Class | Index Number

Name : ________________________________

# METHODIST GIRLS' SCHOOL

Founded in 1887

# PRELIMINARY EXAMINATION 2018
## Secondary 4

Thursday **ADDITIONAL MATHEMATICS** **4047/1**

2 August 2018 **Paper 1** 2 h

**INSTRUCTIONS TO CANDIDATES**
Write your class, index number and name on all the work you hand in.
Write in dark blue or black pen on both sides of the paper.
You may use an HB pencil for any diagrams or graphs.
Do not use staples, paper clips, highlighters, glue or correction fluid.

**Answer all questions.**

Write your answers on the separate Answer Paper provided.
Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.
The use of an approved scientific calculator is expected, where appropriate.
You are reminded of the need for clear presentation in your answers.

At the end of the examination, fasten all your work securely together.
The number of marks is given in brackets [ ] at the end of each question or part question.
The total number of marks for this paper is 80.

***Mathematical Formulae***

## 1. ALGEBRA

*Quadratic Equation*

For the quadratic equation $ax^2 + bx + c = 0$,

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

*Binomial Expansion* $(a+b)^n = a^n + \binom{n}{1}a^{n-1}b + \binom{n}{2}a^{n-2}b^2 + \ldots + \binom{n}{r}a^{n-r}b^r + \ldots + b^n$,

where $n$ is a positive integer and $\binom{n}{r} = \frac{n!}{r!(n-r)!} = \frac{n(n-1)...(n-r+1)}{r!}$.

## 2. TRIGONOMETRY

*Identities*

$$\sin^2 A + \cos^2 A = 1$$

$$\sec^2 A = 1 + \tan^2 A$$

$$\text{cosec}^2 A = 1 + \cot^2 A$$

$$\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$$

$$\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$$

$$\tan(A \pm B) = \frac{\tan A \pm \tan B}{1 \mp \tan A \tan B}$$

$$\sin 2A = 2 \sin A \cos A$$

$$\cos 2A = \cos^2 A - \sin^2 A = 2\cos^2 A - 1 = 1 - 2\sin^2 A$$

$$\tan 2A = \frac{2 \tan A}{1 - \tan^2 A}$$

*Formulae for* $\Delta ABC$

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$a^2 = b^2 + c^2 - 2bc \cos A$$

$$\Delta = \frac{1}{2}ab \sin C$$

**1** The function f is defined, for all values of $x$, by

$$\mathrm{f}(x) = x^2 e^{2x}.$$

Find the values of $x$ for which f is a decreasing function. [4]

**2** A man buys an antique porcelain at the beginning of 2015. After $t$ years, its value, $\$V$, is given by $V = 15\ 000 + 3000e^{0.2t}$.

**(i)** Find the value of the porcelain when the man first bought it. [1]

**(ii)** Find the year in which the value of the porcelain first reached $50 000. [3]

**3** Given the identity $\cos 3x = 4\cos^3 x - 3\cos x$, find the value of $\int_{\frac{\pi}{6}}^{\frac{\pi}{2}} \cos^3 x \ \mathrm{d}x$. [3]

**4** **(i)** Sketch the graph of $y = 4x^{\frac{1}{3}}$ for $x \geq 0$. [2]

The line $y = x$ intersects the curve $y = 4x^{\frac{1}{3}}$ at the points $A$ and $B$.

**(ii)** Show that the perpendicular bisector of $AB$ passes through the point (5, 3). [4]

**5** Solve the following equations:

**(i)** $\log_8 y + \log_2 y = 4$ [2]

**(ii)** $10^{2x+1} = 7(10^x) + 26$ [4]

**6** **(i)** Show that $(\operatorname{cosec} x - 1)(\operatorname{cosec} x + 1)(\sec x - 1)(\sec x + 1) \equiv 1$. [2]

**(ii)** Hence solve $(\operatorname{cosec} x - 1)(\operatorname{cosec} x + 1)(\sec x - 1)(\sec x + 1) = 2\tan^2 2x - 5\sec 2x$ for $0 \leq x \leq 360°$. [4]

**7** The function $\mathrm{f}(x) = \sin^2 x + 2 - 3\cos^2 x$ is defined for $0 \le x \le 2\pi$.

**(i)** Express f($x$) in the form $a + b\cos 2x$, stating the values of $a$ and $b$. [2]

**(ii)** State the period and amplitude of f($x$). [2]

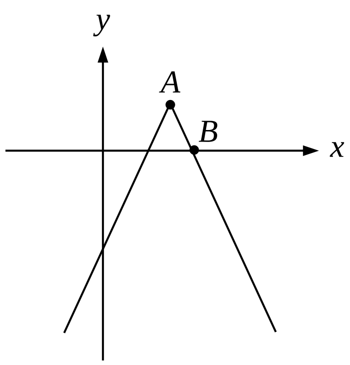
**(iii)** Sketch the graph of $y = \mathrm{f}(x)$ and hence state the number of solutions of the equation $\dfrac{1}{2} - \dfrac{x}{2\pi} + \cos 2x = 0$. [4]

**8** A particle moves in a straight line passes through a fixed point $X$ with velocity 5 m/s. Its acceleration is given by $a = 4 - 2t$, where $t$ is the time in seconds after passing $X$. Calculate

**(i)** the value of $t$ when the particle is instantaneously at rest, [4]

**(ii)** the total distance travelled by the particle in the first 6 seconds. [4]

**9** **(i)** The diagram shows part of the graph of $y = 1 - |2x - 6|$. Find the coordinates of $A$ and $B$. [3]

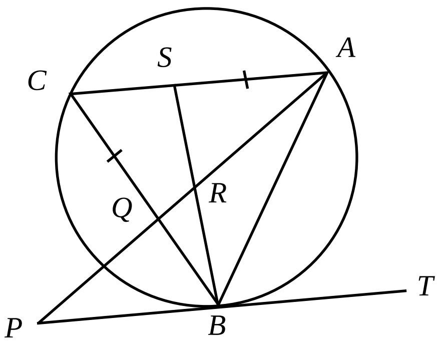



A line of gradient $m$ passes through the point (4, 1).

**(ii)** In the case where $m$ = 2, find the coordinates of the points of intersection of the line and the graph of $y = 1 - |2x - 6|$. [4]

**(iii)** Determine the sets of values of $m$ for which the line intersects the graph of $y = 1 - |2x - 6|$ in two points. [1]

**10** An equilateral triangle *ABC* is inscribed in a circle. *PT* is a tangent to the circle at *B*. It is given that *AS* = *QC*. *PQA* is a straight line and *BS* meets *AQ* at *R*.

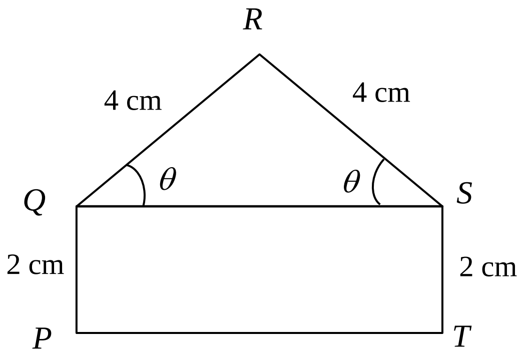



**(i)** Show that *AC* is parallel to *PB*. [2]

**(ii)** Prove that $\Delta ABS$ is congruent to $\Delta CAQ$. [2]

**(iii)** Prove that $\angle PBQ = \angle BRQ$. [3]

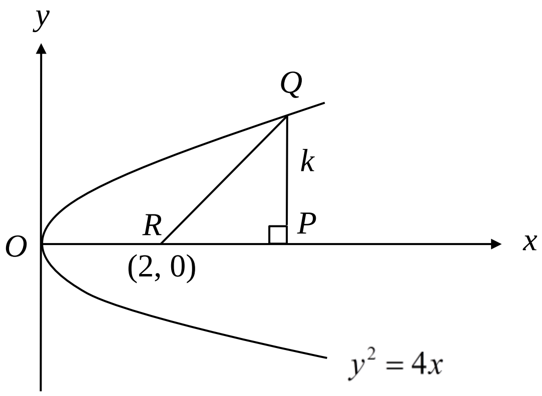
**11** In the diagram, *PQRST* is a piece of cardboard. *PQST* is a rectangle with $PQ = 2$ cm and *QRS* is an isosceles triangle with $QR = RS = 4$ cm. $\angle RSQ = \angle RQS = \theta$ radians.




**(i)** Show that the area, *A* cm$^2$, of the cardboard is given by $A = 8\sin 2\theta + 16\cos\theta$. [3]

**(ii)** Given that $\theta$ can vary, find the stationary value of *A* and determine whether it it is a maximum or a minimum. [6]

**12** **(a)**

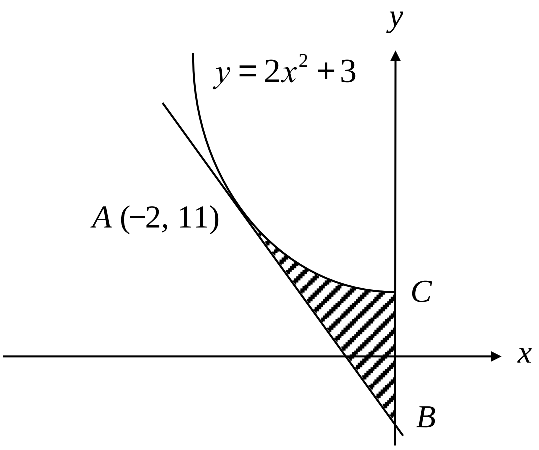



The diagram shows part of a curve $y^2 = 4x$. The point *P* is on the *x*-axis and the point *Q* is on the curve. *PQ* is parallel to the *y*-axis and *k* is units in length. Given *R* is (2, 0), express the area, *A*, of the $\Delta PQR$ in terms of *k* and hence show that $\frac{dA}{dk} = \frac{3k^2 - 8}{8}$.

The point *P* moves along the *x*-axis and the point *Q* moves along the curve in such a way that *PQ* remains parallel to the *y*-axis. *k* increases at the rate of 0.2 units per second.

