Anglo-Chinese School (Barker Road)

PRELIMINARY EXAMINATION 2017

SECONDARY FOUR EXPRESS

ADDITIONAL MATHEMATICS 4047 PAPER 1

2 HOURS

Additional Materials: Answer Paper

READ THESE INSTRUCTIONS FIRST

Write your Centre number, index number and name on all the work you hand in.
Write in dark blue or black pen on both sides of the paper.
You may use a soft pencil for any diagrams or graphs.
Do not use staples, paper clips, highlighters, glue or correction fluid.

Answer **all** the questions.
Write your answers on the separate Answer Paper provided.
Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.
The use of an approved scientific calculator is expected, where appropriate.
You are reminded of the need for clear presentation in your answers.

At the end of the examination, fasten all your work securely together.
Attach the cover page on top of your answer script.
The number of marks is given in brackets [ ] at the end of each question or part question.
The total of the marks for this paper is 80.

Mathematical Formulae

1. ALGEBRA

Quadratic Equation

For the equation $ax^2 + bx + c = 0$,

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Binomial expansion

$$(a+b)^n = a^n + \binom{n}{1} a^{n-1} b + \binom{n}{2} a^{n-2} b^2 + \ldots + \binom{n}{r} a^{n-r} b^r + \ldots + b^n,$$

where n is a positive integer and $\binom{n}{r} = \frac{n!}{r!(n-r)!} = \frac{n(n-1)\ldots(n-r+1)}{r!}$

2. TRIGONOMETRY

Identities

$$\sin^2 A + \cos^2 A = 1$$

$$\sec^2 A = 1 + \tan^2 A$$

$$\operatorname{cosec}^2 A = 1 + \cot^2 A$$

$$\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$$

$$\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$$

$$\tan(A \pm B) = \frac{\tan A \pm \tan B}{1 \mp \tan A \tan B}$$

$$\sin 2A = 2 \sin A \cos A$$

$$\cos 2A = \cos^2 A - \sin^2 A = 2\cos^2 A - 1 = 1 - 2\sin^2 A$$

$$\tan 2A = \frac{2\tan A}{1 - \tan^2 A}$$

Formulae for ΔABC

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$a^2 = b^2 + c^2 - 2bc \cos A$$

$$\Delta = \frac{1}{2} ab \sin C$$

Answer **all** the questions

1 The equation of a curve is $y = 2x^2 - 6x + k$, where k is a constant.

(i) In the case when $k = -20$, find the set of values of x for which $y < 0$. [2]

(ii) In the case when $k = 10$, show that the line $y + 2x = 8$ is a tangent to the curve. [3]

2 **(i)** Given that $u = 2^x$, express $4^x - 2^{x+1} = 3$ as an equation in u. [2]

(ii) Hence find the value of x, correct to 2 decimal places. [3]

(iii) Explain why the equation $4^x - 2^{x+1} = k$ has no solution if $k < -1$. [2]

3 The equation $3x^2 - x + 5 = 0$ has roots α and β.

(i) Find the value of $\alpha^3 + \beta^3$. [5]

(ii) Find a quadratic equation with integer coefficient whose roots are $\frac{1}{\alpha^3}$ and $\frac{1}{\beta^3}$. [3]

4 **(i)** Express $\frac{1 - \sin x \cos x + 2\cos^2 x}{\sin^2 x}$ as a quadratic expression in $\cot x$. [3]

(ii) Hence, using **(i)** solve the equation $1 + 2\cos^2 x = \sin x(3\sin x + \cos x)$ for $0° < x < 360°$. [4]

5 A freshly baked bread, with an initial temperature of 75°C, is left to cool on the rack. The temperature, T°C of the bread, t minutes after it has been placed on the rack is given by $T = 27 + ke^{-nt}$, where k and n are positive constants.

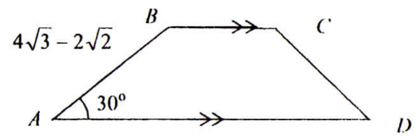
(i) Calculate the value of k. [1]

(ii) If the bread has cooled down by 20°C after 10 minutes, find the value of n. [2]

(iii) Find the least time taken, to the nearest minute, for the bread to have a temperature of less than 30°C. [3]

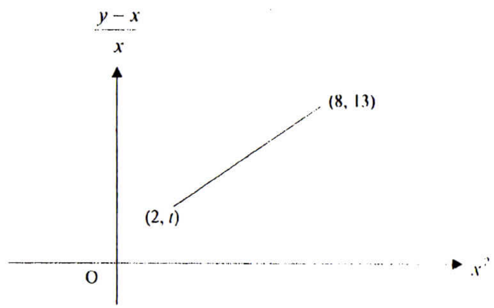
(iv) Explain with the sketch for $T = 27 + ke^{-nt}$, why the temperature of the bread can never reach 27°C. [1]

6



The diagram shows a trapezium $ABCD$ in which side $AB = 4\sqrt{3} - 2\sqrt{2}$ cm and angle $BAD = 30°$. Given that the length of AD is twice the length of BC and that area of the trapezium is $25 + 5\sqrt{6}$ cm², find without the use of a calculator, the length of AD in the form $a\sqrt{2} + b\sqrt{3}$. [4]

7 The diagram shows part of a straight line, passing through $(2, t)$ and $(8, 13)$, drawn to represent the equation $3y = 4x^3 + ax$, where a and t are constants. Find the value of a and t.



