Name: ______________________________ (   )   Class: __________

Fairfield Methodist School
Additional Math
Sec 4E/5NA SA2 2017

Mathematical Formulae

1. ALGEBRA

Quadratic Equation

For the equation $ax^2 + bx + c = 0$,

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Binomial expansion

$$(a+b)^n = a^n + \binom{n}{1}a^{n-1}b + \binom{n}{2}a^{n-2}b^2 + \ldots + \binom{n}{r}a^{n-r}b^r + \ldots + b^n,$$

where n is a positive integer and $\binom{n}{r} = \frac{n!}{r!(n-r)!} = \frac{n(n-1)\ldots(n-r+1)}{r!}$

2. TRIGONOMETRY

Identities

$$\sin^2 A + \cos^2 A = 1$$

$$\sec^2 A = 1 + \tan^2 A$$

$$\operatorname{cosec}^2 A = 1 + \cot^2 A$$

$$\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$$

$$\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$$

$$\tan(A \pm B) = \frac{\tan A \pm \tan B}{1 \mp \tan A \tan B}$$

$$\sin 2A = 2\sin A \cos A$$

$$\cos 2A = \cos^2 A - \sin^2 A = 2\cos^2 A - 1 = 1 - 2\sin^2 A$$

$$\tan 2A = \frac{2\tan A}{1 - \tan^2 A}$$

Formulae for ΔABC

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$a^2 = b^2 + c^2 - 2bc\cos A$$

$$\Delta = \frac{1}{2}ab\sin C$$

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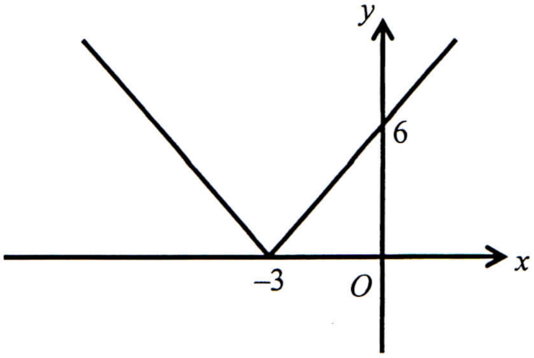
1



(i) The diagram shows the graph of $y = |f(x)|$ passing through (0, 6) and touching the x-axis at (–3, 0). Given that the graph $y = f(x)$ is a straight line, write down the two possible expressions for $f(x)$. [2]

(ii) State the range of values of m for which the line $y = mx$ intersects the graph, $y = |f(x)|$ at 2 distinct points. [1]

2 An isosceles triangle PQR in which $PQ = PR$ has an area of 46 cm^2. Given that its base QR is $\left(8\sqrt{3} - 2\sqrt{2}\right)$ cm, find in surd form,

(i) the height of the triangle, [2]

(ii) the perimeter of the triangle. [3]

3 Express $\dfrac{3x^3 + 6x - 8}{x(x^2 + 2)}$ in partial fractions. [5]

4 **(i)** Given that $p < 1$, show that the roots of the equation $x^2 - 2x + 2 - p = 0$ are not real. [3]

(ii) Find the range of values of k for which the line, $y + kx = 8$ intersects the curve, $x^2 + 4y = 20$. [3]

5 **(i)** Show that $\dfrac{d}{dx}[2x(\ln x - 3)] = 2\ln x - 4$. [2]

(ii) Hence, find $\int_1^8 2\ln x \, dx$, giving your answer in the form of $h \ln 2 + k$, where h and k are constants. [4]

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6 Given that $y = \dfrac{\cos 2x}{e^{2x-1}}$. Find

(i) $\dfrac{dy}{dx}$, [3]

(ii) the equation of the normal at the point where the curve intersects the y-axis. [3]

7 **(a)** State the values between which the principal value of $\tan^{-1} x$ must lie. [1]

(b) Given that $\tan A = -p$ where A is a reflex angle, without the use of a calculator, obtain an expression, in terms of p, for

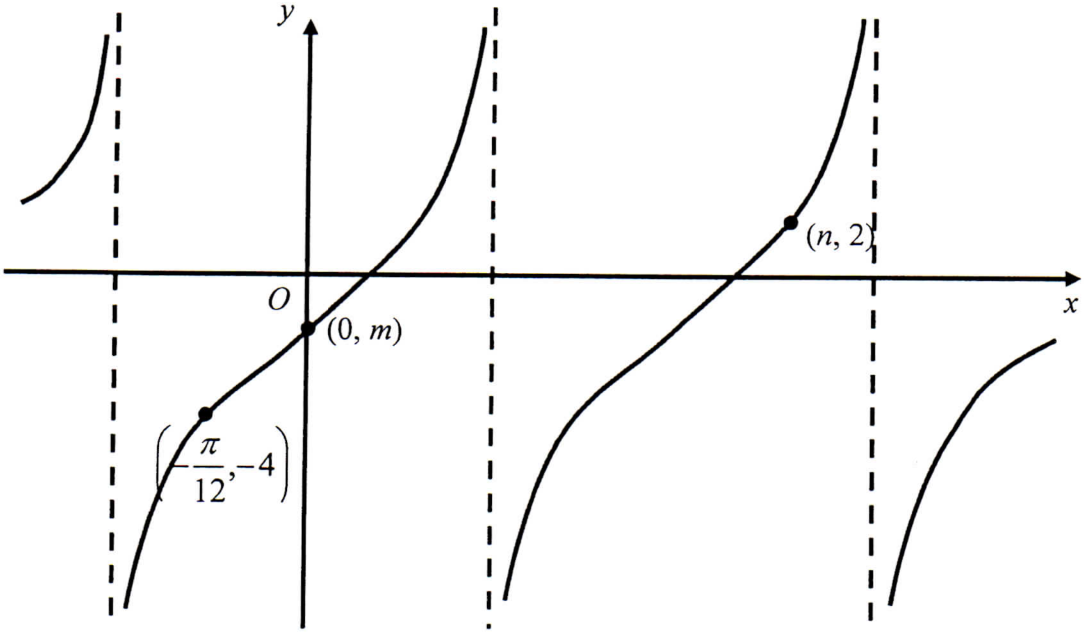
(i) $\sin A$, [1]

(ii) $\sec A$, [1]

(iii) $\cot(-A)$, [1]

(iv) $\tan\,(90 - A)°$. [1]

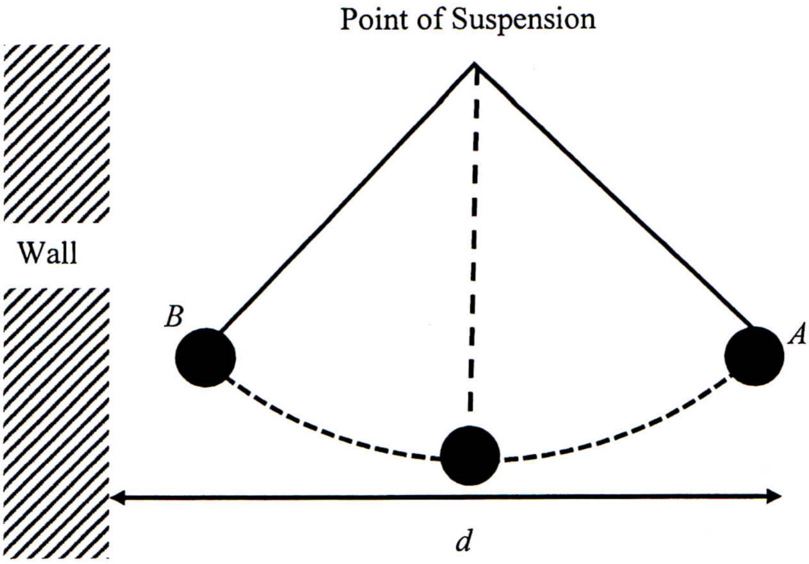
(c)



