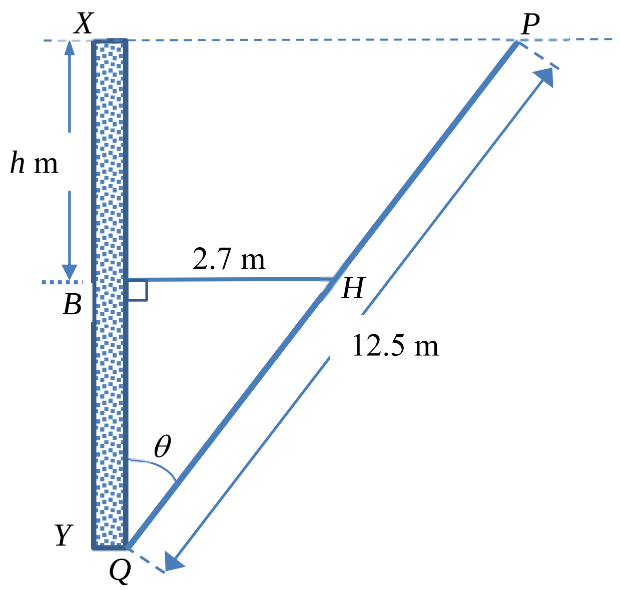
(ii) Find the point of intersection between the graphs. [3]

10 **(i)** Express $\dfrac{x+1}{x(x+3)^2-(x+3)^2}$ in partial fractions. [5]

(ii) Hence find the value of $\int_2^3 \dfrac{x+1}{x(x+3)^2-(x+3)^2}\,dx$ giving your answer to 2 decimal places. [3]

11 **(a)** Show that $\dfrac{d}{d\theta}(\cot\theta) = -\dfrac{1}{\sin^2\theta}$. [2]

(b) In the diagram below, a straight wooden plank *PQ*, of length 12.5 m is supported at an angle θ to a vertical wall *XY* by a taut rope fixed to a hook at *H*.
The length of the rope *BH* from the wall is 2.7 m. The end *P* of the plank is at a vertical height *h* m above *H*.



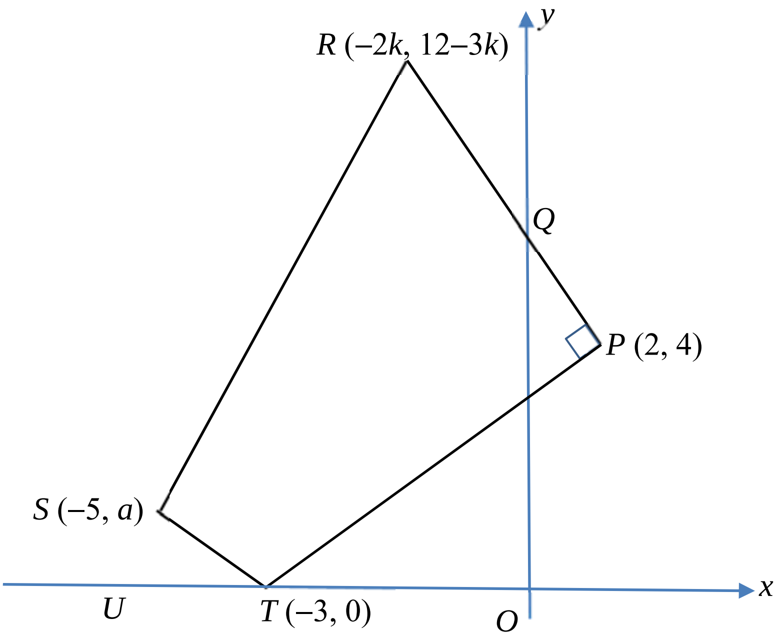
(i) Show that $h = 12.5\cos\theta - \dfrac{2.7\cos\theta}{\sin\theta}$. [2]

(ii) Using part (a), determine the value of $\sin\theta$ for which $\dfrac{dh}{d\theta} = 0$. [2]

(iii) Hence or otherwise, show that as θ varies, *h* attains a maximum value and find this value. [3]

Solutions to this question by accurate drawing will not be accepted.

12 The figure shows a quadrilateral *PTSR* for which *P* is (2, 4), *T* is (−3, 0), *S* is (−5, *a*), *R* is (−2*k*, 12−3*k*) and angle *QPT* is a right angle. *RQP* is a straight line with point *Q* lying on the *y*-axis.



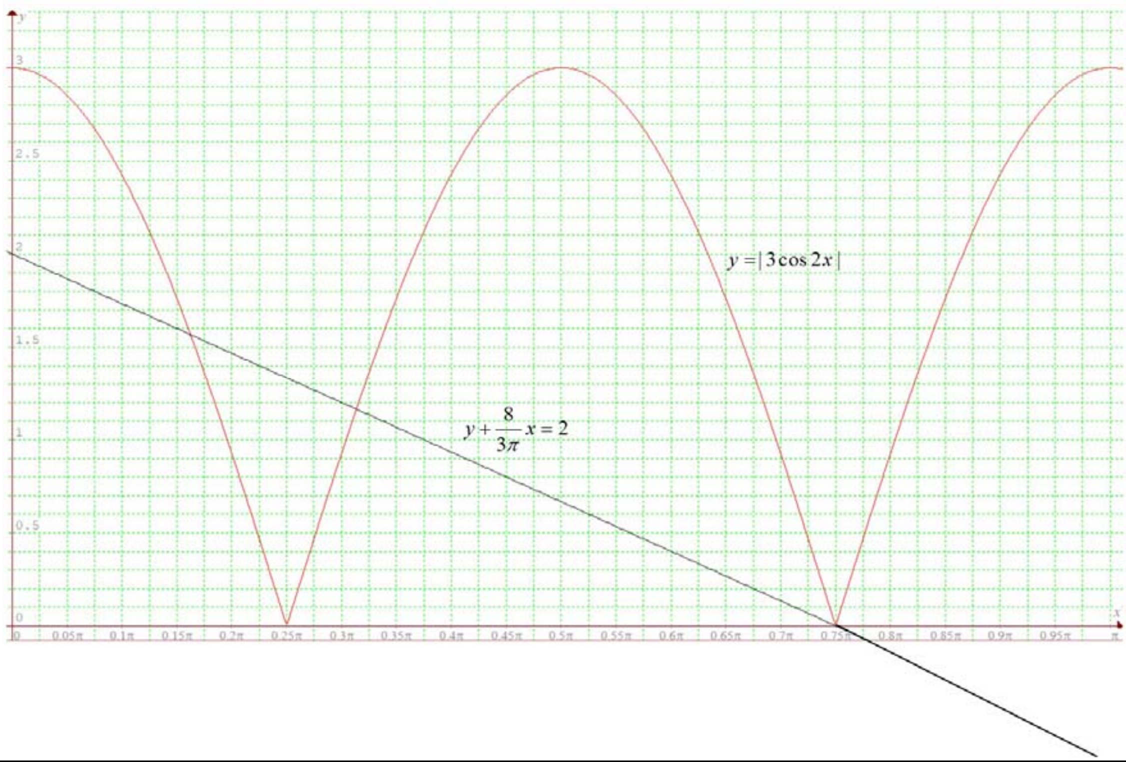
(i) Find the value of *k*. [2]

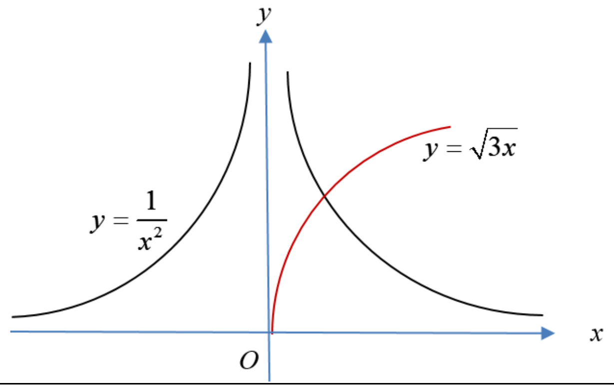
(ii) Given that angle *STU* = 45°, determine the value of *a*. [2]

(iii) A line passing through *Q* and is perpendicular to *TS* cuts the *x*-axis at *V*. Find the value of VR^2. [5]

END OF PAPER

2018 CGS A Math Prelim Paper 1 Answer Key

Qn	Ans Key
1	$m=-1$
2	Yes
3(i)	
3(ii)	1 solution
4(i)	$a=2$; $b=8$
4(ii)	Maximum point
5(ii)	$\sin^2 2x$
5(iii)	$f(x)=\frac{x}{2}-\frac{\sin 4x}{8}-\frac{\pi}{8}$
6(a)	$c=24$; $d=25$
6(b)	$h=(16+7\sqrt{3})\,\text{m}$
7(ii)	$a=-2$; $b=2$; $c=5$
7(iii)	$-2<k<2$
8(i)	$x^2+3x^2\ln x$
8(ii)	$\frac{1}{6}x^3\ln x-\frac{1}{18}x^3+c$
9(a)	3.2

9(b)(i)	
9(b)(ii)	(0.803, 1.55)
10(i)	$\frac{1}{8(x-1)}-\frac{1}{8(x+3)}+\frac{1}{2(x+3)^2}$
10(ii)	0.08
11(b)(ii)	$\sin\theta=\frac{3}{5}$
11(b)(iii)	$h=6.4$ m
12(i)	$k=1$
12(ii)	$a=2$
12(iii)	101.25