[4]

8 **(a)** Given that the first 3 terms in the expansion of $(a-x)(1+3x)^n$ in ascending powers of x is

$$2 + 29x + bx^2 + \dots$$

Find the values of the constants a, b and n. [5]

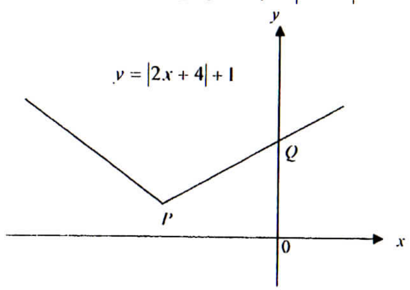
(b) **(i)** Write down the general term in the binomial expansion of $\left(x^2 + \frac{1}{2x^3}\right)^{10}$. [1]

(ii) Write down the power of x in this general term. [1]

(iii) Hence, or otherwise, determine the coefficient of x^{-15} in the binomial expansion of $\left(x^2 + \frac{1}{2x^3}\right)^{10}$. [2]

9 A curve has equation $y = \ln\left(\frac{3x-1}{5-2x}\right)$. The normal to the curve at the point (a, b) is parallel to the line $13y + 5x - 26 = 0$. Given that where $a > 1$, find the values of a and b. [5]

10 The diagram shows part of the graph of $y = |2x+4| + 1$.



(i) Find the coordinates of P and of Q. [3]

A line of gradient m passes through the point (0,3).

(ii) In the case where $m = -1$, find the x-coordinates of the points of intersection between the line and the graph of $y = |2x+4| + 1$. [3]

(iii) Determine the set of values of m for which the line intersects the graph of $y = |2x+4| + 1$ at two points. [2]

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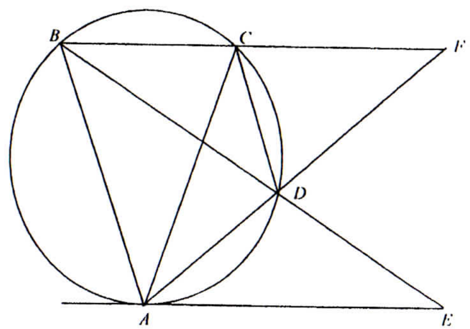
11 A particle starts from rest and moves in a straight line, so that t seconds after leaving O, its velocity, v m/s is given by $v = 28 - 2e^{\frac{t}{3}}$.

(i) Calculate the initial acceleration of the particle. [2]

(ii) Calculate, to 2 decimal places, the displacement of the particle from O when $t = 10$. [4]

(iii) Determine, with explanation, whether the particle will return to O. [1]

12 In the diagram below, the line AE is tangent to the circle at A. The line EB is the angle bisector of angle ABC and cuts the circle at D. The chord AC is the angle bisector of angle BAD and cuts the circle at C. The chords BC and AD are produced to meet at F. The line segments AD and FD are equal.



(i) Prove that angle DAC = angle DAE. [3]

(ii) Prove that triangles ADE and BAE are similar. [2]

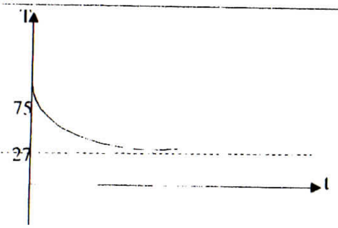
(iii) Prove that C is a mid-point of BF. [2]

(iv) Hence, using **(ii)** and **(iii)**, show that $AF \times AE = 4CD \times DE$. [2]

Anglo-Chinese School (Barker) Sec 4 Prelim 2017 Add Math Paper 1

		Answers	
1	(i)	When $k = -20$	
		$y = 2x^2 - 6x - 20$	
		$2x^2 - 6x - 20 < 0$	
		$(x-5)(x+2) < 0$	
		$-2 < x < 5$	
	(ii)	When $k = 10$	
		$y = 2x^2 - 6x + 10$	
		$y = -2x + 8$	
		$2x^2 - 6x + 10 = -2x + 8$	
		$2x^2 - 4x + 2 = 0$	
		$x^2 - 2x + 1 = 0$	
		$b^2 - 4ac$ $4 - 4(1)(1)$	
		0 (shown) The line is tangent to curve	
2	(i)	$(2^x)^2 - 2^x \times 2 = 3$	
		$u^2 - 2u - 3 = 0$	
	(ii)	$(u+1)(u-3) = 0$	
		$u = 3$ or $u = -1$	
		$2^x = 3$ or $2^x = -1$(NA)	
		$x = \dfrac{\lg 3}{\lg 2}$	
		$x = 1.58$	
	(iii)	$u^2 - 2u - k = 0$	
		For no solution, $b^2 - 4ac < 0$	
		$(-2)^2 - 4(1)(-k) < 0$	
		$4 + 4k < 0$	
		$1 + k < 0$	
		$k < -1$ The equation has no solution if $k < -1$	