The diagram shows part of the graph $y = m + 3\tan 3x$ passing though the points $\left(-\dfrac{\pi}{12}, -4\right)$, $(0, m)$ and $(n, 2)$. Find the value of m and of n. [3]

8 **(a)** Prove that $\frac{1-2\sin x\cos x}{\sin^2 x-\cos^2 x}=\frac{\tan x-1}{\tan x+1}$. [3]

(b)



The distance of the giant pendulum from the wall, *d*, varies from 13 m to 3 m. The giant pendulum swings from *A* to *B* and back to *A* every 12 seconds. The distance of the pendulum from the wall, *d*, is, is modelled by the equation $d = 8 + a\cos b\pi t$, where *a* and *b* are constants and *t* is the time in seconds from the start of motion.

(i) Find the value of *a* and of *b*. [2]

(ii) Hence, sketch the graph of $d = 8 + a\cos b\pi t$ for $0 \le t \le 24$. [2]

(iii) Find the first two times when the pendulum was 10 m away from the wall. [2]

9 **(a)** Solve the following equations.

(i) $4^x = 7^{x-1}$ [3]

(ii) $2\log_4 5x^2 - \log_8 (4-x)^3 = 1 + \log_2 (1-x)$ [6]

(b) Sketch the graph of $y = 2e^{-x}$ and $y = 3 - e^x$ on the same diagram.

Find the *x*-coordinate of the points of intersection of the two graphs. [4]

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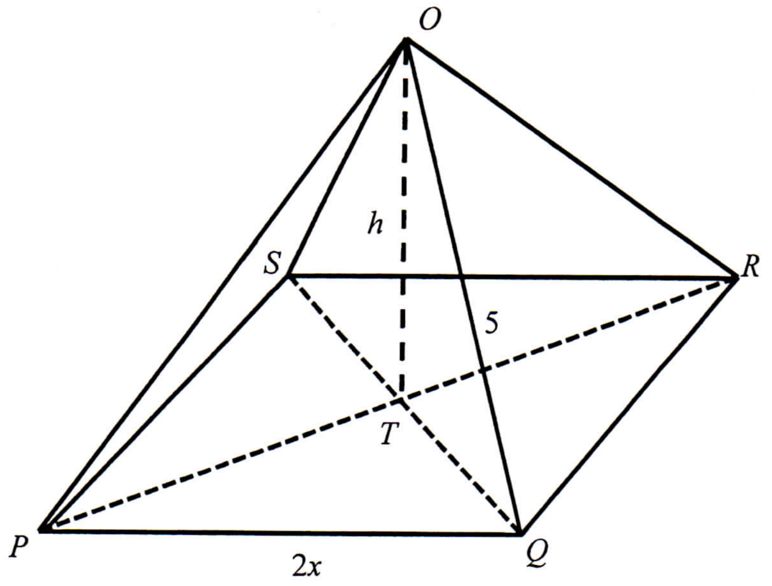
10



A right pyramid has a square base, *PQRS*, with vertex, *O*. *O* is directly above the centre of the base, *T*, as shown in the diagram above.

The lengths of the sides of the base are $2x$ metres and the height is h metres.

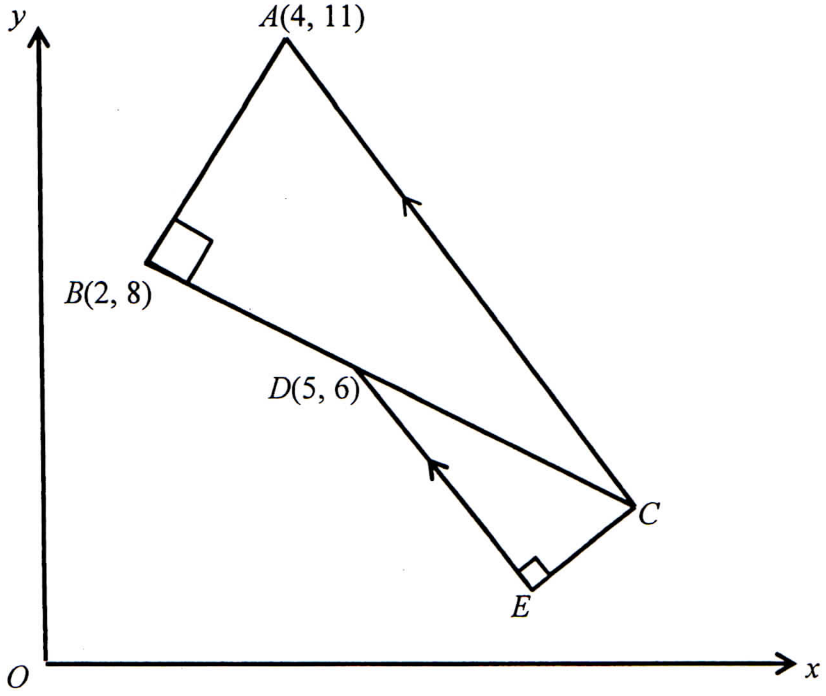
The lengths of the sloping edges, *OP*, *OQ*, *OR* and *OS* are each 5 metres.

(i) Show that the volume of pyramid, $V\text{ m}^3$, is given by $V=\dfrac{4x^2\sqrt{25-2x^2}}{3}$. [3]

(ii) Given that x can vary, find the value of x for which V has a stationary value.
Hence, calculate this stationary value of V. [5]

(iii) By considering the sign of $\dfrac{dV}{dx}$, determine whether this stationary value is a maximum or a minimum. [1]

11 Solutions to this question by accurate drawing will not be accepted.



In the diagram, the points, A, B and D have coordinates (4, 11), (2, 8) and (5, 6) respectively. The point D is the mid-point of BC. The line ED is parallel to CA and angle ABC = angle $CED = 90°$. Find

(i) coordinates of C, [2]

(ii) the coordinates of E, [6]

(iii) the of area of $ABDEC$. [2]

~ End of Paper ~

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1 The variables x and y are related by the equation $y = \frac{5}{2(x-1)^2}$, where $x \neq 1$.

(i) Given that x is decreasing at a rate of 0.2 units per second, find the rate of change of y when $x = 2$. [3]

It is given further that the variable w is such that $\sqrt{w} = y$.

(ii) Show that, when $x = 2$, the rate of change of w is five times the rate of change of y. [3]

2 The trees in a certain forest are dying because of an unknown virus.

The number of trees, N, surviving t years after the onset of the virus is shown in the table below.

t	1	2	3	4	5	6
N	2000	1300	890	590	395	260

The relationship between N and t is thought to be of the form $N = Ab^{-t}$.

(i) Using suitable variables, draw, on graph paper, a straight line graph and hence estimate the value of each of the constants A and b. [6]

(ii) If the trees continue to die in the same way, find the number of trees surviving after 15 years. [1]

3 The coefficient of x^2 in the expansion of $\left(1+\frac{x}{5}\right)^n$, where n is a positive integer, is $\frac{3}{5}$.

