Mathematical Formulae

1. ALGEBRA

Quadratic Equation

For the equation $ax^2 + bx + c = 0$,

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Binomial Expansion

$$(a+b)^n = a^n + \binom{n}{1} a^{n-1} b + \binom{n}{2} a^{n-2} b^2 + \ldots + \binom{n}{r} a^{n-r} b^r + \ldots + b^n$$

where n is a positive integer and $\binom{n}{r} = \frac{n!}{r!(n-r)!} = \frac{n(n-1)\ldots(n-r+1)}{r!}$

2. TRIGONOMETRY

Identities

$$\sin^2 A + \cos^2 A = 1$$

$$\sec^2 A = 1 + \tan^2 A$$

$$\operatorname{cosec}^2 A = 1 + \cot^2 A$$

$$\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$$

$$\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$$

$$\tan(A \pm B) = \frac{\tan A \pm \tan B}{1 \mp \tan A \tan B}$$

$$\sin 2A = 2 \sin A \cos A$$

$$\cos 2A = \cos^2 A - \sin^2 A = 2\cos^2 A - 1 = 1 - 2\sin^2 A$$

$$\tan 2A = \frac{2 \tan A}{1 - \tan^2 A}$$

Formula for ΔABC

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$a^2 = b^2 + c^2 - 2bc \cos A$$

$$\Delta = \frac{1}{2} ab \sin C$$

1 Find the range of values of m such that $y = 3mx^2 + 6x + m$ is always positive. [4]

2 Find the value of x for which $8\log_4 x + (\log_x 4)^2 = 0$. [3]

3 A structured deposit pays a compound interest of r % per annum.

In n years, the principal amount P_0 will become P_n where $P_n = P_0(1 + \frac{r}{100})^n$.

Mandy invests \$20000 and receives \$22497.28 in 3 years.

(i) Find the value of r. [2]

(ii) Find the number of years Mandy has to invest if she wishes to double the principal amount. [2]

4 **(i)** Sketch the graph of $y^2 = 12x$. [2]

(ii) If $x + y = k$ is a normal to the curve $y^2 = 12x$, find the value of k. [3]

5 Given that $y = \frac{2x^2}{4x-3}$, where $x > \frac{3}{4}$,

(i) find an expression for $\frac{dy}{dx}$, [3]

(ii) find the range of values of x for which $y = \frac{2x^2}{4x-3}$ is a decreasing function. [2]

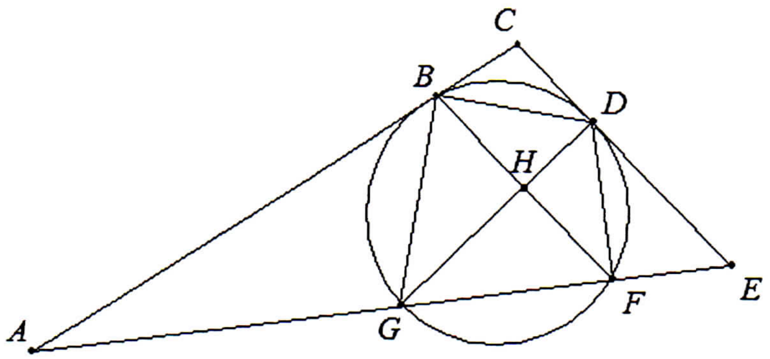
6 **(i)** Prove the identity $(1 + \operatorname{cosec}\theta)(\sec\theta - \tan\theta) \equiv \cot\theta$. [3]

(ii) Hence solve the equation $\sec 2x - \tan 2x = \frac{\sqrt{3}}{1 + \operatorname{cosec} 2x}$ for $0° < x < 360°$. [4]

7 In the diagram, the points B, D, F and G lie on a circle with diameter DG.

Lines AC and CE are tangents to the circle at B and D respectively.

$AGFE$ is a straight line and line BF meets DG at H.



(i) Prove that triangle ABG is similar to triangle AFB. [3]

(ii) Given that angle $GHF = 90°$, show that BF and CE are parallel. [2]

(iii) Using the above results, show that $\frac{AC}{CE} = \frac{AG}{GB}$. [2]

8 **(i)** Sketch the graph of $y = |x-2| - 2$, showing clearly the intercepts on the coordinate axes and the vertex. [2]

(ii) A line $y = mx + c$ is drawn on the same axes.

(a) If $m = 0$, write down the range of values of c such that the line $y = mx + c$ will intersect the graph of $y = |x-2| - 2$ at two distinct points. [1]

(b) If $c = 0$, write down the range of values of m such that the line $y = mx + c$ will intersect the graph of $y = |x-2| - 2$ at two distinct points. [2]

(iii) Solve the equation $|x-2| - 2 = -\frac{1}{2}x + 1$. [3]

9 A particle travels in a straight line such that t seconds after passing through a fixed point O, its acceleration a m/s^2 is given by $a = \dfrac{8}{(t+2)^2}$.