Find the rate of increase of *A* when *k* = 6 units. [5]

**(b)**




The diagram shows part of the curve $y = 2x^2 + 3$.

The tangent to the curve at the point $A(-2, 11)$ intersects the *y*-axis at *B*. Find the area of the shaded region *ABC*. [6]

**~ End of Paper ~**

Class | Index Number

Name : ___________________________________

# METHODIST GIRLS' SCHOOL

Founded in 1887

## PRELIMINARY EXAMINATION 2018
## Secondary 4

Thursday **ADDITIONAL MATHEMATICS** **4047/1**

2 August 2018 **Paper 1** 2 h

**INSTRUCTIONS TO CANDIDATES**
Write your class, index number and name on all the work you hand in.
Write in dark blue or black pen on both sides of the paper.
You may use an HB pencil for any diagrams or graphs.
Do not use staples, paper clips, highlighters, glue or correction fluid.

**Answer all questions.**

Write your answers on the separate Answer Paper provided.
Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.
The use of an approved scientific calculator is expected, where appropriate.
You are reminded of the need for clear presentation in your answers.

At the end of the examination, fasten all your work securely together.
The number of marks is given in brackets [ ] at the end of each question or part question.
The total number of marks for this paper is 80.

***Mathematical Formulae***

## 1. ALGEBRA

*Quadratic Equation*

For the quadratic equation $ax^2 + bx + c = 0$,

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

*Binomial Expansion* $(a+b)^n = a^n + \binom{n}{1}a^{n-1}b + \binom{n}{2}a^{n-2}b^2 + \ldots + \binom{n}{r}a^{n-r}b^r + \ldots + b^n$,

where $n$ is a positive integer and $\binom{n}{r} = \frac{n!}{r!(n-r)!} = \frac{n(n-1)\ldots(n-r+1)}{r!}$.

## 2. TRIGONOMETRY

*Identities*

$$\sin^2 A + \cos^2 A = 1$$

$$\sec^2 A = 1 + \tan^2 A$$

$$\text{cosec}^2 A = 1 + \cot^2 A$$

$$\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$$

$$\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$$

$$\tan(A \pm B) = \frac{\tan A \pm \tan B}{1 \mp \tan A \tan B}$$

$$\sin 2A = 2 \sin A \cos A$$

$$\cos 2A = \cos^2 A - \sin^2 A = 2\cos^2 A - 1 = 1 - 2\sin^2 A$$

$$\tan 2A = \frac{2\tan A}{1 - \tan^2 A}$$

*Formulae for* $\Delta ABC$

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$a^2 = b^2 + c^2 - 2bc \cos A$$

$$\Delta = \frac{1}{2}ab \sin C$$

**1** The function f is defined, for all values of $x$, by

$$f(x) = x^2 e^{2x}.$$

Find the values of $x$ for which f is a decreasing function. [4]

$f(x) = x^2 e^{2x}$

$f¢(x) = e^{2x}(2x) + x^2(2e^{2x})$

$f¢(x) = 2xe^{2x}(1+x)$

For increasing function,

$f¢(x) < 0$

$2xe^{2x}(1+x) < 0$

Since $e^{2x} > 0$

$x(1+x) < 0$

Ans : $-1 < x < 0$

**2** A man buys an antique porcelain at the beginning of 2015. After $t$ years, its value, $$V$, is given by $V = 15\ 000 + 3000e^{0.2t}$.

**(i)** Find the value of the porcelain when the man first bought it. [1]

**(ii)** Find the year in which the value of the porcelain first reached $50 000. [3]

**(i)** at t = 0,

$V = 15\ 000 + 3000e^0 = 18\ 000$

**(ii)** $50\ 000 = 15\ 000 + 3000e^{0.2t}$

$35\ 000 = 3000e^{0.2t}$

$\frac{35}{3} = e^{0.2t}$

$0.2t = \ln\left(\frac{35}{3}\right)$

$t = 12.283...$

Ans : 2027

**3** Given the identity $\cos 3x = 4\cos^3 x - 3\cos x$, find the value of $\int_{\frac{\pi}{6}}^{\frac{\pi}{2}} \cos^3 x \ \mathrm{d}x$. [3]

$$\int_{\frac{\pi}{6}}^{\frac{\pi}{2}} \cos^3 x \ \mathrm{d}x$$

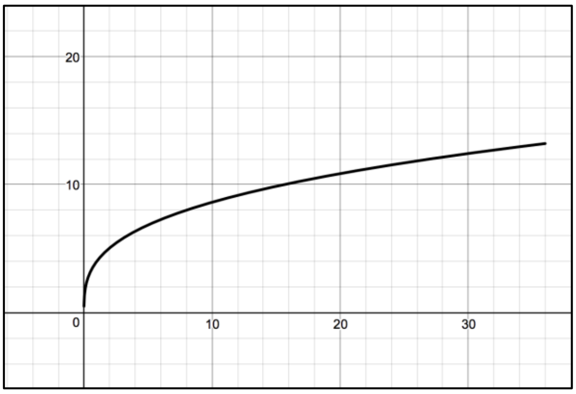
$$= \frac{1}{4}\left[ \int_{\frac{\pi}{6}}^{\frac{\pi}{2}} \left(\cos 3x + 3\cos x\right) \ \mathrm{d}x \right]$$

$$= \frac{1}{4}\left[ \frac{\sin 3x}{3} + 3\sin x \right]_{\frac{\pi}{6}}^{\frac{\pi}{2}}$$

$$= \frac{1}{4}\left[ \left( \frac{-1}{3} + 3 \right) - \left( \frac{1}{3} + \frac{3}{2} \right) \right]$$

$$= \frac{5}{24}$$

**4** **(i)** Sketch the graph of $y = 4x^{\frac{1}{3}}$ for $x \geq 0$. [2]




The line $y = x$ intersects the curve $y = 4x^{\frac{1}{3}}$ at the points *A* and *B*.

**(ii)** Show that the perpendicular bisector of *AB* passes through the point (5, 3). [4]

$x = 4x^{\frac{1}{3}}$

$x - 4x^{\frac{1}{3}} = 0$

$x^{\frac{1}{3}}\left(x^{\frac{2}{3}} - 4\right) = 0$

$x^{\frac{1}{3}} = 0$ or $x^{\frac{2}{3}} = 4$

$x = 0$ or $x = 4^{\frac{3}{2}}$

$x = 0$ or $x = 8$ $(x \geq 0)$

$A(0,0)$, $B(8,8)$

mid-point of *AB* = (4, 4)

gradient *AB* = 1

eqn of perpendicular bisector ,

$y - 4 = -1(x - 4)$

$y = -x + 8$

when $x = 5$, $y = 3$.

Therefore the perpendicular bisector passes through (5, 3).

**5** Solve the following equations:

**(i)** $\log_8 y + \log_2 y = 4$ [2]

**(ii)** $10^{2x+1} = 7(10^x) + 26$ [4]

**(i)** $\log_8 y + \log_2 y = 4$

$$\frac{\log_2 y}{\log_2 8} + \log_2 y = 4$$

$$\frac{\log_2 y}{3} + \log_2 y = 4$$

$$\frac{4}{3}\log_2 y = 4$$

$$\log_2 y = 3$$

$$y = 8$$

**(ii)** $10^{2x+1} = 7(10^x) + 26$

$$10^{2x}(10^1) = 7(10^x) + 26$$

let $p = 10^x$,

$$10p^2 - 7p - 26 = 0$$

$$(10p + 13)(p - 2) = 0$$

$$p = -\frac{13}{10} \quad \text{or} \quad p = 2$$

$$10^x = -\frac{13}{10} \quad \text{or} \quad 10^x = 2$$

$$\text{(NA)} \quad \text{or} \quad x = \lg 2 = 0.301$$

**6** **(i)** Show that $(\operatorname{cosec} x - 1)(\operatorname{cosec} x + 1)(\sec x - 1)(\sec x + 1) \equiv 1$. [2]

**(ii)** Hence solve $(\operatorname{cosec} x - 1)(\operatorname{cosec} x + 1)(\sec x - 1)(\sec x + 1) = 2\tan^2 2x - 5\sec 2x$ for $0 \le x \le 360°$. [4]

**(i)** LHS,

$(\operatorname{cosec} x - 1)(\operatorname{cosec} x + 1)(\sec x - 1)(\sec x + 1)$

$= (\operatorname{cosec}^2 x - 1)(\sec^2 x - 1)$

$= (\cot^2 x)(\tan^2 x)$

$= 1$

**(ii)** $(\operatorname{cosec} x - 1)(\operatorname{cosec} x + 1)(\sec x - 1)(\sec x + 1) = 2\tan^2 2x - 5\sec 2x$

$1 = 2\tan^2 2x - 5\sec 2x$

$2(\sec^2 2x - 1) - 5\sec 2x - 1 = 0$

$2\sec^2 2x - 5\sec 2x - 3 = 0$

$(\sec 2x - 3)(2\sec 2x + 1) = 0$

$\sec 2x = 3$ or $\sec 2x = -\frac{1}{2}$

$\cos 2x = \frac{1}{3}$ or $\cos 2x = -2$

basic angle, $\alpha = 70.529...$ or NA

$2x = \alpha, 360° - \alpha, 360° - \alpha, 720° - \alpha$

$x = 35.3°,\ 144.7°,\ 215.3°,\ 324.7°$

**7** The function $f(x) = \sin^2 x + 2 - 3\cos^2 x$ is defined for $0 \le x \le 2\pi$.

**(i)** Express f($x$) in the form $a + b\cos 2x$, stating the values of $a$ and $b$. [2]

**(ii)** State the period and amplitude of f($x$). [2]

**(iii)** Sketch the graph of $y = f(x)$ and hence state the number of solutions of the equation $\frac{1}{2} - \frac{x}{2\pi} + \cos 2x = 0$. [4]