(i) Find the value of n. [3]

(ii) Using this value of n, find the term independent of x in the expansion of

$$\left(1+\frac{x}{5}\right)^n\left(2-\frac{3}{x}\right)^2.$$ [3]

4 **(a)** Find all the values of x between 0 and 4 for which $\cos(x+\frac{\pi}{6}) = \frac{1}{3}\cos(x-\frac{\pi}{6})$. [4]

(b) **(i)** Show that $2\sin 3x + 2\sin x = 8\sin x\cos^2 x$. [3]

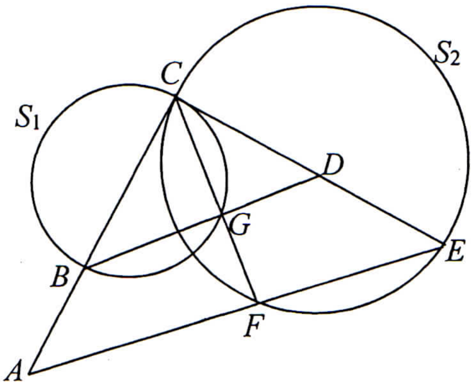
(ii) Hence solve the equation $\sin 3x - 4\sin x = 0$ for $0° < x < 360°$. [4]

5 The roots of the quadratic equation $4x^2+3x+1=0$ are α and β. Find

(i) the value of $\alpha^2+\beta^2$, [3]

(ii) a quadratic equation with roots $\dfrac{\alpha^2}{\beta}$ and $\dfrac{\beta^2}{\alpha}$. [4]

6



In the diagram, not to scale, *BC* and *CE* are diameters of the circles, S_1 and S_2 respectively. *CE* is a tangent to the circle S_1 at *C*. *CF* and *BD* meet at *G*, which lies on the circumference of S_1. *F* lies on the circumference of S_2 with centre at *D*. *CB* produced and *EF* produced meet at *A*. Show that

(a) lines *BD* and *AE* are parallel, [2]

(b) $AC = 2BC$, [3]

(c) triangle *CEF* is similar to triangle *AEC*, [2]

(d) $CF^2 = AF \times EF$. [3]

7 **(a)** It is given that $y=(x+5)(x-1)^2$.

(i) Obtain an expression for $\dfrac{dy}{dx}$ in the form $p(x-q)(x+p)$, where p and q are integers. [3]

(ii) Determine the values of x for which y is an increasing function. [2]

(b) A curve is such that $\mathrm{f}\,''(x)=4\mathrm{e}^{-2x}$. Given that $\mathrm{f}\,'(0)=3$ and the curve passes through the point $\left(2, \dfrac{1}{\mathrm{e}^4}\right)$, find the equation of the curve. [6]

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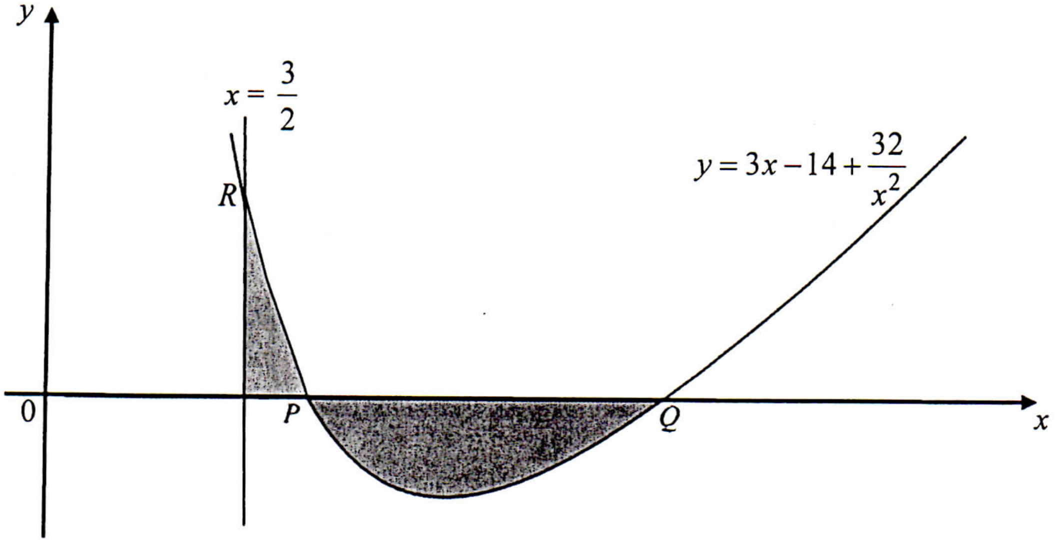
8 **(i)** Show that $x-2$ is a factor of $3x^3-14x^2+32$. [1]

(ii) Hence factorise $3x^3-14x^2+32$ completely. [3]

The diagram below shows part of the curve $y=3x-14+\frac{32}{x^2}$ meeting the x-axis at the points P and Q and the line $x=\frac{3}{2}$ at the point R.



(iii) Find the x-coordinates of P and Q. [2]

(iv) Find the area of the shaded region. [4]

9 The equation of a circle C_1 is $3x^2-30x+75-12y+3y^2=0$.

(i) Find the radius and the coordinates of the centre of C_1. [3]

(ii) Show that the circle C_1 touches the x-axis. [2]

A second circle, C_2, has the same centre as the circle C_1 and a diameter AB. Given that the coordinates of A are (1, 6), find

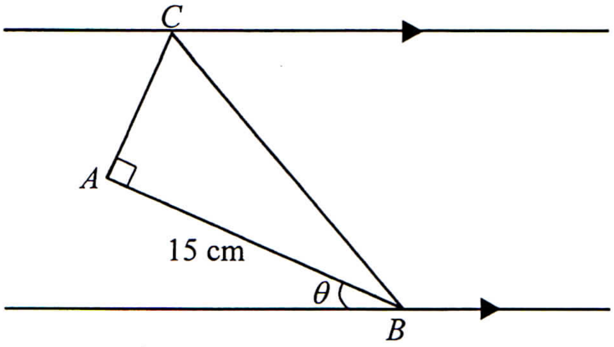
(iii) the equation of the circle C_2, [2]

(iv) the equation of the tangent to C_2, at B. [3]

A point P, which lies on the circle C_2, has the same distance from the x-axis as the point A.

(v) Find the equation of PB. [2]

10 The diagram shows two parallel lines and a right-angled triangle BAC with $AB = 15$ cm, the area of $\Delta ABC = 60$ cm^2 and AB makes an acute angle θ with one of the lines.