3		$\alpha + \beta = \dfrac{1}{3}$, $\quad \alpha\beta = \dfrac{5}{3}$	
		$\alpha^2 + \beta^2 = (\alpha+\beta)^2 - 2\alpha\beta$	
		$= \left(\dfrac{1}{3}\right)^2 - 2\left(\dfrac{5}{3}\right)$	
		$= -\dfrac{29}{9}$	
		$\alpha^3 + \beta^3 = (\alpha+\beta)(\alpha^2 - \alpha\beta + \beta^2)$	
		$\alpha^3 + \beta^3 = \left(\dfrac{1}{3}\right)\left(-\dfrac{29}{9} - \dfrac{5}{3}\right)$	
		$= -\dfrac{44}{27}$	
	(iii)	$\dfrac{1}{\alpha^3} + \dfrac{1}{\beta^3} = \dfrac{\beta^3 + \alpha^3}{\alpha^3\beta^3}$	
		$= -\dfrac{44}{27} \div \left(\dfrac{5}{3}\right)^3$	
		$= -\dfrac{44}{125}$	
		$\left(\dfrac{1}{\alpha^3}\right)\left(\dfrac{1}{\beta^3}\right) = \dfrac{27}{125}$	
		$x^2 + \dfrac{44}{125}x + \dfrac{27}{125} = 0$	
		$125x^2 + 44x + 27 = 0$	
4	(i)	$\dfrac{1}{\sin^2 x} - \dfrac{\cos x}{\sin x} + \dfrac{2\cos^2 x}{\sin^2 x}$	
		$\operatorname{cosec}^2 x - \cot x + 2\cot^2 x$	
		$(\cot^2 x + 1) - \cot x + 2\cot^2 x$	
		$3\cot^2 x - \cot x + 1$	
	(ii)	$1 + 2\cos^2 x = 3\sin^2 x + \sin x\cos x$	
		$1 + 2\cos^2 x - \sin x\cos x = 3\sin^2 x$	
		$\dfrac{1 - \sin x\cos x + 2\cos^2 x}{\sin^2 x} = 3$	
		$3\cot^2 x - \cot x + 1 = 3$	
		$3\cot^2 x - \cot x - 2 = 0$	
		$(3\cot x + 2)(\cot x - 1) = 0$	

		$\cot x = -\frac{2}{3}$ or $\cot x = 1$	
		$\tan x = -\frac{3}{2}$ or $\tan x = 1$	
		$x = 180° - 56.3°, 360° - 56.3°, 45°, 225°$	
		$x = 45°, 123.7°, 225°, 303.7°$ (1 d.p)	
5	(i)	when $t = 0$, $T = 75$	
		$75 = 27 + k$	
		$k = 48$	
	(ii)	when $t = 10$, $T = 55$	
		$55 = 27 + 48e^{-10n}$	
		$28 = 48e^{-10n}$	
		$\frac{7}{12} = e^{-10n}$	
		$-10n = \ln\frac{7}{12}$	
		$n = 0.0539$(3sf)	
	(iii)	$27 + 48e^{-0.0539t} < 30$	
		$48e^{-0.0539t} < 3$	
		$e^{-0.0539t} < \frac{1}{16}$	
		$-0.0539t < \ln\frac{1}{16}$	
		$t > \frac{\ln\frac{1}{16}}{-0.0539}$	
		$t > 51.439$	
		Least time taken = 52 minutes	
	(iv)		
			
		The temperature of the bread can never reach 27°C	

6		Let $AD = 2x \Rightarrow BC = x$	
		$\sin 30° = \frac{h}{4\sqrt{3} - 2\sqrt{2}} \Rightarrow h = 2\sqrt{3} - \sqrt{2}$	
		$\frac{1}{2}(x + 2x)(2\sqrt{3} - \sqrt{2}) = 25 + 5\sqrt{6}$	
		$\frac{3x}{2} = \frac{25 + 5\sqrt{6}}{2\sqrt{3} - \sqrt{2}} \times \frac{2\sqrt{3} + \sqrt{2}}{2\sqrt{3} + 2\sqrt{2}}$	
		$x = \frac{2}{3} \times \frac{50\sqrt{3} + 25\sqrt{2} + 10\sqrt{18} + 5\sqrt{12}}{12 - 2}$	
		$x = \frac{2(50\sqrt{3} + 25\sqrt{2} + 30\sqrt{2} + 10\sqrt{3})}{30}$	
		$x = \frac{(60\sqrt{3} + 55\sqrt{2})}{15}$	
		$AD = \frac{22}{3}\sqrt{2} + 8\sqrt{3}$	

7		$y = \frac{4}{3}x^3 + \frac{a}{3}x$	
		$y - x = \frac{4}{3}x^3 + \frac{a}{3}x - x$	
		$\frac{y - x}{x} = \frac{4}{3}x^2 + \frac{a}{3} - 1$----(1)	
		$\frac{y - x}{x} = \frac{13 - t}{6}x^2 + C$ ---(2)	
		$\frac{13 - t}{6} = \frac{4}{3}$	
		$t = 5$	
		Sub (8,13) into (2)	
		$13 = \frac{13 - t}{6}(8) + C$	
		Sub $t = 5$	
		$13 = \frac{13 - 5}{6}(8) + C$	
		$C = \frac{7}{3}$	
		$\frac{a}{3} - 1 = \frac{7}{3}$	
		$a = 10$	