**(i)** $f(x) = \sin^2 x + 2 - 3\cos^2 x$

$f(x) = \sin^2 x + \cos^2 x + 2 - 4\cos^2 x$

$f(x) = 3 - 2(2\cos^2 x)$

$f(x) = 1 - 2(2\cos^2 x - 1)$

$f(x) = 1 - 2\cos 2x$

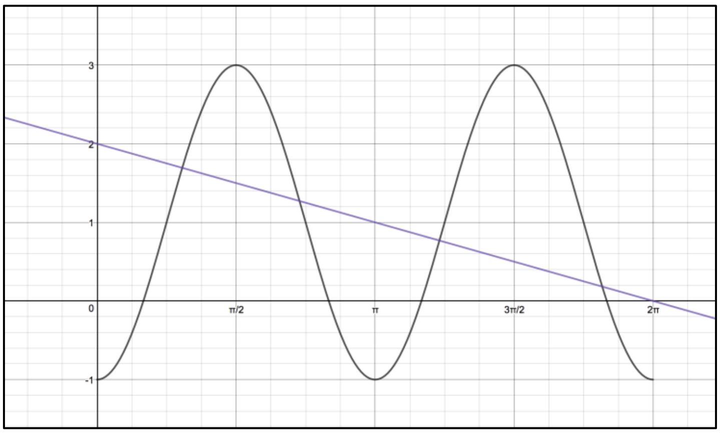
**(ii)** Amplitude = 2

Period = $\frac{2\pi}{2} = \pi$

**(iii)** $\frac{1}{2} - \frac{x}{2\pi} + \cos 2x = 0$

$1 - \frac{x}{\pi} = -2\cos 2x$

$2 - \frac{x}{\pi} = 1 - 2\cos 2x$




No. of solutions = 4

**8** A particle moves in a straight line passes through a fixed point *X* with velocity 5 m/s. Its acceleration is given by $a = 4 - 2t$, where *t* is the time in seconds after passing *X*. Calculate

**(i)** the value of *t* when the particle is instantaneously at rest, [4]

**(ii)** the total distance travelled by the particle in the first 6 seconds. [4]

**(i)** $a = 4 - 2t$

$v = \int (4 - 2t)\ \mathrm{d}t$

$v = 4t - t^2 + c$

at $t = 0$, $v = 5$,

$5 = c$

$v = 4t - t^2 + 5$

at $v = 0$,

$0 = 4t - t^2 + 5$

$t^2 - 4t - 5 = 0$

$(t - 5)(t + 1) = 0$

$t = 5$ or $t = -1$ (NA)

**(ii)** $s = \int (4t - t^2 + 5)\ \mathrm{d}t$

$s = 2t^2 - \frac{t^3}{3} + 5t + c_1$

at $t = 0$, $s = 0$,

$c_1 = 0$

$s = 2t^2 - \frac{t^3}{3} + 5t$

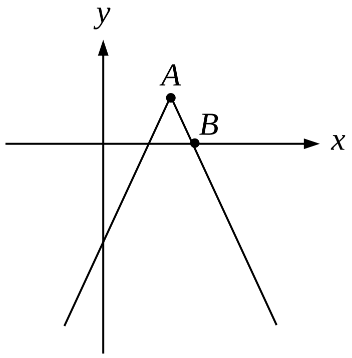
at $t = 0$, $s = 0$

at $t = 5$, $s = \frac{100}{3}$

at $t = 6$, $s = 30$

Total Distance = $\left(2 \times \frac{100}{3}\right) - 30 = 36\frac{2}{3}$

**9** **(i)** The diagram shows part of the graph of $y = 1 - |2x - 6|$. Find the coordinates of $A$ and $B$. [3]




$2x - 6 = 0$

$x = 3$

$A(3, 1)$

$|2x - 6| = 1$

$2x - 6 = 1$ or $2x - 6 = -1$

$x = 3.5$ or $x = 2.5$

$B(3.5, 0)$

A line of gradient $m$ passes through the point (4, 1).

**(ii)** In the case where $m = 2$, find the coordinates of the points of intersection of the line and the graph of $y = 1 - |2x - 6|$. [4]

**(iii)** Determine the sets of values of $m$ for which the line intersects the graph of $y = 1 - |2x - 6|$ in two points. [1]

**(ii)** $y = 2x + c$

at (4, 1),

$1 = 8 + c$

$c = -7$

$y = 2x - 7$

$y = 1 - |2x - 6|$

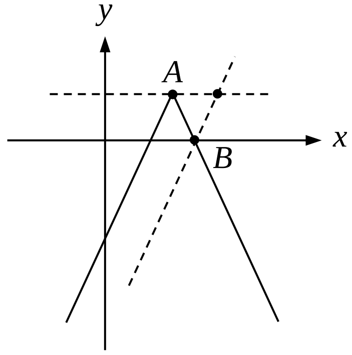
$2x - 7 = 1 - |2x - 6|$

$|2x - 6| = 8 - 2x$

$2x - 6 = 8 - 2x$ or $2x - 6 = -(8 - 2x)$

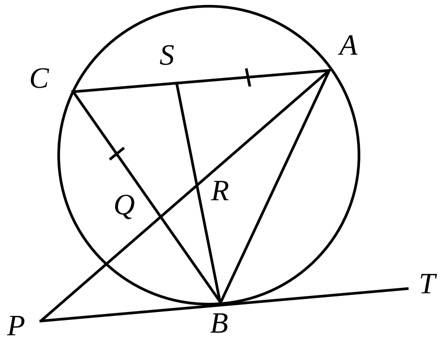
$4x = 14$ or $2x - 6 = -8 + 2x$

$x = 3.5$ or NA




**(iii)** $0 < m < 2$

**10** An equilateral triangle $ABC$ is inscribed in a circle. $PT$ is a tangent to the circle at $B$. It is given that $AS = QC$. $PQA$ is a straight line and $BS$ meets $AQ$ at $R$.




**(i)** Show that $AC$ is parallel to $PB$. [2]

**(ii)** Prove that $\triangle ABS$ is congruent to $\triangle CAQ$. [2]

**(iii)** Prove that $\angle PBQ = \angle BRQ$. [3]

**(i)** $\angle ACB = \angle BAC = 60°$ (equilateral triangle)
$\angle PBC = \angle BAC$ (Alternate Segment Theorem)
Since $\angle PBC = \angle ACB$,
$AC$ is parallel to $PB$ (alternate angle)

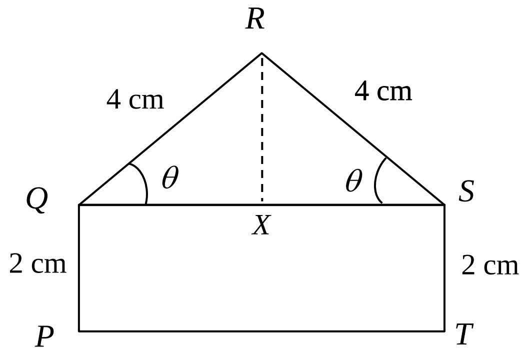
**(ii)** $AS = CQ$ (given)
$\angle BAS = \angle ACQ = 60°$ (equilateral triangle)
$AB = AC$ (sides of a equilateral triangle)
$\therefore \triangle ABS \equiv \triangle CAQ$ (SAS)

**(iii)** let $\angle RBQ = x$,
$\angle RBA = 60° - x$ (equilateral triangle)
$\angle ASB = 180° - (60° - x) - 60° = 60° + x$ (angle sum of triangle)

$\angle RBA = \angle RAS = 60° - x$ ($\triangle ABS \equiv \triangle CAQ$)

$\angle ARS = 180° - (60° + x) - (60° - x) = 60°$ (angle sum of triangle)
$\angle BRQ = 60°$ (vertically opposite angle)
so, $\angle PBQ = \angle BRQ$

**11** In the diagram, *PQRST* is a piece of cardboard. *PQST* is a rectangle with $PQ = 2$ cm and *QRS* is an isosceles triangle with $QR = RS = 4$ cm. $\angle RSQ = \angle RQS = \theta$ radians.




**(i)** Show that the area, $A$ cm$^2$, of the cardboard is given by $A = 8\sin 2\theta + 16\cos\theta$. [3]

**(ii)** Given that $\theta$ can vary, find the stationary value of $A$ and determine whether it it is a maximum or a minimum. [6]

**(i)** $QS = 2(4\cos\theta) = 8\cos\theta$

$RX = 4\sin\theta$

Area, $A = \frac{1}{2}\left(4\sin\theta\right)\left(8\cos\theta\right) + 2\left(8\cos\theta\right)$

$= 16\sin\theta\cos\theta + 16\cos\theta$

$= 8\sin 2\theta + 16\cos\theta$

**(ii)** $\frac{\mathrm{d}A}{\mathrm{d}\theta} = (8\cos 2\theta)(2) + 16(-\sin\theta)$

$\frac{\mathrm{d}A}{\mathrm{d}\theta} = 16(\cos 2\theta - \sin\theta)$

$\frac{\mathrm{d}^2A}{\mathrm{d}\theta^2} = 16(-2\sin 2\theta - \cos\theta)$

For $\frac{\mathrm{d}A}{\mathrm{d}\theta} = 0$,

$\cos 2\theta - \sin\theta = 0$

$1 - 2\sin^2\theta - \sin\theta = 0$

$2\sin^2\theta + \sin\theta - 1 = 0$

$(2\sin\theta - 1)(\sin\theta + 1) = 0$

$\sin\theta = 0.5$ or $\sin\theta = -1$

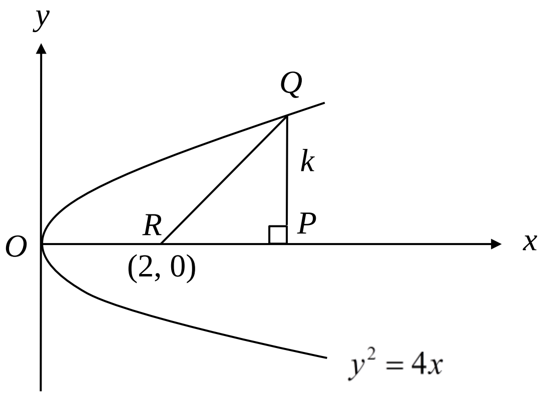
$\theta = \frac{\pi}{6} \approx 0.524$ or NA

For $\theta = \frac{\pi}{6} \approx 0.524$

$\frac{\mathrm{d}^2A}{\mathrm{d}\theta^2} < 0$ => max. area

$A = 12\sqrt{3} = 20.8$

**12** **(a)**




The diagram shows part of a curve $y^2 = 4x$. The point $P$ is on the $x$-axis and the point $Q$ is on the curve. $PQ$ is parallel to the $y$-axis and $k$ is units in length. Given $R$ is (2, 0), express the area, $A$, of the $\Delta PQR$ in terms of $k$ and hence show that $\frac{dA}{dk} = \frac{3k^2 - 8}{8}$.