(i) Show that the distance between the parallel lines, $D = (15 \sin \theta + 8 \cos \theta)$ cm. [3]

(ii) Express D in the form $R \cos (\theta - \alpha)$, where $R > 0$ and $0 < \alpha < \frac{\pi}{2}$. [3]

(iii) Find the greatest possible value of D and the value of θ at which this occurs. [2]

(iv) Find the values of θ for which $D = 16$. [3]

11 A particle starts at a displacement 6 m from O and travels in a straight line so that its velocity, v m/s, is given by $v = -24 \sin 2t$, where t is the time in seconds measured from the start of the motion. Find

(i) the time at which the particle first has a velocity of 4 ms^{-1}, [1]

(ii) the initial acceleration of the particle, [2]

(iii) an expression, in terms of t, for the displacement of the particle from O, [2]

(iv) the maximum displacement of the particle from O, [1]

(v) the total distance travelled by the particle in the first 4 seconds. [3]

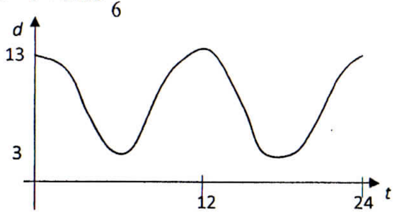
~ End of Paper ~

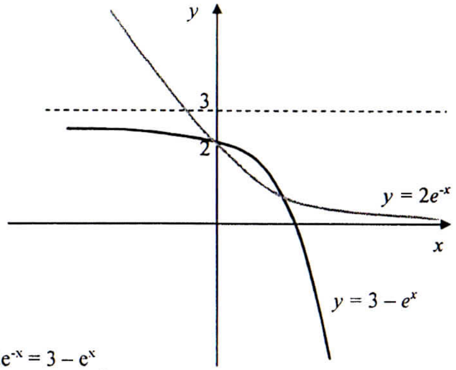
Fairfield Methodist School (Secondary)
Secondary 4 Express Preliminary Examination
Additional Mathematics
Marking Scheme Paper 1

No.	Description	Remarks
1(i)	Gradient of line = -2 or 2 $f(x) = 2x + 6$ or $f(x) = -2x - 6$	B1, B1
1(ii)	$-2 < m < 0$	B1
2(i)	$\frac{1}{2} \times (8\sqrt{3} - 2\sqrt{2}) \times h = 46$, h = height of triangle $(4\sqrt{3} - \sqrt{2}) \times h = 46$ $h = \frac{46}{(4\sqrt{3} - \sqrt{2})} \times \frac{4\sqrt{3} + \sqrt{2}}{4\sqrt{3} + \sqrt{2}}$ $h = \frac{184\sqrt{3} + 46\sqrt{2}}{46}$ $h = 4\sqrt{3} + \sqrt{2}$	M1 A1
2(ii)	$PQ^2 = h^2 + \left[\frac{1}{2}(8\sqrt{3} - 2\sqrt{2})\right]^2$ $PQ^2 = (4\sqrt{3} + \sqrt{2})^2 + (4\sqrt{3} - \sqrt{2})^2$ $PQ^2 = 48 + 8\sqrt{6} + 2 + 48 - 8\sqrt{6} + 2$ $PQ^2 = 100$ $PQ = 10$ Perimeter of triangle $PQR = 2 \times 10 + 8\sqrt{3} - 2\sqrt{2}$ $= (20 + 8\sqrt{3} - 2\sqrt{2})$ cm	M1 A1 A1
3	$\frac{3x^3 + 6x - 8}{x(x^2 + 2)} = 3 - \frac{8}{x(x^2 + 2)}$ $\frac{-8}{x(x^2 + 2)} = \frac{A}{x} + \frac{Bx + C}{x^2 + 2}$ $-8 = A(x^2 + 2) + (Bx + C)(x)$ Using substitution, x = 0, - 8 = 2A A = - 4 When x = 1, -8 = A(3) + B + C -8 = -12 + B + C 4 = B + C ------ (1) When x = -1, -8 = 3A + B – C 4 = B – C ------ (2) (1) + (2): 8 = 2B B = 4 Sub B = 4 into (1), C = 0 Therefore, $\frac{3x^3 + 6x - 8}{x(x^2 + 2)} = 3 - \frac{4}{x} + \frac{4x}{(x^2 + 2)}$	B1 M1 M1 (Substitution / Comparing Coefficient) A1 A1

No.	Description	Remarks
4(i)	$a = 1, b = -2, c = 2 - p$ $b^2 - 4ac = (-2)^2 - 4(1)(2 - p)$ $= 4 - 8 + 4p$ $= -4 + 4p$ If $p < 1$, $4p < 4$ $-4 + 4p < 0$ Since $b^2 - 4ac < 0$, therefore, the roots are not real.	M1 evaluate the discriminant M1 A1 Explanation
4(ii)	$y + kx = 8 \rightarrow y = 8 - kx$ ------(1) $x^2 + 4y = 20$ ------ (2) Substitute (1) into (2) $x^2 + 4(8 - kx) = 20$ $x^2 + 32 - 4kx - 20 = 0$ $x^2 - 4kx + 12 = 0$ $b^2 - 4ac \geq 0$ $(-4k)^2 - 4(1)(12) \geq 0$ $16k^2 - 48 \geq 0$ $k^2 - 3 \geq 0$ $k \leq -\sqrt{3}$ or $k \geq \sqrt{3}$	M1 equate line and curve M1 $b^2 - 4ac \geq 0$ A1
5(i)	$\frac{d}{dx}[2x(\ln x - 3)] = \frac{d}{dx}[2x \ln x - 6x)]$ $= 2\ln x + 2 - 6$ $= 2\ln x - 4$(Shown)	B2
5(ii)	$\int_1^8 2\ln x - 4dx = [2x(\ln x - 3)]_1^8$ $\int_1^8 2\ln x dx - \int_1^8 4dx = [2x(\ln x - 3)]_1^8$ $\int_1^8 2\ln x dx = [2x(\ln x - 3)]_1^8 + \int_1^8 4dx$ $\int_1^8 2\ln x dx = [2x(\ln x - 3)]_1^8 + [4x]_1^8$ $\int_1^8 2\ln x\, dx = [16(\ln 8 - 3) - 2(\ln 1 - 3)] + [4x]_1^8$ $= 16\ln 8 - 48 - 2\ln 1 + 6 + (32 - 4)$ $= 48\ln 2 - 14$	M1 using hence M1 make 2 ln x as subject M1 substitution A1
6(i)	$y = \frac{\cos 2x}{e^{2x-1}}$ $\frac{dy}{dx} = \frac{e^{2x-1}(-2\sin 2x) - (\cos 2x)(2e^{2x-1})}{(e^{2x-1})^2}$ $\frac{dy}{dx} = \frac{-2(\sin 2x + \cos 2x)}{e^{2x-1}}$ or $\frac{dy}{dx} = \frac{-2\sin 2x - 2\cos 2x}{e^{2x-1}}$	M1 for Quotient Rule M1 for differentiating cos 2x and e^{2x} correctly A1
6(ii)	At y-axis, $x = 0$, gradient of tangent, $\frac{dy}{dx} = \frac{-2(0+1)}{(e^{-1})} = -2e$ Therefore, gradient of normal = $\frac{1}{2e}$ When $x = 0$, $y = \frac{\cos 0}{e^{-1}} = e$. Therefore (0, e)	B1 (gradient of normal) B1 (coordinate at y-axis)