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$$\Delta = \frac{1}{2}ab\sin C$$

1 The straight line $y-1=2m$ does not intersect the curve $y=x+\dfrac{m^2}{x}$.

Find the largest integer value of m. [5]

Solutions

$y=2m+1 \quad --(1)$

$y=x+\dfrac{m^2}{x} \quad --(2)$

$(1)=(2)$: $x+\dfrac{m^2}{x}=2m+1$ [M1]

$x^2-2mx-x+m^2=0$

$x^2-(2m+1)x+m^2=0$ [M1] -- simplification

Line does not intersect curve, $b^2-4ac<0$

$[-(2m+1)]^2-4(1)(m^2)<0$ [M1]

$(2m+1+2m)(2m+1-2m)<0$

$4m+1<0$

$m<-\dfrac{1}{4}$ [A1]

The largest integer value of m is -1. [A1]

2 The line $2y+x=5$ intersects the curve $y^2=6-xy$ at the points P and Q.
Determine, with explanation, if the point $(1, 2)$ lies on the line joining the midpoint of PQ and $(3, 1)$. [5]

Solutions

$x=5-2y$ ---- (1)

Sub (1) into $y^2=6-xy$

$y^2=6-(5-2y)y$ [M1] – Substitution

$y^2-5y+6=0$

$(y-3)(y-2)=0$

Hence $y=3$ or $y=2$ [A1]

Correspondingly, $x=5-2(3)$ or $x=5-2(2)$

$x=-1$ or $x=1$

The coordinates of P and Q are $(-1, 3)$ and $(1, 2)$.

Midpoint of $PQ=\left(\dfrac{-1+1}{2}, \dfrac{3+2}{2}\right)=(0, 2.5)$ [A1]

Equation of line joining midpoint of PQ and $(3, 1)$ is $\dfrac{y-1}{2.5-1}=\dfrac{x-3}{0-3}$ [M1]

$y=-\dfrac{1}{2}x+\dfrac{5}{2}$

When $x=1$, $y=-\dfrac{1}{2}(1)+\dfrac{5}{2}=2$

Therefore, the point $(1, 2)$ lies on the line joining midpoint of PQ and $(3, 1)$ [A1] – *conclusion*

Alternative method

Let R be the coordinates of the midpoint of PQ, S be the point $(3, 1)$ and T be the point $(1, 2)$. Find gradient of RT and gradient of RS and conclude that point T lies on RS due to collinearity.

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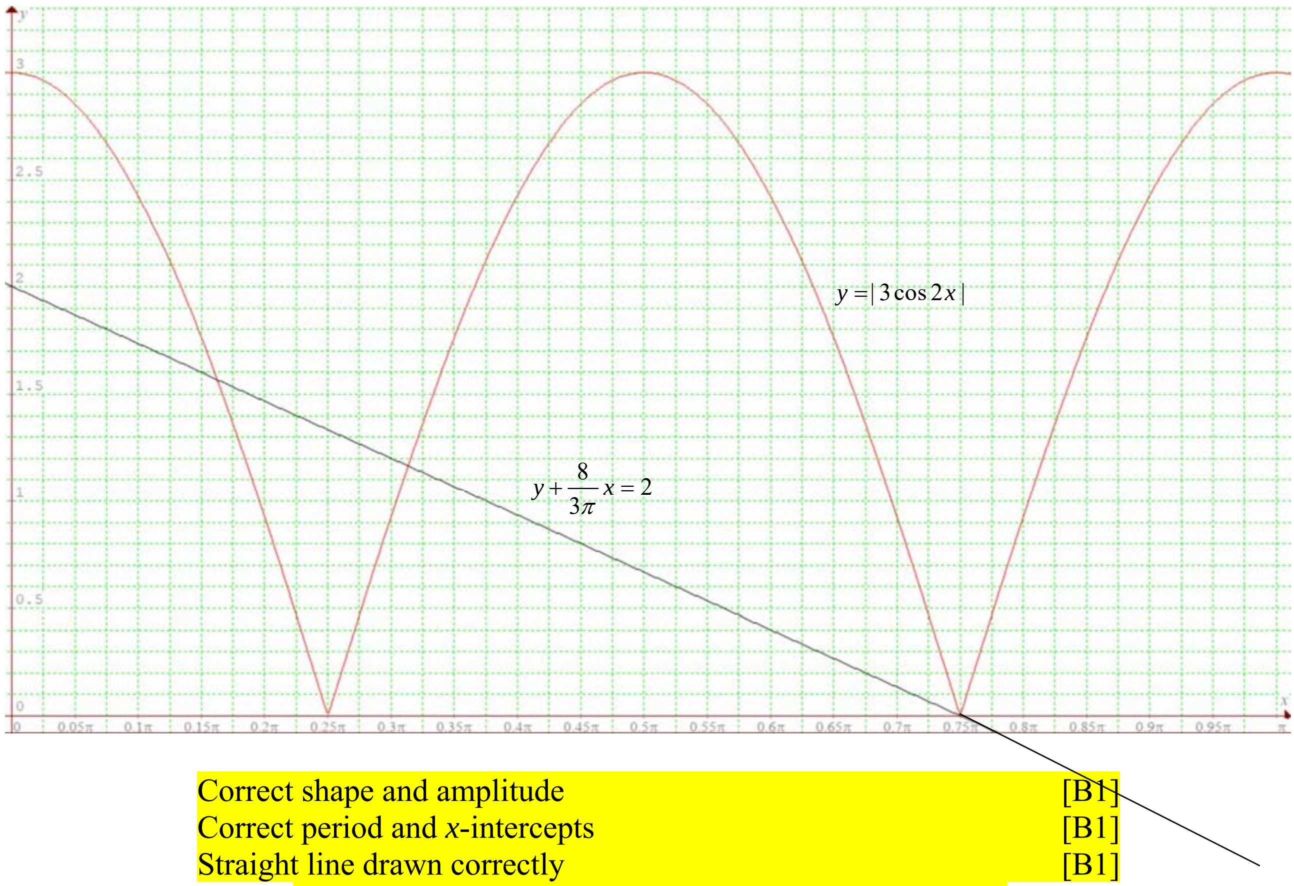
(ii) Hence, showing your working clearly, deduce the number of solutions in $|\cos 2x| - \frac{2}{3} + \frac{8x}{3\pi} = 0$ in the interval $0 \le x \le \pi$. [2]

Solutions

(i)



Correct shape and amplitude [B1]
Correct period and x-intercepts [B1]
Straight line drawn correctly [B1]

Minus 1m if eqn of graphs and/or axes are not labelled.

(ii) $|\cos 2x| - \frac{2}{3} + \frac{8x}{3\pi} = 0$

$|3\cos 2x| = 2 - \frac{8}{\pi}x$

$y = 2 - \frac{8}{\pi}x$ (y-intercept = 2; x-intercept = $2 \div \frac{8}{\pi} = \frac{\pi}{4}$) [M1] – St line NOT required

There is one solution. [A1]

4 **(i)** Find the value of a and of b if the curve $f(x) = ax + \frac{b}{x}$ where $x \neq 0$ has a stationary point at $(-2, -8)$. [4]

(ii) By considering the sign of $f'(x)$, determine the nature of the stationary point. [2]

Solutions

(i) $f(x) = ax + \frac{b}{x}$ Sub $x = -2$, $f(x) = -8$

$-8 = -2a - \frac{b}{2}$

$4a + b = 16$ ---- (1) [B1]

$f'(x) = a - \frac{b}{x^2}$. When $x = -2$, $f'(x) = 0$ [M1] – for $f'(x)$

$0 = a - \frac{b}{4} \Rightarrow b = 4a$ ---- (2)

Sub (2) into (1): $4a + 4a = 16$ [M1] – solve simultaneous equations

$a = 2$

Hence $b = 2(4) = 8$ [A1] – both correct

(ii) $f'(x) = 2 - \frac{8}{x^2}$

x	-2^-	-2	-2^+
Sign of $f'(x)$	+	0	–
Sketch of tangent	/	–	\

[M1] – First derivative test

$(-2, -8)$ is a maximum point. [A1] – Awarded only with correct first derivative test

5 It is given that $\int f'(x)\,dx = \frac{x}{2} - \frac{\sin kx}{8} + c$ where c is a constant of integration, and that $\int_0^{\frac{\pi}{8}} f'(x)\,dx = \frac{\pi}{16} - \frac{1}{8}$.