8	(a)	$(a-x)\left(1^n+\binom{n}{1}(1)(3x)+\binom{n}{2}(1)(3x)^2+\dots\right)$	
		By comparing like terms	
		$a=2$	
		$-x+3anx=29x$	
		$-1+6n=29$	
		$6n=30$	
		$n=5$	
		$a\left(\frac{n(n-1)}{2}\right)(1)(9x^2)-\binom{n}{1}(1)(3x^2)$	
		Sub $a=2,\ n=5$	
		$2\left(\frac{5(4)}{2}\right)(1)(9x^2)-\binom{5}{1}(1)(3x)$	
		$180x^2-15x^2=bx^2$	
		$b=165$	
	(b)(i)	$\binom{10}{r}(x^2)^{10-r}\left(\frac{1}{2x^3}\right)^r$	
	(ii)	x^{20-5r}	
	(iii)	$20-5r=-15$	
		$r=7$	
		$\binom{10}{r}(2^{-r})$	
		$\binom{10}{7}(2^{-7})$	
		$=\frac{15}{16}$	
9		$y=\ln(3x-1)-\ln(5-2x)$	
		$\frac{dy}{dx}=\frac{3}{3x-1}+\frac{2}{5-2x}$	
		$y=-\frac{5}{13}x+2$	
		Gradient of normal $=-\frac{5}{13}$	
		Gradient of tangent $=\frac{13}{5}$	
		$\frac{13}{5}=\frac{3}{3x-1}+\frac{2}{5-2x}$	
		$\frac{13}{5}=\frac{3(5-2x)+2(3x-1)}{17x-5-6x^2}$	
		$\frac{13}{5}=\frac{13}{17x-5-6x^2}$	
		$17x-5-6x^2=5$	
		$6x^2-17x+10=0$	
		$(6x-5)(x-2)=0$	
		$x=\frac{5}{6}$ or $x=2$	
		since $x=a$	
		$a=\frac{5}{6}$ or $a=2$	
		since $a>1$,	
		$a=2$	
		$b=\ln\frac{5}{1}\approx 1.61$	
10	(i)	At $P=(x,1)$	
		$y=\lvert 2x+4\rvert+1$	
		$1=\lvert 2x+4\rvert+1$	
		$\lvert 2x+4\rvert=0$	
		$2x+4=0$	
		$x=-2$	
		$P=(-2,1)$	
		At $Q=(0,y)$	
		$y=\lvert 4\rvert+1$	
		$y=5$	
		$Q=(0,5)$	
	(ii)	$\lvert 2x+4\rvert+1=-x+3$	
		$\lvert 2x+4\rvert=-x+2$	
		$2x+4=-x+2$ or $2x+4=x-2$	
		$x=-\frac{2}{3}$ $\quad x=-6$	
	(iii)	Max $=\frac{3-1}{0-(-2)}=1$	
		Min $=-2$	
		$-2<m<1$	

11	(i)	$a = \frac{2}{3}e^{-\frac{t}{3}}$	
		When $t = 0$, $a = \frac{2}{3}$	
	(ii)	$s = \int 28 - 2e^{-\frac{t}{3}}dt$	
		$s = 28t + 6e^{-\frac{t}{3}} + c$	
		$s = 0, t = 0$	
		$c = -6$	
		$s = 28t + 6e^{-\frac{t}{3}} - 6$	
		$t = 10$	
		$s = 28t + 6e^{-\frac{10}{3}} - 6$	
		$s = 274.21$	
	(iii)	No.	
		For $t \geq 0$, $2e^{-\frac{t}{3}} \leq 2$,	
		$v = 28 - 2e^{-\frac{t}{3}} \leq 26$	
		$\therefore v > 0$	
12	(i)	Let $\angle DAC = a$	
		$\angle DBC = a$ (angles in the same segment)	
		$\angle DBA = a$ (BD is angle bisector of $\angle ABC$)	
		$\angle DAE = \angle DBA = a$ (tangent chord theorem)	
		$\because \angle DAC = \angle DAE$ (proven)	
	(ii)	$\angle DEA = \angle BEA$ (common angle)	
		$\angle DAE = \angle ABE = a$ (tangent chord theorem)	
		ΔADE and ΔBAE are similar (AA property)	
	(iii)	$\angle DCA = \angle DBA = a$ (angles in the same segment)	
		$\angle DCA = \angle CAB = a$	
		$\angle DCA, \angle CAB$ are alternate angles $\therefore CD \parallel BA$	
		$CD \parallel BA$ & D is the midpoint of AF	
		$\therefore C$ is a midpoint of BF (By midpoint theorem)	
	(iv)	ΔADE and ΔBAE are similar	
		$\frac{AE}{BE} = \frac{AD}{AB} = \frac{DE}{AE}$	
		$AD \times CE = BC \times DE$	
		ΔGCD and ΔGBA are similar	

		$\frac{AD}{AB} = \frac{DE}{AE}$	
		$\frac{\frac{1}{2}AF}{2CD} = \frac{DE}{AE}$	
		$AF \times AE = 4CD \times DE$ (shown)	

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Binomial expansion

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where n is a positive integer and $\binom{n}{r} = \frac{n!}{r!(n-r)!} = \frac{n(n-1)\ldots(n-r+1)}{r!}$

2. TRIGONOMETRY

Identities

$$\sin^2 A + \cos^2 A = 1$$

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$$\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$$

$$\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$$

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$$a^2 = b^2 + c^2 - 2bc\cos A$$

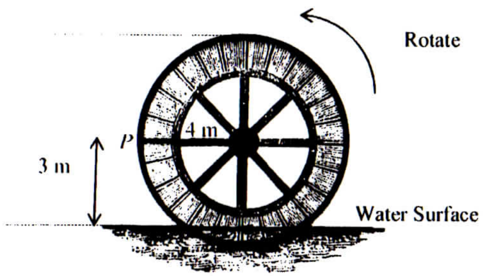
$$\Delta = \frac{1}{2}ab\sin C$$

Answer **all** the questions

1 **(a)** Given that $\lg(3y+2)-4x^2=2$, express y in terms of x. [3]

(b) Solve the equation $2\log_4(8-2x)-\log_2(x-2)=3-\log_2(1+x)$ [5]

2 The diagram shows a water wheel which rotates at 3 revolutions per minute in an anticlockwise direction. At the start of the revolution, a point P on the rim of the wheel is at the height of 3 m above the surface of the water. The radius of the water wheel is 4 m.