No.	Description	Remarks
	Equation of normal: $y-e=\frac{1}{2e}(x-0)$ Or $y=\frac{1}{2e}x+e$	B1 (either one)
7(a)	$-\frac{\pi}{2}<x<\frac{\pi}{2}$ or $-90°<x<90°$	B1
7(b)(i)	$-\frac{p}{\sqrt{p^2+1}}$	B1
7(b)(ii)	$\sqrt{p^2+1}$	B1
7(b)(iii)	$\frac{1}{p}$	B1
7(b)(iv)	$-\frac{1}{p}$	B1
7(c)	When $x=-\frac{\pi}{12}$, $y=-4$ $-4=m+3\tan\left(3\times-\frac{\pi}{12}\right)$ $-4=m+3\ tan\left(-\frac{\pi}{4}\right)$ $m=-1$ When $x=n$, $y=2$ $2=m+3\ tan\ 3n$ When $m=-1$, $2=-1+3\tan 3n$ $3=3\tan 3n$ $1=\tan 3n$ $3n=\frac{\pi}{4},\frac{5\pi}{4}$ $n=\frac{5\pi}{12}$	B1 M1 use their m and (n, 2) A1
8(a)	$\frac{1-2\sin x\cos x}{\sin^2 x-\cos^2 x}=\frac{\tan x-1}{\tan x+1}$ RHS = $\frac{\tan x-1}{\tan x+1}$ $=\frac{\frac{\sin x}{\cos x}-1}{\frac{\sin x}{\cos x}+1}$ $=\frac{\frac{\sin x-\cos x}{\cos x}}{\frac{\sin x+\cos x}{\cos x}}$ $=\frac{\sin x-\cos x}{\sin x+\cos x}\times\frac{\sin x-\cos x}{\sin x-\cos x}$	M1 change to sin x / cos x M1 multiply by 1 for the given form

No.	Description	Remarks
	$=\frac{\sin^2 x+\cos^2 x-2\sin x\cos x}{\sin^2 x-\cos^2 x}$ $=\frac{1-2\sin x\cos x}{\sin^2 x-\cos^2 x}$ OR LHS = $\frac{1-2\sin x\cos x}{\sin^2 x-\cos^2 x}$ $=\frac{\sin^2 x+\cos^2 x-2\sin x\cos x}{(\sin x-\cos x)(\sin x+\cos x)}$ $=\frac{(\sin x-\cos x)^2}{(\sin x-\cos x)(\sin x+\cos x)}$ $=\frac{(\sin x-\cos x)}{(\sin x+\cos x)}\div\frac{\cos x}{\cos x}$ $=\frac{\tan x-1}{\tan x+1}$	AG1 M1 substitute 1 and factorize denominator correctly M1 divide by cos x AG1
8(b)(i)	Amplitude = (13 - 3)/2 = 5 Therefore $a=5$ Or Sub $t=0$, $d=13$, Sub $t=6$, $d=3$ $13=8+a$ $3=8+a\cos\pi$ $a=5$ $a=5$ Period = 12s $Period, 12=\frac{2\pi}{b}-->b=\frac{2\pi}{12}$ Therefore $b=\frac{1}{6}$	B1 B1
8(b)(ii)	$d=8+5\cos\frac{\pi t}{6}$ 	S1 correct shape with maximum and minimum value shown P1 correct period of graph
8(b)(iii)	$d=10$, $10=8+5\cos\frac{\pi t}{6}$ $\cos\frac{\pi t}{6}=\frac{2}{5}$ Basic angle = 1.15927 $\frac{\pi t}{6}=$ 1.15927, 5.1239 t = 2.214, 9.7859 t = 2.21, 9.79 (3 s.f.)	M1 A1

No.	Description	Remarks
9(a)(i)	$4^x = 7^{x-1}$ $\lg 4^x = \lg 7^{x-1}$ $x\lg 4 = (x-1)\lg 7$ $x\lg 4 - x\lg 7 = -\lg 7$ $x(\lg 4 - \lg 7) = -\lg 7$ $x = \dfrac{-\lg 7}{\lg 4 - \lg 7}$ $=3.477225 = 3.48$ (3 s.f.)	M1 applying natural / common logarithm M1 power law A1
9(a)(ii)	$2\log_4 5x^2 - \log_8 (4-x)^3 = 1 + \log_2 (1-x)$ $2\left(\dfrac{\log_2 5x^2}{\log_2 4}\right) - \left(\dfrac{\log_2 (4-x)^3}{\log_2 8}\right) = \log_2 2 + \log_2 (1-x)$ $2\left(\dfrac{\log_2 5x^2}{2}\right) - \left(\dfrac{3\log_2 (4-x)}{3}\right) = \log_2 2 + \log_2 (1-x)$ $\log_2 5x^2 - \log_2 (4-x) = \log_2 2 + \log_2 (1-x)$ $\log_2 \left(\dfrac{5x^2}{4-x}\right) = \log_2 2(1-x)$ $\left(\dfrac{5x^2}{4-x}\right) = 2(1-x)$ $5x^2 = 2(1-x)(4-x)$ $5x^2 = 2(4 - 5x + x^2)$ $5x^2 = 8 - 10x + 2x^2$ $3x^2 + 10x - 8 = 0$ $(3x-2)(x+4) = 0$ $x = \dfrac{2}{3}$ *or* $x = -4$	M1 Change of base M1 Power Law / Quotient Law / Product Law M1 Change to same base for 1 (if don't show, give M1 for Quotient Law M1 form equation M1 for solving for x A1 if write NA no mark
9(b)	 $2e^{-x} = 3 - e^x$ Let $y = e^{-x}$, $y^2 - 3y + 2 = 0$ or $e^{2x} - 3e^x + 2 = 0$ $(y-2)(y-1) = 0$ $y = 2$ or $y = 1$ $x = \ln 2$ or $x = 0$ $= 0.693$ (3 s.f.)	B1 for $y = 2e^{-x}$ correct shape and y-intercept. B1 for $y = 3 - e^x$ for correct shape, asymptote and y-intercept. M1 for quadratic equation A1

<table>
<tr><th>No.</th><th>Description</th><th>Remarks</th></tr>
<tr><td>10(i)</td><td>By Pythagoras Theorem,
$TQ^2 = x^2 + x^2$
$TQ^2 = 2x^2$
By Phytagoras Theorem,
$OQ^2 = OT^2 + TQ^2$
$5^2 = h^2 + 2x^2$
$2x^2 = 25 - h^2$
$h = \sqrt{25 - 2x^2}$</td><td>M1
A1</td></tr>
<tr><td></td><td>Volume of pyramid, V = $\dfrac{1}{3} \times (2x)^2 \times h$
$= \dfrac{1}{3} \times 4x^2 \times \sqrt{25 - 2x^2}$</td><td>AG1</td></tr>
<tr><td>10(ii)</td><td>$\dfrac{dV}{dx} = \sqrt{25-2x^2} \times \dfrac{8x}{3} + \dfrac{4x^2}{3} \times \dfrac{1}{2} \times (25 - 2x^2)^{-\frac{1}{2}} \times (-4x)$
$= \dfrac{(25-2x^2)8x - 8x^3}{3\sqrt{25-2x^2}}$
$= \dfrac{(25-3x^2)8x}{3\sqrt{25-2x^2}}$
When V has a stationary value, $\dfrac{dV}{dx} = 0$
$\dfrac{(25-3x^2)8x}{3\sqrt{25-2x^2}} = 0$
$25 = 3x^2$
$x = \pm\sqrt{\dfrac{25}{3}}$
Since $x > 0$, $x = \dfrac{5}{\sqrt{3}}$ or 2.89 (3s.f.) [2.88675]
Therefore, $V = 32.075 = 32.1$ cm^3 (3 s.f.)</td><td>M2 for product rule
M1 equating to zero
A1
B1</td></tr>
<tr><td>10(iii)</td><td><table>
<tr><td>x</td><td>$\dfrac{5}{\sqrt{3}}^-$</td><td>$\dfrac{5}{\sqrt{3}}$</td><td>$\dfrac{5}{\sqrt{3}}^+$</td></tr>
<tr><td>$\dfrac{dV}{dx}$</td><td>+ve</td><td>0</td><td>-ve</td></tr>
<tr><td>Sketch of tangent</td><td>/</td><td>—</td><td>\</td></tr>
</table>
V has a maximum value.</td><td>B1 table and conclusion</td></tr>
<tr><td></td><td></td><td></td></tr>
<tr><td>11(i)</td><td>Let coordinate C be (x, y)</td><td></td></tr>
</table>