The particle comes to instantaneous rest when $t = 2$.

(i) Find the initial velocity of the particle. [3]

(ii) Find the distance from O at which the particle comes to instantaneous rest. [3]

(iii) Show that the particle is again at O at some instant during the 6th second. [2]

10 Given that $y = 5\sin^2 t - 3\cos^2 t$,

(i) express y in the form $a\cos 2t + b$, where a and b are integers. [3]

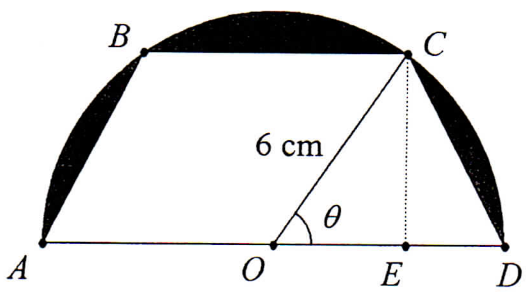
(ii) Sketch the graph of y for $0 \le t \le 2\pi$. [3]

(iii) The equation $y = 5\sin^2 t - 3\cos^2 t$ models the motion of a whale jumping in and out of the ocean. y, in metres, is the vertical displacement of the whale from the sea level and t is the time in seconds.

Find the duration the whale is first out of the water. [3]

11 The diagram below shows a semicircle $OABCD$ with radius 6 cm and centre O.

$ABCD$ is a symmetrical trapezium with height CE and angle COE is θ radians.



(i) Show that the shaded area, A cm^2, is given by $A = 18\pi - 18\sin 2\theta - 36\sin\theta$. [3]

(ii) Find the minimum value of A, leaving your answer in the exact form in terms of surds and π. [7]

12 **(i)** Sketch the graph of $y = \ln(2x+1)$ for $-\frac{1}{2} < x \leq 4$. [3]

(ii) A point P moves along the curve $y = \ln(2x+1)$ such that the x-coordinate of P increases at a constant rate of 0.005 units/s. Find the x-coordinate of P at the instant the y-coordinate is increasing at 0.002 units/s. [3]

(iii) Make x the subject in the equation $y = \ln(2x+1)$.

Hence find the area bounded by the curve $y = \ln(2x+1)$, the x-axis and the line $x = 4$. [4]

END OF PAPER

Answer **all** the questions

1. The points P and Q have coordinates $(\sqrt{3}, 2\sqrt{3})$ and $(\sqrt{5}, 4\sqrt{5})$ respectively. Show that the gradient of PQ can be expressed as $a + b\sqrt{15}$, stating the value of the integers a and b. [5]

2. A viral post on a social media platform spreads to its users according to the equation

$$x = \frac{1}{1 + e^{-kt}},$$

where x is the proportion of users that received the post in their feed at time t minutes and k is a positive constant.

(i) Find the rate of spread of the viral post in terms of k. [2]

(ii) Show that $\frac{dx}{dt} = kx(1 - x)$. [3]

(iii) Find the values of x when $\frac{dx}{dt} = 0$. Explain the significance of these values of x. [2]

3. (i) Given $y = 2\cos x \sin 2x - \sin x \cos 2x$, show that $\frac{dy}{dx} = k\cos x \cos 2x$, stating the value of k. [3]

(ii) Hence, show that $\int_0^{\frac{\pi}{4}} (5\cos x \cos 2x + \sec^2 x)\, dx = \frac{5\sqrt{2}}{3} + 1$. [4]

4. (i) Given that the term independent of x in the binomial expansion of $\left(x - \frac{k}{x^2}\right)^9$ is -672, find the value of the integer k. [4]

(ii) Using the value of k found in part (i), find the coefficient of x^7 in the expansion of $(1 + 3x)\left(x - \frac{k}{x^2}\right)^9$. [4]

5. Points P and Q lie on a curve with equation $y = -x^2 + 4x - 3$. The tangent to the curve at P has gradient 2. The points P and Q are equidistant from the line of the symmetry of the curve.

(i) Find the coordinates of P and of Q. [4]

The normal to the curve at point Q meets the curve again at R.