The height, h m, of point P above the water surface is given as $h=a\sin(\frac{\pi}{b}t)+c$, where t is the time in seconds.

(i) State the values of a, b and c. [3]

(ii) Find the time, t, where point P first emerge from the water. [3]

3 **(i)** Show that $\frac{d}{dx}[e^{2x}(2x+1)]=e^{2x}(4x+4)$. [2]

(ii) Hence, or otherwise, evaluate $\int_0^1 2xe^{2x}\,dx$. [4]

4 The cubic polynomial f(x) is such that the coefficient of x^3 is 4 and the constant term is -3. When $x+1$ is a factor of f(x), the quadratic factor is px^2+qx+r. It is given that f(x) leaves a remainder of -20 when divided by $x-1$.

(i) Find the values of p, q and r. [3]

(ii) Solve f(x) =0. [2]

(iii) Hence solve the equation f(-x) = 0. [1]

5 It is given that $\frac{x^3-x^2-4x+1}{x^2-4}=ax+b+\frac{c}{x^2-4}$

(i) Find the values of a, b and c. [3]

(ii) Hence, using partial fractions and the values of a, b and c obtained in part (i), find $\int\frac{x^3-x^2-4x+1}{x^2-4}dx$. [6]

6 **(a)** The curves $y=a(\sqrt[3]{x})$ and $y=\frac{2a}{k}\left(\frac{1}{x^2}\right)$ meets at the point (1,5) where a and k are constants.

(i) Find the value of a and of k. [2]

(ii) On the same axes, sketch the two curves, for $x>0$. [2]

(b) Given that $y=(2x+\tan x)^2$ and that $\frac{dy}{dx}=a\pi+b\sqrt{3}$ when $x=\frac{\pi}{3}$, find the value of a and of b. [4]

7 A curve has the equation $y = \mathrm{f}(x)$, where $\mathrm{f}(x) = \dfrac{3(x-1)}{5x+3}$ for $x > 0$.

(i) Find an expression for $\mathrm{f}'(x)$. [2]

(ii) Explain why the curve has no stationary points. [1]

(iii) Show with full workings, determine whether the gradient function of the curve is an increasing or decreasing function for $x > 0$. [2]

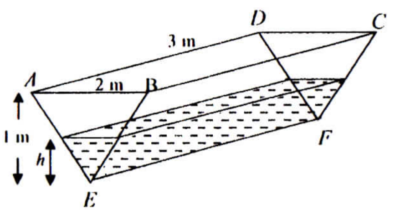
8 The points (1, 10) and (7, 10) are on the circumference of a circle whose centre, C, lies above the x-axis. The line $y = 1$ is tangent to the circle.

(i) Find the coordinates of C. [3]

(ii) Find the equation of the circle in the form $x^2 + y^2 + px + qy + r = 0$, where p, q and r are integers. [2]

(iii) Find the equations of the tangents to the circle parallel to the y-axis. [2]

9



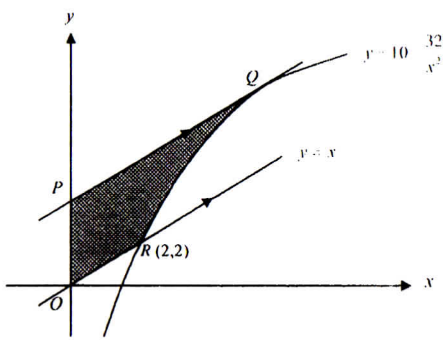
The above diagram shows a trough of 3m long and 1m deep with $ABCD$ horizontal. Its cross section is an isosceles triangle of base 2m with its vertex downwards. The empty trough is filled with water at the rate of 0.03 m^3/s.

(i) If the depth of the water at time t seconds is h m, show that the volume of water is $3h^2$ m^3. [1]

(ii) Hence, find the rate at which the water level is rising after the water has been running for 25s. [4]

Anglo-Chinese School (Barker Road)

10

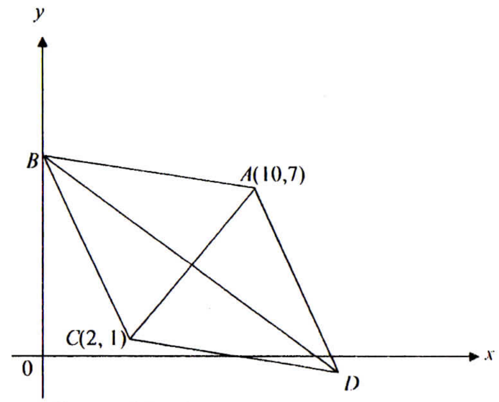


The diagram shows part of the curve and two parallel lines OR and PQ. The line OR intersects the curve at the point $R(2,2)$ and the line PQ is a tangent to the curve at the point Q.