The point $P$ moves along the $x$-axis and the point $Q$ moves along the curve in such a way that $PQ$ remains parallel to the $y$-axis. $k$ increases at the rate of 0.2 units per second.

Find the rate of increase of $A$ when $k = 6$ units. [5]

$$y^2 = 4x$$

at $Q$, $k^2 = 4x$

$$x = \frac{k^2}{4}$$

$$A = \frac{1}{2}(k)\left(\frac{k^2}{4} - 2\right)$$

$$A = \frac{k^3}{8} - k$$

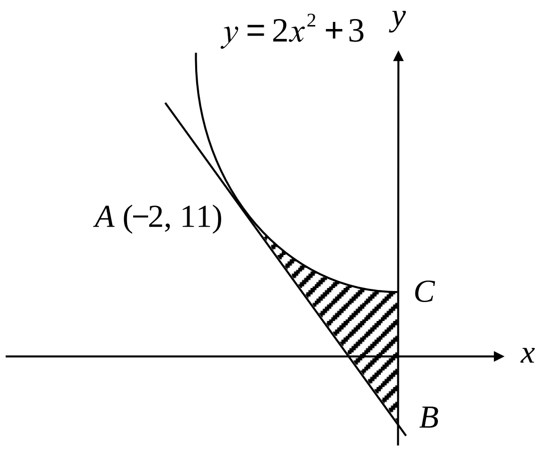
$$\frac{dA}{dk} = \frac{3k^2}{8} - 1$$

$$\frac{dA}{dk} = \frac{3k^2 - 8}{8}$$

$$\frac{dA}{dt} = \frac{dA}{dk} \times \frac{dk}{dt}$$

at $p = 6$, $\frac{dA}{dt} = \left(\frac{3(6)^2 - 8}{8}\right) \times 0.2 = 2.5$

**(b)**




The diagram shows part of the curve $y = 2x^2 + 3$.

The tangent to the curve at the point $A(-2, 11)$ intersects the *y*-axis at *B*. Find the area of the shaded region *ABC*. [6]

$\frac{dy}{dx} = 4x$

at *A*, $m = -8$

let *B* (0, *y*)

$m_{AB} = \frac{11 - y}{-2 - 0}$

$-8 = \frac{11 - y}{-2}$

$y = -5$

*B* (0, –5)

at *C*,

$y = 2(0)^2 + 3 = 3$

*C*(3, 0)

eqn *AB*

$y = -8x - 5$

Area = $\int_{-2}^{0} \left[\left(2x^2 + 3\right) - \left(-8x - 5\right)\right]$

$= \int_{-2}^{0} \left[2x^2 + 8x + 8\right]$

$= \left[\frac{2x^3}{3} + 4x^2 + 8x\right]_{-2}^{0}$

$= 0 - \left[\frac{-16}{3} + 16 - 16\right] = \frac{16}{3}$

**~ End of Paper ~**

***Mathematical Formulae***

## 1. ALGEBRA

*Quadratic Equation*

For the quadratic equation $ax^2 + bx + c = 0$,

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

*Binomial Expansion* $(a+b)^n = a^n + \binom{n}{1}a^{n-1}b + \binom{n}{2}a^{n-2}b^2 + \ldots + \binom{n}{r}a^{n-r}b^r + \ldots + b^n$,

where $n$ is a positive integer and $\binom{n}{r} = \frac{n!}{r!(n-r)!} = \frac{n(n-1)...(n-r+1)}{r!}$.

## 2. TRIGONOMETRY

*Identities*

$$\sin^2 A + \cos^2 A = 1$$

$$\sec^2 A = 1 + \tan^2 A$$

$$\operatorname{cosec}^2 A = 1 + \cot^2 A$$

$$\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$$

$$\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$$

$$\tan(A \pm B) = \frac{\tan A \pm \tan B}{1 \mp \tan A \tan B}$$

$$\sin 2A = 2 \sin A \cos A$$

$$\cos 2A = \cos^2 A - \sin^2 A = 2\cos^2 A - 1 = 1 - 2\sin^2 A$$

$$\tan 2A = \frac{2\tan A}{1 - \tan^2 A}$$

*Formulae for* $\Delta ABC$

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$a^2 = b^2 + c^2 - 2bc \cos A$$

$$\Delta = \frac{1}{2} ab \sin C$$

**1.** The equation $2x^2 + px + 3 = 0$ , where $p > 0$, has roots α and β.

**(i)** Given that $\alpha^2 + \beta^2 = 1$ , show that $p = 4$. **[3]**

**(ii)** Find the value of $\alpha^3 + \beta^3$. **[2]**

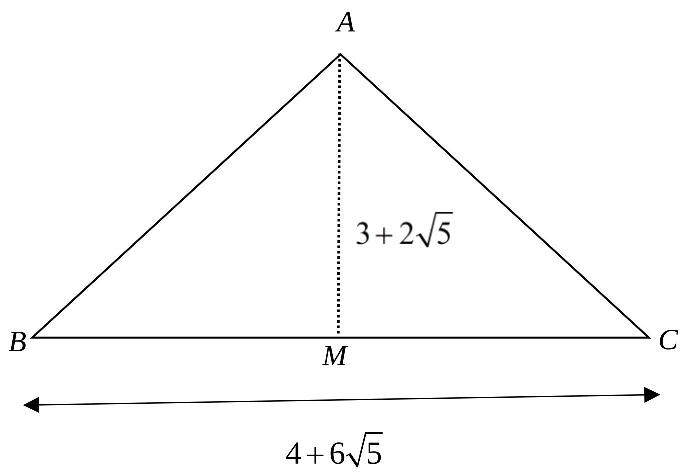
**(iii)** Find a quadratic equation with roots $\dfrac{2\alpha}{\beta^2}$ and $\dfrac{2\beta}{\alpha^2}$. **[3]**

**2.** **(a)** Find the term independent of $x$ in the expansion of $2x\left(2x - \dfrac{1}{x^2}\right)^8$. **[4]**

**(b)** The first 3 terms in the binomial expansion $(1+kx)^n$ are $1 + 5x + \dfrac{45}{4}x^2 + ...$

Find the value of $n$ and of $k$. **[4]**

**3.**




The diagram shows an isosceles triangle $ABC$, where $AB = AC$. The point $M$ is the mid-point of $BC$. Given that $AM = (3+2\sqrt{5})cm$ and $BC = (4+6\sqrt{5})cm$.

**Without the use of a calculator,** find

**(i)** the area of triangle $ABC$, **[2]**

**(ii)** $AB^2$, **[3]**

**(iii)** $\sin \angle BAC$ , giving your answer in the form $\dfrac{p+q\sqrt{5}}{r}$ where $p$, $q$ and $r$ are positive integers. **[3]**

**4.** **(i)** Given that $\dfrac{6x^3-15x^2+6x-5}{2x^2-x}=ax+b+\dfrac{c}{2x^2-x}$ , where $a$, $b$ and $c$ are integers,

express $\dfrac{6x^3-15x^2+6x-5}{2x^2-x}$ in partial fractions. **[5]**

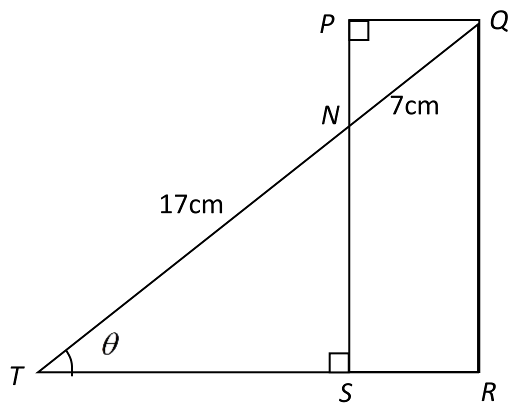
**(ii)** **Hence** find $\int \dfrac{6x^3-15x^2+6x-5}{2x^2-x}\,dx$. **[3]**

**5.** The term containing the highest power of $x$ and the term independent of $x$ in the polynomial $f(x)$ are $2x^4$ and -3 respectively. It is given that $(2x^2+x-1)$ is a quadratic factor of $f(x)$ and the remainder when $f(x)$ is divided by $(x - 1)$ is 4.

**(i)** Find an expression for f(x) in descending powers of x, **[5]**

**(ii)** Explain why the equation $f(x) = 0$ has only 2 real roots and state the values. **[4]**

**6.** *PQRS* is a rectangle. A line through *Q*, intersects *PS* at *N* and *RS* produced at *T*, where $QN$= 7cm, $NT$ = 17cm, $\angle NTS = \theta$, and $\theta$ varies.