(i) Show that $k = 4$. [2]

(ii) Hence find $f'(x)$, expressing your answer in $\sin^2 px$, where p is a constant. [2]

(iii) Find the equation of the curve $y = f(x)$ given that the point $\left(\frac{\pi}{4}, 0\right)$ lies on the curve. [2]

Solutions

(i) $\int_0^{\frac{\pi}{8}} f'(x)\,dx = \frac{\pi}{16} - \frac{1}{8}$

$\frac{\frac{\pi}{8}}{2} - \frac{\sin k\left(\frac{\pi}{8}\right)}{8} = \frac{\pi}{16} - \frac{1}{8}$ [M1]

$\sin\left(\frac{k\pi}{8}\right) = 1$

$\frac{k\pi}{8} = \frac{\pi}{2}$ [A1]

$k = 4$ (shown)

(ii) $\int f'(x)\,dx = \frac{x}{2} - \frac{\sin 4x}{8} + c$

$f'(x) = \frac{1}{2} - \frac{1}{8}(4\cos 4x)$ [M1] -- Differentiation

$= \frac{1}{2} - \frac{1}{2}\cos 4x$

$= \frac{1}{2} - \frac{1}{2}(1 - 2\sin^2 2x)$

$= \sin^2 2x$ [A1] – Upon correct application of double angle formula

(iii) $\int f'(x)\,dx = f(x) = \frac{x}{2} - \frac{\sin 4x}{8} + c$

At $\left(\frac{\pi}{4}, 0\right)$, $0 = \frac{\pi}{8} - 0 + c$ [M1]

$c = -\frac{\pi}{8}$

$f(x) = \frac{x}{2} - \frac{\sin 4x}{8} - \frac{\pi}{8}$ [A1]

6 **(a)** The length of each side of a square of area $\left(49 + 20\sqrt{6}\right)$ m^2 can be expressed in the form $\left(\sqrt{c} + \sqrt{d}\right)$ m where c and d are integers and $c < d$.
Find the value of c and of d. [3]

(b) A parallelogram with base equals to $\left(4 - \sqrt{12}\right)$ m has an area of $\left(22 - \sqrt{48}\right)$ m^2.
Find, without using a calculator, the height of the parallelogram in the form $\left(p + q\sqrt{3}\right)$ m. [3]

Solutions

(a) $\left(\sqrt{c} + \sqrt{d}\right)^2 = 49 + 20\sqrt{6}$

$c + d + 2\sqrt{cd} = 49 + 20\sqrt{6}$ [M1] – correct expansion

$c + d = 49$

$d = 49 - c$ ----- (1)

$2\sqrt{cd} = 20\sqrt{6} \;\Rightarrow\; cd = 600$ ---- (2) [M1] – compare rational and irrational terms

Sub (1) into (2),

$c(49-c) = 600$

$c^2 - 49c + 600 = 0$

$(c-25)(c-24) = 0$

Since $c < d$, $c = 24$, $d = 25$ [A1] – Both correct

(b) Height $= \dfrac{22-4\sqrt{3}}{4-2\sqrt{3}} \cdot \dfrac{4+2\sqrt{3}}{4+2\sqrt{3}}$ [M1] – Rationalise denominator

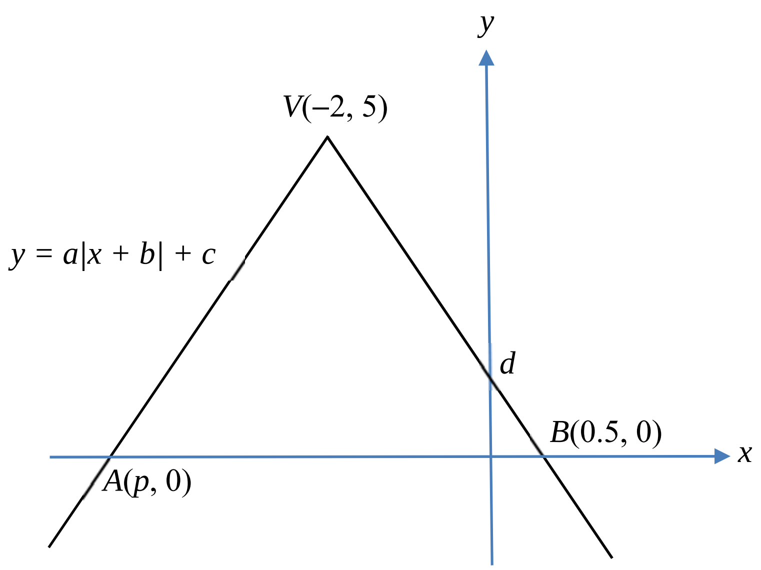
$= \dfrac{(22-4\sqrt{3})(4+2\sqrt{3})}{4^2-4(3)}$

$= \dfrac{1}{4}\left(88+44\sqrt{3}-16\sqrt{3}-24\right)$ [M1] – correct expansion

$= \dfrac{1}{4}\left(64+28\sqrt{3}\right)$

$= (16+7\sqrt{3})$ m [A1]

7 The diagram shows part of the graph $y = a\,|\,x+b\,| + c$. The graph cuts the x-axis at $A\,(p,\,0)$ and at $B\,(0.5,\,0)$. The graph has a vertex point at $V\,(-2,\,5)$ and y-intercept, d.



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(ii) Determine the value of each of a, b and c. [4]

(iii) State the set of values of k for which the line $y = kx + d$ intersects the graph at two distinct points. [2]

Solutions

(i) $\frac{p+0.5}{2}=-2$ [B1]

$p=-4.5$

(ii) y-coordinate of vertex point, c = 5 [B1]

$b=2$ [B1]

$y=a|x+b|+c$

$y=a|x+2|+5$

At B, $0=a|0.5+2|+5$ [M1]

$a=-2$ [A1]

(iii) Gradient of $AV=\frac{5-0}{-2+4.5}=2$

Gradient of $VB=-2$ [B1] – Any one

Hence $-2<k<2$ [B1]

8 **(i)** Differentiate $x^3 \ln x$ with respect to x. [2]

(ii) Hence find $\int \frac{x^2 \ln x}{2}\,dx$. [4]

Solutions

(i) $\frac{d}{dx}\left(x^3 \ln x\right)=x^3\left(\frac{1}{x}\right)+(\ln x)(3x^2)$ [M1] – Product Rule

$=x^2+3x^2\ln x$ [A1]

(ii) $\frac{d}{dx}\left(x^3 \ln x\right)=x^2+3x^2\ln x$

$\frac{x^2\ln x}{2}=\frac{1}{6}\frac{d}{dx}(x^3\ln x)-\frac{x^2}{6}$ [M1]

$\int \frac{x^2 \ln x}{2}\,dx=\frac{1}{6}x^3\ln x-\frac{1}{6}\int x^2\,dx$

[M1] [M1]

$=\frac{1}{6}x^3\ln x-\frac{1}{18}x^3+c$ [A1]

9 **(a)** If $32^y \times 5^{4y}=2^{4y+4}\times 5^{3y-1}$, determine the value of 10^y. [3]

(b) **(i)** Sketch on the same axes, the graphs of $y=x^{-2}$ and $y=\sqrt{3x}$. [2]

(ii) Find the point of intersection between the graphs. [3]

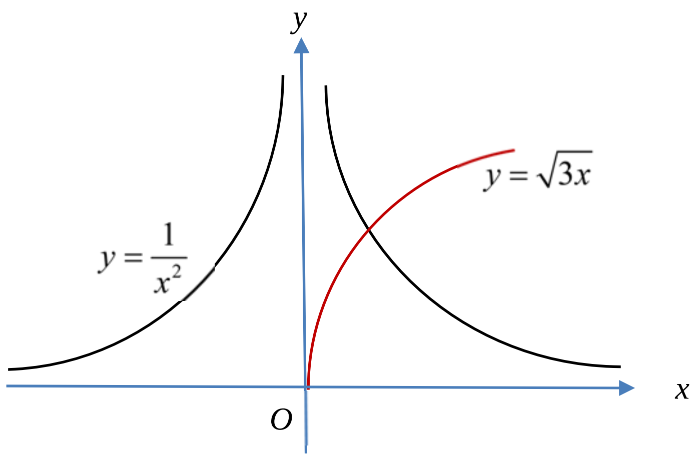
Solutions

$32^y \times 5^{4y} = 2^{4y+4} \times 5^{3y-1}$

$2^{5y} \times 5^{4y} = 2^{4y}(2^4) \times 5^{3y}\left(\frac{1}{5}\right)$ [M1] -- splitting

$\frac{2^{5y}}{2^{4y}} \times \frac{5^{4y}}{5^{3y}} = (2^4) \times \left(\frac{1}{5}\right)$ [M1] – using Laws of Indices

$2^y \times 5^y = 10^y = \frac{16}{5}$ [A1]



[B1][B1] – axes and eqns must be labelled.
Graph does not level off for $y = \sqrt{3x}$.

$\frac{1}{x^2} = \sqrt{3x}$

$\frac{1}{x^4} = 3x$ [M1] – square both sides

$x = \sqrt[5]{\frac{1}{3}} = 0.80274$ [A1]

$y = \frac{1}{0.80274^2} = 1.55$

The point of intersection is (0.803, 1.55). [A1] – 3 s.f.