(i) Find the coordinates of P and of Q. [4]

(ii) Find the area of the shaded region $OPQR$. [6]

11 The diagram shows a rhombus $ABCD$ where A (10,7) and C (2, 1). B is a point on the y-axis and is equidistant from A and C.

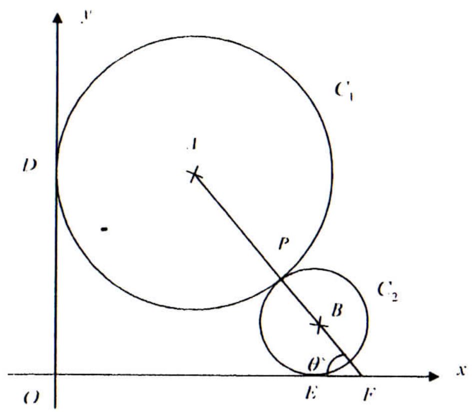


(i) Find the coordinates of B and D. [4]

(ii) Find the area of $ABCD$. [2]

(iii) If point P lies on AC extended such that $PC : PA = 1 : 5$. Find the coordinates of P. [3]

12 In the diagram, two circles C_1 and C_2 whose centres are A and B respectively, touch each other at P. The radii of C_1 and C_2 are 4 units and 1 unit respectively. C_1 touches the y-axis at D and C_2 touches the x-axis at E. The line AB joining the centres of C_1 and C_2 meets the x-axis at F and $\angle BFO = \theta°$.



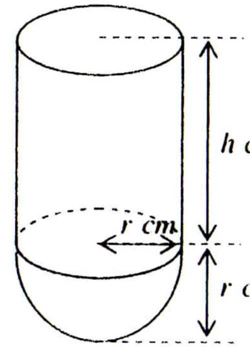
(i) Express OD and OE in terms of θ. [2]

(ii) Hence, or otherwise, show that $DE^2 = 40\cos\theta + 10\sin\theta + 42$. [3]

(iii) Express DE^2 in the form $R\cos(\theta - \alpha) + 42$ where $R > 0$ and α is acute. [3]

(iii) Find the maximum value of ***DE*** and the value of θ at which the maximum value occurs. [3]

13 The diagram below shows an open container. It consists of a hemisphere of radius r cm, and a cylinder of radius r cm and height h cm. The hemisphere is fixed to the end of the cylinder and the volume of the container is 800 cm^3.



(i) Show that $h = \dfrac{2400 - 2\pi r^3}{3\pi r^2}$. [2]

The container is made of some thin metal sheets. The cost of metal sheets for the cylindrical surface is \$1.50 per cm^2 and the cost of metal sheets for hemispherical surface is \$3.20 per cm^2.

(ii) Let \$$C$ be the cost of making the container. Show that $C = 4.4\pi r^2 + \dfrac{2400}{r}$. [3]

(iii) Find the value of h and r such that the total cost of constructing the container is minimum. [5]

Anglo-Chinese (Barker) Prelim 2017 4E Add Math P2 Answer

1(a)	$\lg(3y+2)-4x^2=2$	
	$\lg(3y+2)=4x^2+2$	
	$10^{2+4x^2}=3y+2$	
	$3y=10^{2+4x^2}-2$	
	$y=\frac{1}{3}\left(10^{2+4x^2}-2\right)$	
(b)	$2\log_4(8-2x)-\log_2(x-2)=3-\log_2(1+x)$	
	$2\left[\frac{\log_2(8-2x)}{\log_2 2^2}\right]-\log_2(x-2)=\log_2 2^3-\log_2(1+x)$	
	$\log_2(8-2x)-\log_2(x-2)=\log_2\frac{8}{(1+x)}$	
	$\log_2\frac{8-2x}{x-2}=\log_2\frac{8}{(1+x)}$	
	$\frac{8-2x}{x-2}=\frac{8}{(1+x)}$	
	$(8-2x)(1+x)=8(x-2)$	
	$8+8x-2x-2x^2=8x-16$	
	$-2x^2-2x+24=0$	
	$x^2+x-12=0$	
	$(x-3)(x+4)=0$	
	$x=3 \quad x=-4(\text{NA})$	
	$\therefore x=3$	

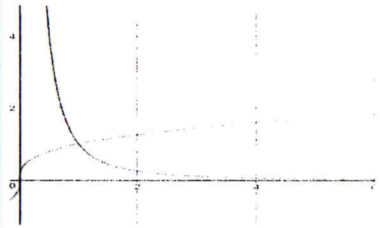
2(i)	$a=-4$	
	$b=10$	
	$c=3$	
(ii)	$h=-4\sin(\frac{\pi}{10}t)+3$	
	$h=0$	
	$-4\sin(\frac{\pi}{10}t)+3=0$	
	$\sin(\frac{\pi}{10}t)=\frac{3}{4}$	
	$\frac{\pi}{10}t=\sin^{-1}\frac{3}{4}$	
	$\frac{\pi}{10}t=0.84806, \pi-0.84806$	
	$t=\frac{10(\pi-0.84806)}{\pi}$	
	$t=7.30$ (3sf)	