**(i)** Show that the perimeter of *PQRS*, *P* cm, is given by $P = 14\cos\theta + 48\sin\theta$. **[2]**

**(ii)** Express *P* in the form of $R\cos(\theta-\alpha)$ and state the value of *R* and $\alpha$ in degree. **[3]**

**(iii)** Without evaluating $\theta$, justify with reasons if *P* can have a value of 48 cm. **[1]**

**(iv)** Find the value of *P* for which *QR* =12 cm. **[2]**

**7.** Variables $x$ and $y$ are related by the equation $\frac{x+sy}{t} = xy$, where $s$ and $t$ are constants.

The table below shows the measured values of $x$ and $y$ during an experiment.

| | | | | | |
|---|---|---|---|---|---|
| ***x*** | 1.00 | 1.50 | 2.00 | 2.50 | 3.00 |
| ***y*** | 0.48 | 0.65 | 0.85 | 1.00 | 1.13 |

**(i)** On graph paper, draw a straight line graph of $\frac{x}{y}$ against $x$, using a scale of 4 cm to represent 1 unit on the $x$ – axis. The vertical $\frac{x}{y}$ – axis should start at 1.5 and have a scale of 1 cm to 0.1 units. **[3]**

**(ii)** Determine which value of $y$ is inaccurate and estimate its correct value. **[1]**

**(iii)** Use your graph to estimate the value of $s$ and of $t$. **[2]**

**(iv)** By adding a suitable straight line on the **same axes**, find the value of $x$ and $y$ which satisfy the following pair of simultaneous equations.

$$\frac{x+sy}{t} = xy$$

$$5y - 2x = 2xy\,.$$ **[3]**

**8.** The equation of a circle $C_1$, is $x^2 + y^2 - 2x - y - 10 = 0$.

**(i)** Find the centre and the radius of the circle. **[3]**

**(ii)** The equation of a tangent to the circle $C_1$ at the point $A$ is $y + 2x = k$, where $k > 0$.

Find the value of the constant $k$. **[4]**

A second circle $C_2$ has its centre at point $A$ and its lowest point $B$ lies on the x-axis.

**(iii)** Find the equation of the circle $C_2$. **[2]**

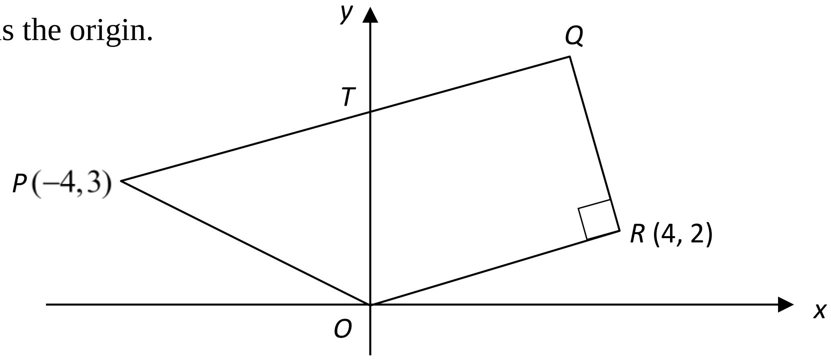
**9.** **(a)** The curve $y = \frac{2x-5}{1-2x}$ passes through the point $A$ where $x = 1$.

**(i)** Find the equation of the normal to the curve at the point $A$. **[4]**

**(ii)** Find the acute angle the tangent makes with the positive $x$-axis. **[2]**

**9.** **(b)** The curve $y = f(x)$ is such that $f''(x) = 3(e^x - e^{-3x})$ and the point $P(0, 2)$ lies on the curve. Given that the gradient of the curve at $P$ is 5, find the equation of the curve. **[6]**

**10.** The diagram (not drawn to scale) shows a trapezium *OPQR* in which *PQ* is parallel to *OR* and $\angle ORQ = 90°$. The coordinates of *P* and *R* are $(-4, 3)$ and $(4, 2)$ respectively and *O* is the origin.




**(i)** Find the coordinates of *Q*. **[3]**

**(ii)** *PQ* meets the *y*-axis at *T*. Show that triangle *ORT* is isosceles. **[2]**

**(iii)** Find the area of the trapezium *OPQR*. **[2]**

**(iv)** *S* is a point such that *ORPS* forms a parallelogram, find the coordinates of *S*. **[2]**

**11.** **(a)** Given that $y = x^2\sqrt{2x+1}$, show that $\dfrac{dy}{dx} = \dfrac{x(5x+2)}{\sqrt{2x+1}}$. **[3]**

**(b)** Hence

**(i)** find the coordinates of the stationary points on the curve $y = x^2\sqrt{2x+1}$ and determine the nature of these stationary points. **[5]**

**(ii)** evaluate $\displaystyle\int_1^5 \frac{5x^2 + 2x - 3}{\sqrt{2x+1}}\, dx$. **[4]**

~~ ***End of Paper*** ~~

# METHODIST GIRLS' SCHOOL

Founded in 1887

**PRELIMINARY EXAMINATION 2018**
**Secondary 4**

| | | |
|---|---|---|
| Thursday | **ADDITIONAL MATHEMATICS** | **4047/02** |
| 3 Aug 2018 | **Paper 2** | 2 h 30 min |

**READ THESE INSTRUCTIONS FIRST**

Write your class, index number and name on all the work you hand in.
Write in dark blue or black pen on both sides of the paper.
You may use an HB pencil for any diagrams or graphs.
Do not use staples, paper clips, glue or correction fluid.

Answer **all** the questions**.**
Write your answers on the separate Answer Paper provided.
Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.
The use of an approved scientific calculator is expected, where appropriate.
You are reminded of the need for clear presentation in your answers.

At the end of the examination, fasten all your work securely together.
The number of marks is given in brackets [ ] at the end of each question or part question.
The total number of marks for this paper is 100.

***Mathematical Formulae***

## 1. ALGEBRA

*Quadratic Equation*

For the quadratic equation $ax^2 + bx + c = 0$, $x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

*Binomial Expansion* $(a+b)^n = a^n + \binom{n}{1}a^{n-1}b + \binom{n}{2}a^{n-2}b^2 + \ldots + \binom{n}{r}a^{n-r}b^r + \ldots + b^n$,

where $n$ is a positive integer and $\binom{n}{r} = \dfrac{n!}{r!(n-r)!} = \dfrac{n(n-1)...(n-r+1)}{r!}$.

## 2. TRIGONOMETRY

*Identities*

$$\sin^2 A + \cos^2 A = 1$$

$$\sec^2 A = 1 + \tan^2 A$$

$$\cos ec^2 A = 1 + \cot^2 A$$

$$\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$$

$$\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$$

$$\tan(A \pm B) = \frac{\tan A \pm \tan B}{1 \mp \tan A \tan B}$$

$$\sin 2A = 2 \sin A \cos A$$

$$\cos 2A = \cos^2 A - \sin^2 A = 2\cos^2 A - 1 = 1 - 2\sin^2 A$$

$$\tan 2A = \frac{2\tan A}{1 - \tan^2 A}$$

$$\sin A + \sin B = 2 \sin \frac{1}{2}(A+B) \cos \frac{1}{2}(A-B)$$

$$\sin A - \sin B = 2 \cos \frac{1}{2}(A+B) \sin \frac{1}{2}(A-B)$$

$$\cos A + \cos B = 2 \cos \frac{1}{2}(A+B) \cos \frac{1}{2}(A-B)$$

$$\cos A - \cos B = -2 \sin \frac{1}{2}(A+B) \sin \frac{1}{2}(A-B)$$

*Formulae for* $\Delta ABC$

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$a^2 = b^2 + c^2 - 2bc \cos A$$

$$\Delta = \frac{1}{2}ab \sin C$$

**1.** The equation $2x^2 + px + 3 = 0$ , where $p > 0$, has roots α and β.

**(i)** Given that $\alpha^2 + \beta^2 = 1$ , show that $p = 4$. **[3]**

**(ii)** Find the value of $\alpha^3 + \beta^3$. **[2]**

**(iii)** Find a quadratic equation with roots $\frac{2\alpha}{\beta^2}$ and $\frac{2\beta}{\alpha^2}$. **[3]**

**(i)** $\alpha + \beta = -\frac{p}{2}$ *and* $\alpha\beta = \frac{3}{2}$

$\alpha^2 + \beta^2 = 1$

$(\alpha + \beta)^2 - 2\alpha\beta = 1$

$\frac{p^2}{4} - 3 = 1$

$p^2 = 16$

$p = 4$ *or* $p = -4$

Since $p > 0$, $p = 4$ (Shown)

**(ii)** $\alpha^3 + \beta^3 = (\alpha + \beta)(\alpha^2 - \alpha\beta + \beta^2)$

$= -2(1 - \frac{3}{2})$

$= 1$

**(iii)** $\frac{2\alpha}{\beta^2} + \frac{2\beta}{\alpha^2} = \frac{2\left(\alpha^3 + \beta^3\right)}{\alpha^2\beta^2}$

$= \frac{8}{9}$

$\frac{2\alpha}{\beta^2} \times \frac{2\beta}{\alpha^2} = \frac{4}{\alpha\beta}$

$= \frac{8}{3}$

Required quadratic equation : $x^2 - \frac{8}{9}x + \frac{8}{3} = 0$ or $9x^2 - 8x + 24 = 0$

**2.** **(a)** Find the term independent of $x$ in the expansion of $2x\left(2x-\frac{1}{x^2}\right)^8$. **[4]**

**(b)** The first 3 terms in the binomial expansion $(1+kx)^n$ are $1+5x+\frac{45}{4}x^2+...$

Find the value of $n$ and of $k$. **[4]**

**(a)** For $\left(2x-\frac{1}{x^2}\right)^8$, $T_{r+1}=\binom{8}{r}(2x)^{8-r}\left(-\frac{1}{x^2}\right)^r$

For $x^{-1}$, $8-r-2r=-1$

$r = 3$

Coefficient of $x^{-1}=\binom{8}{3}(2)^5(-1)^3$ = -1792

Term independent of $x$ in $2x\left(2x-\frac{1}{x^2}\right)^8=-3584$.