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(ii) Hence find the value of $\int_2^3 \frac{x+1}{x(x+3)^2-(x+3)^2}\,dx$ giving your answer to 2 decimal places. [3]

Solutions

(i) $\frac{x+1}{x(x+3)^2-(x+3)^2} = \frac{x+1}{(x-1)(x+3)^2} = \frac{A}{x-1} + \frac{B}{x+3} + \frac{C}{(x+3)^2}$ [M1]

$x+1 = A(x+3)^2 + B(x-1)(x+3) + C(x-1)$

Sub $x=-3$: $-2=C(-4) \Rightarrow C=\frac{1}{2}$ [A1]

Sub $x=1$: $2=A(16) \Rightarrow A=\frac{1}{8}$ [A1]

Sub $x=0$: $1=\left(\frac{1}{8}\right)(9)+B(-1)(3)+\left(\frac{1}{2}\right)(-1) \Rightarrow B=-\frac{1}{8}$ [A1]

$$\frac{x+1}{x(x+3)^2-(x+3)^2}=\frac{x+1}{(x-1)(x+3)^2}=\frac{1}{8(x-1)}-\frac{1}{8(x+3)}+\frac{1}{2(x+3)^2}$$ [A1]

(ii) $\int_2^3 \frac{x+1}{x(x+3)^2-(x+3)^2}\,dx=\int_2^3 \frac{1}{8(x-1)}-\frac{1}{8(x+3)}+\frac{1}{2(x+3)^2}\,dx$

$$=\left[\frac{1}{8}\ln(x-1)-\frac{1}{8}\ln(x+3)+\frac{1}{2}\cdot\frac{(x+3)^{-1}}{-1}\right]_2^3$$

[M1] –Any [M1]

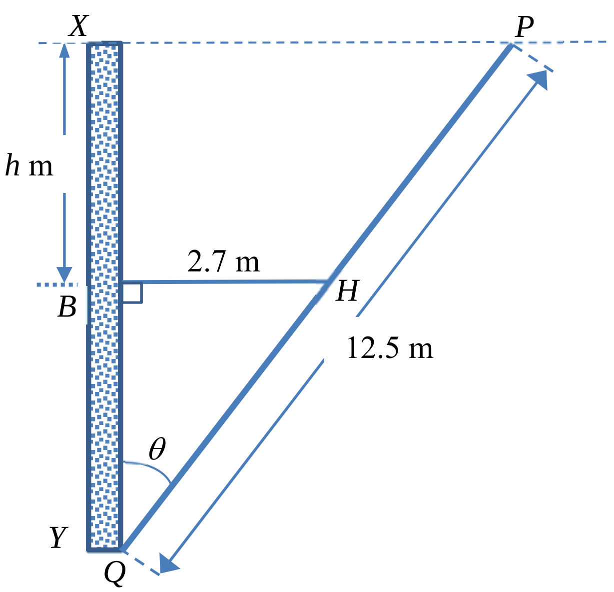
$$=\left[\frac{1}{8}\ln\left(\frac{x-1}{x+3}\right)-\frac{1}{2}\cdot\frac{1}{(x+3)}\right]_2^3$$

$$=\frac{1}{8}\ln\frac{1}{3}-\frac{1}{12}-\left(\frac{1}{8}\ln\frac{1}{5}-\frac{1}{10}\right)$$

$= 0.08$ (to 2 d.p.) [A1]

11 **(a)** Show that $\frac{d}{d\theta}(\cot\theta)=-\frac{1}{\sin^2\theta}$. [2]

(b) In the diagram below, a straight wooden plank PQ, of length 12.5 m is supported at an angle θ to a vertical wall XY by a taut rope fixed to a hook at H.
The length of the rope BH from the wall is 2.7 m. The end P of the plank is at a vertical height h m above H.



(i) Show that $h = 12.5\cos\theta - \dfrac{2.7\cos\theta}{\sin\theta}$. [2]

(ii) Using part (a), determine the value of $\sin\theta$ for which $\dfrac{dh}{d\theta} = 0$. [2]

(iii) Hence or otherwise, show that as θ varies, h attains a maximum value and find this value. [3]

Solutions

(a) $\dfrac{d}{d\theta}(\cot\theta) = \dfrac{d}{d\theta}\left(\dfrac{\cos\theta}{\sin\theta}\right)$

$= \dfrac{\sin\theta(-\sin\theta) - \cos\theta(\cos\theta)}{\sin^2\theta}$ [M1] – Quotient Rule

$= \dfrac{-(\sin^2\theta + \cos^2\theta)}{\sin^2\theta}$ [A1] -- $\sin^2\theta + \cos^2\theta = 1$

$= -\dfrac{1}{\sin^2\theta}$ (shown)

Method 2

$\dfrac{d}{d\theta}(\cot\theta) = \dfrac{d}{d\theta}(\tan\theta)^{-1}$

$= (-1)(\tan\theta)^{-2}(\sec^2\theta)$ [M1] – Chain Rule

$= (-1)\left(\dfrac{\cos^2\theta}{\sin^2\theta}\right)\left(\dfrac{1}{\cos^2\theta}\right)$

$= -\dfrac{1}{\sin^2\theta}$ [A1]

(b)(i) $\cos\theta = \dfrac{XY}{12.5} \Rightarrow XY = 12.5\cos\theta$

$\tan\theta = \dfrac{2.7}{BY} \Rightarrow BY = \dfrac{2.7}{\tan\theta} = \dfrac{2.7\cos\theta}{\sin\theta}$ [M1] -- either XY or BY

$h = XY - BY$

$= 12.5\cos\theta - \dfrac{2.7\cos\theta}{\sin\theta}$ [A1] – clear working above

(ii) $\dfrac{dh}{d\theta} = -12.5\sin\theta - 2.7\left(\dfrac{d}{d\theta}\left(\dfrac{\cos\theta}{\sin\theta}\right)\right)$

$= -12.5\sin\theta + \dfrac{2.7}{\sin^2\theta}$ [M1]

$\dfrac{dh}{d\theta} = 0 \Rightarrow -12.5\sin\theta + \dfrac{2.7}{\sin^2\theta} = 0$

$\sin\theta = \sqrt[3]{\dfrac{2.7}{12.5}}$

$= 0.6$ [A1]

(iii)

$\sin\theta = \dfrac{3}{5}$ giving rise to $\cos\theta = \dfrac{4}{5}$ [M1]

$\dfrac{d^2h}{d\theta^2} = -12.5\cos\theta - \dfrac{5.4\cos\theta}{\sin^3\theta}$

$= -12.5\left(\dfrac{4}{5}\right) - \dfrac{5.4\left(\dfrac{4}{5}\right)}{\left(\dfrac{3}{5}\right)^3} = -30 < 0$ [M1] – verify max

Max $h = 12.5(0.8) - \dfrac{2.7(0.8)}{(0.6)} = 6.4$ m [A1]

Alternative method

$\theta = 36.870^0$

$\dfrac{d^2h}{d\theta^2} = -12.5\cos\theta + 2.7(-2)(\sin\theta)^{-3}(\cos\theta)$

$= -12.5\cos\theta - \dfrac{5.4\cos\theta}{\sin^3\theta}$ [M1] – first or second derivative test

When $\theta = 36.870^0$,

$\dfrac{d^2h}{d\theta^2} = -12.5\cos 36.870^o - \dfrac{5.4\cos 36.870^o}{\sin^3 36.870^o} = -30.0 < 0$ [M1] – verify max

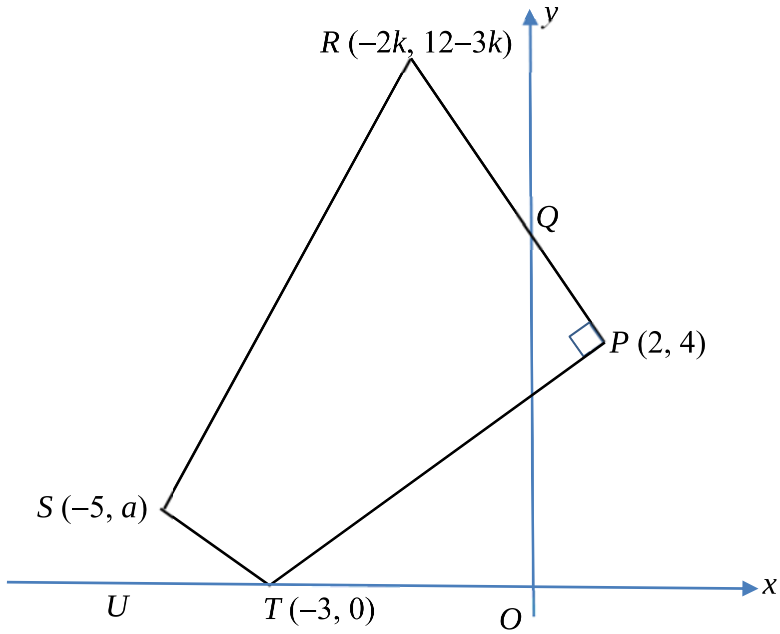
h is maximum when θ is 36.870°.

Maximum $h = 12.5\cos 36.870^o - \dfrac{2.7\cos 36.870^o}{\sin 36.870^o}$

$= 6.40$ m [A1]

Solutions to this question by accurate drawing will not be accepted.

12 The figure shows a quadrilateral $PTSR$ for which P is $(2, 4)$, T is $(-3, 0)$, S is $(-5, a)$, R is $(-2k, 12-3k)$ and angle QPT is a right angle. RQP is a straight line with point Q lying on the y-axis.