No.	Description	Remarks
	$\left(\frac{x+2}{2}, \frac{y+8}{2}\right) = (5,6)$ x = 8, y = 4 ➔ Coordinate C is (8, 4)	M1 A1 Coordinate of C
11(ii)	Gradient of AC = $\frac{11-4}{4-8} = -\frac{7}{4}$ Gradient of DE = $-\frac{7}{4}$ (parallel to AC) Gradient of CE = $\frac{4}{7}$ Equation of CE: $y - 4 = \frac{4}{7}(x - 8)$ $y = \frac{4}{7}x - \frac{4}{7}$ ------(1) Equation of DE: $y - 6 = -\frac{7}{4}(x - 5)$ ----- (2) Sub (1) into (2): $\frac{4}{7}x - \frac{4}{7} - 6 = -\frac{7}{4}x + \frac{35}{4}$ $x = \frac{33}{5}$ When $x = \frac{33}{5}$ or $6\frac{3}{5}$, $y = \frac{16}{5}$ or $3\frac{1}{5}$ Coordinate E is $\left(\frac{33}{5}, \frac{16}{5}\right)$	B1 gradient of DE B1 gradient of CE M1 (follow through) B1 M1 substituition A1
11(iii)	Area of *ABDEC* = $\frac{1}{2}\begin{vmatrix} 4 & 2 & 5 & \frac{33}{5} & 8 & 4 \\ 11 & 8 & 6 & \frac{16}{5} & 4 & 11 \end{vmatrix}$ = $\frac{1}{2}\lvert 32+12+16+26.4+88-(22+40+39.6+25.6+16)\rvert$ $=15.6$ units^2	M1 A1

Fairfield Methodist School (Secondary)
2017 Secondary 4 Express Preliminary Examination
Additional Mathematics
Marking Scheme Paper 2

No.	Description	Remarks
1(i)	$y=\dfrac{5}{2(x-1)^2}=\dfrac{5}{2}(x-1)^{-2}$ $\dfrac{dx}{dt}=-0.2 units/s$ $\dfrac{dy}{dx}=\dfrac{-5}{(x-1)^3}$ When $x=2$, $\dfrac{dy}{dt}=\dfrac{dy}{dx}\times\dfrac{dx}{dt}$ $=\dfrac{-5}{(2-1)^3}\times(-0.2)$ $=1 units/s$	 M1(dy/dx) M1 (connected rate0 A1
1(ii)	$\sqrt{w}=y$ $w=y^2$ $\dfrac{dw}{dy}=2y$ $\dfrac{dw}{dt}=\dfrac{dw}{dy}\times\dfrac{dy}{dt}$ When $x=2$, $y=\dfrac{5}{2(2-1)^2}$ $=2.5$ $\dfrac{dw}{dt}=2(2.5)\times\dfrac{dy}{dt}$ $=5\dfrac{dy}{dt}$ Thus, when $x=2$, w is increasing at 5 times the rate of y. (shown)	 M1 (dw/dy) M1 (connected rate) } } }AG1 }
2(i)	$\begin{array}{|c|c|c|c|c|c|c|} \hline t & 1 & 2 & 3 & 4 & 5 & 7 \\ \hline \lg N & 3.30 & 3.11 & 2.95 & 2.77 & 2.60 & 2.41 \\ \hline \end{array}$ See attach for straight line graph $\lg A=3.48$ $A=10^{3.48}$ ≈ 3019.95 $=3020(3sf)$ **Accepts A: 2950 – 3090**	B1 (3sf) (table) Points –P1 Straight line – S1 B1

No.	Description	Remarks
	$gradient=\dfrac{-0.6}{3.4}$ $=-0.17647$ $\Rightarrow -\lg b=-0.17647$ $\lg b=0.17647$ $b=10^{0.17647}$ ≈ 1.50130 $=1.50(3sf)$ **Accepts: 1.41 – 1.55**	M1 (gradient) A1
2(ii)	$t=15$ $\lg N=-0.17647(15)+3.48$ ≈ 0.83295 $N\approx 10^{0.83295}$ ≈ 6.807 $=6 trees$ **Accepts: 4 to 17 trees**	 B1
3(i)	$(1+\dfrac{x}{5})^n$ $1+\dfrac{n}{5}x+\dfrac{n(n-1)}{2}\left(\dfrac{x}{5}\right)^2+\ldots$ Coeff of $x^2=\dfrac{3}{5}$ $\dfrac{n(n-1)}{2\times 25}=\dfrac{3}{5}$ $n^2-n=30$ $n^2-n-30=0$ $(n+5)(n-6)=0$ $n=-5\,(NA)\ or\ 6$	 M1 (equation) M1 (factorisation) A1
3(ii)	$(1+\dfrac{x}{5})^6(2-\dfrac{3}{x})^2$ $=[1+\dfrac{6}{5}x+15\left(\dfrac{x^2}{25}\right)+\ldots)(4-\dfrac{12}{x}+\dfrac{9}{x^2}]$ $=[4+\dfrac{6}{5}x(\dfrac{-12}{x})+\dfrac{15}{25}x^2(\dfrac{9}{x^2})+\ldots]$ $=[4-\dfrac{72}{5}+\dfrac{135}{25}+\ldots]$ $=-5$ The term independent of x is -5.	 M1 (expansion till x^2 term) M1 (identify product of terms that will give independent terms) A1

No.	Description	Remarks
4(a)	$\cos(x+\frac{\pi}{6})=\frac{1}{3}\cos(x-\frac{\pi}{6})$. $3\cos(x+\frac{\pi}{6})=\cos(x-\frac{\pi}{6})$	
	$3(\cos x\cos\frac{\pi}{6}-\sin x\sin\frac{\pi}{6})=\cos x\cos\frac{\pi}{6}+\sin x\sin\frac{\pi}{6}$	M1 [cos(*A*+*B*)]
	$3(\cos x.\frac{\sqrt{3}}{2}-\sin x.\frac{1}{2})=\cos x.\frac{\sqrt{3}}{2}+\sin x.\frac{1}{2})$ $\frac{3\sqrt{3}}{2}\cos x-\frac{3}{2}\sin x-\frac{\sqrt{3}}{2}\cos x-\frac{1}{2}\sin x=0$ $\sqrt{3}\cos x-2\sin x=0$ $\sqrt{3}-2\tan x=0$	M1 (Special $\angle$s)
	$\tan x=\frac{\sqrt{3}}{2}$ $basic\angle=0.71372$ $\therefore x=0.71372, 3.8553$ $=0.714, 3.86$ *(3sf)*	M1 (tan *x*) A1
4b(i)	Show: $2\sin 3x+2\sin x=8\sin x\cos^2 x$ LHS $=2\sin 3x+2\sin x$ $=2(\sin 3x+\sin x)$ $=2[\sin(2x+x)+\sin x]$	
	$=2[\sin 2x\cos x+\cos 2x\sin x+\sin x]$	M1[sin(A+B)]
	$=2[2\sin x\cos^2 x+(2\cos^2 x-1)\sin x+\sin x]$	M1(cos2x= 2cos^{2}x-1)
	$=2[2\sin x\cos^2 x+2\sin x\cos^2 x-\sin x+\sin x]$ $=8\sin x\cos^2 x$ =RHS (Shown)	} } AG1 }
4b(ii)	$\sin 3x-4\sin x=0$----------(1) From b(i) $\sin 3x=4\sin x\cos^2 x-\sin x$ ----------(2) Sub (2) into (1): $4\sin x\cos^2 x-\sin x-4\sin x=0$	M1
	$4\sin x\cos^2 x-5\sin x=0$ $\sin x(4\cos^2 x-5)=0$ $\sin x=0$ *or* $\cos^2 x=\frac{5}{4}(NA)$	M1(factorise out sinx) A1 (NA for cos^{2}x or cosx)
	$x=0°, 180°, 360°$ For $0°<x<360°$, $\therefore x=180°$	A1(only 1 answer)