**(b)** $(1+kx)^n=1+\binom{n}{1}kx+\binom{n}{2}k^2x^2+....$

$= 1+nkx+\frac{n(n-1)k^2}{2}x^2+...$

Comparing coefficients : $nk = 5$ ……….. (1)

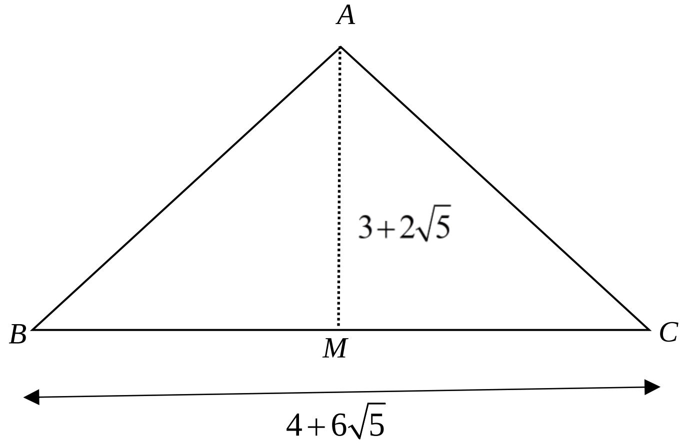
$\frac{n(n-1)k^2}{2}=\frac{45}{4}$

$2n^2k^2 - 2nk^2 = 45$……….. (2)

Subst (1) in (2) : $50 - 10k = 45$

$\therefore k=\frac{1}{2}$ and $n = 10$

**3.**




The diagram shows an isosceles triangle *ABC*, where *AB* = *AC*. The point *M* is the mid-point of *BC*. Given that *AM* = $(3+2\sqrt{5})cm$ and $BC = (4+6\sqrt{5})cm$.

**Without the use of a calculator,** find

**(i)** the area of triangle *ABC*, **[2]**

**(ii)** $AB^2$, **[3]**

**(iii)** $\sin\angle BAC$, giving your answer in the form $\frac{p+q\sqrt{5}}{r}$ where *p*, *q* and *r* are positive integers. **[3]**

**(i)** Area of triangle *ABC* = $\frac{1}{2}(4+6\sqrt{5})(3+2\sqrt{5})$

$$=(2+3\sqrt{5})(3+2\sqrt{5})$$
$$=(36+13\sqrt{5})\ cm^2$$

**(ii)** $AB^2 = (3+2\sqrt{5})^2 + (2+3\sqrt{5})^2$

$$=9+12\sqrt{5}+20+4+12\sqrt{5}+45$$
$$=(78+24\sqrt{5})\ cm^2$$

**(iii)** $\frac{1}{2}(78+24\sqrt{5})\ \sin\angle BAC = 36+13\sqrt{5}$

$$\sin\angle BAC = \frac{36+13\sqrt{5}}{39+12\sqrt{5}}$$

$$=\frac{36+13\sqrt{5}}{39+12\sqrt{5}}\times\frac{39-12\sqrt{5}}{39-12\sqrt{5}}$$
$$=\frac{1404-432\sqrt{5}+507\sqrt{5}-780}{801}$$
$$=\frac{624+75\sqrt{5}}{801}$$
$$=\frac{208+25\sqrt{5}}{267}$$

**4.** **(i)** Given that $\dfrac{6x^3-15x^2+6x-5}{2x^2-x}=ax+b+\dfrac{c}{2x^2-x}$ , where *a*, *b* and *c* are integers,

express $\dfrac{6x^3-15x^2+6x-5}{2x^2-x}$ in partial fractions. **[5]**

**(ii)** **Hence** find $\int \dfrac{6x^3-15x^2+6x-5}{2x^2-x}\,dx$ . **[3]**

**(i)** Using long division, $\dfrac{6x^3-15x^2+6x-5}{2x^2-x}=3x-6-\dfrac{5}{2x^2-x}$

Let $\dfrac{-5}{x(2x-1)}=\dfrac{A}{x}+\dfrac{B}{2x-1}$

$-5=A(2x-1)+Bx$

Put $x = 0$ : $A = 5$

Put $x=\dfrac{1}{2}$ : $\dfrac{1}{2}B=-5$

$B = -10$

$\therefore \dfrac{6x^3-15x^2+6x-5}{2x^2-x}=3x-6+\dfrac{5}{x}-\dfrac{10}{2x-1}$

**(ii)** $\int \dfrac{6x^3-15x^2+6x-5}{2x^2-x}dx=\int (3x-6+\dfrac{5}{x}-\dfrac{10}{2x-1})dx$

$=\dfrac{3x^2}{2}-6x+5\ln x-5\ln(2x-1)+C$

**5.** The term containing the highest power of $x$ and the term independent of $x$ in the polynomial $f(x)$ are $2x^4$ and -3 respectively. It is given that $(2x^2 + x - 1)$ is a quadratic factor of $f(x)$ and the remainder when $f(x)$ is divided by $(x - 1)$ is 4.

**(i)** Find an expression for $f(x)$ in descending powers of $x$, **[5]**

**(ii)** Explain why the equation $f(x) = 0$ has only 2 real roots and state the values. **[4]**

**(i)** $f(x) = (2x^2 + x - 1)(x^2 + bx + 3)$

$f(1) = 4$

$2(4 + b) = 4$
$b = -2$

$f(x) = (2x^2 + x - 1)(x^2 - 2x + 3)$

$= 2x^4 - 4x^3 + 6x^2 + x^3 - 2x^2 + 3x - x^2 + 2x - 3$
$= 2x^4 - 3x^3 + 3x^2 + 5x - 3$

**(ii)** $f(x) = (2x^2 + x - 1)(x^2 - 2x + 3)$

$= (2x - 1)(x + 1)(x^2 - 2x + 3)$
$(2x - 1)(x + 1)(x^2 - 2x + 3) = 0$

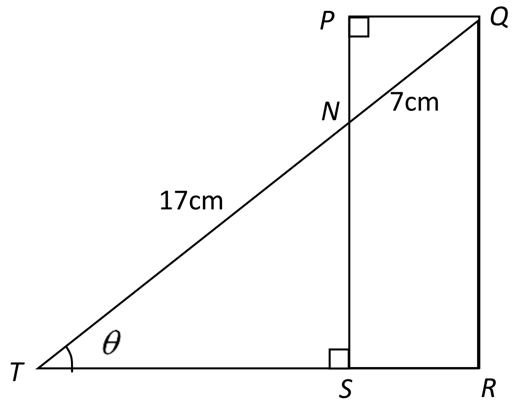
$x = \frac{1}{2} \quad or \quad x = -1$

$x^2 - 2x + 3 = 0$

$D = (-2)^2 - 4(1)(3) = -8 < 0$

$\therefore f(x) = 0$ has only 2 real roots (Shown)

**6.** *PQRS* is a rectangle. A line through *Q*, intersects *PS* at *N* and *RS* produced at *T*, where *QN*= 7cm, *NT* = 17cm, $\angle NTS = \theta$, and $\theta$ varies.




**(i)** Show that the perimeter of *PQRS*, *P* cm, is given by $P = 14\cos\theta + 48\sin\theta$. **[2]**

**(ii)** Express *P* in the form of $R\cos(\theta - \alpha)$ and state the value of *R* and $\alpha$ in degrees. **[3]**

**(iii)** Without evaluating $\theta$, justify with reasons if *P* can have a value of 48 cm **[1]**

**(iv)** Find the value of *P* for which *QR* =12 cm. **[3]**

**(i)** $P = 2(7\cos\theta) + 2(24\sin\theta)$

$= 14\cos\theta + 48\sin\theta$

$14\cos\theta + 48\sin\theta = R\cos(\theta - \alpha) = R\cos\theta\cos\alpha + R\sin\theta\sin\alpha$

$R\cos\alpha = 14$ and $R\sin\alpha = 48$

$R = \sqrt{14^2 + 48^2} = \sqrt{2500} = 50$

$\tan\alpha = \frac{48}{14}$

$\alpha = 73.74°$

$= 73.7°$

$14\cos\theta + 48\sin\theta = 50\cos(\theta - 73.74°)$

**(ii)** Since maximum value of *P* = 50, *P* can have a value of 48 cm.

**Or** $\cos(\theta - 73.74°) = \frac{48}{50} < 1$, *P* can have a value of 48 cm.

When *QR*=12cm, $\sin\theta = \frac{12}{24} = \frac{1}{2}$

$\theta = 30°, 150°$ (NA $\because \theta < 90°$)

$\therefore P = 50\cos(30° - 73.74°)$

= 36.1 cm (3sf)

**7.** Variables $x$ and $y$ are related by the equation $\frac{x+sy}{t}=xy$, where $s$ and $t$ are constants.

The table below shows the measured values of $x$ and $y$ during an experiment.

| | | | | | |
|---|---|---|---|---|---|
| ***x*** | 1.00 | 1.50 | 2.00 | 2.50 | 3.00 |
| ***y*** | 0.48 | 0.65 | 0.85 | 1.00 | 1.13 |

**(i)** On graph paper, draw a straight line graph of $\frac{x}{y}$ against $x$, using a scale of 4 cm to represent 1 unit on the $x$ – axis. The vertical $\frac{x}{y}$ – axis should start at 1.5 and have a scale of 1 cm to 0.1 units. **[3]**

**(ii)** Determine which value of $y$ is inaccurate and estimate its correct value. **[1]**

**(iii)** Use your graph to estimate the value of $s$ and of $t$. **[2]**

**(iv)** By adding a suitable straight line on the **same axes**, find the value of $x$ and $y$ which satisfy the following pair of simultaneous equations.

$$\frac{x+sy}{t}=xy$$

$$5y-2x=2xy\,.$$ **[3]**

**(i)** $x+sy=xyt$

$$\frac{x}{y}=tx-s$$

Gradient = $t$ and $\frac{x}{y}-\text{int}\,ercept=-s$

**(ii)** Incorrect value of $y$ = 0.65.

From graph, correct value of $\frac{x}{y}=2.2$

Estimated correct value of $y$ = 0.68

**(iii)** From the graph,
$s$ = -1.75 (-1.82 ~ -1.72)
t = 0.3 (0.28 ~ 0.32)

**(iv)** Draw the line : $\frac{x}{y}=-x+\frac{5}{2}$

From graph, $x$ =0.575 (0.55 ~ 0.60)

and $\frac{x}{y}=1.93(1.92\ \sim\ 1.95)\Rightarrow y=0.30$

**8.** The equation of a circle $C_1$, is $x^2 + y^2 - 2x - y - 10 = 0$.

**(i)** Find the centre and the radius of the circle. **[3]**

**(ii)** The equation of a tangent to the circle $C_1$ at the point $A$ is $y + 2x = k$, where $k > 0$.

Find the value of the constant $k$. **[4]**

A second circle $C_2$ has its centre at point $A$ and its lowest point $B$ lies on the $x$-axis.