(ii) Find the area of the triangle PQR. [6]

6. The equation of a curve is $y = x\sqrt{8 - x^2}$.
   (i) Find the coordinates of the stationary points of the curve. [5]
   (ii) Determine the nature of these stationary points. [5]

7. The lines $y = 8$ and $4x + 3y = 30$ are tangent to a circle C at the points $(-1, 8)$ and $(3, 6)$ respectively.
   (i) Show that the equation of C is $x^2 + y^2 + 2x - 6y - 15 = 0$. [5]
   (ii) Explain whether or not the x-axis is tangent to C. [3]
   (iii) The points Q and R also lie on the circle, and the length of the chord QR is 2. Calculate the shortest distance from the centre of C to the chord QR. [2]

8. (i) Find the remainder when $8x^3 - 4x^2 - 2x - 3$ is divided by $2x + 1$. [2]
   (ii) Hence, factorise completely the cubic polynomial $8x^3 - 4x^2 - 2x + 1$. [4]
   (iii) Express $\dfrac{8x}{(x+1)(x-1)^2}$ as a sum of 3 partial fractions. [4]

9. The mass of radioactive iodine can be modelled by the equation $m = m_0 k^t$ where m grams is the mass after t days and m_0 and k are constants. Measured values of m and t are given in the following table.

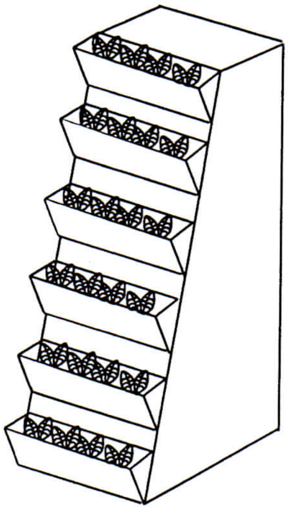
t	2	4	6	8
m	14.72	9.42	6.03	3.86

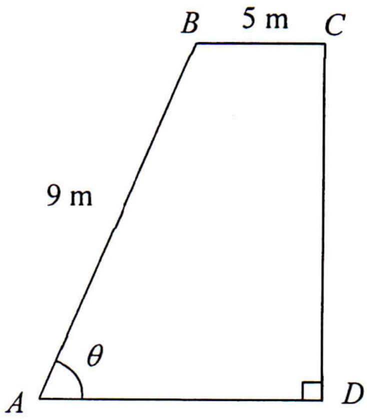
   (i) Using suitable variables, draw, on graph paper, a straight line graph and hence estimate m_0 and k to 1 decimal place. [6]
   (ii) Using your values of m_0 and k in (i), calculate the value of m when $t = 10$. [2]

   The half-life of a radioactive substance is the time it takes for a mass of the substance to reduce to half its mass.
   (iii) Explain how another straight line can be drawn on your diagram to lead to the estimate of the half-life of iodine. Draw this line and estimate this value. [3]

10. The roots of the quadratic equation $-2x^2 + 3x - 7 = 0$ are α and β.
   (i) Find the value of $\alpha^2 + \beta^2$ and hence deduce the nature of the roots. [4]
   (ii) Find the value of $\alpha^3 + \beta^3$. [3]
   (iii) Find a quadratic equation with integer coefficients whose roots are $\dfrac{1}{\alpha}$ and $\dfrac{1}{\beta}$. [4]

11.





A vertical farming tower is used to grow plants in land-scarce Singapore. The tower is placed on rooftops of high-rise buildings to allow the plants to absorb sunlight. The diagram shows the cross-section of the tower which consists of a trapezium *ABCD*. Angle $BAD = \theta$, angle $ADC = 90^\circ$ and the lengths of *AB* and *BC* are 9 m and 5 m respectively.

(i) Show that *L* m, the total perimeter of *ABCD*, can be expressed in the form

$$p + q\sin\theta + r\cos\theta,$$

where *p*, *q* and *r* are constants to be found. [2]

(ii) Express *L* in the form $p + R\cos(\theta - \alpha)$, where $R > 0$ and $0 < \alpha < 90^\circ$. [3]

(iii) State the maximum value *L*. [1]

(iv) Find θ if $L = 22$ m. Explain why a tower with this value of *L* should not be built. [5]

END OF PAPER

2017 4E5N Amaths P1
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No	Solution

1 [4] See N08

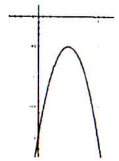
$y = 3mx^2 + 6x + m > 0 \Rightarrow b^2 - 4ac < 0 \Rightarrow (6)^2 - 4(3m)(m) < 0 \Rightarrow 36 - 12m^2 < 0 \Rightarrow m^2 - 3 > 0$

$\Rightarrow (m+\sqrt{3})(m-\sqrt{3}) > 0 \Rightarrow m < -\sqrt{3}$ (rejected as $m > 0$ for U shaped graph) , $\mathbf{m > \sqrt{3}}$

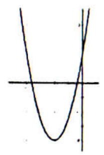
** Note : NO answer marks if y write $m^2 > 3 \Rightarrow m > \pm\sqrt{3}$