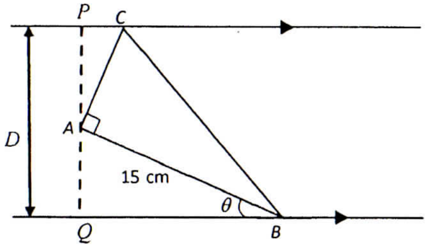
No.	Description	Remarks
5(i)	$4x^2+3x+1=0$ $\alpha+\beta=\frac{-3}{4}$	B1
	$\alpha\beta=\frac{1}{4}$	B1
	$\alpha^2+\beta^2=(\alpha+\beta)^2-2\alpha\beta$ $=\left(\frac{-3}{4}\right)^2-2\left(\frac{1}{4}\right)$ $=\frac{1}{16}$	B1
5(ii)	New sum of roots: $\frac{\alpha^2}{\beta}+\frac{\beta^2}{\alpha}$ $=\frac{\alpha^3+\beta^3}{\alpha\beta}$ $=\frac{(\alpha+\beta)(\alpha^2-\alpha\beta+\beta^2)}{\alpha\beta}$	M1
	$=\frac{\left(\frac{-3}{4}\right)\left(\frac{1}{16}-\frac{1}{4}\right)}{\frac{1}{4}}$ $=\frac{9}{16}$	A1
	New Product of roots: $\frac{\alpha^2}{\beta}\times\frac{\beta^2}{\alpha}$ $=\frac{(\alpha\beta)^2}{\alpha\beta}$ $=\frac{\left(\frac{1}{4}\right)^2}{\frac{1}{4}}$ $=\frac{1}{4}$	B1

No.	Description	Remarks
	∴The quadratic equation is $x^2 - \frac{9}{16}x + \frac{1}{4} = 0$ *or* $16x^2 - 9x + 4 = 0$	B1
6(a)	$\angle CGB = 90^\circ$ (rt. $\angle$ in a semicircle) $\angle CGD = 90^\circ$ (adj. $\angle$s on a straight line) $\angle CFE = 90^\circ$ (rt. $\angle$ in a semicircle) Since $\angle CGD = \angle CFE = 90^\circ$, Using the **converse property of corresponding angles are equal**, BD is parallel to AE (shown)	} } B1 ($\angle CGD$ & $\angle CFE$) } } } B1 }
6(b)	$\angle CDB = \angle CEA$ (corresponding $\angle$s, $BD \parallel AE$) $\angle CBO = \angle CAE$ (corresponding $\angle$s, $BD \parallel AE$) By AA test, triangle CDB is similar to triangle CEA. $\therefore \frac{CD}{CE} = \frac{BC}{AC} = \frac{1}{2}$ ⇒ AC = 2BC (shown)	} }B1 B1 } } } AG1 }
6(c)	$\angle CEF = \angle AEC$ (Common $\angle$) $\angle CFE = 90^\circ$ (rt. $\angle$ in a semicircle) $\angle ACE = 90^\circ$ (tan ⊥ radius) $\therefore \angle CFE = \angle AEC$ By AA test, triangle CEF is similar to triangle AEC (Shown).	} } }B1 } }B1 }
6(d)	**Method 1:** Using similar triangles in (c) $\frac{CE}{EF} = \frac{AE}{EC}$ $CE^2 = AE \times EF$ --------(1) In triangle CEF, by Pythagoras Theorem $CE^2 = CF^2 + EF^2$ --------(2) Sub (1) into (2): $AE \times EF = CF^2 + EF^2$ $CF^2 = EF(AE - EF)$ $CF^2 = EF \times AF(shown)$	 M1 M1 } }AG1 } }

No.	Description	Remarks
	Method 2: $\angle CFE = 90^\circ$ (rt. $\angle$ in a semicircle) $\angle CFA = 90^\circ$ (adj. $\angle$s on a straight line) $\therefore \angle CFE = \angle CFA$ $\angle FCE = \angle CBG$ (Alternate Segment Theorem) $\angle CBG = \angle CAF$ (corr. $\angle$s, $BD \parallel AE$) $\therefore \angle FCE = \angle CAF$ By AA test, triangle FCE is similar to triangle FAC Using similar triangles, $\frac{CF}{EF} = \frac{AF}{CF}$ $CF^2 = AF \times EF(shown)$	 B1 B1 AG1
7(ai)	$y = (x+5)(x-1)^2$ $\frac{dy}{dt} = (x+5)2(x-1) + (x-1)^2(1)$ $= (x-1)[2(x+5) + x - 1]$ $= (x-1)(3x+9)$ $= 3(x-1)(x+3)$	 M2 (Product/Chain Rule) A1
7(aii)	y is an increasing function $\frac{dy}{dt} > 0$ $\Rightarrow 3(x-1)(x+3) > 0$ $(x-1)(x+3) > 0$ $x > 1$ *or* $x < -3$	 M1 A1
7b	$f'(x) = \int 4e^{-2x} dx$ $= \frac{4e^{-2x}}{-2} + c$ $= -2e^{-2x} + c$ When x = 0, f '(x) = 3 3 = -2 + c c = 5 $f'(x) = -2e^{-2x} + 5$ $f(x) = \int -2e^{-2x} + 5dx$ $= e^{-2x} + 5x + d$ At $\left(2, \frac{1}{e^4}\right)$, $\frac{1}{e^4} = e^{-4} + 10 + d$ d =-10 Equation of curve: $f(x) = e^{-2x} + 5x - 10$	 B1 M1 A1 B1 M1 A1