Find the equation of the circle $C_2$. **[2]**

**(i)** $x^2 + y^2 - 2x - y - 10 = 0$

$(x-1)^2 - 1 + \left(y - \frac{1}{2}\right)^2 - \frac{1}{4} - 10 = 0$

$(x-1)^2 + \left(y - \frac{1}{2}\right)^2 = 11\frac{1}{4}$

$\therefore$ centre of circle = $\left(1, \frac{1}{2}\right)$ and radius = $\frac{\sqrt{45}}{2} = \frac{3\sqrt{5}}{2}$ *units*

**(ii)** $x^2 + (k-2x)^2 - 2x - (k-2x) - 10 = 0$

$5x^2 - 4kx + k^2 - 2x - k + 2x - 10 = 0$

$5x^2 - 4kx + k^2 - k - 10 = 0$

Since line is a tangent to the circle, Discriminant = 0

$(-4k)^2 - 4(5)(k^2 - k - 10) = 0$

$-4k^2 + 20k + 200 = 0$

$k^2 - 5k - 50 = 0$

$k = 10 \quad or \quad k = -5 \ (NA \because k > 0)$

**(iii)** When $k$ = 10, $5x^2 - 40x + 80 = 0$

$x^2 - 8x + 16 = 0$

$\therefore x = 4 \quad and \quad y = 2$

$A(4, \ 2)$

Since lowest point lies on $x$-axis, radius of circle $C_2$ = 2 units

Equation of circle $C_2$ : $(x-4)^2 + (y-2)^2 = 4$.

**9.** **(a)** The curve $y = \dfrac{2x-5}{1-2x}$ passes through the point $A$ where $x = 1$.

**(i)** Find the equation of the normal to the curve at the point $A$. **[4]**

**(ii)** Find the acute angle the tangent makes with the positive $x$-axis. **[2]**

**(a)(i)**

$$y = \frac{2x-5}{1-2x}$$

$$\frac{dy}{dx} = \frac{(1-2x)(2)-(2x-5)(-2)}{(1-2x)^2}$$

$$= \frac{2-4x+4x-10}{(1-2x)^2}$$

$$= \frac{-8}{(1-2x)^2}$$

$$m_{\tan gent} = -8$$

$$m_{normal} = \frac{1}{8}$$

$$y = 3$$

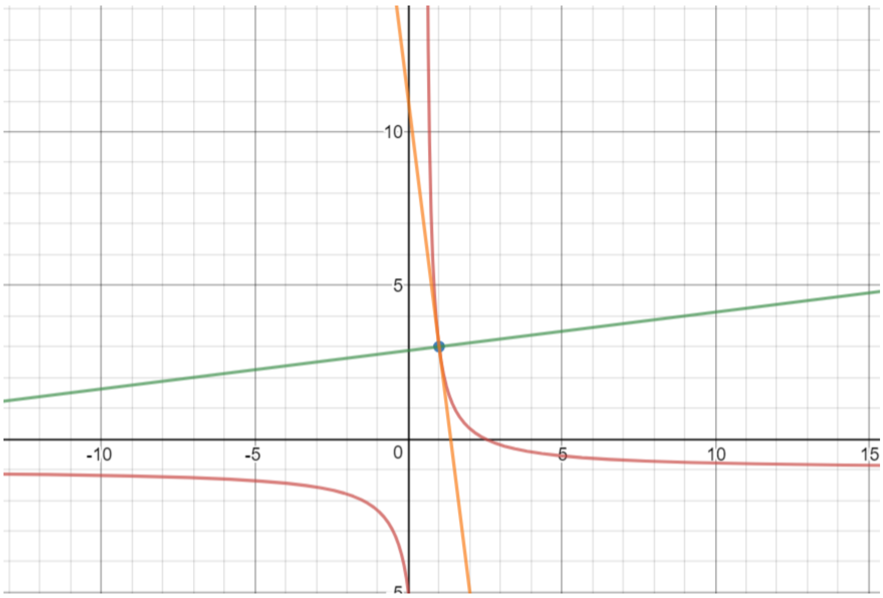
$$y - 3 = \frac{1}{8}(x-1)$$

$$y = \frac{1}{8}x + \frac{23}{8} \quad or\ 8y = x + 23$$

**(ii)**

$$\tan\theta = 8$$

$$\theta = 82.9^0 \quad \text{or} \quad 1.45rad$$



**9.** **(b)** The curve $y = f(x)$ is such that $f''(x) = 3(e^x - e^{-3x})$ and the point $P(0, 2)$ lies on the curve. Given that the gradient of the curve at $P$ is 5, find the equation of the curve. **[6]**

$f'(x) = 3e^x + e^{-3x} + C$, where $C$ is an arbitrary constant.

$f'(0) = 5$

$3e^0 + e^0 + C = 5$
$C = 1$

$\therefore f'(x) = 3e^x + e^{-3x} + 1$

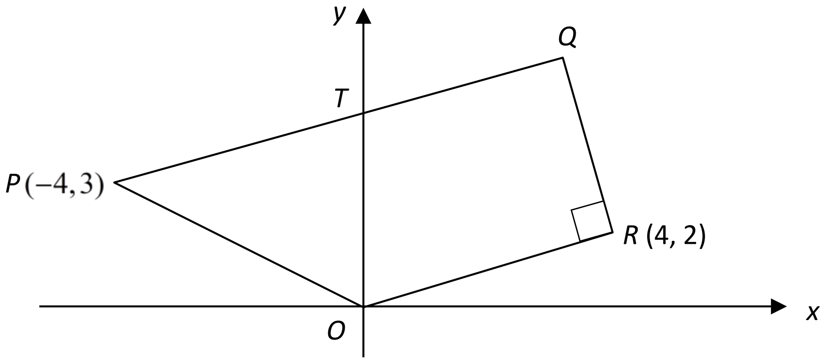
$f(x) = \int (3e^x + e^{-3x} + 1)dx$

$= 3e^x - \frac{e^{-3x}}{3} + x + D$, where $D$ is an arbitrary constant.

$f(0) = 2$
$3 - \frac{1}{3} + 0 + D = 2$
$D = -\frac{2}{3}$

Equation of curve : $y = 3e^x - \frac{1}{3e^{3x}} + x - \frac{2}{3}$.

**10.** The diagram (not drawn to scale) shows a trapezium *OPQR* in which *PQ* is parallel to *OR* and $\angle ORQ = 90°$. The coordinates of *P* and *R* are $(-4, 3)$ and (4, 2) respectively and *O* is the origin.




**(i)** Find the coordinates of *Q*. **[3]**

**(ii)** *PQ* meets the *y*-axis at *T*. Show that triangle *ORT* is isosceles. **[2]**

**(iii)** Find the area of the trapezium *OPQR*. **[2]**

**(iv)** *S* is a point such that *ORPS* forms a parallelogram, find the coordinates of *S*.

**[2]**

**(i)** Gradient of *PQ* = gradient of *OR*= 0.5

Eqn of PQ: $y-3=\frac{1}{2}(x+4)$

$y=\frac{1}{2}x+5$ --------(1)

Gradient of QR = $-2$

Eqn of QR: $y-2=-2(x-4)$

$y=-2x+10$ ------(2)

(1)=(2)

$-2x+10=\frac{1}{2}x+5$

$\frac{5}{2}x=5$

$x=2$

$y=-2(2)+10=6$

$\therefore Q(2,6)$

**(ii)** In eqn (1), let $x = 0$, $y = 5$, $\therefore OT = 5units$

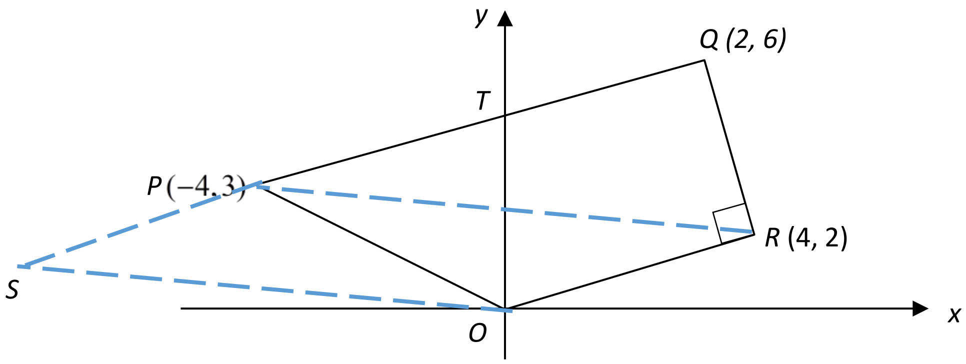
$$RT = \sqrt{(4-0)^2 + (2-5)^2}$$
$$RT = \sqrt{25} = 5$$

Since $OT = RT = 5$ units

$\therefore \Delta ORT$ is isosceles.

Area of trapezium $OPQR$

$$= \frac{1}{2}\begin{vmatrix} 0 & -4 & 2 & 4 & 0 \\ 0 & 3 & 6 & 2 & 0 \end{vmatrix}$$
$$= \frac{1}{2}\left|-24 + 4 - 24 - 6\right|$$
$$= \frac{1}{2}\left|-50\right|$$
$$= 25units^2$$




**(iii)** Let $S\ (a, b)$

Midpoint of $RS$ = Midpoint of $OP$

$$\left(\frac{a+4}{2}, \frac{b+2}{2}\right) = \left(-\frac{4}{2}, \frac{3}{2}\right)$$
$$a + 4 = -4 \quad \& \quad b + 2 = 3$$
$$a = -8 \qquad\qquad b = 1$$

Hence coordinates of $S\ (-8, 1)$

**11.** **(a)** Given that $y = x^2\sqrt{2x+1}$, show that $\dfrac{dy}{dx} = \dfrac{x(5x+2)}{\sqrt{2x+1}}$. **[3]**

**(a)** $y = x^2\sqrt{2x+1}$

$$\frac{dy}{dx} = x^2[\frac{1}{2}(2x+1)^{-\frac{1}{2}}(2)] + 2x(2x+1)^{\frac{1}{2}}$$

$$= x(2x+1)^{-\frac{1}{2}}(x+4x+2)$$

$$= x(5x+2)(2x+1)^{-\frac{1}{2}}$$

$$\therefore \frac{dy}{dx} = \frac{x(5x+2)}{\sqrt{2x+1}} \text{ (shown)}$$

**(b)** Hence

**(i)** find the coordinates of the stationary points on the curve $y = x^2\sqrt{2x+1}$ and determine the nature of these stationary points. **[5]**

**(ii)** evaluate $\int_0^4 \dfrac{5x^2+2x-3}{\sqrt{2x+1}}\, dx$ . **[4]**

**(b)(i)** For stationary points, $\dfrac{dy}{dx} = \dfrac{x(5x+2)}{\sqrt{2x+1}} = 0$

$x = 0$ or $x = -\dfrac{2}{5}$

Stationary points are (0, 0) and $(-\dfrac{2}{5}, 0.0716)$

Using 1st derivative test :

| $x$ | -0.5 | -0.4 | -0.3 | -0.1 | 0 | 0.1 |
|---|---|---|---|---|---|---|
| $\dfrac{dy}{dx}$ | >0 | 0 | <0 | < 0 | 0 | >0 |
| Sketch of tangent | / | — | \ | \ | — | / |

$(-\dfrac{2}{5}, 0.0716)$ is a maximum point and (0, 0) is a minimum point.