Correct case :
m = 2 ($m > \sqrt{3}$)
$y = 6x^2 + 6x + 2$

Incorrect case 1 :
m = -2 ($m < -\sqrt{3}$)
$y = -6x^2 + 6x - 2$



Incorrect case 2 :
m = 1 ($-\sqrt{3} < m < \sqrt{3}$)
$y = 3x^2 + 6x + 1$



2 [3]

$8\log_4 x + (\log_x 4)^2 = 0 \Rightarrow 8\log_4 x + (\frac{\log_4 4}{\log_4 x})^2 = 0 \Rightarrow 8\log_4 x + \frac{1}{(\log_4 x)^2} = 0$

$\Rightarrow 8(\log_4 x)^3 + 1 = 0 \Rightarrow (\log_4 x)^3 = -\frac{1}{8} \Rightarrow \log_4 x = -\frac{1}{2} \Rightarrow x = 4^{-1/2} = \frac{1}{2}$

3 [4]

i) $P_n = P_0(1+\frac{r}{100})^n \Rightarrow 22497.28 = 20000(1+\frac{r}{100})^3 \Rightarrow (1+\frac{r}{100})^3 = 1.124864$, (or 1.125 4sf)

$\Rightarrow 1+\frac{r}{100} = 1.04 \Rightarrow r = 4$ (accept 4.00 3sf)

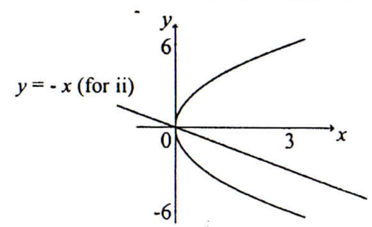
ii) $P_0(1+\frac{4}{100})^n = 2P_0 \Rightarrow 1.04^n = 2 \Rightarrow n = \frac{\lg 2}{\lg 1.04} = 17.7$ years (3sf) or 18 years

4 [5]

i) Sketch the graph of $y^2 = 12x$.

x	0	0.75	3
y	0	±3	±6

(Table is for reference only)



ii) If $y = -x + k$ is a normal to the curve $y^2 = 12x$

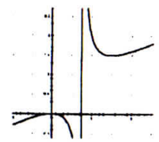
$\Rightarrow$ gradient of normal = -1 $\Rightarrow$ gradient of tangent = 1

$\Rightarrow$ normal is at a point on the **positive y section**. $y = \sqrt{12}x^{1/2} \Rightarrow \frac{dy}{dx} = \frac{\sqrt{12}}{2\sqrt{x}} = 1$

$\Rightarrow 2\sqrt{x} = \sqrt{12} \Rightarrow 4x = 12 \Rightarrow x = 3$ and $y = 6$ $\quad \therefore k = x + y = 3 + 6 = 9$

5 [5]

Given that $y = \frac{2x^2}{4x-3}$, where $x > \frac{3}{4}$,



i) $\frac{dy}{dx} = \frac{(4x-3)(4x)-(2x^2)(4)}{(4x-3)^2} = \frac{8x^2-12x}{(4x-3)^2} = \frac{4x(2x-3)}{(4x-3)^2}$.

ii) For decreasing function, $\frac{dy}{dx} = \frac{4x(2x-3)}{(4x-3)^2} < 0 \Rightarrow 4x(2x-3) < 0$ since $(4x-3)^2 > 0$

$\Rightarrow 0 < x < 1\frac{1}{2}$ and $x > \frac{3}{4} \Rightarrow \therefore \frac{3}{4} < x < 1\frac{1}{2}$ (No answer mark if the lower limit is 0)

6 [7] See N15

i) LHS $= (1+\operatorname{cosec}\theta)(\sec\theta - \tan\theta) = (1+\frac{1}{\sin\theta})(\frac{1}{\cos\theta} - \frac{\sin\theta}{\cos\theta})$

$= (\frac{1+\sin\theta}{\sin\theta})(\frac{1-\sin\theta}{\cos\theta}) = \frac{1-\sin^2\theta}{\sin\theta\cos\theta} = \frac{\cos^2\theta}{\sin\theta\cos\theta} = \frac{\cos\theta}{\sin\theta} = \cot\theta =$ RHS

ii) Solve $\cot 2x = \sqrt{3} \Rightarrow \tan 2x = \frac{1}{\sqrt{3}}$, $0° < 2x < 720°$

$\alpha = 30° \Rightarrow 2x = 30°, 210°, 390°, 570° \Rightarrow x = 15°, 105°, 195°, 285°$

7 [7]

i) $\angle ABG = \angle AFB$ (alternate segment theorem) , $\angle BAG = \angle FAB$ (same angle)

$\therefore \triangle ABG$ is similar to $\triangle AFB$ (all corresponding angles are equal)

ii) BF is perpendicular to GD (given), CE is perpendicular to GD (tan ⊥ rad) $\therefore BF // CE$