3(i)	$\frac{d}{dx}\left(e^{2x}(2x+1)\right)$	
	$= 2e^{2x} + (2x+1)(2e^{2x})$	
	$= 2e^{2x}(2 + 4x + 2)$	
	$= 2e^{2x}(4x+4)$ (shown)	
(ii)	$\int_0^1 4xe^{2x} + 4e^{2x}dx = \left[e^{2x}(2x+1)\right]$	
	$2\int_0^1 2xe^{2x} dx = \left[e^{2x}(2x+1)\right]_0^1 - \int_0^1 4e^{2x}dx$	
	$\int_0^1 2xe^{2x} dx = \frac{1}{2}\left[e^{2x}(2x+1)\right]_0^1 - \frac{1}{2}\int_0^1 4e^{2x}dx$	
	$= \frac{1}{2}\left[e^{2x}(2x+1)\right]_0^1 - \frac{1}{2}\left[2e^{2x}\right]_0^1$	
	$= \frac{1}{2}\left[3e^2 - 1\right] - \frac{1}{2}\left[2e^2 - 2\right]$	
	$= \frac{1}{2}\left[3e^2 - 2e^2 - 1 + 2\right]$	
	$= \frac{1}{2}\left[e^2 + 1\right]$	
	$=4.19$	

4(i)	$f(x) = (x+1)(px^2 + qx + r)$	
	$p = 4$	
	$r = -3$	
	$f(1) = -20$	
	$(2)(p+q+r) = -20$	
	$(2)(4+q-3) = -20$	
	$(2)(q+1) = -20$	
	$q = -11$	
(ii)	$f(x) = (x+1)(4x^2 - 11x - 3)$	
	$f(x) = 0$	
	$(x+1)(4x+1)(x-3) = 0$	
	$x = -1, -\frac{1}{4}, 3$	
(iii)	$(-x+1)(-4x+1)(-x-3) = 0$	
	$x = 1, \frac{1}{4}, -3$	

128

5(i)	By long division, $\frac{x^3-x^2-4x+1}{x^2-4} = x-1-\frac{3}{x^2-4}$	
	$a=1, b=-1, c=-3$	
(ii)	$-\frac{3}{x^2-4} = \frac{A}{x+2}+\frac{B}{x-2}$	
	$-3 = A(x-2)+B(x+2)$	
	Let $x=-2, A=\frac{3}{4}$	
	Let $x=2, B=-\frac{3}{4}$	
	$\int x-1+\frac{3}{4(x+2)}-\frac{3}{4(x-2)}\,dx$	
	$=\frac{1}{2}x^2-x+\frac{3}{4}\ln(x+2)-\frac{3}{4}\ln(x-2)+c$	

6a(i)	sub $x=1$, $y=5$ into $y=a\sqrt[3]{x}$	
	$a=5$	
	sub $x=1$, $y=5$, $a=5$ into $y=\frac{2a}{kx^2}$	
	$k=2$	
6a(ii)		
6b	$\frac{dy}{dx}=2(2x+\tan x)(2+\sec^2 x)$	
	When $x=\frac{\pi}{3}$	
	$\frac{dy}{dx}=2(2\left(\frac{\pi}{3}\right)+\tan\frac{\pi}{3})(2+\sec^2\frac{\pi}{3})$	
	$\frac{dy}{dx}=2\left(\frac{2\pi}{3}+\sqrt{3}\right)\left(2+\frac{1}{\left(\frac{1}{2}\right)^2}\right)$	
	$\frac{dy}{dx}=\left(\frac{4\pi}{3}+2\sqrt{3}\right)(6)$	
	$\frac{dy}{dx}=8\pi+12\sqrt{3} \Rightarrow \quad a=8, b=12$	

7(i)	$f'(x) = \dfrac{3(5x+3) - 3(x-1)(5)}{(5x+3)^2}$	
	$= \dfrac{9+15}{(5x+3)^2}$	
(ii)	$f'(x) > 0$ since $(5x+3)^2 > 0, x > 0$	
	$f'(x) \neq 0$, the curve has no stationary points	
(ii)	$f''(x) = -48(5x+3)^{-3}(5)$	
	$= \dfrac{-240}{(5x+3)^3}$	
	For $x > 0$, $f''(x) < 0$, the gradient function is a decreasing function.	

8(i)		
	Midpoint = $\left(\dfrac{1+9}{2}, \dfrac{10+10}{2}\right)$	
	$= (4, 10)$	
	Let the centre be $(4, y)$	
	$(4-4)^2 + (y-1)^2 = (4-1)^2 + (y-10)^2$	
	$y^2 - 2y + 1 = 9 + y^2 - 20y + 100$	
	$18y = 108$	
	$y = 6$	
	Centre (4,6)	
(ii)	Radius = 5 units	
	$(x-4)^2 + (y-6)^2 = 25$	
	$x^2 + y^2 - 8x - 12y + 27 = 0$	
(iv)	$x = -1$	
	$x = 9$	

129

9(i)	Let the base of the water be x	
	$\frac{x}{2}=\frac{h}{1}$	
	$x=2h$	
	$V=\frac{1}{2}(2h)(h)(3)$	
	$V=3h^2$ (shown)	
(ii)	$\frac{dV}{dt}=0.03$	
	$\frac{dV}{dh}=6h$	
	When $t=25$	
	$V=25\times0.03=0.75\,\text{m}^3$	
	$3h^2=0.75$	
	$h=0.5$	
	$\frac{dh}{dt}=\frac{dh}{dV}\times\frac{dV}{dt}$	
	$=\frac{1}{6h}\times0.03$	
	$=\frac{1}{6(0.5)}\times0.03$	
	$\frac{dh}{dt}=0.01m/s$	