(i) Find the value of k. [2]

(ii) Given that angle $STU = 45^\circ$, determine the value of a. [2]

(iii) A line passing through Q and is perpendicular to TS cuts the x-axis at V. Find the value of VR^2. [5]

Solutions

(i) Gradient of $PT = \frac{4}{5}$

Gradient of PR, $\frac{12-3k-4}{-2k-2} = -\frac{5}{4}$ [M1]

$$4(8-3k) = 5(2k+2)$$

$$-22k = -22$$

$$k = 1$$ [A1]

(ii) angle $STU = 45^\circ \Rightarrow$ gradient of $ST = -1$ [M1]

$$\frac{a-0}{-5+3} = -1$$ [A1]

$$a = 2$$

(iii) Equation of PR is $y-4=-\frac{5}{4}(x-2)$ [M1]

$$-4(y-4)=5(x-2)$$
$$4y+5x=26$$

At Q, $x=0$

$4y=26 \Rightarrow y=6.5$

$Q(0, 6.5)$ [A1]

Equation of line passing through Q and perpendicular to TS is

$y-6.5=\frac{-1}{-1}(x-0)$

$y=x+6.5$ [M1]

At V, $y=0$. Hence $x=-6.5$

$V(-6.5, 0)$ [A1]

$VR^2=(-2+6.5)^2+9^2$

$=101.25$ [A1]

END OF PAPER

Name:	Register No.:	Class:

CRESCENT GIRLS' SCHOOL
SECONDARY FOUR
PRELIMINARY EXAMINATION

ADDITIONAL MATHEMATICS **4047/02**

Paper 2

17 August 2018

2 hours 30 minutes

Additional Materials: Answer Paper
Mark Sheet

READ THESE INSTRUCTIONS FIRST

Write your name, register number and class in the spaces provided at the top of this page and on all the work you hand in.
Write in dark blue or black pen.
You may use a soft pencil for any diagrams or graphs.
Do not use paper clips, highlighter, glue or correction fluid.

Answer **all** the questions.
Write your answers on the separate Answer Paper provided.
Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.
The use of an electronic calculator is expected, where appropriate.
You are reminded of the need for clear presentation in your answers.

At the end of the examination, fasten all your work and mark sheet securely together.
The number of marks is given in brackets [ ] at the end of each question or part question.
The total number of marks for this paper is 100.

Mathematical Formulae

1. ALGEBRA

Quadratic Equation

For the equation $ax^2 + bx + c = 0$,

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Binomial expansion

$$(a+b)^n = a^n + \binom{n}{1} a^{n-1} b + \binom{n}{2} a^{n-2} b^2 + \ldots + \binom{n}{r} a^{n-r} b^r + \ldots + b^n,$$

where n is a positive integer and $\binom{n}{r} = \frac{n!}{r!(n-r)!} = \frac{n(n-1)\ldots(n-r+1)}{r!}$

2. TRIGONOMETRY

Identities

$$\sin^2 A + \cos^2 A = 1$$
$$\sec^2 A = 1 + \tan^2 A$$
$$\operatorname{cosec}^2 A = 1 + \cot^2 A$$
$$\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$$
$$\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$$
$$\tan(A \pm B) = \frac{\tan A \pm \tan B}{1 \mp \tan A \tan B}$$
$$\sin 2A = 2 \sin A \cos A$$
$$\cos 2A = \cos^2 A - \sin^2 A = 2\cos^2 A - 1 = 1 - 2\sin^2 A$$
$$\tan 2A = \frac{2 \tan A}{1 - \tan^2 A}$$

Formulae for ΔABC

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$
$$a^2 = b^2 + c^2 - 2bc \cos A$$
$$\Delta = \frac{1}{2} ab \sin C$$

1 **(i)** Write down and simplify the first four terms in the expansion $\left(2x-\dfrac{p}{x^2}\right)^5$ in descending powers of x, where p is a non-zero constant. [3]

(ii) Given that the coefficient of x^{-1} in the expansion $\left(4x^3-1\right)\left(2x-\dfrac{p}{x^2}\right)^5$ is $-160p^2$, find the value of p. [4]

2 Variables x and y are related by the equation $y=ax^b+3$ where a and b are constants. When $\lg(y-3)$ is plotted against $\lg x$, a straight line is obtained. The straight line passes through $(-2.5,\ 8)$ and $(3.5,\ -4)$. Find

(i) the value of a and of b, [5]

(ii) the coordinates of the point on the line when $x=10^6$. [3]

3 **(a)** Given that $x=\log_3 a$ and $y=\log_3 b$, express $\log_3\dfrac{\sqrt{b^5}}{27a^4}$ in terms of x and y. [3]

(b) Solve the equation $\log_2(5x+3)^2-\log_{5x+3}2=1$. [5]

4 **(i)** The roots of the equation $2x^2+px-8=0$, where p is a constant, are α and β. The roots of the equation $4x^2-24x+q=0$, where q is a constant, are $\alpha+2\beta$ and $2\alpha+\beta$. Find the values of p and q. [6]

(ii) Hence form the quadratic equation whose roots are α^3 and β^3. [3]

5 The equation of a circle C is $x^2+y^2-12x-8y-13=0$.

(i) Find the centre and radius of C. [3]

(ii) Find the equation of the line which passes through the centre of C and is perpendicular to the line $4x+7y=117$. [3]

(iii) Show that the line $4x+7y=117$ is a tangent to C and state the coordinates of the point where the line touches C. [5]

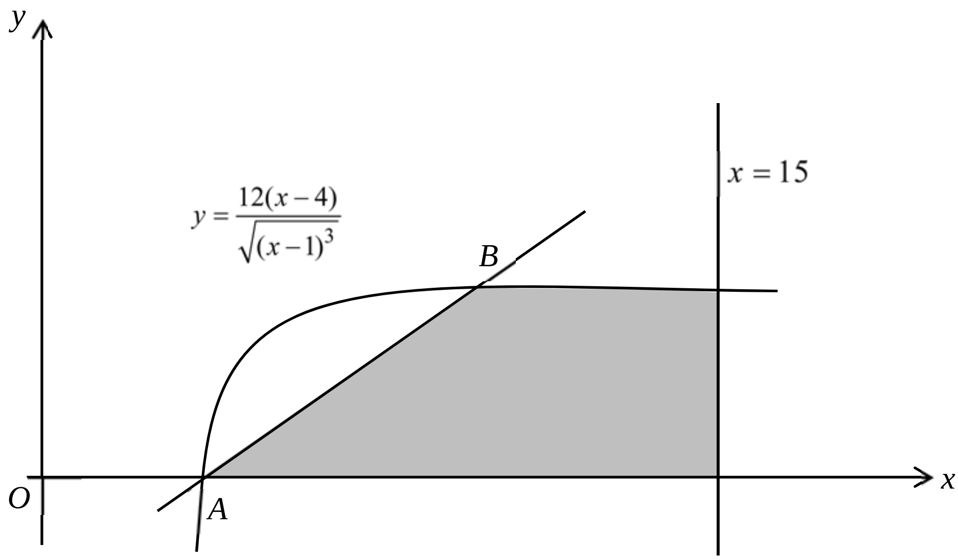
6 **(a)** A car travelling on a straight road passes through a traffic light X with speed of 90 m/s. The acceleration, a m/s^2 of the car, t seconds after passing X, is given by $a = 20 - 8t$. Determine with working whether the car is travelling towards or away from X when it is travelling at maximum speed. [4]

(b) A particle moving in a straight line such that its displacement, s m, from the fixed point O is given by $s = 7\sin t - 2\cos 2t$, where t is the time in seconds, after passing through a point A.

(i) Find the value of t when the particle first comes to instantaneous rest. [5]

(ii) Find the total distance travelled by the particle during the first 4 seconds of its motion. [3]

7 **(i)** Show that $\dfrac{\mathrm{d}}{\mathrm{d}x}\left(\dfrac{x+2}{\sqrt{x-1}}\right) = \dfrac{x-4}{2\sqrt{(x-1)^3}}$. [3]



The diagram shows the line $x = 15$ and part of the curve $y = \dfrac{12(x-4)}{\sqrt{(x-1)^3}}$. The curve intersect the x-axis at the point A. The line through A with gradient $\dfrac{4}{9}$ intersects the curve again at the point B.

(ii) Verify that the y-coordinate of B is $2\dfrac{2}{3}$. [4]

(iii) Determine the area of the region bounded by the curve, the x-axis, the line $x = 15$ and the line AB. [4]

8 A curve has equation given by $y = \dfrac{e^{4x-3}}{8e^{2x}}$.

(i) Show that $\dfrac{dy}{dx} = \dfrac{e^{2x-3}}{4}$. [2]

(ii) Given that x is decreasing at a rate of $4e^2$ units per second, find the exact rate of change of y when $x = 1$. [3]

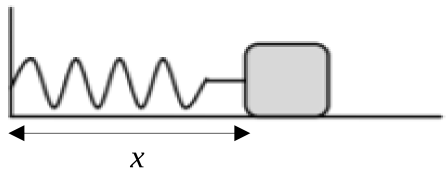
(iii) The curve passes through the y-axis at P. Find the equations of the tangent and normal to the curve at point P. [4]

(iv) The tangent and normal to the curve at point P meets the x-axis at Q and R respectively. Show that the area of the triangle PQR is $\dfrac{1+16e^6}{512e^9}$ units^2 . [3]

9 **(a)** Prove that $\operatorname{cosec}^4 x - \cot^4 x = 2\operatorname{cosec}^2 x - 1$. [3]

(b) Solve the equation $6\tan 2x + 1 = \cot 2x$, for the interval $0 \le x \le 180°$. [5]

(c)

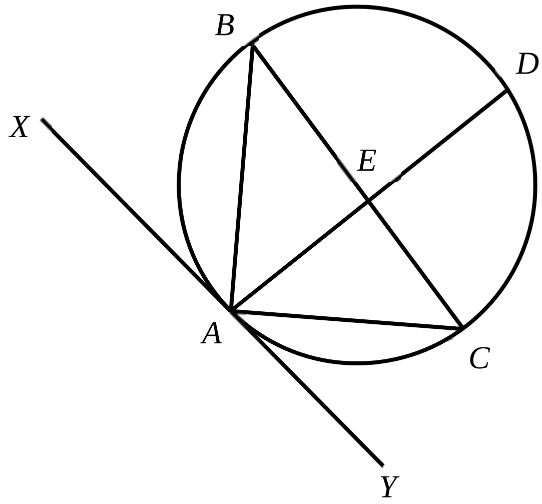


An object is connected to the wall with a spring that has a original horizontal length of 20 cm. The object is pulled back 8 cm past the original length and released. The object completes 4 cycles per second.