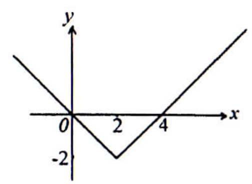
iii) $\frac{AC}{CE} = \frac{AB}{BF}$ ($\triangle ACE$ and $\triangle ABF$ are similar)

$\frac{AB}{BF} = \frac{AG}{GB}$ ($\triangle AFB$ and $\triangle ABG$ are similar) $\therefore \frac{AC}{CE} = \frac{AG}{GB}$ (shown)

* Note : To subtract max of 1 mark if reasons are missing or incorrect

8 [8]

i) Sketch the graph of $y = |x-2| - 2$



No	Solution

8 [8] See N15, SP4047

ii) a) For $y=c$ to intersect $y=|x-2|-2$ at 2 points $\Rightarrow c>-2$

b) For $y=mx$ to intersect $y=|x-2|-2$ at 2 points $\Rightarrow -1<m<1$

iii) $|x-2| = -\frac{1}{2}x+3 \Rightarrow x-2 = -\frac{1}{2}x+3$ or $x-2 = \frac{1}{2}x-3$

$\Rightarrow \frac{3}{2}x=5 \Rightarrow x=3\frac{1}{3}$ or $\frac{1}{2}x=-1 \Rightarrow x=-2$

9 [8] See N14, SP4047

Given $a=\frac{8}{(t+2)^2}$, when $t=0$, $s=0$ and when $t=2$, $v=0$.

i) **Find v when $t=0$.** $v = \int 8(t+2)^{-2}dt = \frac{8(t+2)^{-1}}{(-1)(1)}+c_1 = \frac{-8}{t+2}+c_1$

Since when $t=2$, $v=0$, $-2+c_1=0 \Rightarrow c_1=2 \Rightarrow v=\frac{-8}{t+2}+2$

$\therefore$ when $t=0$, the initial velocity = -2 m/s (particle was travelling **left**)

ii) **Find s when $v=0$ ($t=2$)** $s=\int\frac{-8}{t+2}+2\,dt=-8\ln(t+2)+2t+c_2$

When $t=0$, $s=0$, $-8\ln 2+c_2=0 \Rightarrow c_2=8\ln 2 \Rightarrow s=-8\ln(t+2)+2t+8\ln 2$

When $t=2$, $v=0$ at $s=-8\ln 4+4+8\ln 2=4-8\ln 2=-1.55$ (3sf) (**U turn** here)

iii) **Show between $t=5$ and $t=6$, $s=0$.** (notice not possible to solve $s=0$ for t)

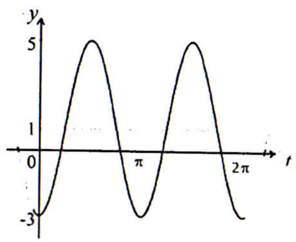
At $t=5$, $s=-0.022<0$ (on the **left** of O) At $t=6$, $s=0.910>0$ (on the **right** of O)

$\therefore$ the particle is again at O at some instant during the 6th second. (shown)

10 [9] See N15

i) $y=5\sin^2 t-3\cos^2 t=5(\frac{1-\cos 2t}{2})-3(\frac{1+\cos 2t}{2})=-4\cos 2t+1$

ii) Sketch the graph of $y=-4\cos 2t+1$ for $0\le t\le 2\pi$.



iii) Solve $-4\cos 2t+1=0$

$\Rightarrow \cos 2t=\frac{1}{4} \Rightarrow 2t=1.318,\ 2\pi-1.318$

$\Rightarrow t=0.6591,\ 2.4826$

$\Rightarrow$ Duration the whale is first out of water = 2.4826-0.6591=1.82 s

No	Solution

11 [10] See N15

i) In $\triangle COE$, $\sin\theta=\frac{CE}{6} \Rightarrow CE=6\sin\theta$ and $\cos\theta=\frac{OE}{6} \Rightarrow OE=6\cos\theta$

Shaded area = semicircle – trapezium = $\frac{1}{2}\pi(6)^2-\frac{1}{2}(12+2(6\cos\theta))(6\sin\theta)$

$=18\pi-36\sin\theta\cos\theta-36\sin\theta=18\pi-18\sin 2\theta-36\sin\theta$

ii) For min A, $\frac{dA}{d\theta}=-36\cos 2\theta-36\cos\theta=0 \Rightarrow (\div -36)\quad \cos 2\theta+\cos\theta=0$