**(ii)**
$$\int_1^5 \frac{5x^2+2x-3}{\sqrt{2x+1}}\, dx = \int_1^5 \frac{x(5x+2)}{\sqrt{2x+1}} dx - 3\int_1^5 (2x+1)^{-\frac{1}{2}} dx$$

$$= [x^2\sqrt{2x+1}]_1^5 - 3\left[\sqrt{2x+1}\right]_1^5$$

$$= 76.4$$

| Qn | Answer Key | | Qn | Answer Key |
|---|---|---|---|---|
| **1(ii)** | $\alpha^3+\beta^3=1$ | | **7(iii)** | From the graph,<br>$s$ = -1.75 (-1.82 ~ -1.72)<br><br>t = 0.3 (0.28 ~ 0.32) |
| **(iii)** | $9x^2-8x+24=0$ | | **7(iv)** | $\frac{x}{y}=1.93(1.92 \sim 1.95) \Rightarrow y=0.30$ |
| **2(a)** | Term independent of $x$ in<br>$2x\left(2x-\frac{1}{x^2}\right)^8=-3584$. | | **8(i)** | centre of circle = $\left(1,\frac{1}{2}\right)$<br><br>radius = $\frac{\sqrt{45}}{2}=\frac{3\sqrt{5}}{2}$ *units* |
| **2(b)** | $\therefore k=\frac{1}{2}$ and $n$ = 10 | | **8(ii)** | $k$ = 10 |
| **3(i)** | $(36+13\sqrt{5})\ cm^2$ | | **8(iii)** | Equation of circle $C_2$ :<br>$(x-4)^2+(y-2)^2=4$. |
| **3(ii)** | $(78+24\sqrt{5})\ cm^2$ | | **9(ai)** | $y=\frac{1}{8}x+\frac{23}{8}$ *or* $8y=x+23$ |
| **3(iii)** | $\frac{208+25\sqrt{5}}{267}$ | | **9(aii)** | $\tan\theta=8$<br>$\theta=82.9^0$ or $1.45rad$ |
| **4(i)** | $3x-6+\frac{5}{x}-\frac{10}{2x-1}$ | | **9(b)** | Equation of curve :<br>$y=3e^x-\frac{1}{3e^{3x}}+x-\frac{2}{3}$. |
| **4(ii)** | $\frac{3x^2}{2}-6x+5\ln x-5\ln(2x-1)+C$ | | **10(i)** | $Q(2,6)$ |
| **5(i)** | $f(x)=2x^4-3x^3+3x^2+5x-3$ | | **10(ii)** | Since $OT$ = $RT$ = 5 units<br><br>$\therefore \Delta ORT$ is isosceles. |
| **5(ii)** | $x^2-2x+3=0$<br>$D=(-2)^2-4(1)(3)=-8<0$<br>$f(x)$ = 0 has only 2 real roots (Shown) | | **10(iii)** | Area of trapezium $OPQR$<br>$=25units^2$ |
| **6(ii)** | $14\cos\theta+48\sin\theta=50\cos(\theta-73.74°)$ | | **10(iv)** | $S(-8,1)$ |
| **6(iii)** | Since maximum value of $P$ = 50, $P$ can have a value of 48 cm.<br><br>**Or** $\cos(\theta-73.74°)=\frac{48}{50}<1$, $P$ can have a value of 48 cm. | | **11(bi)** | $(-\frac{2}{5},0.0716)$ is a maximum point and<br>(0, 0) is a minimum point. |
| **6(iv)** | 36.1 cm (3sf) | | **11(bii)** | 76.4 |
| **7 (ii)** | Incorrect value of $y$ = 0.65.<br>Estimated correct value of $y$ = 0.68 | | | |

Candidate Name ..........

Subject 2018 A Math Prelim Exam Paper 2

Question No. 7

Centre Number | Index Number

(i) $x + sy = xyt$

$\frac{x}{y} = tx - s$

Gradient $= t$, $\frac{x}{y}$-int $= -s$

(ii) Incorrect value of $y = 0.65$

Correct value of $\frac{x}{y} = 2.2$

$\therefore$ Correct value of $y = 0.68$

(iii) From the graph, $-s = 1.75$

$\therefore s = -1.75$

Gradient $= t = \frac{2.65 - 2.35}{3 - 2}$

$= 0.3$

| $x$ | 1.00 | 1.50 | 2.00 | 2.50 | 3.00 |
|---|---|---|---|---|---|
| $\frac{x}{y}$ | 2.08 | 2.31 | 2.35 | 2.50 | 2.65 |

(iv) $5y - 2x = 2xy$

$5 - \frac{2x}{y} = 2x$

$\frac{2x}{y} = -2x + 5$

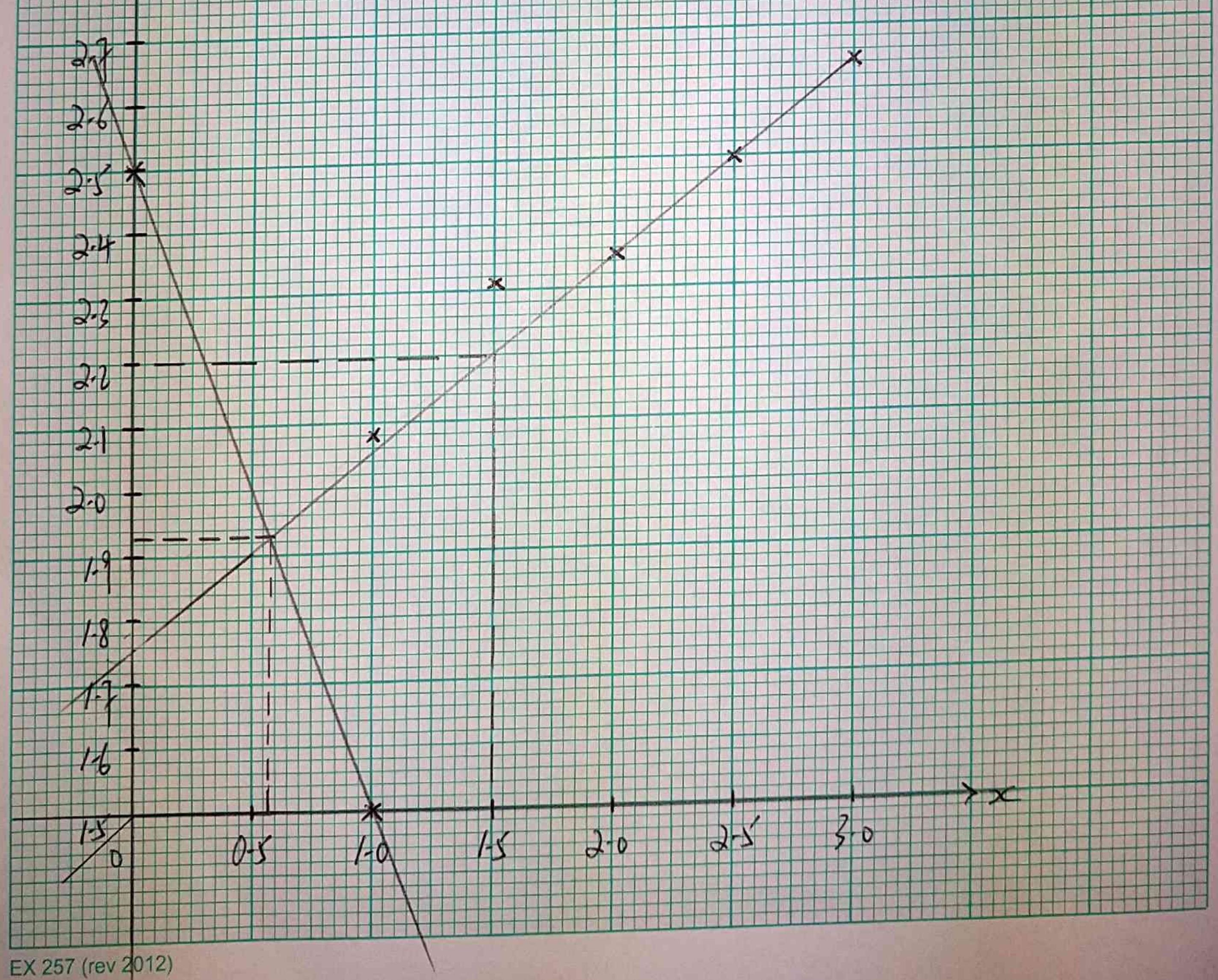
$\frac{x}{y} = -x + \frac{5}{2}$

| $x$ | 0 | 1 |
|---|---|---|
| $\frac{x}{y}$ | 2.5 | 1.5 |

$\therefore x = 0.575$

$\frac{x}{y} = 1.93$

$\therefore y = \frac{0.575}{1.93} = 0.30$



EX 257 (rev 2012)