(i) Given that the function $x = 8\cos(a\pi t) + b$, where x is the horizontal distance, in centimetres, of the object from the wall and t is the time in seconds after releasing the object, find the values of a and b. [2]

(ii) Find the duration of time for each cycle such that the object is more than 27 cm from the wall. [3]

10



Given that *AD* and *BC* are straight lines, *AC* bisects angle *DAY* and *AB* bisects angle *DAX*, show that

(i) $AC^2 = EC \times BC$, [3]

(ii) *BC* is a diameter of the circle, [3]

(iii) *AD* and *BC* are perpendicular to each other. [3]

END OF PAPER

Answer Key for Paper 2

1(i)	$32x^5 - 80px^2 + \frac{80p^2}{x} - \frac{40p^3}{x^4} + \ldots$
(ii)	$p = 0.5$
2(i)	$a = 1000,\ b = -2$
(ii)	$(6, -9)$
3(a)	$\frac{5}{2}y - 4x - 3$
(b)	$x = -0.459$ or -0.2
4(i)	$p = -4$, $q = 16$
(ii)	$x^2 - 32x - 64 = 0$
5(i)	Centre $= (6,\ 4)$, Radius $= \sqrt{65}$ units
(ii)	$4y = 7x - 26$
(iii)	$(10,\ 11)$
6(a)	Travelling away from X
(b)(i)	$\frac{\pi}{2}$ s
(b)(ii)	25.0 m
7(iii)	21.0 units^2
8 (ii)	$-e$ units/s
(iii)	$y = \frac{x}{4e^3} + \frac{1}{8e^3}$, $y = -4e^3x + \frac{1}{8e^3}$
9(b)	$x = 9.2°,\ 76.7°,\ 99.2°,\ 166.7°$
(c)(i)	$a = 8$, $b = 20$
(c)(ii)	0.0402 s

Name:	Register No.:	Class:

CRESCENT GIRLS' SCHOOL
SECONDARY FOUR
PRELIMINARY EXAMINATION

ADDITIONAL MATHEMATICS — 4047/02

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17 August 2018

2 hours 30 minutes

Additional Materials: Answer Paper, Mark Sheet

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(ii) Given that the coefficient of x^{-1} in the expansion $\left(4x^3-1\right)\left(2x-\frac{p}{x^2}\right)^5$ is $-160p^2$, find the value of p. [4]

Solution:

(i)

$$\left(2x-\frac{p}{x^2}\right)^5=(2x)^5+5(2x)^4\left(-\frac{p}{x^2}\right)+10(2x)^3\left(-\frac{p}{x^2}\right)^2+10(2x)^2\left(-\frac{p}{x^2}\right)^3+\dots \quad \text{[M1]}$$

$$=32x^5-80px^2+\frac{80p^2}{x}-\frac{40p^3}{x^4}+\dots \quad \text{[A2]}$$

(ii)

$$\left(4x^3-1\right)\left(2x-\frac{p}{x^2}\right)^5=\left(4x^3-1\right)\left(32x^5-80px^2+\frac{80p^2}{x}-\frac{40p^3}{x^4}+\dots\right) \quad \text{[M1]}$$

Coefficient of $x^{-1}=4\left(-40p^3\right)+(-1)\left(80p^2\right)$ [M1]

$=-160p^3-80p^2$

$-160p^3-80p^2=-160p^2$ [M1]

$80p^2\left(2p-1\right)=0$

$p=0\ (NA)$ or $p=0.5$ [A1]

2 Variables x and y are related by the equation $y = ax^b + 3$ where a and b are constants. When $\lg(y-3)$ is plotted against $\lg x$, a straight line is obtained. The straight line passes through $(-2.5,\ 8)$ and $(3.5,\ -4)$. Find

(i) the value of a and of b, [5]

(ii) the coordinates of the point on the line when $x = 10^6$. [3]

Solution:

(i) $y = ax^b + 3$

$y - 3 = ax^b$

$\lg(y-3) = \lg a + b\lg x$ [M1]

Gradient = $\dfrac{8-(-4)}{-2.5-3.5}$ [M1]

$= -2$

$b = -2$ [A1]

Sub $\lg x = -2.5$, $\lg(y-3) = 8$ and $b = -2$,

$8 = -2(-2.5) + \lg a$ [M1]

$\lg a = 3$

$a = 10^3 = 1000$ [A1]

(ii) $\lg(y-3) = -2\lg x + 3$

$x = 10^6$

$\lg x = 6$ [M1]

$\lg(y-3) = -2(6) + 3 = -9$ [M1]

Coordinates = $(6,\ -9)$ [A1]

3 **(a)** Given that $x = \log_3 a$ and $y = \log_3 b$, express $\log_3 \dfrac{\sqrt{b^5}}{27a^4}$ in terms of x and y. [3]

(b) Solve the equation $\log_2 (5x+3)^2 - \log_{5x+3} 2 = 1$. [5]

Solution

(a)

$$\log_3 \frac{\sqrt{b^5}}{27a^4} = \log_3 \sqrt{b^5} - \log_3 27 - \log_3 a^4$$ [M1]

$$= \frac{5}{2}\log_3 b - 3 - 4\log_3 a$$ [M1]

$$= \frac{5}{2}y - 4x - 3$$ [A1]

(b)

$$\log_2 (5x+3)^2 - \log_{5x+3} 2 = 1$$

$$2\log_2 (5x+3) - \frac{\log_2 2}{\log_2 (5x+3)} = 1$$ [M1]

$$2\left[\log_2 (5x+3)\right]^2 - 1 = \log_2 (5x+3)$$

$$2\left[\log_2 (5x+3)\right]^2 - \log_2 (5x+3) - 1 = 0$$ [M1]

Let $y = \log_2 (5x+3)$.

$$2y^2 - y - 1 = 0$$

$$(2y+1)(y-1) = 0$$ [M1]

$y = -0.5$ or $y = 1$

$\log_2 (5x+3) = -0.5$ $\quad\quad$ $\log_2 (5x+3) = 1$ [M1]

$5x + 3 = 2^{-0.5}$ $\quad\quad$ $5x + 3 = 2$

$x = -0.459$ $\quad\quad$ $x = -0.2$ [A1]

4 **(i)** The roots of the equation $2x^2 + px - 8 = 0$, where p is a constant, are α and β. The roots of the equation $4x^2 - 24x + q = 0$, where q is a constant, are $\alpha + 2\beta$ and $2\alpha + \beta$. Find the values of p and q. [6]

(ii) Hence form the quadratic equation whose roots are α^3 and β^3. [3]

Solution:

(i) $2x^2 + px - 8 = 0$

$\alpha + \beta = -\dfrac{p}{2}, \quad \alpha\beta = -4$ [B1]

$4x^2 - 24x + q = 0$

$\alpha + 2\beta + 2\alpha + \beta = 6$ [M1]

$3(\alpha + \beta) = 6$

Sub $\alpha + \beta = -\dfrac{p}{2}$,

$-\dfrac{p}{2} = 2 \quad \Rightarrow \quad p = -4$ [A1]

$(\alpha + 2\beta)(2\alpha + \beta) = \dfrac{q}{4}$ [M1]

$2(\alpha^2 + \beta^2) + 5\alpha\beta = \dfrac{q}{4}$

$2\left[(\alpha + \beta)^2 - 2\alpha\beta\right] + 5\alpha\beta = \dfrac{q}{4}$ [M1]

$2(\alpha + \beta)^2 + \alpha\beta = \dfrac{q}{4}$

Sub $\alpha + \beta = 2,\ \alpha\beta = -4,$

$2(2)^2 - 4 = \dfrac{q}{4}$

$q = 16$ [A1]

(ii) $\alpha^3 + \beta^3 = (\alpha + \beta)(\alpha^2 - \alpha\beta + \beta^2)$ [M1]

$= (\alpha + \beta)\left[(\alpha + \beta)^2 - 3\alpha\beta\right]$

$= 2\left[2^2 - 3(-4)\right]$

$= 32$

$(\alpha\beta)^3 = (-4)^3 = -64$ [M1]