$2\cos^2\theta+\cos\theta-1=0 \Rightarrow (2\cos\theta-1)(\cos\theta+1)=0$

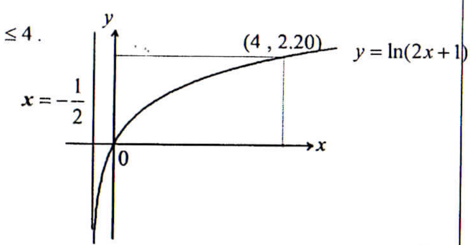
$\Rightarrow \cos\theta=\frac{1}{2}$, so $\theta=\frac{\pi}{3}$ or $\cos\theta=-1$, so $\theta=\pi$ (rejected as θ is acute)

$\frac{d^2A}{d\theta^2}=72\sin 2\theta+36\sin\theta=72\sin\frac{2\pi}{3}+36\sin\frac{\pi}{3}=\frac{72\sqrt{3}}{2}+\frac{36\sqrt{3}}{2}=54\sqrt{3}>0 \Rightarrow$ min A

$\therefore$ min A = $18\pi-18\sin\frac{2\pi}{3}-36\sin\frac{\pi}{3}=18\pi-18\frac{\sqrt{3}}{2}-36\frac{\sqrt{3}}{2}=18\pi-27\sqrt{3}$ cm^2.

12 [10]

i) Sketch the graph of $y=\ln(2x+1)$ for $-\frac{1}{2}<x\le 4$.



ii) Given $\frac{dx}{dt}=0.005$ and $\frac{dy}{dt}=0.002$, find x. $\frac{dy}{dx}=\frac{2}{2x+1}$

$\therefore \frac{dy}{dt}=\frac{2}{2x+1}\cdot\frac{dx}{dt} \Rightarrow 0.002=\frac{2}{2x+1}\cdot 0.005 \Rightarrow \frac{2}{2x+1}=\frac{2}{5} \Rightarrow 2x+1=5 \Rightarrow x=2$

iii) $y=\ln(2x+1) \Rightarrow 2x+1=e^y \Rightarrow x=\frac{e^y-1}{2}$

Required area $= (4)(\ln 9) - \int_0^{\ln 9}\frac{e^y-1}{2}dy = 4\ln 9-[\frac{1}{2}e^y-\frac{1}{2}y]_0^{\ln 9}$

$= 4\ln 9-[\frac{1}{2}e^{\ln 9}-\frac{\ln 9}{2}-\frac{1}{2}+0] = 4\ln 9-[4-\frac{\ln 9}{2}]=\frac{9\ln 9}{2}-4=5.89$ units^2 (3sf)

2017 4E5N Amaths P2

Commonwealth

2017 Prelim A Math P2 Mark Scheme

1.	Gradient $=\dfrac{2\sqrt{3}-4\sqrt{5}}{\sqrt{3}-\sqrt{5}}$ $=\dfrac{(2\sqrt{3}-4\sqrt{5})(\sqrt{3}+\sqrt{5})}{3-5}$ $=\dfrac{-14-2\sqrt{15}}{-2}$ $=7+\sqrt{15}$ $\Rightarrow a=7, b=1$
2i	$\dfrac{dx}{dt}=-(1+e^{-kt})^{-2}(-ke^{-kt})$ $=\dfrac{ke^{-kt}}{(1+e^{-kt})^2}$
2ii	$kx(1-x)=k\left(\dfrac{1}{1+e^{-kt}}\right)\left(1-\dfrac{1}{1+e^{-kt}}\right)$ $=\dfrac{ke^{-kt}}{1+e^{-kt}}$ $=\dfrac{dx}{dt}$ (shown)
2iii	when $\dfrac{dx}{dt}=0$, $kx(1-x)=0 \Rightarrow x=0$ or $x=1$ The rate of spread is never zero until all users receive the post.
3i	$\dfrac{d}{dx}2\cos x\sin 2x-\sin x\cos 2x$ $=-2\sin x\sin 2x+4\cos x\cos 2x-\cos x\cos 2x+2\sin x\sin 2x$ $=3\cos x\cos 2x$ $\Rightarrow k=3$
3ii	$\int_0^{\frac{\pi}{4}} 5\cos x\cos 2x+\sec^2 x\,dx$ $=\left[\dfrac{5}{3}(2\cos x\sin 2x-\sin x\cos 2x)+\tan x\right]_0^{\frac{\pi}{4}}$ $=\dfrac{5}{3}\left(2\left(\dfrac{1}{\sqrt{2}}\right)-0\right)+1-0$ $=\dfrac{5\sqrt{2}}{3}+1$ (Shown)
4i	$T_{r+1}=\binom{9}{r}x^{9-r}\left(\dfrac{-k}{x^2}\right)^r$ $=\binom{9}{r}(-k)^r x^{9-3r}$ Let $9-3r=0 \Rightarrow r=3$ Term independent of x: $T_4=-\binom{9}{3}k^3=-672$ $k=2$
4ii	Let $9-3r=6 \Rightarrow r=1$ $T_2=9x^8(-\dfrac{2}{x^2})=-18x^6$ Coefficient of x^7 $=3(-18)$ $=-54$
5i	$\dfrac{dy}{dx}=-2x+4$ Let $\dfrac{dy}{dx}=2$ $\Rightarrow x=1$ When $x=1$, $y=0 \Rightarrow P(1,0)$ Line of symmetry: $x=2$ $\Rightarrow Q(3,0)$