10(i)	$y=10-32x^{-2}$	
	$\frac{dy}{dx}=64x^{-3}$	
	$\frac{64}{x^3}=1$	
	$x^3=64$	
	$x=4$	
	When $x=4, y=8$	
	$Q=(4,8)$	
	$P=(0,y)$	
	$\frac{4-0}{8-y}=1$	
	$4=8-y$	
	$y=4$	
	$\therefore P=(0,4)$	
	Area of trapezium = $\frac{1}{2}(4+8)(4)=24$	
	Area of triangle = $\frac{1}{2}(2)(2)=2$	
	Area of under curve = $\int_2^4 10-32x^{-2}dx$	
	$=\left[10x-\frac{32x^{-1}}{-1}\right]_2^4$	
	$=\left[10x+\frac{32}{x}\right]_2^4$	

	$= 48 - 36$	
	$= 12$	
	Shaded area = $24 - 12 - 2$	
	$= 10$	

11(i)	Since D is equidistant from A and C. $\therefore$ DB is perpendicular bisector of AC.	
	Midpoint of AC = $\left(\frac{10+2}{2}, \frac{7+1}{2}\right)$	
	$= (6,4)$	
	Gradient of $AC = \frac{1-7}{2-10}$	
	$= \frac{3}{4}$	
	Gradient of $BD = -\frac{4}{3}$	
	Equation of BD, $y - 4 = -\frac{4}{3}(x-6)$	
	$y = -\frac{4}{3}x + 12$	
	At the y-axis, $x = 0$	
	$y = 12$	
	$\therefore B\,(0,12)$	
	Midpoint of DB = Midpoint of A = $(6,4)$	
	$D\,(0,12)$, $B(x,y)$	
	$\left(\frac{0+x}{2}, \frac{12+y}{2}\right) = (6,4)$	
	$\therefore \frac{x}{2} = 6$ $\quad\quad$ $\therefore \frac{12+y}{2} = 4$	
	$x = 12$ $\quad\quad$ $y = -4$	
	$\therefore D\,(12, -4)$	

11(ii)	Area ABCD = $\frac{1}{2}\begin{vmatrix} 10 & 0 & 2 & 12 & 10 \\ 7 & 12 & 1 & -4 & 7 \end{vmatrix}$	
	$\frac{1}{2}\left\|(120-8+84)-(24+12-40)\right\|$	
	$=\frac{1}{2}(200)$	
	=100 units	
11(iii)	Let $P\ (a, b)$	
	$\left(\frac{10+4a}{5}, \frac{7+4b}{5}\right)=(2,1)$	
	$\therefore \frac{10+4a}{5}=2$	
	$10+4a=10$	
	$a=0$	
	$\therefore \frac{7+4b}{5}=1$	
	$7+4b=5$	
	$b=-\frac{1}{2}$	
	$\therefore P\ (0, -\frac{1}{2})$	

12(i)	$OD = 5\sin\theta + 1$	
	$OE = 5\cos\theta + 4$	
(ii)	$DE^2 = (5\sin\theta+1)^2 + (5\cos\theta+4)^2$	
	$= 25\sin^2\theta + 10\sin\theta + 1 + 25\cos^2\theta + 40\cos\theta + 16$	
	$= 40\cos\theta + 10\sin\theta + 25(\sin^2\theta + \cos^2\theta) + 17$	
	$= 40\cos\theta + 10\sin\theta + 42$ (shown)	
(iii)	$40\cos\theta + 10\sin\theta + 42 = R\cos(\theta-\alpha) + 42$	
	$R\cos\alpha = 40$	
	$R\sin\alpha = 10$	
	$R^2 = 40^2 + 10^2$	
	$R = \sqrt{1700}$	
	$\tan\alpha = \frac{10}{14}$	
	$\alpha = 14''$	
	$DE^2 = \sqrt{1700}\cos(\theta - 14'') + 42$	
	Max $DE^2 = \sqrt{1700} + 42$	
	Max $DE = 9.12$	
	$\cos(\theta - 14'') = 1$	
	$\theta = 14^\circ$	

13(i)	$\pi r^2 h + \frac{2}{3}\pi r^3 = 800$	
	$3\pi r^2 h + 2\pi r^3 = 2400$	
	$h = \frac{2400 - 2\pi r^3}{3\pi r^2}$	
(ii)	$C = 1.5(2\pi r h) + 3.2(2\pi r^2)$	
	$C = 1.5(2\pi r\left(\frac{2400 - 2\pi r^3}{3\pi r^2}\right)) + 3.2(2\pi r^2)$	
	$C = \frac{2400 - 2\pi r^3}{r} + 6.4\pi r^2$	
	$C = \frac{2400}{r} - 2\pi r^2 + 6.4\pi r^2$	
	$C = 4.4\pi r^2 + \frac{2400}{r}$ (shown)	
(iii)	$\frac{dC}{dr} = 8.8\pi r - 2400r^{-2}$	
	$\frac{dC}{dr} = 8.8\pi r - \frac{2400}{r^2}$	
	$\frac{dC}{dr} = 0$ $8.8\pi r = \frac{2400}{r^2}$	
	$r^3 = \frac{2400}{8.8\pi}$	
	$r = 4.43$ (3sf)	
	Sub $r = 4.429$, $h = 10.0$ $\frac{d^2C}{dr^2} = 8.8\pi + 4800r^{-3}$ Sub $r = 4.429$, $\frac{d^2C}{dr^2} > 0 \Rightarrow$ mini cost	