$\therefore x^2 - 32x - 64 = 0$ [A1]

5 The equation of a circle C is $x^2+y^2-12x-8y-13=0$.

(i) Find the centre and radius of *C*. [3]

(ii) Find the equation of the line which passes through the centre of *C* and is perpendicular to the line $4x+7y=117$. [3]

(iii) Show that the line $4x+7y=117$ is a tangent to *C* and state the coordinates of the point where the line touches *C*. [5]

Solution:

(i) $x^2+y^2-12x-8y-13=0$

$(x-6)^2+(y-4)^2-36-16-13=0$ [M1]

$(x-6)^2+(y-4)^2=65$

Centre $=(6,\ 4)$ [A1]

Radius $=\sqrt{65}$ units [A1]

(ii) For $4x+7y=117$,

Gradient of the line $=-\dfrac{4}{7}$

Gradient of the line passing through $C=\dfrac{7}{4}$ [M1]

Equation of the line:

$y-4=\dfrac{7}{4}(x-6)$ [M1]

$4y-16=7x-42$

$4y=7x-26$ [A1]

(iii) $4x+7y=117$ ----- (1)

$4y=7x-26 \quad \Rightarrow \ y=\dfrac{7}{4}x-\dfrac{26}{4}$ ----- (2)

Sub (2) into (1):

$4x+7\left(\dfrac{7}{4}x-\dfrac{26}{4}\right)=117$ [M1]

$16.25x=162.5$

$x=10$ [M1]

$y=11$

Distance between $(10,\ 11)$ and centre of circle $=\sqrt{(10-6)^2+(11-4)^2}$ [M1]

$=\sqrt{65}$ units

Since distance from the point and the centre of circle equals to the radius, the line is a tangent to the circle. [A1]

Coordinates of the point = $(10,\ 11)$ [A1]

Alternative Solution:

$4x+7y=117 \quad \Rightarrow \; x=\dfrac{117-7y}{4}$ ----- (1)

$x^2+y^2-12x-8y-13=0$ ----- (2)

Sub (1) into (2):

$\left(\dfrac{117-7y}{4}\right)^2+y^2-12\left(\dfrac{117-7y}{4}\right)-8y-13=0$ [M1]

$\dfrac{13689-1638y+49y^2}{16}+y^2-351+21y-8y-13=0$

$13689-1638y+49y^2+16y^2-5616+336y-128y-208=0$

$65y^2-1430y+7865=0$ [M1]

$y^2-22y+121=0$

$b^2-4ac=(-22)^2-4(1)(121)$ [M1]

$\quad\quad\;\; =0$

Since $b^2-4ac=0$, the line is a tangent to *C*. [A1]

$y^2-22y+121=0$

$(y-11)^2=0$

$y=11$

$x=10$

Coordinate of the point = $(10,\ 11)$ [A1]

6 **(a)** A car travelling on a straight road passes through a traffic light X with speed of 90 m/s. The acceleration, a m/s^2 of the car, t seconds after passing X, is given by $a = 20 - 8t$. Determine with working whether the car is travelling towards or away from X when it is travelling at maximum speed. [4]

(b) A particle moving in a straight line such that its displacement, s m, from the fixed point O is given by $s = 7\sin t - 2\cos 2t$, where t is the time in seconds, after passing through at a point A.

(i) Find the value of t when the particle first comes to instantaneous rest. [5]

(ii) Find the total distance travelled by the particle during the first 4 seconds of its motion. [3]

Solution:

(a) $a = 20 - 8t$

$v = \int 20 - 8t \text{ dt} = 20t - 4t^2 + c$, where c is a constant [M1]

When $t = 0,\ v = 90,\ c = 90.$

$\therefore v = 20t - 4t^2 + 90$

When car is travelling at max speed, $a = 0$.

$a = 20 - 8t \quad \Rightarrow \quad t = 2.5$ [M1]

$v = 20(2.5) - 4(2.5)^2 + 90 = 115$

$s = \int 20t - 4t^2 + 90 \text{ dt} = 10t^2 - \frac{4}{3}t^3 + 90t + d$, where d is a constant [M1]

When $t = 0,\ s = 0,\ d = 0.$

$\therefore s = 10t^2 - \frac{4}{3}t^3 + 90t$

When $t = 2.5$, $s = 10(2.5)^2 - \frac{4}{3}(2.5)^3 + 90(2.5) = 266\frac{2}{3}$

Since $s > 0$ and $v > 0$, the car is travelling away from X at maximum speed. [A1]

Alternative Solution:

When the car is at instantaneous rest, $v = 0$.

$20t - 4t^2 + 90 = 0$

$$t = \frac{-20 \pm \sqrt{(-20)^2 - 4(-4)(90)}}{2(-4)} = -2.8619 \text{ or } 7.8619$$ [M1]

Since there is no change of direction from $t = 0$ to $t = 7.86$ s, [B1]

the car is travelling away from X at maximum speed.

(b)(i) $s = 7\sin t - 2\cos 2t$

$v = 7\cos t + 4\sin 2t$ [M1]

When the particle is at instantaneous rest, $v = 0$.

$7\cos t + 4\sin 2t = 0$ [M1]

$7\cos t + 8\sin t\cos t = 0$

$\cos t\left(7 + 8\sin t\right) = 0$ [M1]

$\cos t = 0$ or $\sin t = -\dfrac{7}{8}$

$t = \dfrac{\pi}{2},\ \dfrac{3\pi}{2}$ $t = 4.2069,\ 5.2177$ [A1]

Time when particle first comes to instantaneous rest $= \dfrac{\pi}{2}$ s [A1]

(b)(ii) When $t = 0,\ s = -2$.

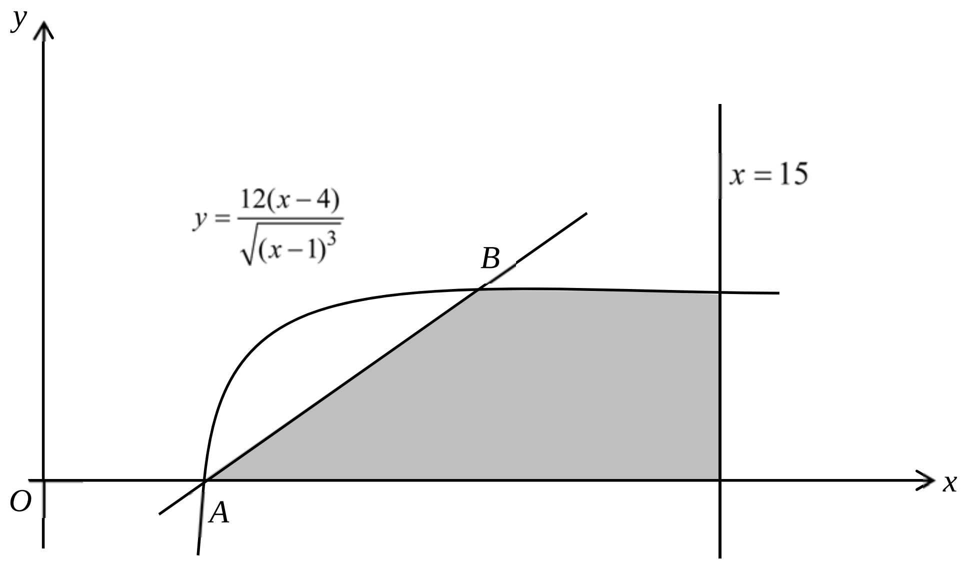
When $t = \dfrac{\pi}{2},\ s = 9$.

When $t = 4,\ s = -5.0066$. [M1]

Total distance travelled $=2+2\left(9\right) + 5.0066$ [M1]

$= 25.0$ m [A1]

7 **(i)** Show that $\dfrac{d}{dx}\left(\dfrac{x+2}{\sqrt{x-1}}\right) = \dfrac{x-4}{2\sqrt{(x-1)^3}}$. [3]



The diagram shows the line $x = 15$ and part of the curve $y = \dfrac{12(x-4)}{\sqrt{(x-1)^3}}$. The curve intersect the x-axis at the point A. The line through A with gradient $\dfrac{4}{9}$ intersects the curve again at the point B.