5ii	When $x=3$, $\frac{dy}{dx}=-2$ Gradient of normal at $Q=\frac{1}{2}$ Equation of normal at Q: $y=\frac{1}{2}x-\frac{3}{2}$
	Let $-x^2+4x-3=\frac{1}{2}x-\frac{3}{2}$ $2x^2+7x-3 \quad =0$ $(2x-1)(x-3) \quad =0$ $x=\frac{1}{2}$ or $x=3$ When $x=\frac{1}{2}$, $y=-\frac{5}{4} \Rightarrow R(\frac{1}{2},-\frac{5}{4})$ Area of □PQR $=\frac{1}{2}\begin{vmatrix}1 & 0.5 & 3 & 1\\ 0 & -1.25 & 0 & 0\end{vmatrix}$ $=\frac{5}{4}$ units^2 Or Area $=\frac{1}{2}\times 2\times\frac{5}{4}=\frac{5}{4}$units^2
6i	$\frac{dy}{dx} \quad =\sqrt{8-x^2}-\frac{x^2}{\sqrt{8-x^2}}$ $=\frac{8-2x^2}{\sqrt{8-x^2}}$ Let $\frac{dy}{dx}=0$ $\Rightarrow x=-2$ or 2 Coordinates of stationary points = $(2,4),(-2,-4)$

6ii	$\frac{d^2y}{dx^2}=\frac{2x^3-24x}{\sqrt{(8-x^2)^3}}$ $\frac{d^2y}{dx^2}\|_{x=2}=-4<0$ $\frac{d^2y}{dx^2}\|_{x=-2}=4>0$ By second derivative test, $(2,4)$ is a max turning point, $(-2,-4)$ is a min turning point. Alternative solution:

x	0	2	3
$\frac{dy}{dx}$	-ve	0	+ve

x	-3	-2	-1
$\frac{dy}{dx}$	+ve	0	-ve

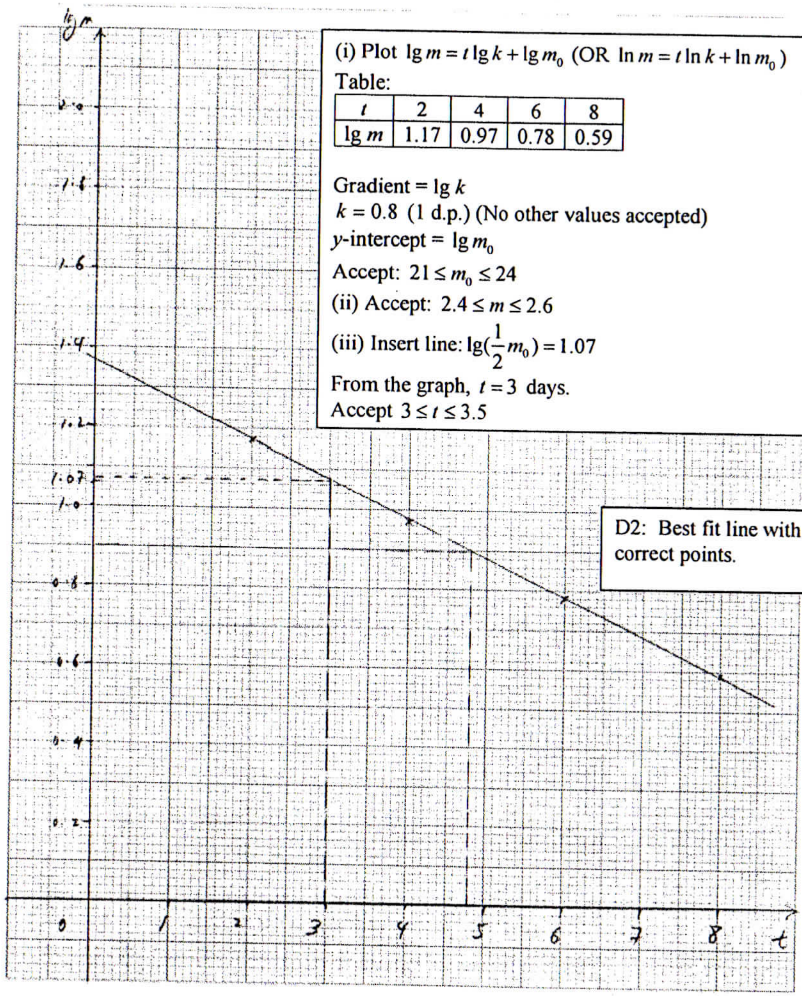
By first derivative test,
$(2,4)$ is a max turning point,
$(-2,-4)$ is a min turning point.