No.	Description	Remarks
8(i)	$f(x)=3x^3-14x^2+32$ $f(2)=3(2)^3-14(2)^2+32$ $=0$ $\therefore (x-2)$ is a factor of f(x)	} }B1 }
8(ii)	$3x^3-14x^2+32=(x-2)(3x^2+px-16)$ Comparing coefficient: $x:\ \ 0=-16-2p$ $p=-8$ $3x^3-14x^2+32=(x-2)(3x^2-8x-16)$ $=(x-2)(3x+4)(x-4)$	} }M1 (long division, comparing coefficients or synthetic division }A2
8(iii)	$y=3x-14+\dfrac{32}{x^2}$ At P and Q, $y=0$ $3x-14+\dfrac{32}{x^2}=0$ $3x^3-14x^2+32=0$ From (ii) $(x-2)(3x+4)(x-4)=0$ $x=2, 4$ or $-4/3$ $\therefore$ x- coordinate of P is 2, x-coordinate of Q is 4.	B1 B1
8(iv)	Area of the shaded region $=\int_{1.5}^{2}3x-14+32x^{-2}dx+\left\|\int_{2}^{4}3x-14+32x^{-2}dx\right\|$ $=\left[\dfrac{3x^2}{2}-14x-\dfrac{32}{x}\right]_{1.5}^{2}+\left\|\dfrac{3x^2}{2}-14x-\dfrac{32}{x}\right\|_{2}^{4}$ $=\left[(\dfrac{3(2)^2}{2}-14(2)-\dfrac{32}{2})-(\dfrac{3(1.5)^2}{2}-14(1.5)-\dfrac{32}{1.5}\right]$ $+\left\|(\dfrac{3(4)^2}{2}-14(4)-\dfrac{32}{4})-(\dfrac{3(2)^2}{2}-14(2)-\dfrac{32}{2}\right\|$ $=\left[(-38)-(-38\dfrac{23}{24})\right]+\left\|(-40)-(038)\right\|$ $=\dfrac{23}{24}+2$ $=2\dfrac{23}{24}$ OR 2.96 units^2 (3sf)	M1 M1 (either one) } } }M1 } } } } A1

No.	Description	Remarks
9(i)	$C_1:\ 3x^2-30x+75-12y+3y^2=0$ $x^2+y^2-10x-4y+25=0$ Centre of circle is (5, 2) Radius of circle $=\sqrt{5^2+2^2-25}$ =2 units	B1 M1 A1
9(ii)	**Method 1:** At x-axis, y = 0 , sub into equation of C₁ $x^2+0^2-10x-4(0)+25=0$ $x^2-10x+25=0$ $(x-5)^2=0$ $x=5$ Since there is **only one solution**, therefore the circle touches the x-axis. (Shown) **Method 2:** As **y-coordinate of the centre** of C₁ **is 2** and **radius** of the circle **is also 2 units**, thus the circle C₁ touches the x-axis. (Shown).	} }B1 } B1 B1 B1
9(iii)	Centre of C₂ is (5, 2) Radius of C₂ $=\sqrt{(5-1)^2+(2-6)^2}$ $=\sqrt{32}$ $=4\sqrt{2}$ Or 5.6568 ≈ 5.66 units. $\therefore$ equation of circle C₂ is $(x-5)^2+(y-2)^2=(4\sqrt{2})^2$ $(x-5)^2+(y-2)^2=32$ Or $x^2+y^2-10x-4y-2=0$	*ft* 1(use wrong centre from (i)) A1
9(iv)	Radius of C₂ = $4\sqrt{2}$ AB is the diameter, centre = (5, 2) Let point B be (x, y)	

No.	Description	Remarks
	$\frac{x+1}{2}=5$ $x=9$ $\frac{y+6}{2}=2$ $y=-2$ $\therefore$ B = (9, −2) Gradient of line joining A and (5, 2) $=\frac{4}{-4}$ $=-1$ Gradient of the tangent at B = 1 Equation of the tangent at B is $y-(-2)=1(x-9)$ $y+2=x-9$ or $y=x-11$	*ft* 1 (use wrong centre from (i)) *ft* 1 A1
9(v)	Let P be $(x, 6)$ That is $(x-5)^2+(6-2)^2=32$ $(x-5)^2=32-16$ $x-5=4$ *or* -4 $x=9$ *or* 1 *(NA)* Equation of PB is $x=9$.	M1 A1
10(i)	Area $\Delta ABC=60$ $\frac{1}{2}\times 15\times AC=60$ $AC=8$  	M1 (AC)

No.	Description	Remarks
	$D=AP+AQ$ In ΔAQB: $\sin\theta=\frac{AQ}{15}$ $AQ=15\sin\theta$ In ΔAPC: $\cos\theta=\frac{AP}{8}$ $AP=8s\cos\theta$ $\therefore$ D = 15sinθ + 8cosθ (shown)	M1 (AQ) } } }AG1 } } }
10(ii)	Let $D=R\cos\theta\cos\alpha+R\sin\theta\sin\alpha$ $8\cos\theta+15\sin\theta=R\cos\theta\cos\alpha+R\sin\theta\sin\alpha$ Compare coefficients: $\cos\theta:8=R\cos\alpha$,,,,,,,,,,,(1) $\sin\theta:15=R\sin\alpha$............(2) (2) ÷ (1) $\tan\alpha=\frac{15}{8}$ $\alpha=\tan^{-1}\left(\frac{15}{8}\right)$ $=1.08084$ $(1)^2+(2)^2$: $R=\sqrt{8^2+15^2}$ $=17$ $\therefore$ D = 17cos(θ − 1.08)	M1 (for α) M1 (for R) A1
10(iii)	D = 17cos(θ − 1.08084) Greatest value of D = 17 It occurs when cos(θ − 1.08084)= 1 That is θ = 1.08084 = 1.08 (3 sf)	B1 B1
10(iv)	17cos(θ − 1.0808) =16 $\cos(\theta-1.08084)=\frac{16}{17}$ Basic angle = 0.34470	M1

No.	Description	Remarks
	$-1.08084 < \theta - 1.08084 < 0.48996$ $\theta - 1.08084 = 0.34470, -0.34470$ $\theta = 1.43, 0.736$	A2
11(i)	$v = -24\sin 2t$ when V = 4 m/s $-24\sin 2t = 4$ $\sin 2t = \frac{-1}{6}$ Basic angle = 0.167448 $2t = \pi + 0.167448$ t = 1.65452 =1.65 s	B1
11(ii)	Initial Acceleration, $t = 0$ $a = \frac{dv}{dt}$ $= -24\cos 2t \times 2$ $= -48\cos 2t$ When $t = 0$, Initial acceleration = −48cos0 = −48 m/s^2	M1 A1
11(iii)	$S = \int V\,dt$ $= \int -24\sin 2t\ dt$ $= \frac{-24(-\cos 2t)}{2} + c$ $= 12\cos 2t + c$ When t = 0, s = 6 6 =12cos0 + c C = −6 ∴ S = 12cos2t − 6	M1 A1
11(iv)	Maximum displacement, $V = 0$ $-24\sin 2t = 0$ $\sin 2t = 0$ $2t = 0, \pi, 2\pi$ $t = 0, \frac{\pi}{2}, \pi$ ∴ max S = 12cos2(π/2) − 6 = 12cos π − 6 = − 18 m	B1

No.	Description	Remarks
	Methods 2: S = 12cos2t − 6 By observation, max S = − 18 m	B1
11(v)	$t = 0$, s = 6 m $t = \frac{\pi}{2}$, s = −18 m $t = \pi$, s = 6 m $t = 4$, s = −7.7460 Total distance travelled = (6 + 18) × 2 + (6 + 7.746) = 61.746 = 61.7 m (3sf) **Method 2:** $\left\lvert\int_0^{\frac{\pi}{2}} -24\sin 2t\ dt\right\rvert + \int_{\frac{\pi}{2}}^{\pi} -24\sin 2t\ dt + \left\lvert\int_{\pi}^{4} -24\sin 2t\ dt\right\rvert$ $= [12\cos 2t]_0^{\frac{\pi}{2}} + [12\cos 2t]_{\frac{\pi}{2}}^{\pi} + [12\cos 2t]_{\pi}^{4}$ = 61.746 = 61.7 m (3sf)	} } }B1 (all correct) } } M1 A1 M2 (Any two correct) A1