(ii) Verify that the y-coordinate of B is $2\dfrac{2}{3}$. [4]

(iii) Determine the area of the region bounded by the curve, the x-axis, the line $x = 15$ and the line AB. [4]

Solution:

(i)

$$\frac{d}{dx}\left(\frac{x+2}{\sqrt{x-1}}\right) = \frac{\sqrt{x-1} - (x+2)\left[\frac{1}{2}(x-1)^{-\frac{1}{2}}\right]}{x-1} \qquad \text{[M1]}$$

$$= \frac{\left[\frac{1}{2}(x-1)^{-\frac{1}{2}}\right]\left[2x-2-x-2\right]}{x-1} \qquad \text{[M1]}$$

$$= \frac{x-4}{2\sqrt{(x-1)^3}} \qquad \text{[A1]}$$

(ii) $A = (4,\ 0)$

Equation of AB: $y = \frac{4}{9}(x-4)$ ---- (1) [M1]

$y = \frac{12(x-4)}{\sqrt{(x-1)^3}}$ ---- (2)

(1) = (2):

$\frac{4}{9}(x-4) = \frac{12(x-4)}{\sqrt{(x-1)^3}}$ [M1]

$(x-4)(x-1)^{\frac{3}{2}} = 27(x-4)$

$(x-4)\left[(x-1)^{\frac{3}{2}} - 27\right] = 0$ [M1]

$x = 4$ or $(x-1)^{\frac{3}{2}} = 27$

$x = 10$

Sub $x = 10$ in (1):

$y = \frac{4}{9}(10-4) = 2\frac{2}{3}$ [A1]

$y-$coordinate of $B = 2\frac{2}{3}$ (shown)

(iii) Area $= \frac{1}{2}\left(2\frac{2}{3}\right)(10-4) + \int_{10}^{15} \frac{12(x-4)}{\sqrt{(x-1)^3}}\ \mathrm{d}x$ [M1]

$= 8 + 24\int_{10}^{15} \frac{x-4}{2\sqrt{(x-1)^3}}\ \mathrm{d}x$ [M1]

$= 8 + 24\left[\frac{x+2}{\sqrt{x-1}}\right]_{10}^{15}$ [M1]

$= 8 + 24\left(\frac{17}{\sqrt{14}} - \frac{12}{\sqrt{9}}\right)$

$= 21.0$ units^2 [A1]

8 A curve has equation given by $y = \dfrac{e^{4x-3}}{8e^{2x}}$.

(i) Show that $\dfrac{dy}{dx} = \dfrac{e^{2x-3}}{4}$. [2]

(ii) Given that x is decreasing at a rate of $4e^2$ units per second, find the exact rate of change of y when $x = 1$. [3]

(iii) The curve passes through the y-axis at P. Find the equations of the tangent and normal to the curve at point P. [4]

(iv) The tangent and normal to the curve at point P meets the x-axis at Q and R respectively. Show that the area of the triangle PQR is $\dfrac{1+16e^6}{512e^9}$ units^2. [3]

Solution:

(i)

$$y = \frac{e^{2x-3}}{8}$$ [M1]

$$\frac{dy}{dx} = \frac{e^{2x-3}}{4}$$ [A1]

(ii)

$$\frac{dy}{dt} = \frac{dy}{dx} \times \frac{dx}{dt}$$

$$= \frac{e^{2x-3}}{4} \times \left(-4e^2\right)$$ [M1]

$$= -e^{2x-1}$$ [M1]

When $x = 1$, $\dfrac{dy}{dt} = -e$ units/s [A1]

(iii)

When $x = 0$, $y = \dfrac{1}{8e^3}$

Gradient of tangent at $P = \dfrac{1}{4e^3}$ [M1]

Equation of tangent at P:

$$y - \frac{1}{8e^3} = \frac{1}{4e^3}(x) \quad \Rightarrow \quad y = \frac{x}{4e^3} + \frac{1}{8e^3}$$ [M1]

Gradient of normal at $P = -4e^3$ [M1]

Equation of normal at P:

$$y - \frac{1}{8e^3} = -4e^3(x) \quad \Rightarrow \quad y = -4e^3 x + \frac{1}{8e^3}$$ [A1]

(iv) Equation of tangent at P: $y = \dfrac{x}{4e^3} + \dfrac{1}{8e^3}$

When $y = 0,\ x = -\dfrac{1}{2}$. $\quad \therefore Q = \left(-\dfrac{1}{2},\ 0\right)$

Equation of normal at P: $y = -4e^3 x + \dfrac{1}{8e^3}$

When $y = 0,\ x = \dfrac{1}{32e^6}$. $\quad \therefore R = \left(\dfrac{1}{32e^6},\ 0\right)$ [M1]

Area of triangle $PQR = \dfrac{1}{2}\left(\dfrac{1}{8e^3}\right)\left[\dfrac{1}{32e^6} - \left(-\dfrac{1}{2}\right)\right]$ [M1]

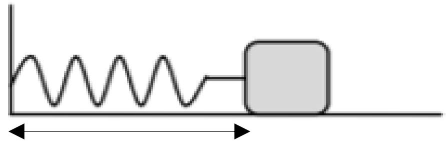
$= \dfrac{1}{16e^3}\left(\dfrac{1+16e^6}{32e^6}\right)$ [A1]

$= \dfrac{1+16e^6}{512e^9}$ units^2

9 **(a)** Prove that $\operatorname{cosec}^4 x - \cot^4 x = 2\operatorname{cosec}^2 x - 1$. [3]

(b) Solve the equation $6\tan 2x + 1 = \cot 2x$, for the interval $0 \le x \le 180°$. [5]

(c)



An object is connected to the wall with a spring that has a original horizontal length of 20 cm. The object is pulled back 8 cm past the original length and released. The object completes 4 cycles per second.

(i) Given that the function $x = 8\cos(a\pi t) + b$, where x is the horizontal distance, in centimetres, of the object from the wall and t is the time in seconds after releasing the object, find the values of a and b. [2]

(ii) Find the duration of time for each cycle such that the object is more than 27 cm from the wall. [3]

Solution:

(a)

$\text{LHS} = (\operatorname{cosec}^2 x - \cot^2 x)(\operatorname{cosec}^2 x + \cot^2 x)$ [B1]

$= \operatorname{cosec}^2 x + \cot^2 x$ [B1]

$= \operatorname{cosec}^2 x + \operatorname{cosec}^2 x - 1$ [B1]

$= 2\operatorname{cosec}^2 x - 1$

$= \text{RHS}$

(b)

$6\tan 2x + 1 = \cot 2x$

$6\tan^2 2x + \tan 2x - 1 = 0$ [M1]

$(3\tan 2x - 1)(2\tan 2x + 1) = 0$ [M1]

$0 \le x \le 360° \quad \Rightarrow \quad 0 \le 2x \le 720°$

$\tan 2x = \frac{1}{3}$ or $\tan 2x = -\frac{1}{2}$ [M1]

$\alpha = 18.435°$ $\qquad$ $\alpha = 26.565°$

$2x = 18.435°, 198.43°$ $\qquad$ $2x = 153.43°, 333.43°$

$x = 9.2°, 99.2°$ (1 dp) $\qquad$ $x = 76.7°, 166.7°$ (1 dp) [A2]

(c)(i)

$b = 20$ [B1]

$\text{Period} = \frac{2\pi}{a\pi}$

$\frac{1}{4} = \frac{2\pi}{a\pi} \quad \Rightarrow \quad a = 8$ [B1]

(c)(ii) $27 = 8\cos(8\pi t) + 20$

$\cos(8\pi t) = \frac{7}{8}$ [M1]

$\alpha = 0.50536$

$8\pi t = 0.50536$ [M1]

$t = 0.020107$

Duration of time $= 0.020107 \times 2$

$= 0.0402$ s [A1]

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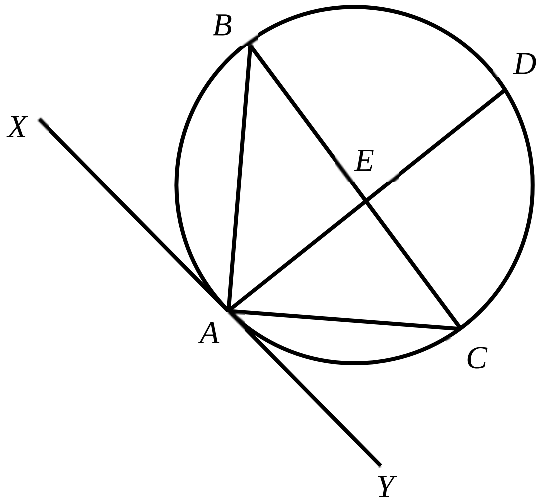

Given that AD and BC are straight lines, AC bisect angle DAY and AB bisects angle DAX, show that

(i) $AC^2 = EC \times BC$, [3]

(ii) BC is a diameter of the circle, [3]

(iii) AD and BC are perpendicular to each other. [3]

Solution:

(i) $\angle BCA = \angle ACE$ (Common angle)

$\angle ABC = \angle CAY$ (Angles in the alternate segments) [B1]

$= \angle EAC$ (AC bisects $\angle DAY$)

$\therefore \Delta BAC$ and ΔAEC are similar. [B1]

$\dfrac{AC}{EC} = \dfrac{BC}{AC}$ (corresponding sides of similar triangles) [B1]

$AC^2 = EC \times BC$ (shown)

(ii) $\angle CAY = \angle EAC$ (AC bisects $\angle DAY$)

$\angle BAX = \angle EAB$ (AB bisects $\angle BAX$) [B1]

$\angle BAX + \angle EAB + \angle EAC + \angle CAY = 180°$ (angles on a straight line) [B1]

$2\angle EAB + 2\angle EAC = 180°$

$\angle EAB + \angle EAC = \angle BAC = 90°$

Since $\angle BAC = 90°$, BC is a diameter of the circle. [B1]

(iii) $\angle ABE = \angle CAY$ (Angles in the alternate segments)

$\angle CAY = \angle EAC$ (AC bisects $\angle BAY$)

$\therefore \angle ABE = \angle EAC$ [B1]

$\angle EAB + \angle EAC = \angle EAB + \angle ABE = 90°$ (from (ii))

$\angle AEB = 90°$ (sum of $\angle$s in a triangle) [B1]

$\therefore$ AD and BC are perpendicular. [B1]

END OF PAPER