7i	x-coordinate = 1 Let centre of C = $(-1,b)$ $\frac{6-b}{3+1}=\frac{3}{4}$ $b=3$ Centre = $(-1,3)$, Radius = $\sqrt{(-1-3)^2+(3-6)^2}=5$ units Equation of circle: $(x+1)^2+(y-3)^2=25$ or $x^2+y^2+2x-6y-15=0$
7ii	When $y=0$, $(x+1)^2+9=25 \Rightarrow x=3$ or -5 Since the circle meets the x-axis at 2 distinct points, the x-axis is not tangent to the circle.

7iii	Shortest distance $= \sqrt{5^2 - 1^2}$ $= \sqrt{24}$ $= 4.90$ units
8i	Let $f(x) = 8x^3 - 4x^2 - 2x - 3$ $f\left(-\frac{1}{2}\right) = -4$ By remainder theorem, remainder $= -4$
8ii	Let $g(x) = 8x^3 - 4x^2 - 2x + 1$ From (i), $f(x) = (2x+1)Q(x) - 4$ $\Rightarrow f(x) + 4 = (2x+1)Q(x)$ $\Rightarrow g(x) = (2x+1)Q(x)$ $\Rightarrow g(x)$ has a factor of $2x+1$ By long division/synthetic division/comparing coefficients, $g(x) = (2x+1)(4x^2 - 4x + 1)$ $= (2x+1)(2x-1)^2$
8iii	$\frac{8x}{(x+1)(x-1)^2} = \frac{A}{x+1} + \frac{B}{x-1} + \frac{C}{(x-1)^2}$ By comparing/substitution, $A = -2$ $B = 2$ $C = 4$ $\frac{8x}{(x+1)(x-1)^2} = -\frac{2}{x+1} + \frac{2}{x-1} + \frac{4}{(x-1)^2}$
9	Refer to graph paper
10i	$\alpha + \beta = \frac{3}{2}$ $\alpha\beta = \frac{7}{2}$ $\alpha^2 + \beta^2 = (\alpha+\beta)^2 - 2\alpha\beta = -\frac{19}{4}$ Since $\alpha^2 + \beta^2 < 0$, the roots are not real.

10ii	$\alpha^3 + \beta^3 = (\alpha+\beta)(\alpha^2 - \alpha\beta + \beta^2)$ $= \left(\frac{3}{2}\right)\left(-\frac{19}{4} - \frac{7}{2}\right)$ $= -\frac{99}{8}$
10iii	$\frac{1}{\alpha} + \frac{1}{\beta} = \frac{\alpha+\beta}{\alpha\beta} = \frac{3}{7}$ $(\frac{1}{\alpha})(\frac{1}{\beta}) = \frac{2}{7}$ equation: $x^2 - \frac{3}{7}x + \frac{2}{7} = 0$ $\Rightarrow 7x^2 - 3x + 2 = 0$
11i	$CD = 9\sin\theta$ $AD = 5 + 9\cos\theta$ $L = AD + CD + AB + BC$ $= 9\sin\theta + 9\cos\theta + 19$
11ii	$R = \sqrt{9^2 + 9^2} = 9\sqrt{2}$ $\alpha = \tan^{-1} 1 = 45^\circ$ $L = 19 + 9\sqrt{2}\cos(\theta - 45^\circ)$
11iii	Max $L = 19 + 9\sqrt{2} = 31.7$ m
11iv	$19 + 9\sqrt{2}\cos(\theta - 45^\circ) = 22$ $\cos(\theta - 45^\circ) = 0.2357$ $\alpha = 76.367^\circ$ $\theta = 121.4^\circ$ or $328.6^\circ (NA)$ The angle θ should not be obtuse as the plants will not be able to absorb sunlight. Hence $L = 22$ is not a suitable value to build this tower.

Q9



(i) Plot $\lg m = t \lg k + \lg m_0$ (OR $\ln m = t \ln k + \ln m_0$)

Table:

t	2	4	6	8
$\lg m$	1.17	0.97	0.78	0.59

Gradient = $\lg k$

$k = 0.8$ (1 d.p.) (No other values accepted)

y-intercept = $\lg m_0$

Accept: $21 \le m_0 \le 24$

(ii) Accept: $2.4 \le m \le 2.6$

(iii) Insert line: $\lg(\frac{1}{2} m_0) = 1.07$

From the graph, $t = 3$ days.

Accept $3 \le t \le 3.5$

D2: Best fit line with correct points.